Leibniz-Institut für Astrophysik Potsdam

AIP

Universität Potsdam

Digital Image Processing IV

Image Enhancement in the Spatial Domain

apl. Prof. Dr. Carsten Denker

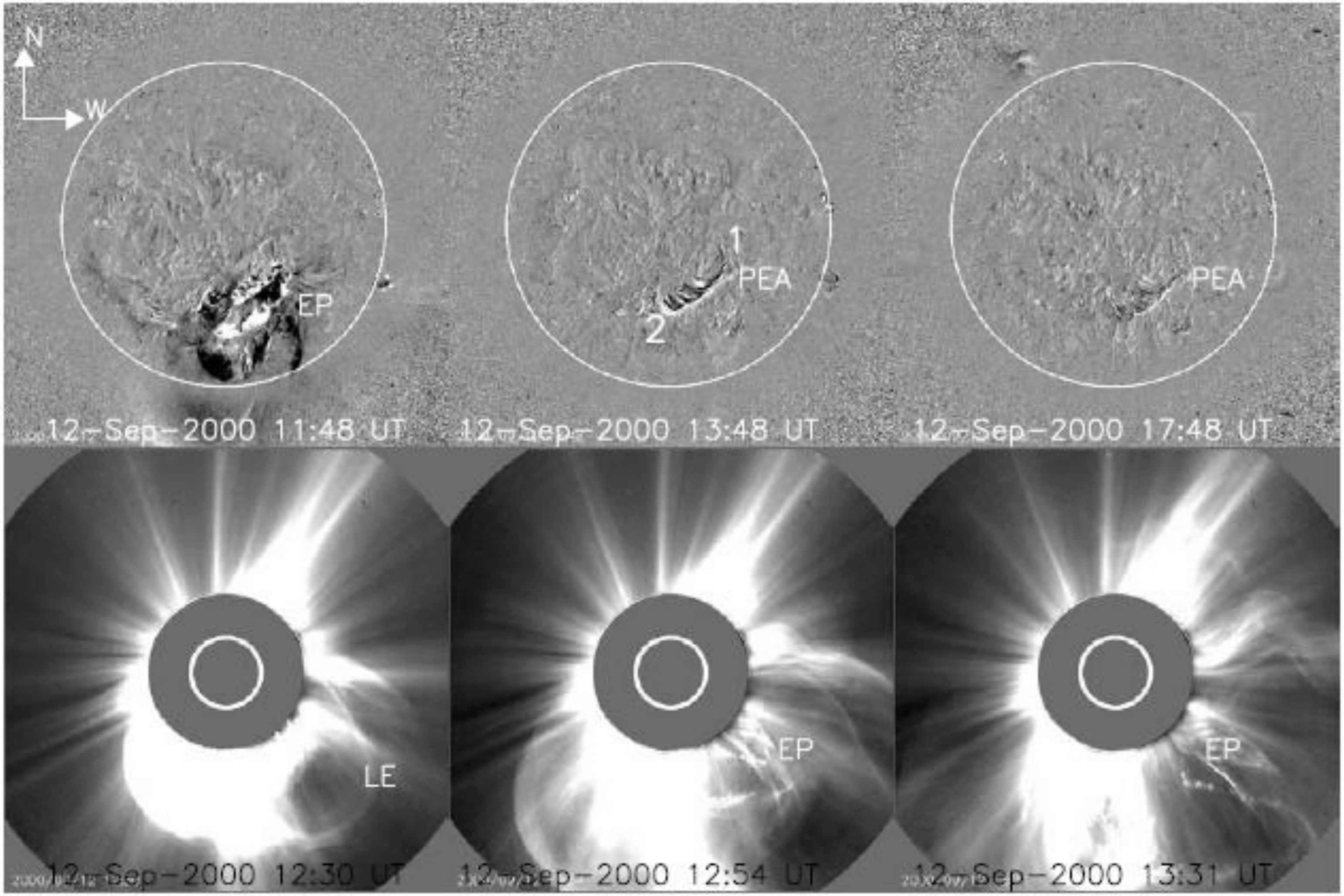
Enhancement Using Arithmetic and Logic Operations

- Operations are performed on a *pixel-by-pixel* basis
- Arithmetic operations
  ⇨ subtraction, addition, division, and multiplication
- Logic operations: **`AND`**, **`OR`**, and **`NOT`**
  ⇨ *masking* or *region-of-interest* (ROI) processing
- Gray-level vs. binary masks
- Image averaging
- Difference images
  - Visualization of temporal evolution
  - X-ray diagnostics using contrast enhancing means in medicine
  - Sliding average
  - Segmentation and feature tracking

www.roentgenkainberger.at

Coronal Dimming

Tripathi *et al.* (2004)



Averages

- Additive noise

$$g(x,y) = f(x,y) + \eta(x,y)$$

- For each coordinate *(x, y)* the noise is uncorrelated with a zero mean.

