PACF Notebook

April 12, 2020

```
In [339]: df <- read.csv('southern_osc.csv')

          # just looking at the top five rows of the data...
          df[1:5,]
```

Date	T_i
Jan-51	1.5
Feb-51	0.9
Mar-51	-0.1
Apr-51	-0.3
May-51	-0.7

```
In [340]: apply_year <- function(year_2) {

              if (year_2 <= 99 && year_2 >= 50) {
                  year_4 <- as.integer(paste('19', as.character(year_2), sep=''))
              }

              else if (year_2 < 10) {
                  year_4 <- as.integer(paste('200', as.character(year_2), sep=''))
              }

              else {
                  year_4 <- as.integer(paste('20', as.character(year_2), sep=''))
              }

              return (year_4)
          }
```

0.1 Faffing with dates...

```
In [341]: # string manipulation
          df$Month <- substr(df$Date, 1, 3)
          df$Year2 <- as.integer(substr(df$Date, 5, 6))

          # getting the year 4's from the year 2's
          df$Year4 <- apply(df['Year2'], 1, apply_year)
```

```
# turning that into a proper date (first of the month)
df$Date <- as.Date(paste('01', df$Month, df$Year4, sep='-'), format = '%d-%b-%Y')
```

0.2 Quick double check to ensure I've got the right min...

```
In [342]: min(df$Year4)
```

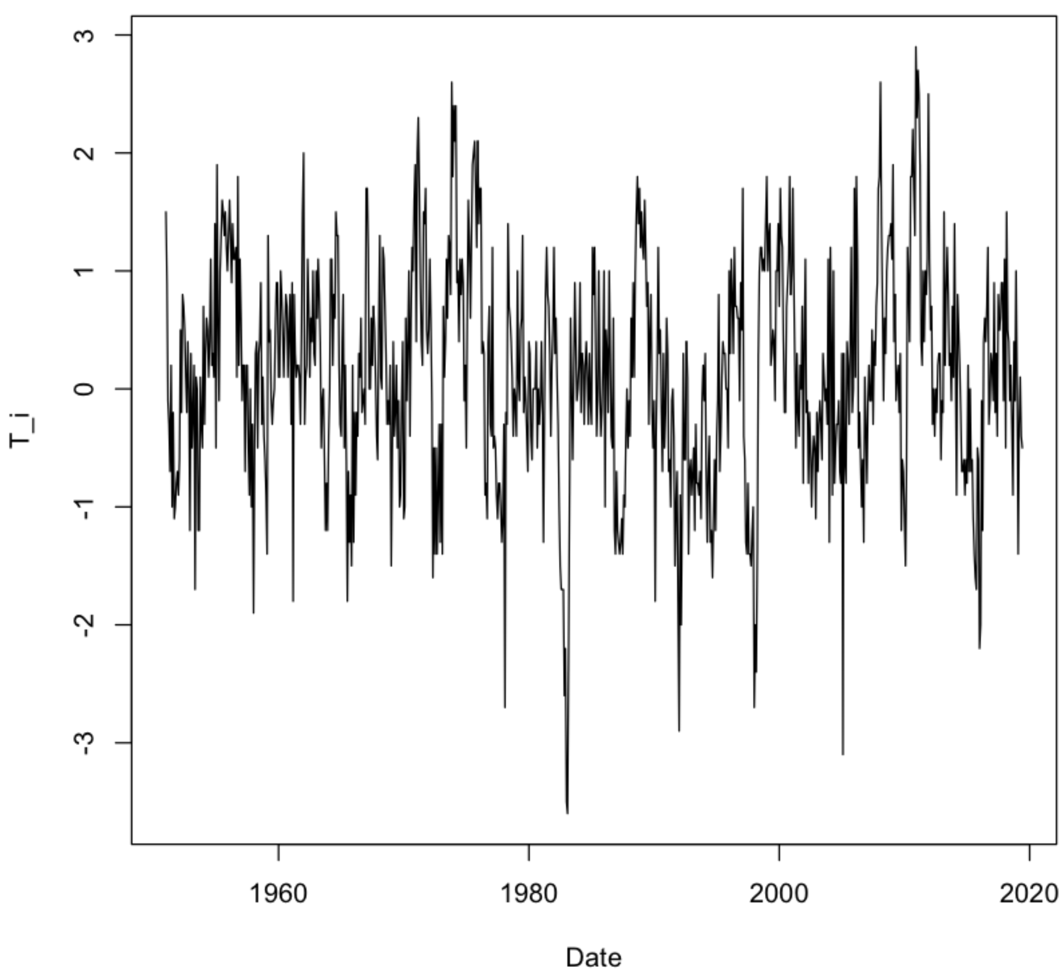
1951

0.3 Getting rid of the non-needed columns

```
In [343]: df <- df[,c('Date','T_i')]
```

0.4 Plotting

```
In [344]: plot(df$Date, df$T_i, type='l', xlab = 'Date', ylab = 'T_i')
```



0.5 Now on to the actual (partial) autocorrelation bit...

```
In [345]: df[1:5,]
```

Date	T_i
1951-01-01	1.5
1951-02-01	0.9
1951-03-01	-0.1
1951-04-01	-0.3
1951-05-01	-0.7

0.5.1 Key assumption in an AR(1) model:

$$Y_t = \phi Y_{t-1} + \epsilon_t,$$

is that the variance of Y_{t-1} expresses the variance by **all** values older than Y_{t-1}.

