Comparison of Types of Drill Bit

May 12, 2020

```
In [19]: library(MASS)
```

1 Lecture 9: Randomised Complete Block Design

1.1 Example: Comparison of Types of Drill Bit

Looking to compare the hardness levels of 4 types of drill bit.

I.e. each drill bit type will be it's own treatment group, and we have 4 treatment groups. Looking at treatment effects τ_i, where $i = 1, 2, 3, 4$.

Is there external variation in experimental conditions that may require blocking? * Yes, the strip metal used to test the hardness of the drill bits comes from four different places * *The strips of metal come from 4 different companies.*

(it's a bit like wanting to test four different fertilisers over four different fields, each with their own properties... instead of using each field for each fertiliser, you block by field, and run all four of the fertilisers within each block / field)

- Therefore we'll want to control for the variation in the different sources of strip metal by running a randomised complete block design, where each source of strip metal is blocked, so we run a two-way ANOVA, where the treatment factor (factor A) is the drill bit type, and the blocking factor (factor B) is the metal strip source.

1.1.1 Randomised complete blocked experiment design

Characterised by:

- Exactly one observation is performed for each treatment per block
- The order that the treatments are processed **within each block** is randomised. Note: the order that the data appears in the table of results is clearly **not** the order that the data was collected / experiment ran.
- Have a treatments (in this case $a = 4$ drill bits) and b blocks (in this case $b = 4$ sources of strip metal).
- **Assuming there is no interaction between the treatment and the blocks**

1.1.2 Randomised complete blocked experiment model:

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon ij,$$

where $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$.
Method: Run two-way ANOVA.

```
In [1]: hardness <- c(9.3,9.4,9.2,9.7,
                      9.4,9.3,9.4,9.6,
                      9.6,9.8,9.5,10.0,
                      10.0,9.9,9.7,10.2)
```

1.2 Coding company of strip metal origin and type of tip.

- Data entered column-wise first
- Want to code up the companies (factor B) first
  - Will do 4*A, then 4*B, ...
- For the type of tip, we'll have 1,2,3,4 repeated 4 times

```
In [2]: company <- factor(c(rep('A',4),rep('B',4),rep('C',4),rep('D',4)))
```

```
In [5]: tip <- factor(c(rep(1:4,4)))
```

```
In [6]: drill <- data.frame(hardness, company, tip)
```

```
In [7]: rm(company, tip, hardness)
        attach(drill)
```

```
In [8]: drill
```

hardness	company	tip
9.3	A	1
9.4	A	2
9.2	A	3
9.7	A	4
9.4	B	1
9.3	B	2
9.4	B	3
9.6	B	4
9.6	C	1
9.8	C	2
9.5	C	3
10.0	C	4
10.0	D	1
9.9	D	2
9.7	D	3
10.2	D	4

1.3 Running two-way ANOVA

Running this without an interaction term as an assumption behind the randomised complete blocked design is that there is no interaction between the treatments and the blocks.

```
In [10]: # Note, if you try and add an interaction term here,
         # you'd need an extra load of degrees of freedom, and there aren't enough to spare
         # so you can't produce an MS_R, and  thus can't compute F values
         drill.aov <- aov(hardness ~ tip + company)

         summary(drill.aov)
```

```
            Df Sum Sq Mean Sq F value   Pr(>F)
tip          3  0.385 0.12833   14.44 0.000871 ***
company      3  0.825 0.27500   30.94 4.52e-05 ***
Residuals    9  0.080 0.00889
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

1.4 Hypothesis

We're interested in the following hypothesis:

H_0: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = 0$

We can reject this hypothesis at the 0.1% level of significance as our F-value, drawn from an $F_{3,9}$ distribution under the null hypothesis has a p-value that's highly significant.

