

PDT Assignment

Deadline: Monday, 9th December, 2019

Total marks: [30]. Marks are shown in boxes []. There are two questions on this assignment.

1. Let $S \subseteq \mathbb{R}^2$ be the set $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x\}$, and let $f_{X,Y}$ be the probability density function

$$f_{X,Y}(x, y) = \begin{cases} cxy & \text{if } (x, y) \in S; \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the value of c . [2]
- (b) Find $\mathbb{P}(X \geq 2Y)$. [4]
- (c) Find the marginal probability density functions of X and Y . [4]
- (d) Find the conditional probability density function of Y given $X = x$, and express the conditional expectation of Y given X as a function of X . [4]
- (e) Are X and Y independent? Given a reason for your answer. [2]
- (f) Let

$$U = \frac{X + Y}{2} \quad \text{and} \quad V = \frac{X - Y}{2}.$$

Find the joint probability density function of U and V . [4]

2. A flowering tree produces n flowers with probability $(1 - p)p^n$ for $n = 0, 1, 2, \dots$ and $p \in (0, 1)$. Each flower independently produces a fruit with probability $q \in (0, 1)$.
- (a) Obtain the probability generating function of the number of fruits produced. [4]
 - (b) Hence, or otherwise, determine the probability mass function of the number of fruits produced. [3]
Hint: Consider using the Taylor series: $(1 - x)^{-1} = \sum_{n=0}^{\infty} x^n$.
 - (c) Determine the probability that the tree has n flowers, given it produces f fruits. [3]

Important Note:

- Please read the current version of the *Mathematics & Statistics Coursework Policy*. Copies can be obtained from the course website, or in hardcopy from the programme administrator.