L7 Revision

June 1, 2020

0.1 Lecture 7: PACF

Main sections of L7:

- Introducing PACF;
- The three (iterative) steps of model building
  - Model **Identification**
    - Candidate p, d, q combinations for ARIMA
    - Plot data - differencing needed?
    - ACF and PACF - clear cut offs?
  - Parameter estimation
    - $\hat{\phi}_k, \hat{\theta}_i$
  - Model **adequecy**
    - Satisfying model fit and diagnostic plots
    - "Does the model fit the data"?
    - AIC?
- White noise processes
  - White noise sample ACF $\sim \text{NID}(0, \frac{1}{T})$
  - AR(p) process: sample PACF when $u > p, \sim \text{NID}(0, \frac{1}{T})$

In [5]:
```
price <- c(5.8, 6.1, 5.4, 6.2, 5.0, 4.6, 5.8, 5.1, 5.3, 5.1, 4.8, 5.3, 6.8, 9.0, 8.6,
9.0, 7.4, 6.4, 4.8, 3.9, 3.9, 5.6, 5.7, 7.5, 7.3, 7.4, 7.5, 9.7, 6.1, 6.0, 5.7, 5.0,
4.2, 4.6, 5.9, 5.4, 5.4, 5.4, 5.6, 7.6, 7.4, 5.4, 5.1, 6.9, 7.5, 5.9, 6.2, 5.6, 5.8,
5.6, 6.6, 4.8, 5.2, 4.5, 4.4, 5.3, 5.0, 6.4, 7.8, 8.5, 5.6, 7.1, 7.1, 8.0, 7.3, 5.7,
4.8, 4.3, 4.4, 5.7, 4.7, 4.1, 4.1, 4.7, 7.0, 8.7, 6.2, 5.9, 5.4, 6.3, 4.9, 5.5, 5.4,
4.7, 4.1, 4.6, 4.8, 4.5, 4.7, 4.8, 5.4, 6.0, 5.1, 6.5, 6.2, 4.6, 4.5, 4.0, 4.1, 4.7,
5.1, 5.2, 5.3, 4.8, 5.0, 6.2, 6.4, 4.7, 4.1, 3.9, 4.0, 4.9, 4.9, 4.8, 5.0, 4.9, 4.9,
5.4, 5.6, 5.0, 4.5, 5.0, 7.2, 6.1)
```

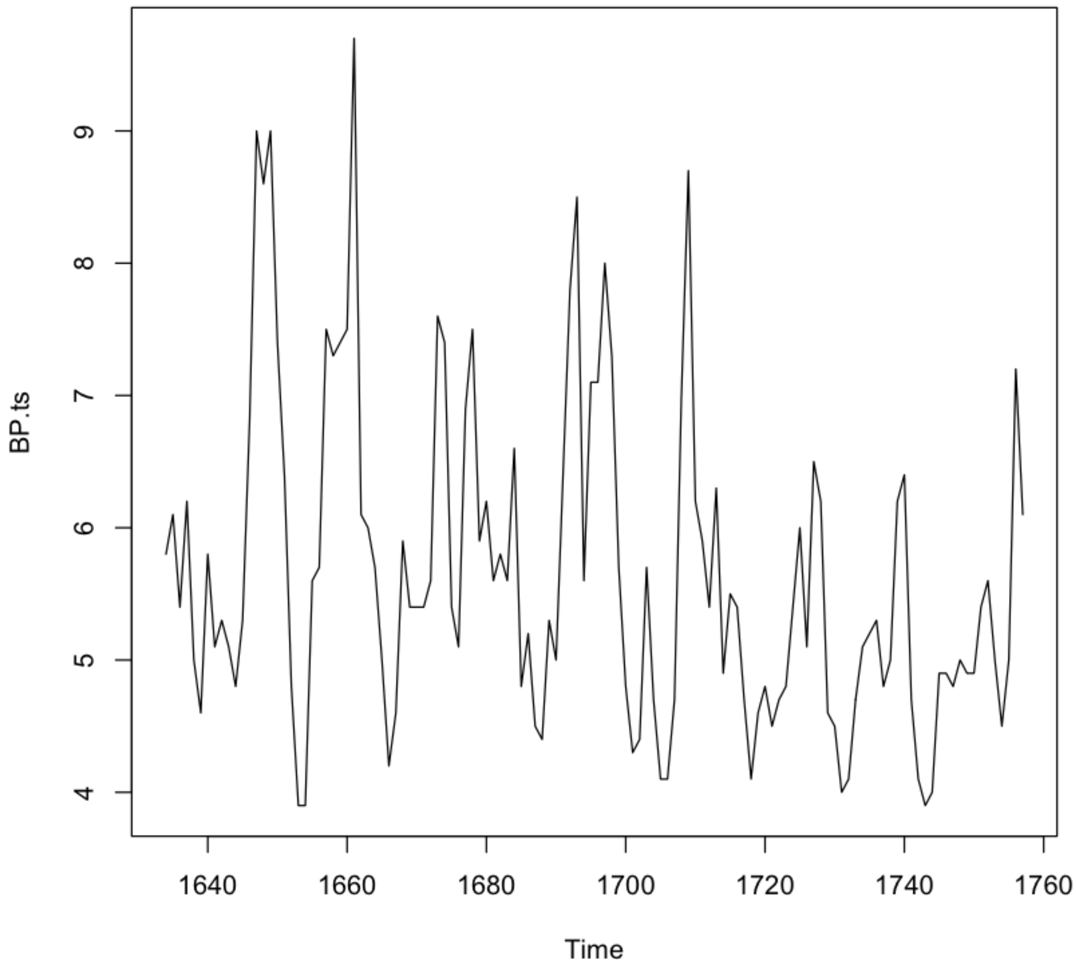
In [6]:
```
# use time series function to transform it into time series format (auto indexes the t
BP.ts <- ts(price, start=1634, frequency = 1)
```