```
In [11]: pf(14.44, 3, 9, lower.tail=FALSE)
```

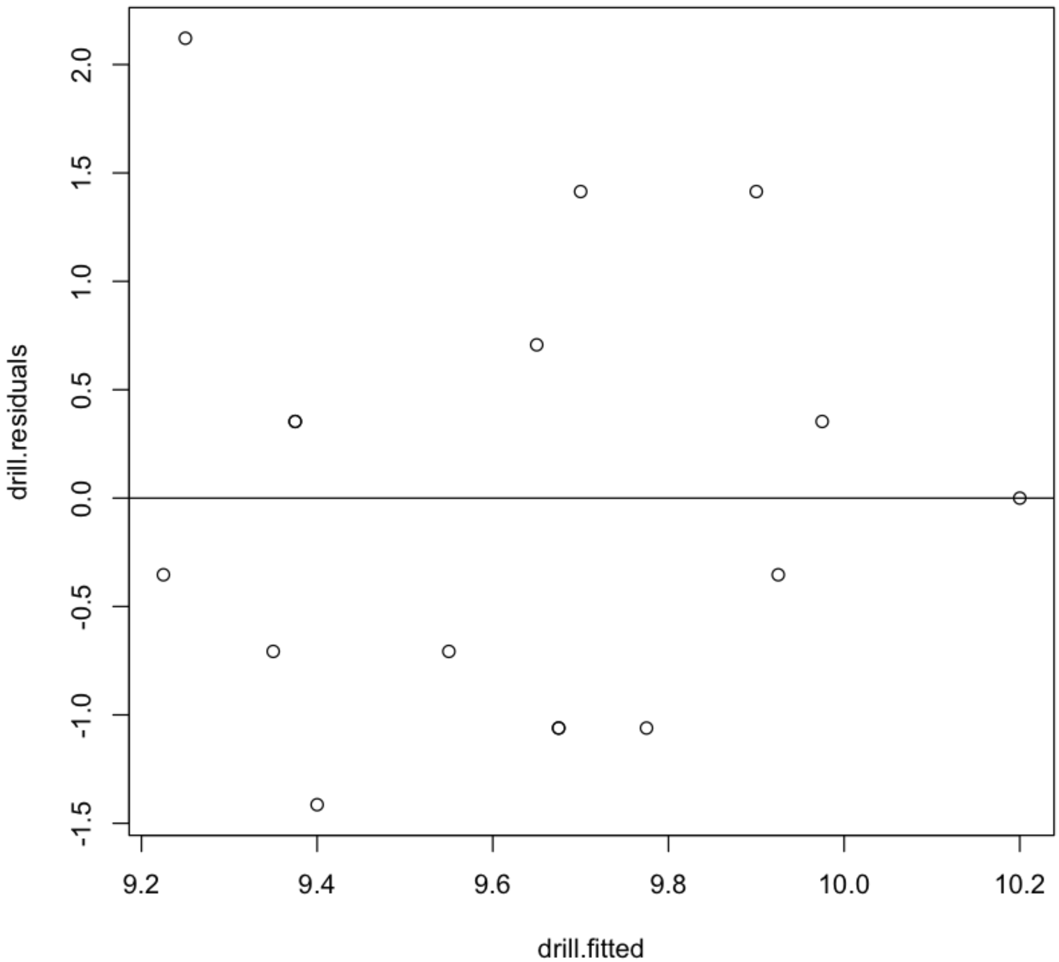
0.00087072043789626

1.5 Check for model adequecy

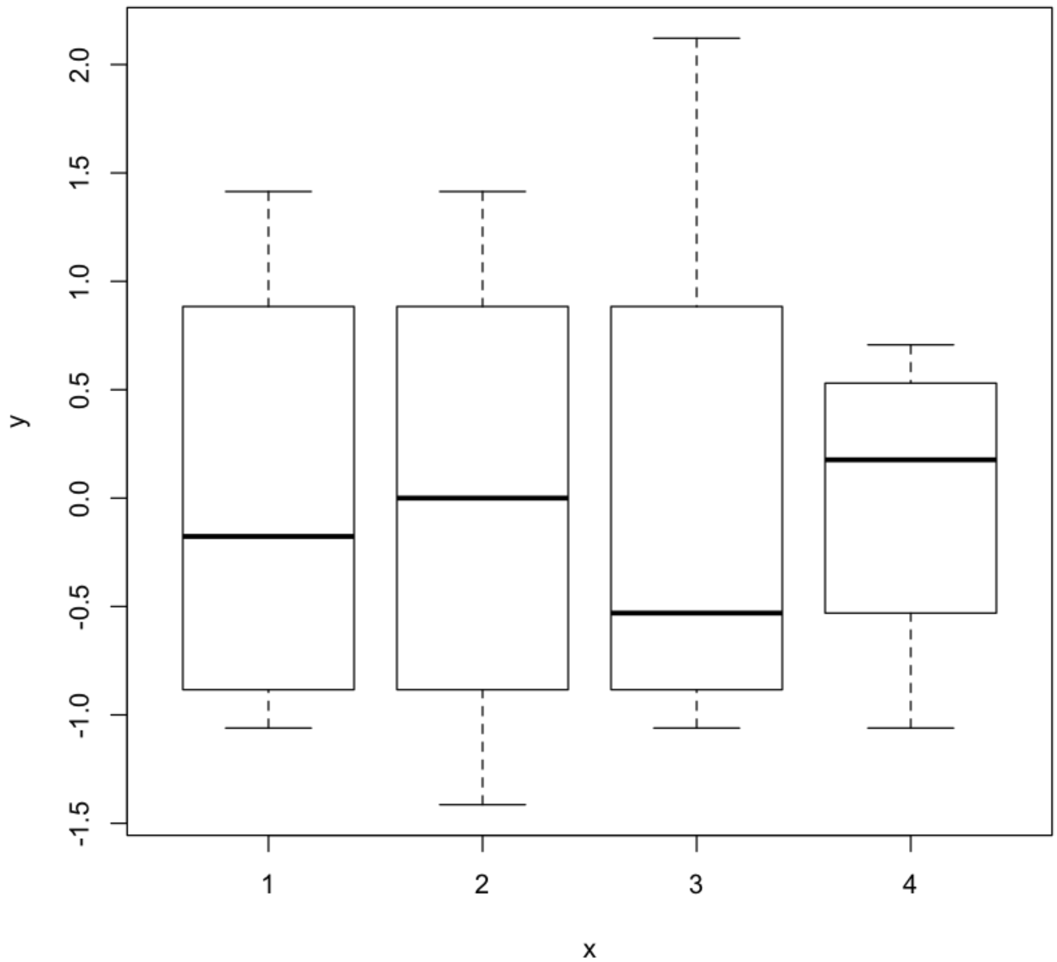
1. Residuals Vs fitted values
2. Residuals Vs factors