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

- Expected value

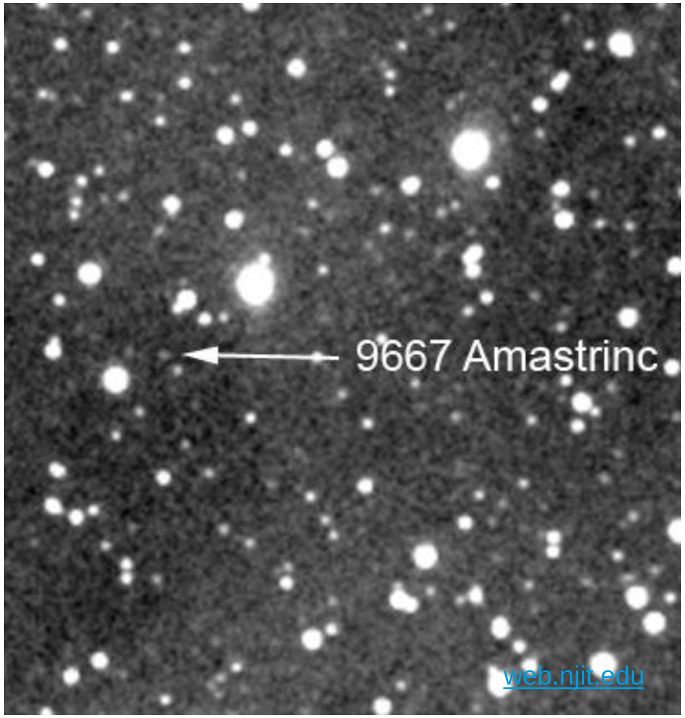
$$E\{\bar{g}(x,y) = f(x,y)\}$$

- Variance

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

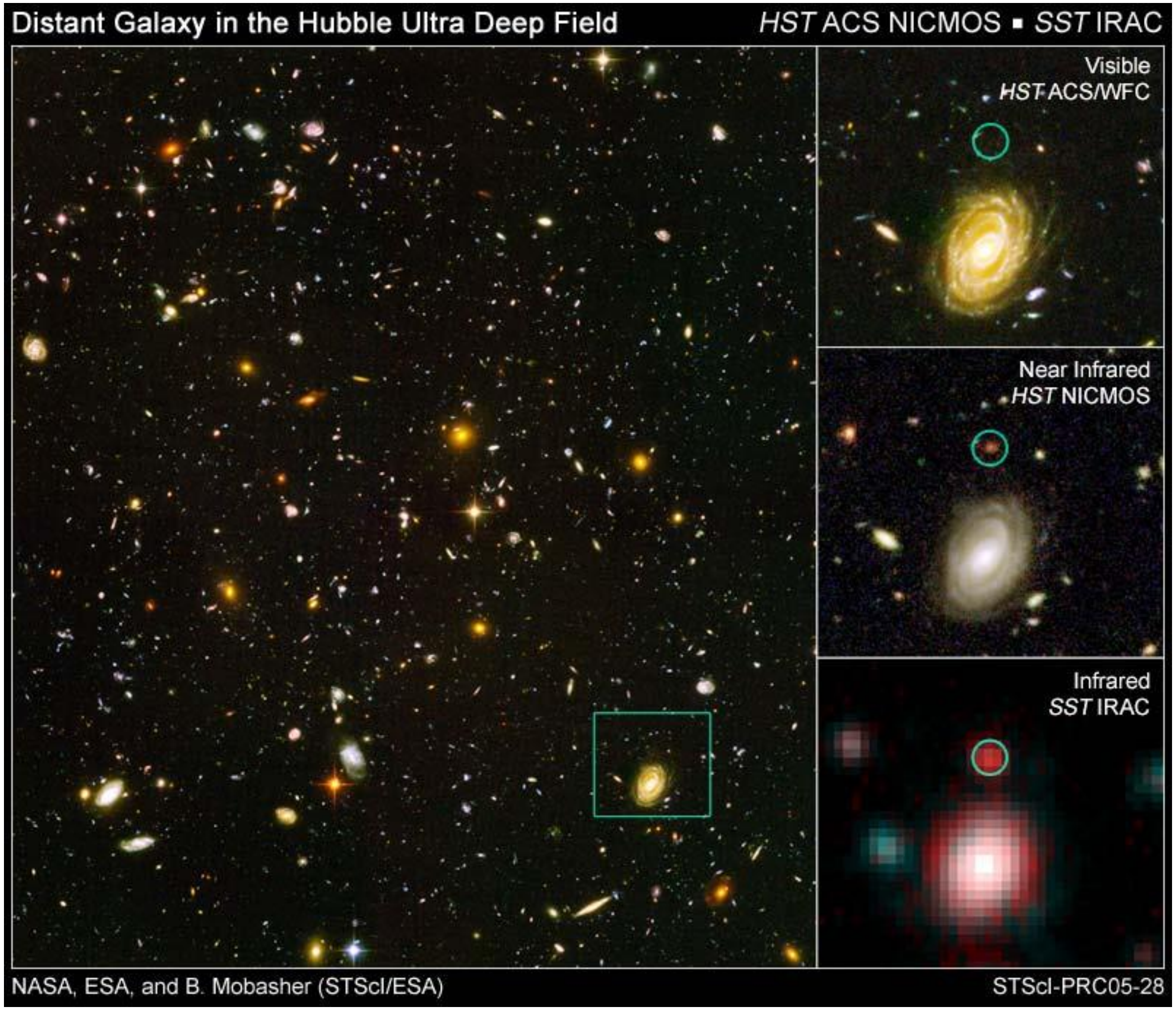
- Standard deviation

$$\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$$



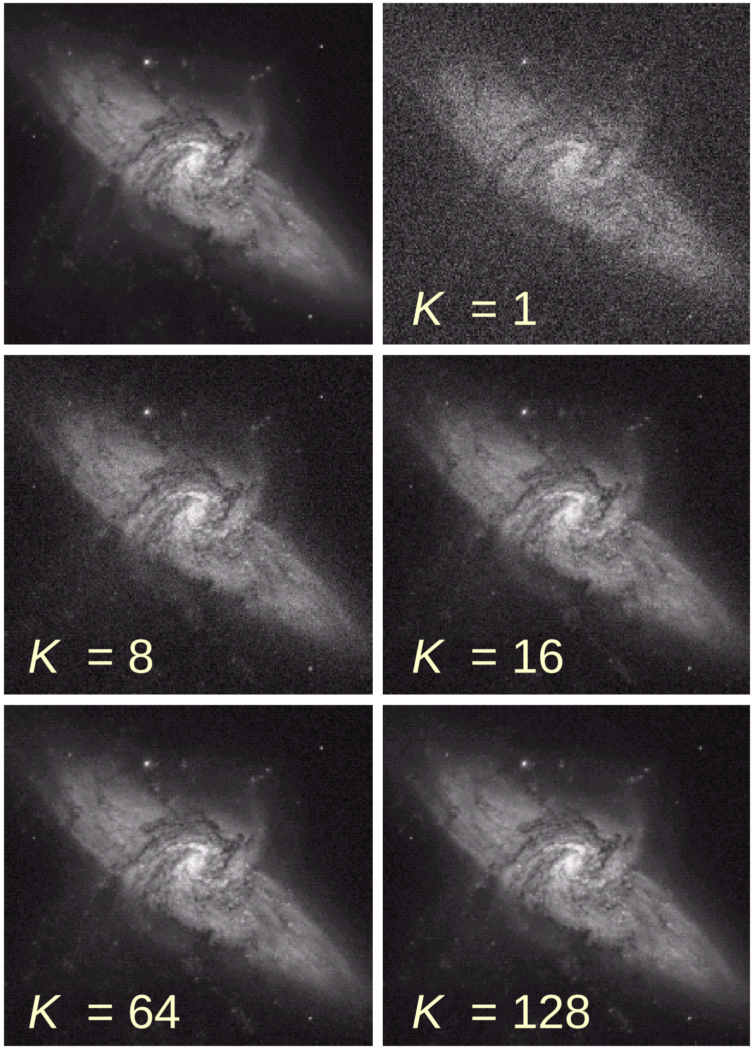
- Careful alignment before adding individual images ⇨ artificats or loss of image information
- Faint astronomical objects ⇨ long exposure times (often many hours or even days)

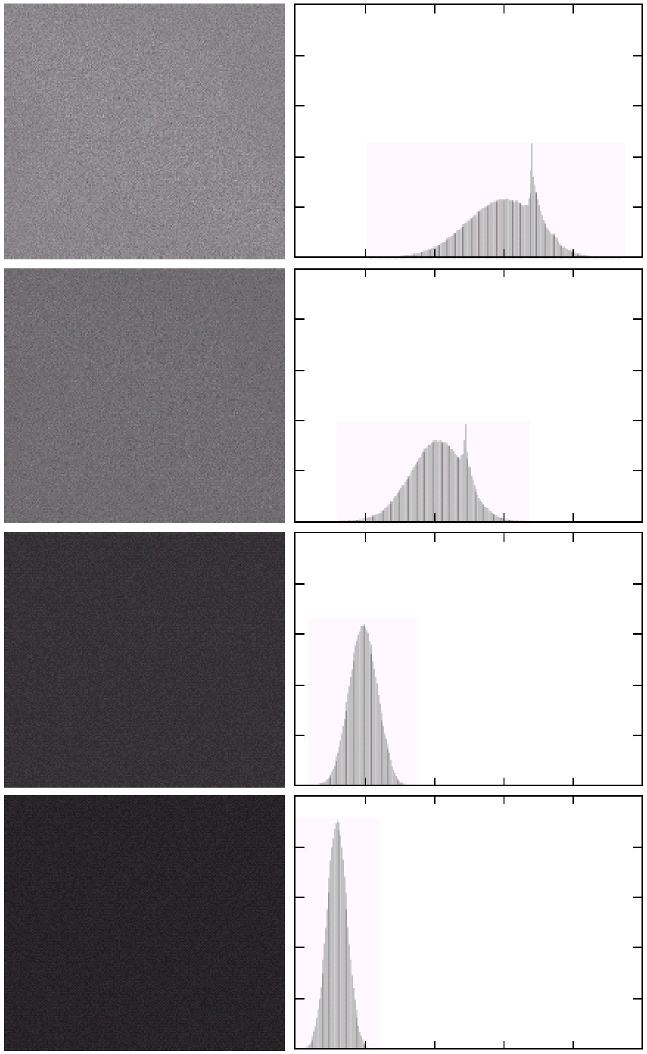
Hubble Ultra Deep Field (HUDF)



- 2003 September 3 – 2004 January 16
- Southwest of Orion
- 1/10th of the lunar diameter
- 800 single exposures
- 400 orbits
- 11.3 days with ACS
- 4.5 days with NICMOS
- Extremely faint galaxies out to a distance of 13 billion lightyears

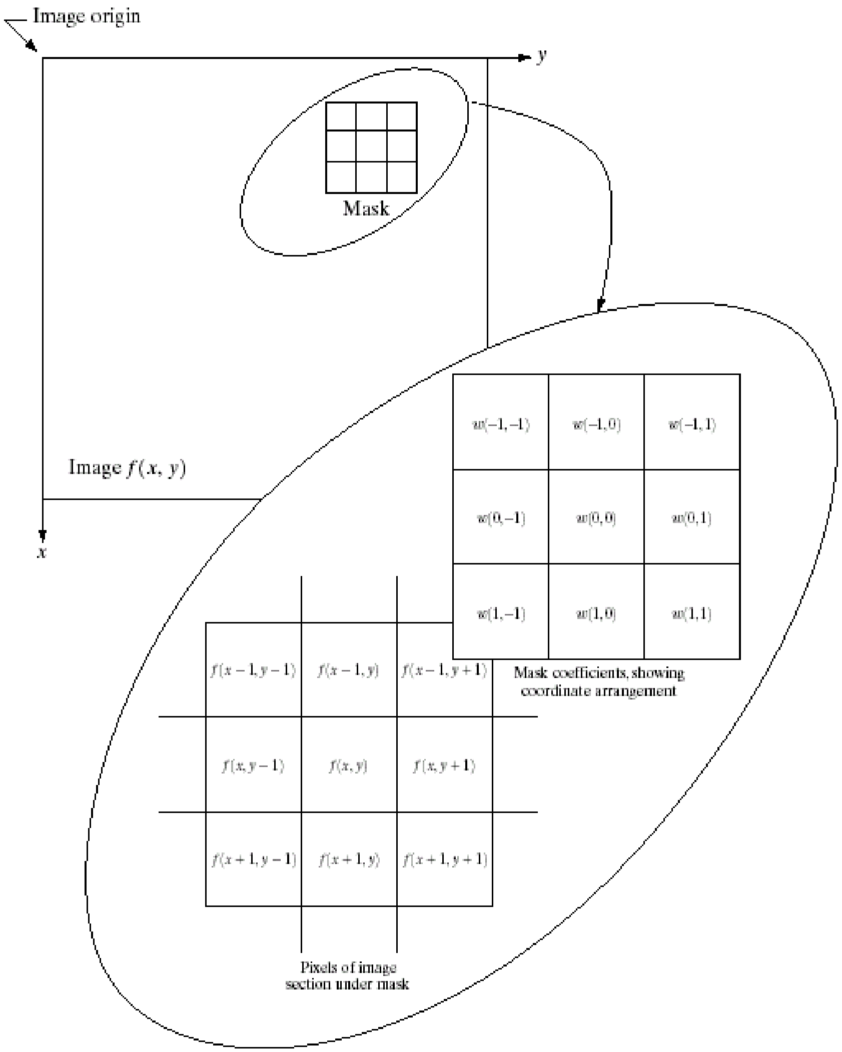
Rauschen in astronomischen Bildern





Hubble Space Telescope image of the twin galaxy NGC 3314

Basics of Spatial Filtering



- Local operators
- The elements of filters, kernels, or masks are called coefficients.
- Linear filters (with $m \times n$ coefficients, m and n odd)

$$
\begin{aligned}
R = & w(-1,-1) f(x-1, y-1) + w(-1,0) f(x-1, y) \\
& + \cdots + w(0,0) f(x, y) + \cdots \\
& + w(1,0) f(x+1, y) + w(1,1) f(x+1, y+1)
\end{aligned}
$$

- General linear filter applied to $M \times N$-pixel images

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

- Convolution with a kernel

$$R = \sum_{i=1}^{mn} w_i z_i$$

Smoothing Spatial Filters

- Blurring and noise reduction (removal of small details, reducing sharp transitions, bridging of gaps, …)
- Averaging or low-pass filters
- Spatial filters in which all coefficients are equal are called *box filters*
- Filters with *weighted averages*
- Order-statistic filters (e.g. median filter ⇨*impulse* or *salt-and-pepper noise*)

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1&1&1\\\hline 1&1&1\\\hline 1&1&1\\\hline\end{array}\quad\text{and}\quad\frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1&2&1\\\hline 2&4&2\\\hline 1&2&1\\\hline\end{array}$$