In [7]:
```
# quick summary
summary(BP.ts)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  3.900   4.800   5.400   5.652   6.200   9.700
```

0.2 Timeseries Plot

```
In [9]: plot(BP.ts)
```



0.3 ACF for bread prices

- Geometric decrease, showing either AR(p) or ARMA(p,q) process
- Not linearly decreasing which would indicate a non-stationary process

Looking at the white noise confidence interval:

- Recall that for sample ACF: r_τ, if underlying process is white noise process, then $r_\tau \sim \text{NID}(0, \frac{1}{T})$

- Therefore the $100(1-\alpha)\%$ confidence interval is given by:

  $$0 \pm z_{0.025} \times \frac{1}{\sqrt{T}}$$

```
In [17]: z.0.025 <- qnorm(0.025, lower.tail = FALSE)
         z.0.025
```

1.95996398454005

```
In [19]: T <- length(BP.ts)

         se <- 1 / sqrt(T)

         se
```

0.0898026510133875

0.3.1 95% confidence interval calculation for white noise process

Note, this is also the same as a PACF 95% confidence interval for a AR(p) process for $\hat{\phi}_{uu}$ where $u > p$.

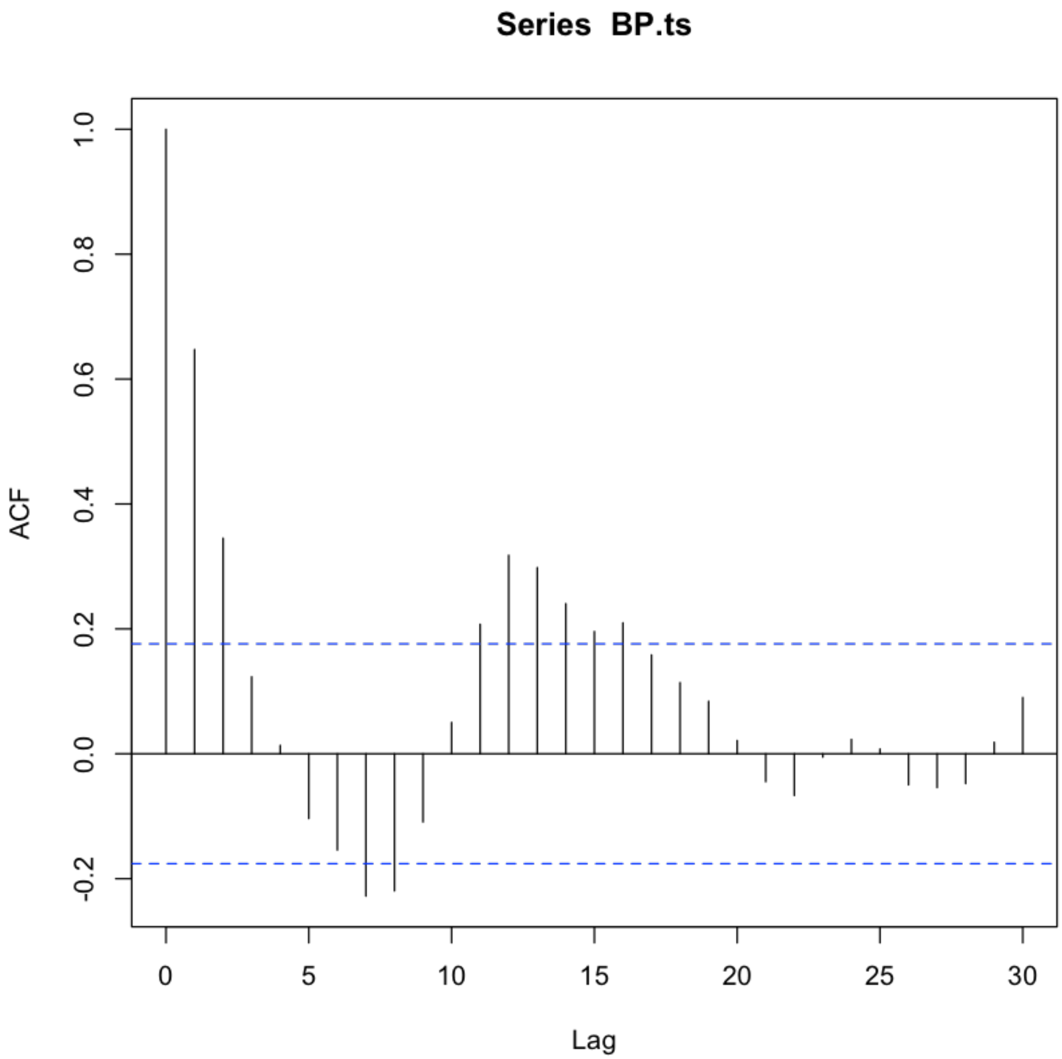
```
In [21]: CI.95 <- 0 + c(-z.0.025*se, z.0.025*se)
         CI.95
```