```
In [20]: drill.residuals <- stdres(drill.aov)
         drill.fitted <- fitted(drill.aov)
```

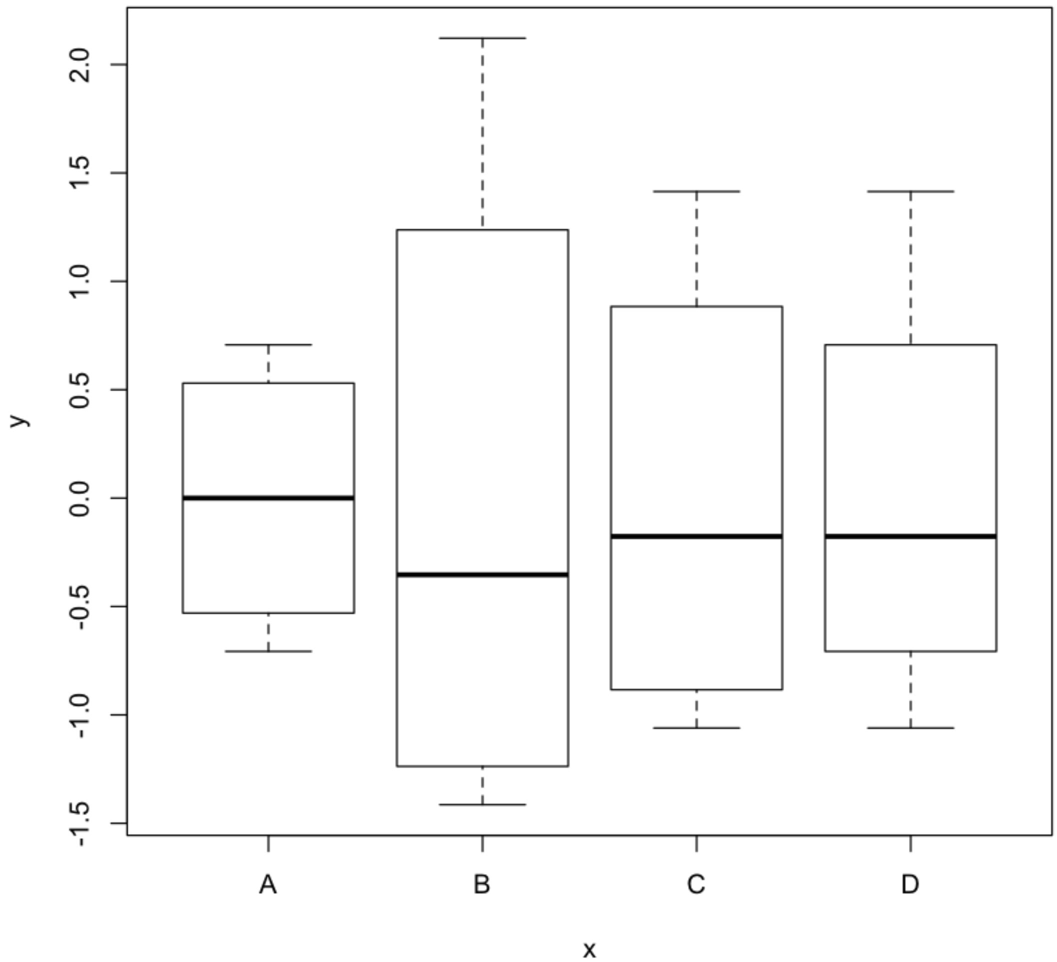
```
In [21]: plot(drill.fitted, drill.residuals)
         abline(h=0)
```




