Statistical Inference

Exercises 1

1. A random sample $X_1, X_2, \ldots, X_n$ is taken from the distribution with p.d.f.

$$f(x; \lambda, \mu) = \sqrt{\frac{\lambda}{2\pi x^3}} \exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right) \qquad x > 0,$$

where $\lambda > 0$ and $\mu > 0$. Write down the likelihood function and find a vector-valued sufficient statistic of dimension 2 for (λ, μ).

2. Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ denote a random sample of size n from the Poisson distribution with mean θ, where the parameter $\theta > 0$ is unknown. Consider the statistic

$$T(\mathbf{X}) = \sum_{i=1}^{n} X_i.$$

(i) Show that T has a Poisson distribution with mean $n\theta$.

(ii) By showing that the conditional distribution of X given T is multinomial and does not depend upon θ, deduce that T is sufficient for θ.

(iii) Use the Factorization Theorem to provide another proof that T is sufficient for θ.

3. Show that the family of $N(\mu, \sigma^2)$ distributions belongs to the exponential family with natural parameters

$$\eta_1 = \frac{1}{\sigma^2}, \qquad \eta_2 = \frac{\mu}{\sigma^2}.$$