1. -0.176009961702459 2. 0.176009961702459

0.4 ACF

- Can clearly see that several lags have autocorrelations that fall outside of the 95% confidence interval for a white noise process, so we can reject the hypothesis that our underlying process is white noise.
- Geometric decrease in the trend and no severe cut off in the ACF suggests this is not an MA(q) process, most likely an AR(p) process or an ARMA(p,q) process.
- The evidence to suggest this is a stationary, rather than non-stationary process, is the geometric rather than linear decrease in the ACF with increased lags.

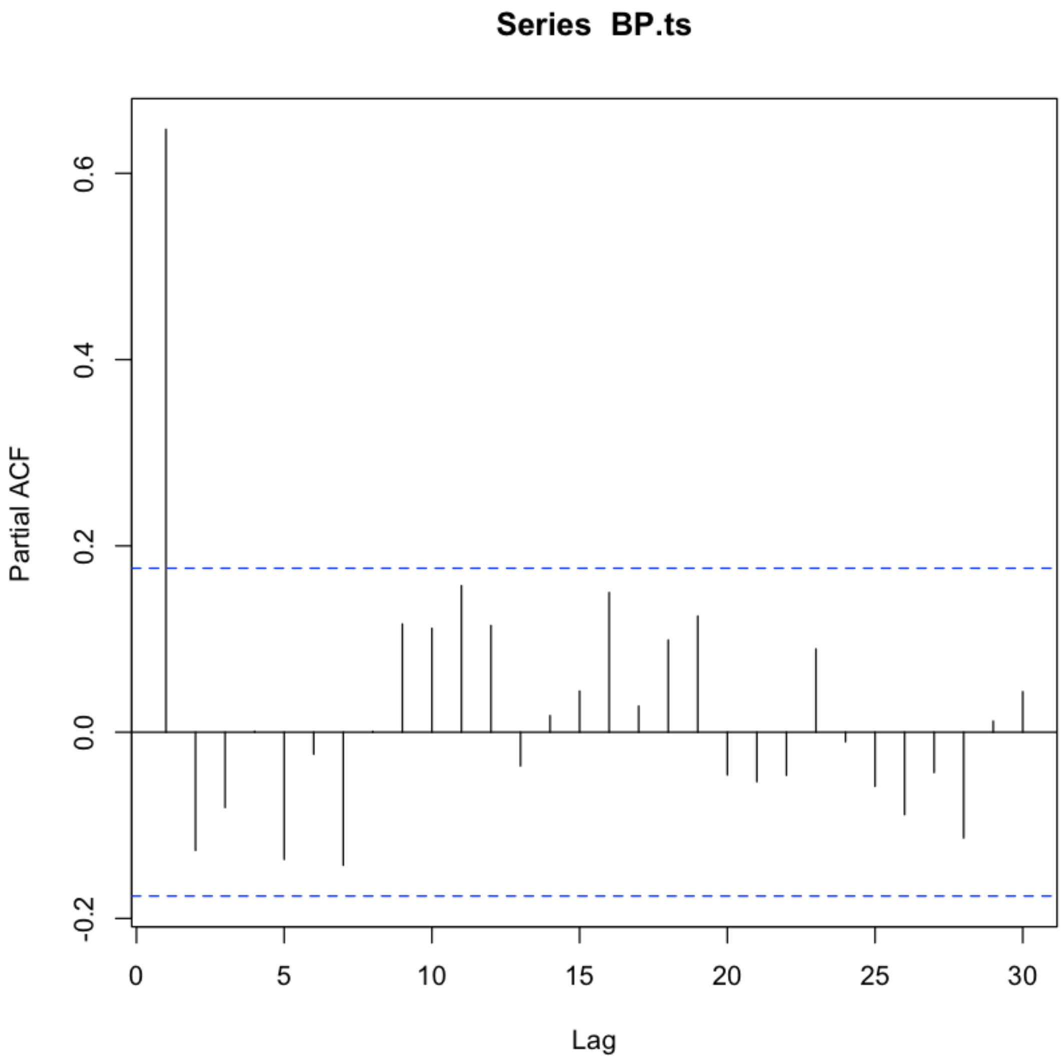
```
In [22]: acf(BP.ts, lag=30)
```



0.5 PACF

- Sample PACF has cut off at $\tau = 1$, indicating that we are dealing with an AR(1) process.
- For an AR(1) model, the pth parameter is the only parameter - ϕ - which is equal to $\phi_{uu} = \phi_{pp} = \phi_p = \phi_1$.
- Therefore our estimate for the AR(1) parameter is 0.647.
- Note! This is also the same as the $r_\tau = r_1$ sample autocorrelation value, since the partial autocorrelation and the sample **partial** autocorrelation are the same when there are no intervening variables.

```
In [11]: pacf(BP.ts, lag=30)
```




```
In [23]: pacf(BP.ts, lag=30, plot = FALSE)
```

```
Partial autocorrelations of series BP.ts, by lag

     1      2      3      4      5      6      7      8      9     10     11
 0.647 -0.127 -0.081  0.001 -0.137 -0.024 -0.143  0.001  0.116  0.111  0.157
    12     13     14     15     16     17     18     19     20     21     22
 0.115 -0.036  0.018  0.044  0.150  0.028  0.099  0.125 -0.046 -0.053 -0.046
    23     24     25     26     27     28     29     30
 0.089 -0.010 -0.058 -0.089 -0.043 -0.114  0.012  0.043
```

0.6 The Akaike Information Criteria (AIC)

Is a goodness of fit statistic that provides a tradeoff between the fit to the data and the number of parameters in the model (i.e. it's a score that will favour parsimony for equally performing models).

$AIC \approx T \ln \hat{\sigma}^2 + 2n + \text{constant}$

where

- n is the number of parameters fit in the model,
- $\hat{\sigma}^2$ is the estimated white noise variance,
- T is the length of the observed series.