```
In [22]: plot(drill$tip, drill.residuals)
```




```
In [23]: plot(drill$company, drill.residuals)
```



1.6 Can go through the adequcy plots and say we've seen mean zero and constant variance for everything so we endorse the model that:

- Passes the F-test, rejecting the overall null.
- Passes the adequcy plots.

There are significant differences between the drill bits, and the model is adequete to describe these differences.

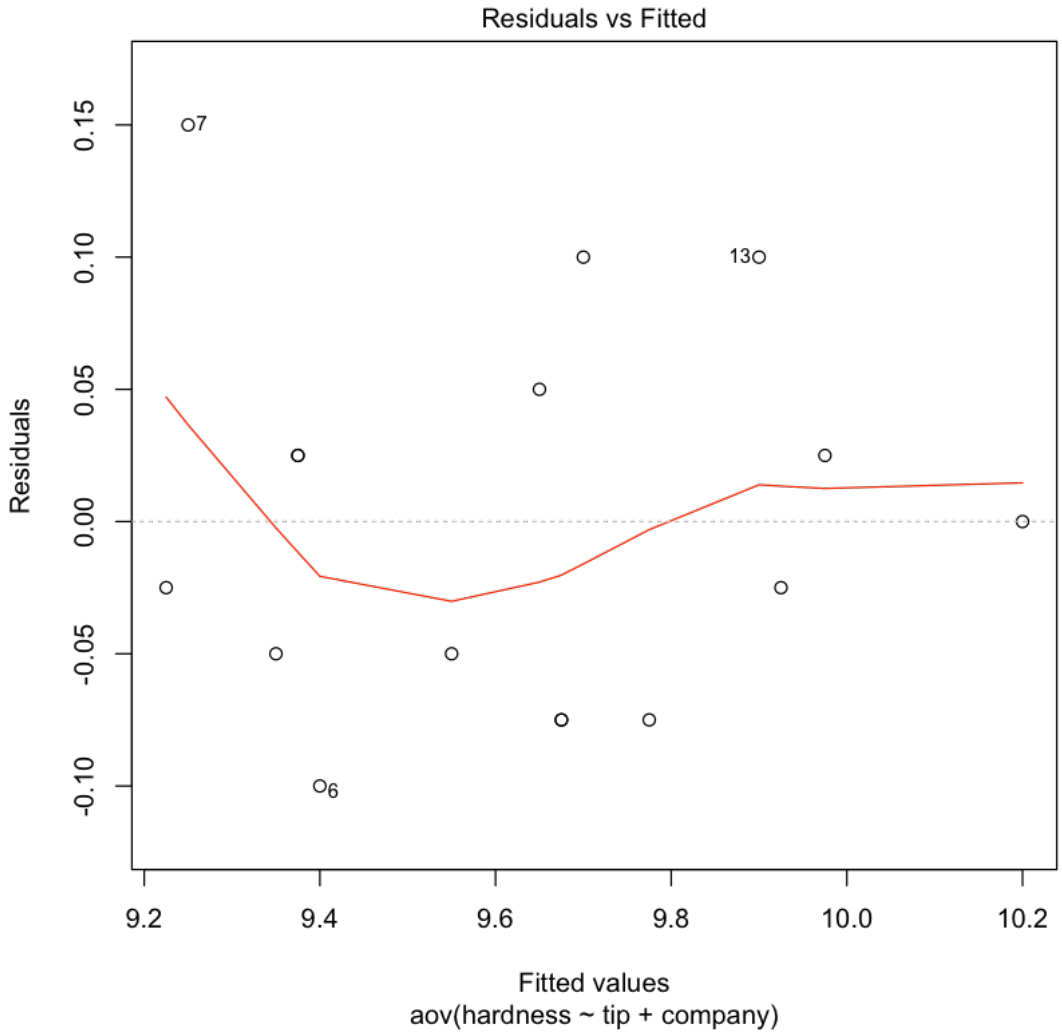
Can then look at the treatment effect tables to discuss these differences between treatment groups (i.e. between drill bits).

```
In [25]: model.tables(drill.aov, type='effects')
```

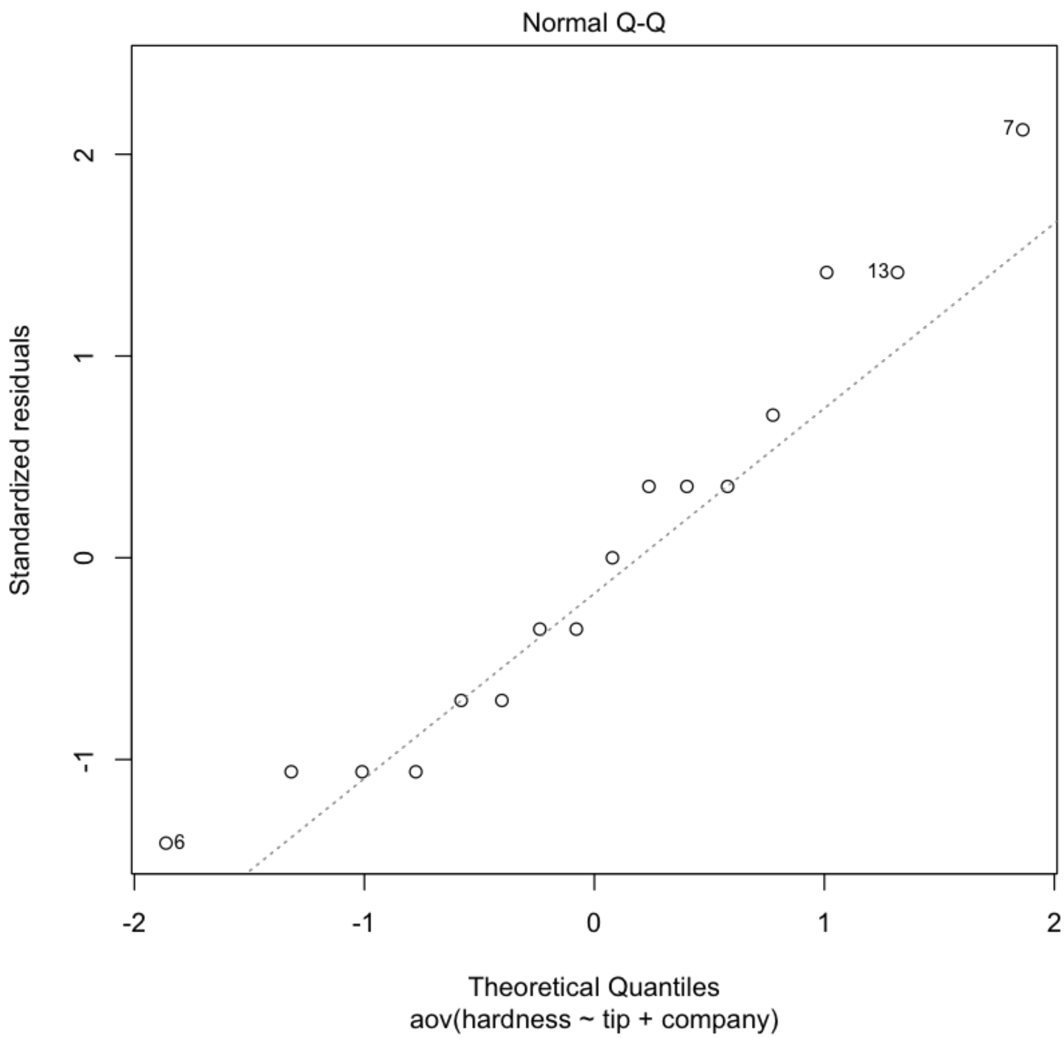
```
Tables of effects

 tip
tip
     1      2      3      4
-0.050 -0.025 -0.175  0.250

 company
company
     A      B      C      D
-0.225 -0.200  0.100  0.325
```

1.7 Final note, can quickly produce the residuals Vs fitted values plots by just hitting `plot(model.aov)`

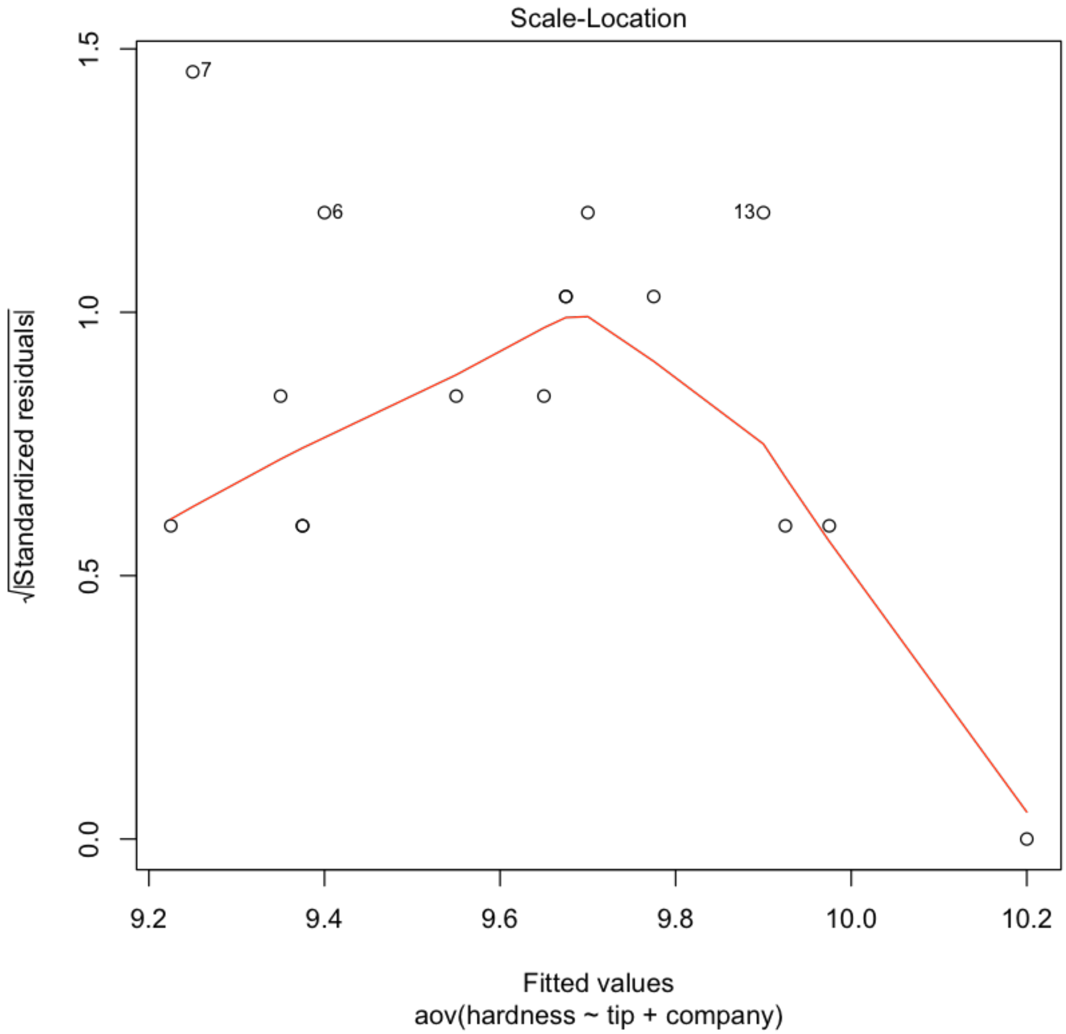
```
In [26]: plot(drill.aov)
```

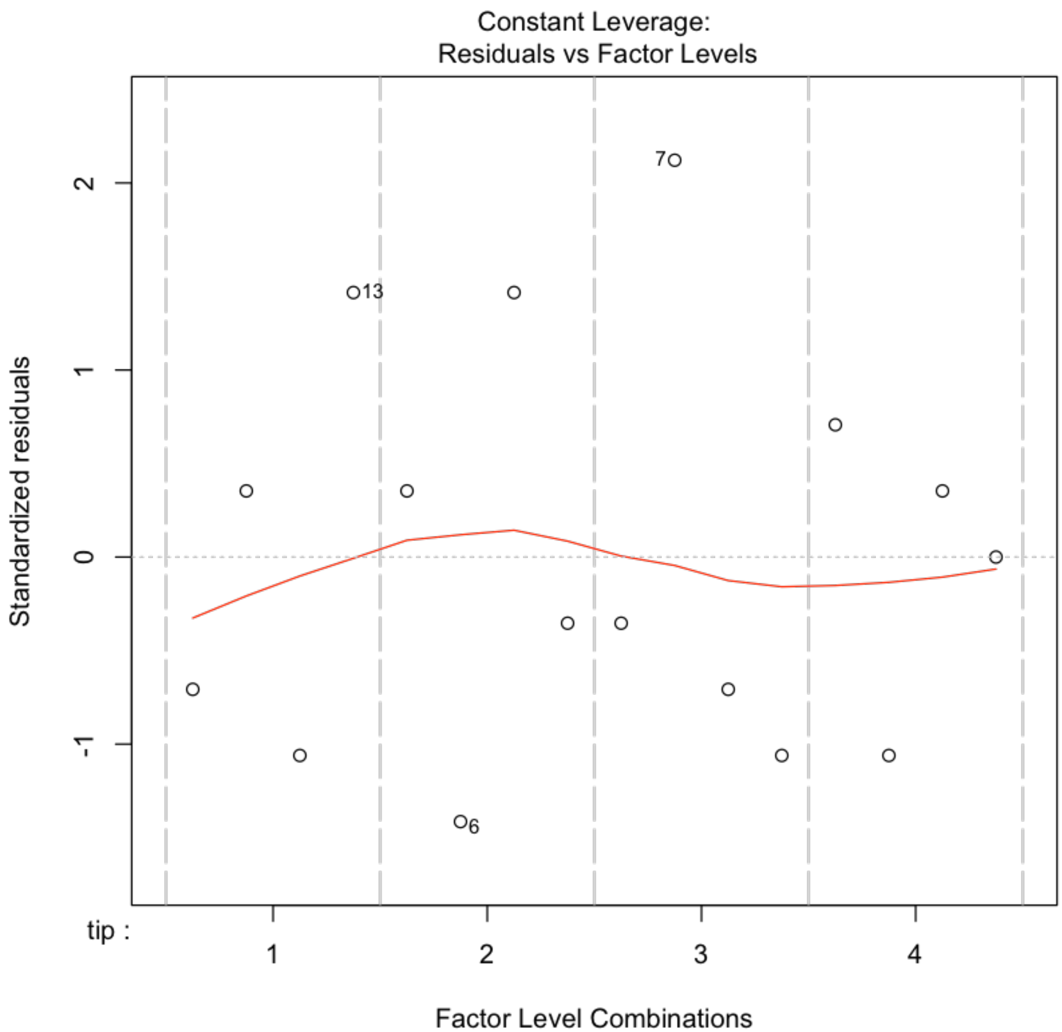


Residuals vs Fitted
Residuals
0.15
0.10
0.05
0.00
-0.05
-0.10
7
13
6
9.2
9.4
9.6
9.8
10.0
10.2
Fitted values
aov(hardness ~ tip + company)



Normal Q-Q
Standardized residuals
Theoretical Quantiles
aov(hardness ~ tip + company)
-2
-1
0
1
2
7
13
6


Scale-Location
7
6
13
√|Standardized residuals|
0.0
0.5
1.0
1.5
9.2
9.4
9.6
9.8
10.0
10.2
Fitted values
aov(hardness ~ tip + company)


Constant Leverage:
Residuals vs Factor Levels
Standardized residuals
2
1
0
-1
7
13
6
tip :
1
2
3
4
Factor Level Combinations