Probability and Distribution Theory: Exercises 2

1. Find conditions on the constants c (and d) $\in \mathbb{R}$ such that the following are density functions:

   (a)

   $$f_X(x) = \begin{cases} cx^{-d} & x > 1 \\ 0 & x \leq 1. \end{cases}$$

   (b)

   $$f_X(x) = ce^x(1 + e^x)^{-2} \qquad \text{for } x \in \mathbb{R}.$$

2. Show that if X has density function $f_X(\cdot)$ with $f_X(x) = 0$ when $x < 0$, and distribution function F_X, then

   $$\mathbb{E}[X] = \int_0^\infty [1 - F_X(x)]dx.$$

   Hint: consider expressing as a double integral and then changing the order of integration.

3. Airlines find that each passenger who reserves a seat fails to turn up with probability 0.1 independently of the other passengers. So *Symphony Airways* always sells 10 tickets for their 9 seat aeroplane while *Harmony Airways* always sells 20 tickets for their 19 seat aeroplane. Which is more often overbooked?