Models with the smallest AIC are viewed more favourably
You want the estimated white noise variance to be as low as possible, as this is the random part of the model.

If this is zero, then it means the systematic part is bang on with its predictions.

If this is high, then it means the systematic part (used for prediction) is lost amongst the noise.

0.6.1 Interesting practical tip:

AIC often over-favours models with a higher number of parameters.

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1 Fitting ARIMA models

Models fit in R estimate parameters using MLE

When fitting ARMA model, $d = 0$, R **includes** a process mean term.

When fitting an ARIMA model (i.e. with a non-zero differencing term), R **does not include** a process mean term.

1.1 ARMA(1,0) = AR(1)

- Note the closeness of the ϕ parameter estimate to the 0.647 estimate we read off the ACF earlier.
- **CRUCIAL**: *THE INTERCEPT IS THE PROCESS MEAN*

Model fit below:

$$Y_t = 0.6429 \times (Y_{t-1} - 5.6608) + \epsilon_t$$

In [26]:
```
ar.1 <- arima(BP.ts, order = c(1,0,0))

ar.1
```

```
Call:
arima(x = BP.ts, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.6429     5.6608
s.e.  0.0678     0.2307

sigma^2 estimated as 0.8655:  log likelihood = -167.26,  aic = 340.52
```

1.2 Can calculate a 95% confidence interval for the parameters provided using a t-distribution, since we're estimating the variance.

In [32]:
```
T <- length(BP.ts)

# we've used up three degrees of freedom:
df <- T - 3

#ănumber of standard errors for 95% confidence interval
t.0.025.141 <- qt(0.025, df, lower.tail = FALSE)

phi.1 <- 0.6429

se <- 0.0678
```

In [34]:
```
# 95% confidence interval for phi.1
phi.1 + c(-t.0.025.141*se, t.0.025.141*se)
```

1. 0.508672016902136 2. 0.777127983097864

1.3 ARIMA(2,0,0) = AR(2)

In [35]:
```
ar.2 <- arima(BP.ts, order=c(2,0,0))

ar.2
```

```
Call:
arima(x = BP.ts, order = c(2, 0, 0))

Coefficients:
         ar1      ar2  intercept
      0.7231  -0.1235     5.6546
s.e.  0.0888   0.0892     0.2051

sigma^2 estimated as 0.8521:  log likelihood = -166.31,  aic = 340.62
```

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2 Model comparison

- AR(1) has smaller (better) AIC than AR(2).

In [ ]: