B.Sc./Grad. Dip.: Probability Models and Time Series
MAS programmes: Stochastic Models and Time Series
SPRING 2020

PMTS - Assignment C/SMTS - Assignment 2

Deadline: Thursday, 30th April, 2020

Total marks: [25]. Marks are shown in boxes [ ]. There are 2 questions in this assignment.

1. Consider a process $\{Y_t\}$ that satisfies the model equation

$$Y_t = \frac{21}{5}Y_{t-1} - \frac{19}{5}Y_{t-2} + \frac{3}{5}Y_{t-3} + \epsilon_t, \qquad t \in \mathbb{Z}.$$

   (a) Find the equation satisfied by the process, $\{W_t\}$, of first differences, where $W_t = Y_t - Y_{t-1}$. [2]

   (b) Is $\{W_t\}$ a stationary process? [3]

2. Let $\{Z_t\}$ be an ARIMA(1,0,3) process which satisfies the relation

$$(1-\phi L)Z_t = \epsilon_t + \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + \theta_3\epsilon_{t-3}, \qquad t \in \mathbb{Z},$$

where L is the *lag operator* such that $LZ_t = Z_{t-1}$, $\{\epsilon_t\}$ is a *white noise* process such that the ϵ_t are NID$(0, \sigma^2)$, ϕ is the *autoregressive* parameter, and θ_1, θ_2, and θ_3, are *moving average* parameters. Let $z_1, z_2, \ldots, z_T$ denote an observed realization of the process up to time T. Consider the forecasting problem with T the origin of the forecasts, h the lead time and $\hat{z}_T(h)$ the *corresponding minimum mean square error forecast*, for lead times $h \geq 1$.

(a) Deduce that an iteration scheme for the forecasts can be characterized by the following relations:

$$\begin{aligned}
\hat{z}_T(1) &= \phi z_T + \theta_1\epsilon_T + \theta_2\epsilon_{T-1} + \theta_3\epsilon_{T-2} \\
\hat{z}_T(2) &= \phi\hat{z}_T(1) + \theta_2\epsilon_T + \theta_3\epsilon_{T-1} \\
\hat{z}_T(3) &= \phi\hat{z}_T(2) + \theta_3\epsilon_T \\
\hat{z}_T(h) &= \phi\hat{z}_T(h-1), \qquad h \geq 4.
\end{aligned}$$

[8]

(b) By working with an appropriately explicit expression for Z_{T+2}, derive an explicit formula for each of the following:

i) the *forecast error* at lead time $h = 2$, formulated in terms of one or more members of the white noise process; [6]

ii) the *forecast error variance* at lead time $h = 2$, namely $V(2)$ (involving nothing more than σ^2, θ_1, and ϕ). [2]

(c) Consider a stationary process $\{Y_t\}$ that satisfies the following model equation

$$Y_t = \phi Y_{t-1} + \epsilon_t + \omega_1\epsilon_{t-1} + \omega_2\epsilon_{t-2} + \omega_3\epsilon_{t-3}, \qquad t \in \mathbb{Z},$$

where ϕ, ω_1, ω_2, and ω_3, are parameters.

Suppose that $\hat{y}_T(2) = 12.904$, the estimated white noise variance $\hat{\sigma}^2 = 0.297$, and that the estimated values of ϕ, ω_1, ω_2, and ω_3, are 0.56, -1.383, 0.533, and -0.0625, respectively.

Stating any assumptions you make, and with reference to appropriate percentage points from statistical tables, find a 99% prediction interval for Y_{T+2}. [4]

Important Note:

• Please read the current version of the *Mathematics & Statistics Coursework Policy*. Copies can be obtained from the course website, or in hardcopy from the programme administrator.