- Weighted average filter

$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)\,f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)}$$

- Gauss filter are isotropic and decrease monotonically ⇨ apodisation

$$w_{mn}=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{m^2+n^2}{2\sigma^2}\right)$$

- Binomial filter

$$\binom{n}{k}=\frac{n!}{k!(n-k)!}\quad\text{with }\ n\geq k$$

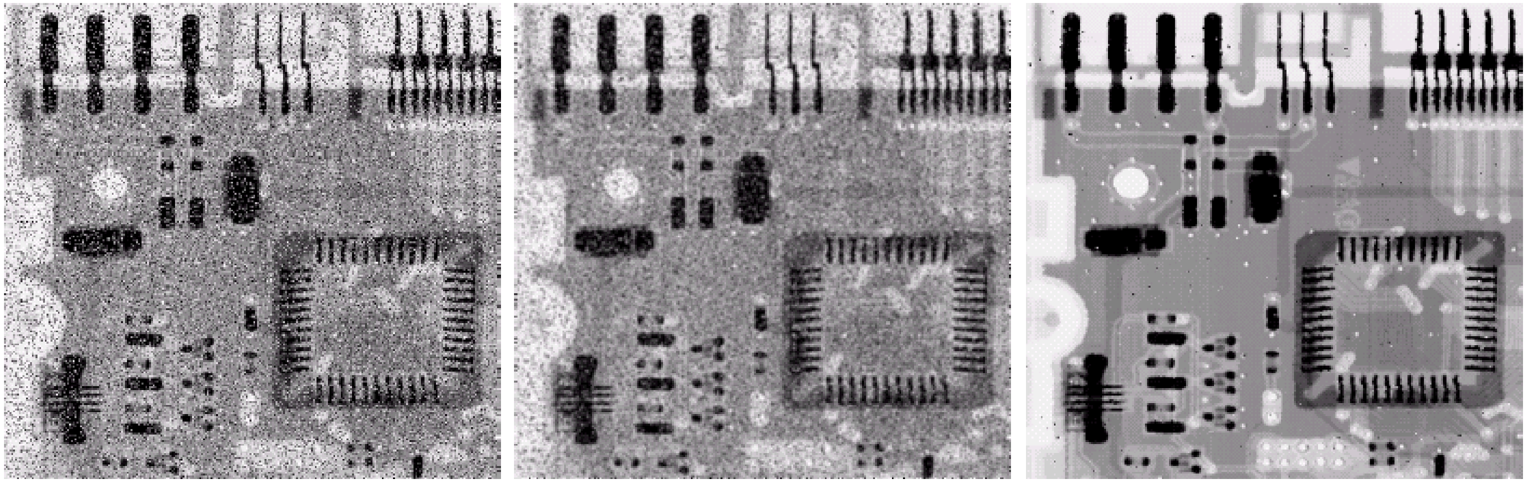
$$\frac{1}{4^4}\times\begin{array}{|c|c|c|c|c|}\hline 1&4&6&4&1\\\hline 4&16&24&16&4\\\hline 6&24&36&24&6\\\hline 4&16&24&16&4\\\hline 1&3&6&4&1\\\hline\end{array}$$

- Smoothing spatial filters are often used in combination with thresholds to identify objects.

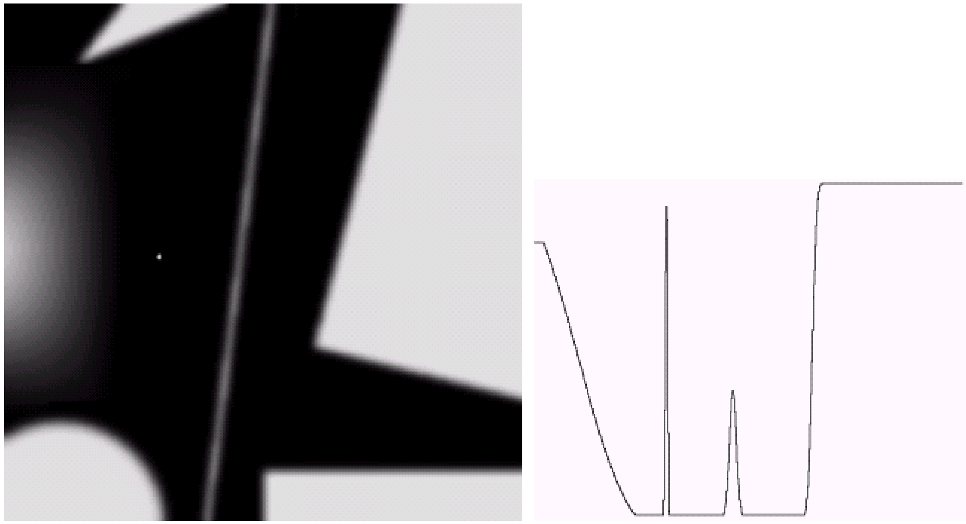
Box Filter

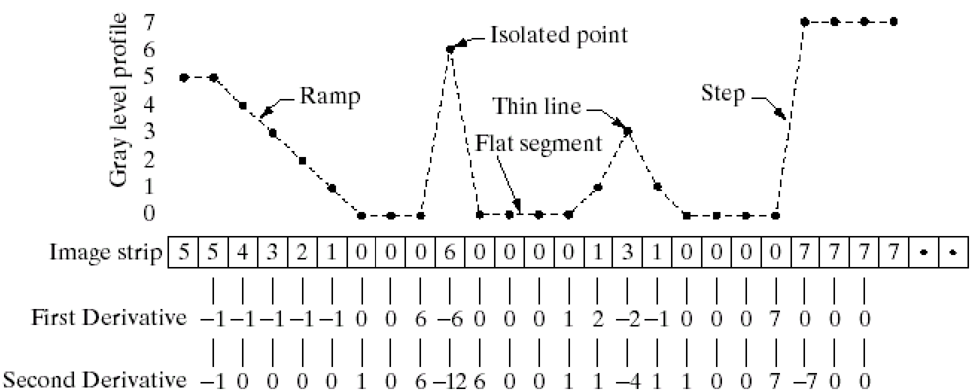
Median Filter

- Non-linear filter, which replaces the gray value of a pixel by the median of its neighborhood.
- Median (50th percentile) ⇨ sort all gray values in a neighborhood and select the value in the middle.
- Effectively suppresses salt-and-pepper (white and black artifacts in a grayscale images)
- Minimum- (0th percentile) and maximum filter (100th percentile)



Sharpening Filters





- Highlight fine details and enhance details, which have been blurred
- Averaging ⇔ integration
- Sharpening ⇔ differentiation
- Strength of the response of a derivative operator is proportional to the degree of discontinuity (enhance edges and noise but deemphasize areas with slowly varying gray levels)
- First- and second-order derivative operators

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Second-Order Differential Operators

- Isotropic operators
  - Do not depend on the direction of the discontinuities
  - Invariant with respect to rotation
- Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f = \left[f(x+1,y) + f(x-1,y) + (x,y+1) + f(x,y-1) - 4f(x,y) \right]$$

- The Laplacian is linear.

- Various implementations of the Laplacian

0	1	0
1	–4	1
0	1	0

1	1	1
1	–8	1
1	1	1

0	–1	0
–1	4	–1
0	–1	0

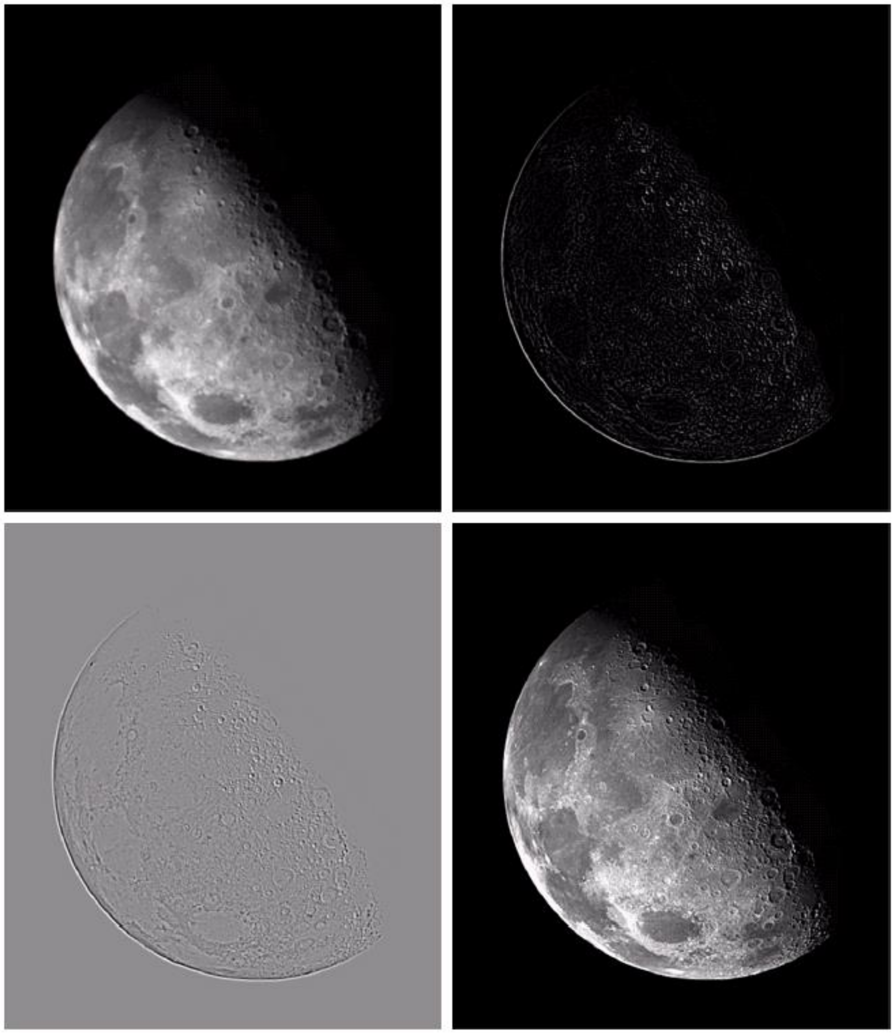
–1	–1	–1
–1	8	–1
–1	–1	–1

- Edge enhancement an inclusion of the background

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) \\ f(x,y) + \nabla^2 f(x,y) \end{cases}$$

for $w_0 < 0$ respectively $w_0 > 0$

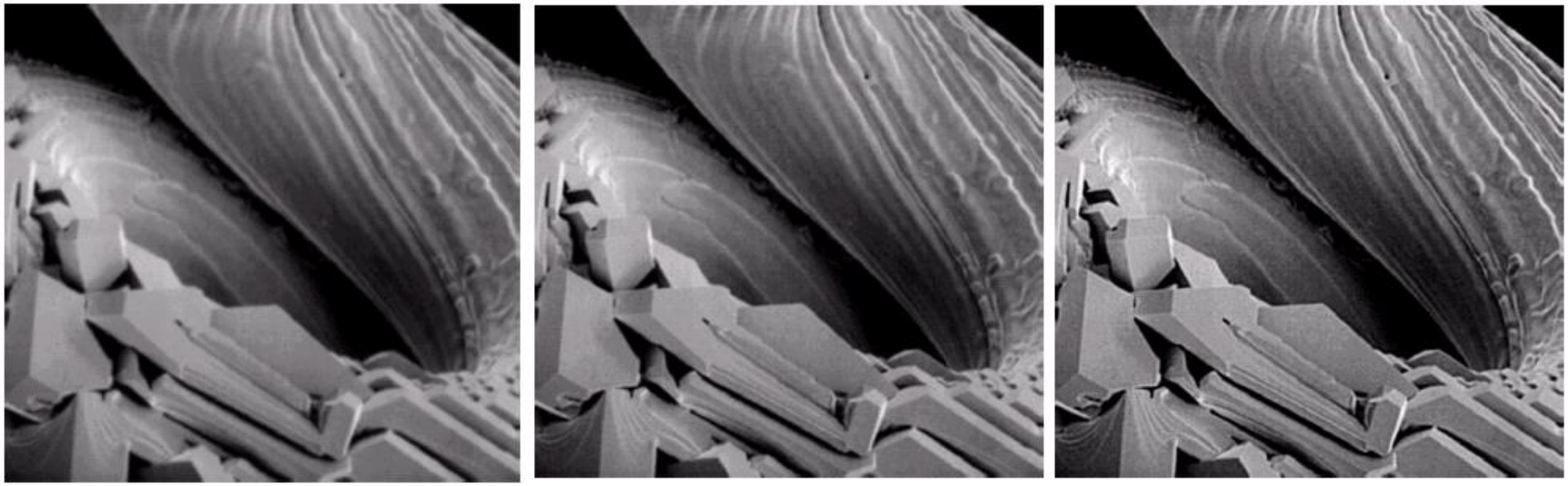
Contrast Enhancement



$$
\begin{aligned}
g(x,y) &= f(x,y) - \nabla^2 f \\
&= f(x,y) - \big[f(x+1,y) + f(x-1,y) \\
&\quad + (x,y+1) + f(x,y-1) - 4f(x,y) \big] \\
&= 5f(x,y) - \big[f(x+1,y) + f(x-1,y) \\
&\quad + (x,y+1) + f(x,y-1) \big]
\end{aligned}
$$

0	–1	0
–1	5	–1
0	–1	0

Laplacian with and without Diagonal Elements



0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

- Laplacian with diagonal elements creates sharper contours.
- Linear operators ⇨ Laplacian + unit-center mask

0	0	0
0	1	0
0	0	0

Unsharp Masking – "High-Boost" Filter

- Unsharp masking has its origins in photography, where an unsharp negative image is combined with a positive image

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

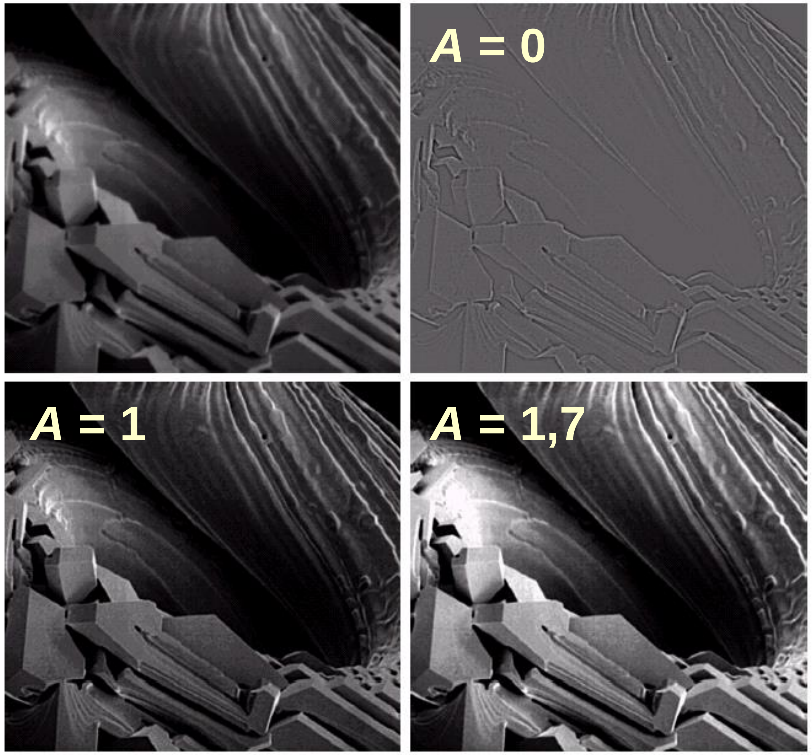
- High-boost filter

$$\begin{aligned} f_{\text{hb}}(x,y) &= Af(x,y) - \bar{f}(x,y) \\ &= (A-1)\,f(x,y) + f(x,y) - \bar{f}(x,y) \\ &= (A-1)\,f(x,y) - f_s(x,y) \end{aligned}$$

- If the value of *A* slowly becomes greater than unity, then the edge enhancement slowly diminishes.

$$f_{\text{hb}}(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) \\ Af(x,y) + \nabla^2 f(x,y) \end{cases}$$

for $w_0 < 0$ respectively $w_0 > 0$



0	–1	0
–1	$A+4$	–1
0	–1	0

–1	–1	–1
–1	$A+8$	–1
–1	–1	–1

Gradient Operators

- Gradient

$$\nabla \boldsymbol{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

- Gradient magnitude

$$\begin{aligned} \nabla f &= \left| \nabla \boldsymbol{f} \right| \\ &= \left[G_x^2 + G_y^2 \right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right] \end{aligned}$$

- The gradient is a linear operator, but not ist magnitude (square root).

- Partial derivatives are not invariant with respect to rotation, but their magnitude is invariant.
- Approximation for faster computation

$$\nabla f \approx |G_x| + |G_y|$$

- Roberts operator

$$G_x = (z_9 - z_5) \text{ and } G_y = (z_8 - z_6)$$

$$\nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2}$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

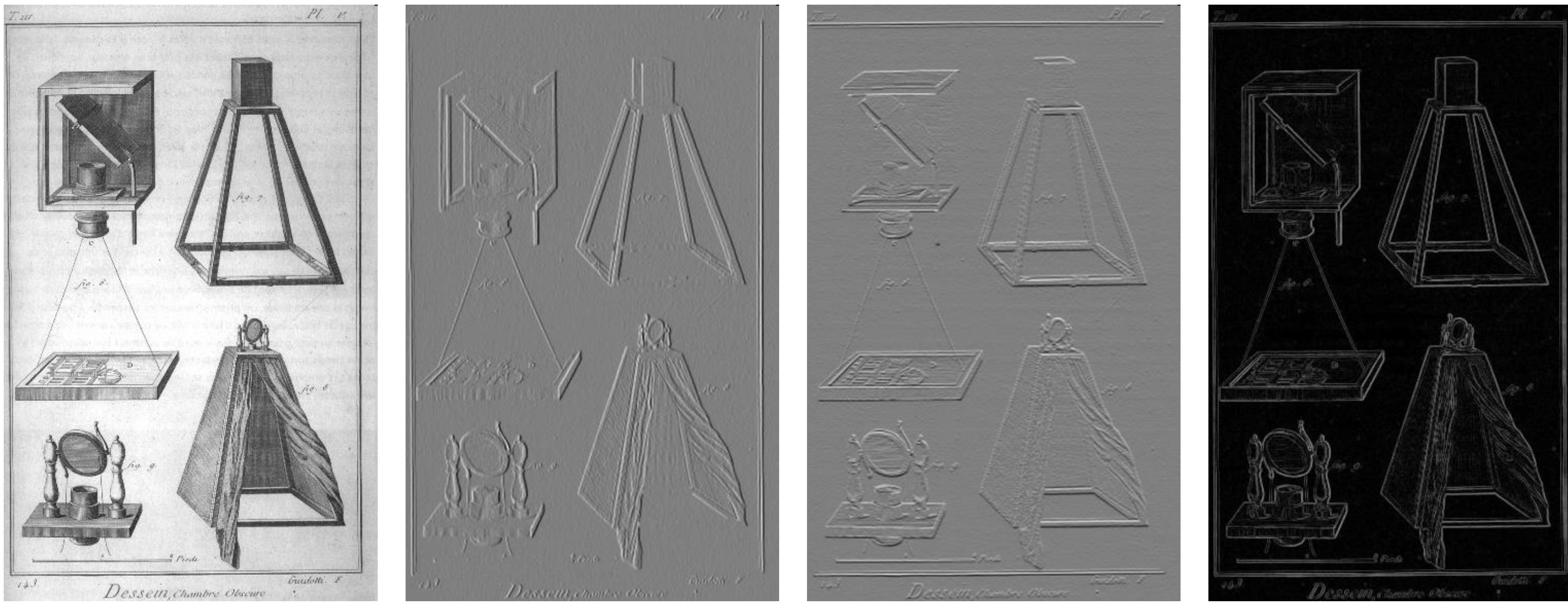
- Sobel operator

$$\begin{aligned} \nabla_S f \approx & \left| (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \right| \\ & + \left| (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7) \right| \end{aligned}$$

