[6]

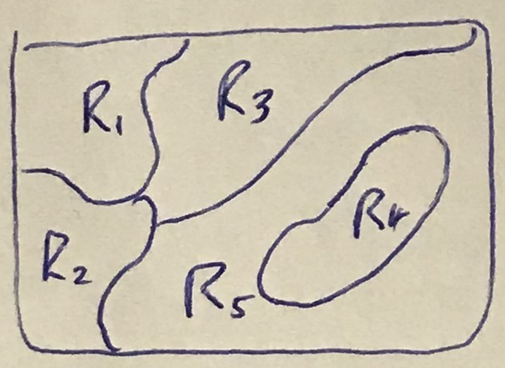
$$\mathcal{X} = \{\text{all possible inputs } x\}$$

$$R_j = \{x : j = \operatorname{argmin} \sum_{i=1}^{k} c(j|i)\pi_i f_i(x)\}$$

$\therefore R_j \subseteq \mathcal{X}$, and indeed $\mathcal{X} = \bigcup_{j=1}^{k} R_j$

with $R_i \cap R_j = \emptyset$.

Pictorially:



$= \mathcal{X}$ say with $K = 5$.

$$\mathbb{P}(\text{error}) := \mathbb{P}(C=1 \wedge X \notin R_1) + \mathbb{P}(C=2 \wedge X \notin R_2) +$$
$$\mathbb{P}(C=3 \wedge X \notin R_3) + \cdots + \mathbb{P}(C=K \wedge X \notin R_K)$$

$$= \pi_1 \int_{\mathcal{X}-R_1} f_1(x)dx + \pi_2 \int_{\mathcal{X}-R_2} f_2(x)dx + \cdots + \pi_K \int_{\mathcal{X}-R_K} f_K(x)dx \quad .$$