–1	0	1
–2	0	2
–1	0	1

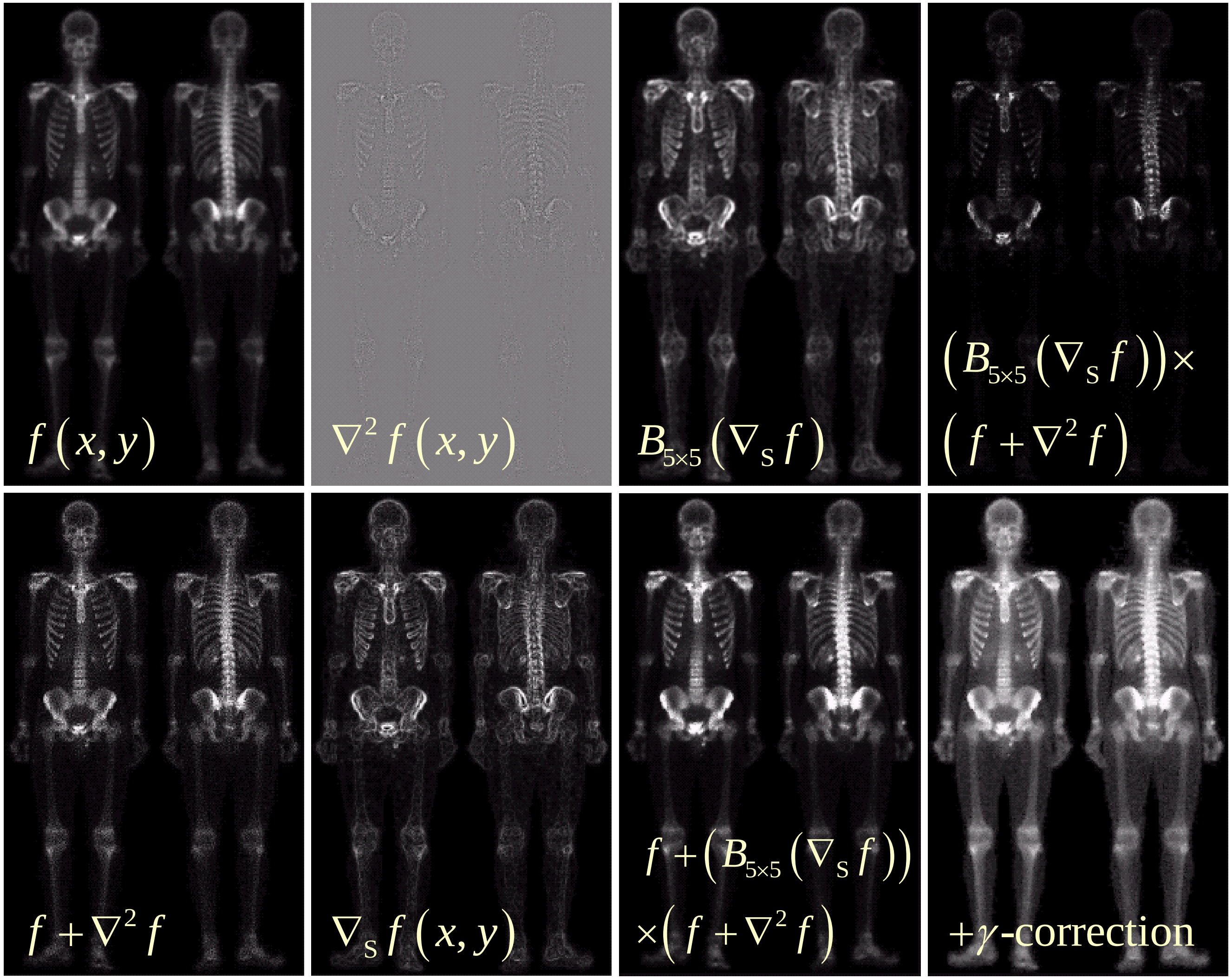
–1	–2	–1
0	0	0
1	2	1

Sobel Operator



de.wikipedia.org

Combination of Different Methods



1D Gaussian Derivatives

1D Gaussian $G(x,\sigma) = e^{-\frac{x^2}{2\sigma^2}}$

Derivatives $G_x(x,\sigma) = -\frac{x}{\sigma^2} G(x,\sigma) = -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$

$$G_{xx}(x,\sigma) = \frac{x^2 - \sigma^2}{\sigma^4} G(x,\sigma) = \frac{x^2 - \sigma^2}{\sigma^4} e^{-\frac{x^2}{2\sigma^2}}$$

$$G_{xxx}(x,\sigma) = \frac{x^3 - x\sigma^2}{\sigma^6} G(x,\sigma) = \frac{x^3 - x\sigma^2}{\sigma^6} e^{-\frac{x^2}{2\sigma^2}}$$

2D Gaussian Derivatives

2D Gaussian $$G(x,y,\sigma) = e^{-\frac{(x^2+y^2)}{2\sigma^2}} = e^{-\frac{x^2}{2\sigma^2}} \cdot e^{-\frac{y^2}{2\sigma^2}}$$

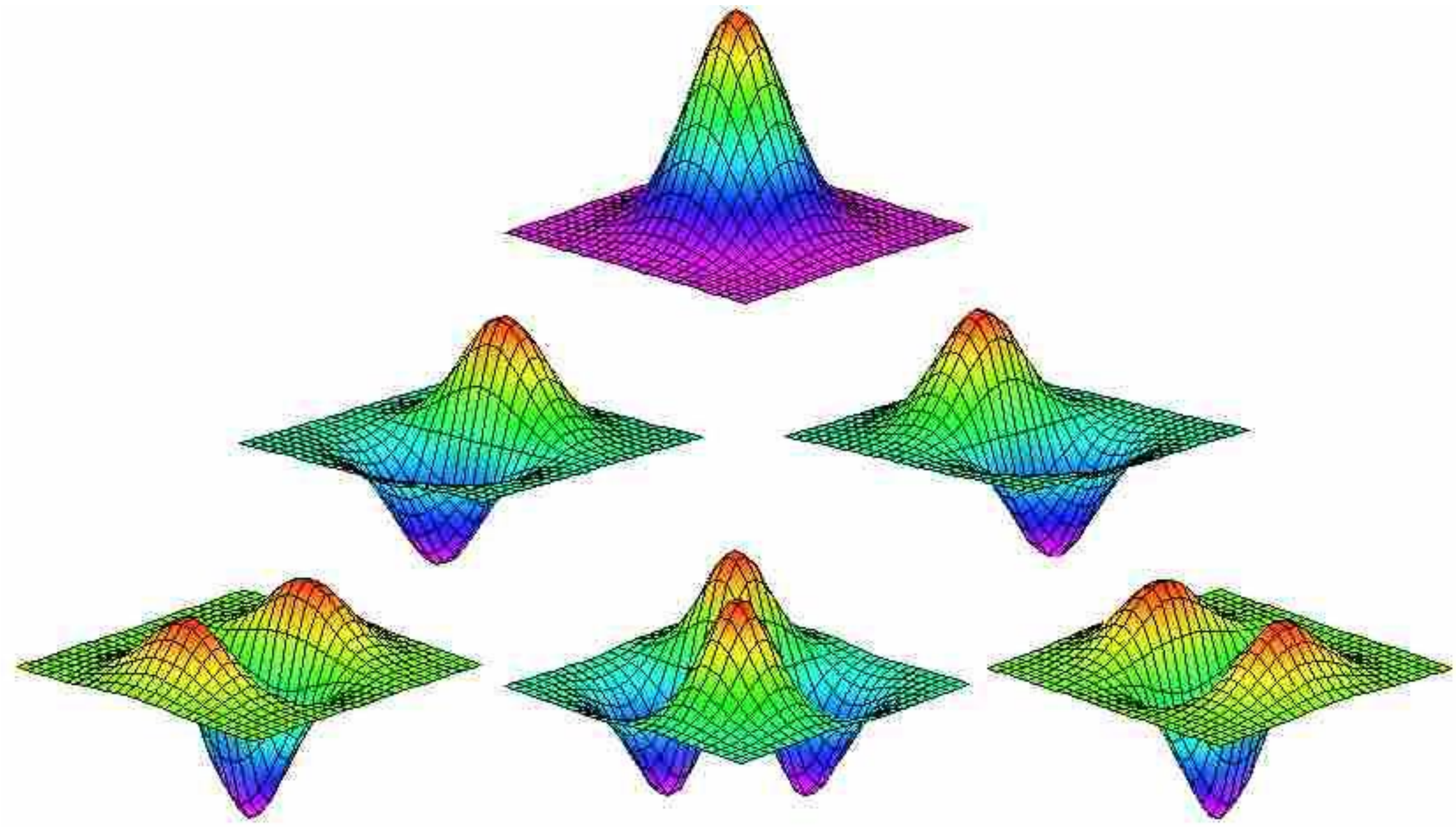
Derivatives $$G_x(x,\sigma) = -\frac{x}{\sigma^2} G(x,y,\sigma) = -\frac{x}{\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$G_{xx}(x,\sigma) = \frac{x^2-\sigma^2}{\sigma^4} G(x,y,\sigma) = \frac{x^2-\sigma^2}{\sigma^4} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$G_{xy}(x,\sigma) = \frac{xy}{\sigma^4} G(x,y,\sigma) = \frac{xy}{\sigma^4} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$G_{xxx}(x,\sigma) = \frac{x^3-x\sigma^2}{\sigma^6} G(x,y,\sigma) = \frac{x^3-x\sigma^2}{\sigma^6} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

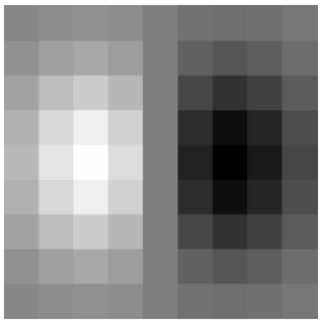
2D Gaussian Derivatives



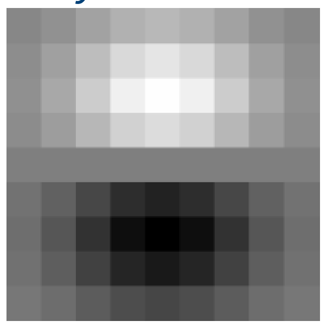
www.ijbem.org

Discrete Forms of the Derivatives

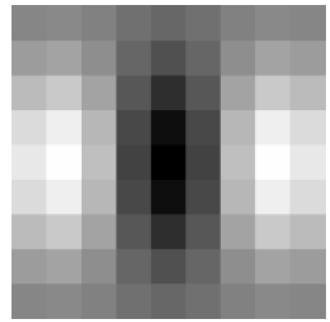
G_x



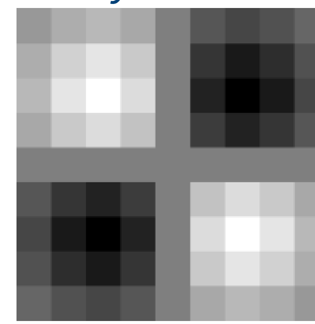
G_y

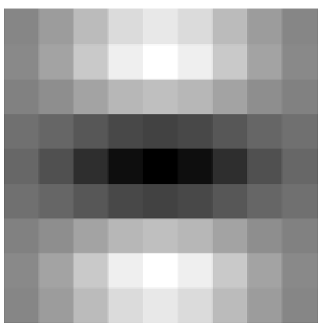


G_{xx}

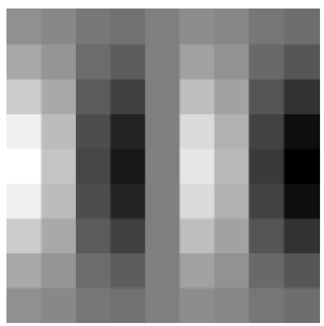


G_{xy}

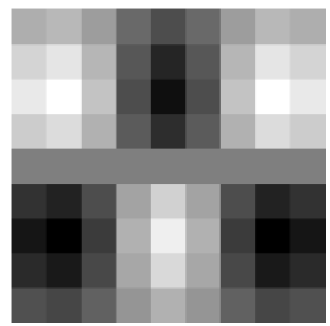




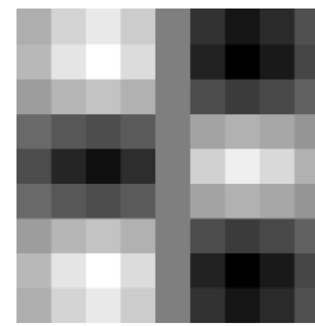
G_{yy}



G_{xxx}



G_{xxy}



G_{xxx}