If this is not true, then the **AR(1)** model does not accurately reflect the process $\{Y_t\}$, and you will need to explore an **AR(2)** model which includes Y_{t-2}, since you need the extra term to provide additional, *unexplained variance* in Y_t.

0.5.2 AR(2) model:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t.$$

The idea here is that the Y_{t-2} will now capture all of the variance from earlier values of Y_{t-i} ($i \geq 2$) not captured by Y_{t-1}.

0.5.3 Remember, a condition of a stationary model is that the variance of Y_t and Y_{t-1} should be the same, because these are the Covariances of:

- $\text{Cov}(Y_{t-1}, Y_{t-1})$
- $\text{Cov}(Y_t, Y_t)$

And both of these just depend on the lag between them, **not** t!!
So $Var(Y_t) = Var(Y_{t-1}) = \gamma_0$

0.6 Why does any of this matter?

- We want to find out what p is in our AR(p) model.
- If all of the variance of previous Y_t values is explained by Y_{t-1}, then AR(1) is great, otherwise you need to look at AR(2), AR(3), AR(x), ...
- So we want to know how important the variance from a previous Y_{t-i} value is to predict Y_t.
- The correlation between two random variables tells us how much and in what direction the variance in one r.v. impacts the other. Covariance is the joint variability of two random variables. Correlation is just a normalised measure of covariance, scaled for the variance of each r.v.

- What we actually figure out the correlation between:
    1. The amount of variance in T_i that is **not explained** by T_{i-1}.
    1. Is basically what's missing from what's already in the AR(1) model
    2. The amount of variance in T_{i-2} that is **not explained** by T_{i-1}
    2. Is basically what's additionally provided by T_{i-2} that **isn't already provided by the intervening variables**.

1. could by high, but 2. could be low, because essentially a significant chunk of variance in Y_t is coming from an external feature - not itself from a previous point in time.

0.7 Key thing to remember:

There's this indirect... partial... method of finding the correlation between T_i and T_{i-2}, because if you did a direct Cov(T_i, T_{i-2}) then you'd get covariance that included the effect of the intervening T_{i-1}.

So it's (1): Find out the amount of T_i variance not covered by T_{i-1}

(1) are the residuals of the regession of T_i on T_{i-1}

Then (2): Find out the variance of T_{i-2} that's not covered / explained by T_{i-1}.

(2) are the residuals of the regression of T_{i-2} on T_{i-1}

The Covariance between (1) and (2) is the partial covariance between T_{i-2} and. T_i

0.8 Formalising this a bit more:

The partial autocorrelation (PAC) of T_i with a τ lagged version of itself - i.e. $T_{t-\tau}$ - is a correlation between the following variables:

1. The amount of variance in T_i not explained by T_{i-1}, T_{i-2}, ..., $T_{i-\tau+1}$

   so the amount of variance in T_i not explained by all of the interveening T_i's, up to $T_{i-\tau}$

2. The amount of variance in $T_{i-\tau}$ not explained by all of the intervening T_i's inbetween *but not including* $T_{i-\tau}$ and T_i.

0.9 How to implement on real data:

- Will guage the variance between one lag and another by creating a linear regression model between $y = T_i$ and $x = T_{i-1}$, and then use that model to predict $\hat{y}$, and the variance in the residuals gives you the variance contributed by T_{i-1} towards T_i

```
In [346]: # applying shifts by hand
          df['T_i_1'] <- c(NA, df['T_i'][1:dim(df['T_i'])[1]-1,])
          df['T_i_2'] <- c(NA, df['T_i_1'][1:dim(df['T_i_1'])[1]-1,])

          # removing the NA's
          df <- df[3:dim(df)[1],]
```

```
In [347]: df[1:10,]
```

	Date	T_i	T_i_1	T_i_2
3	1951-03-01	-0.1	0.9	1.5
4	1951-04-01	-0.3	-0.1	0.9
5	1951-05-01	-0.7	-0.3	-0.1
6	1951-06-01	0.2	-0.7	-0.3
7	1951-07-01	-1.0	0.2	-0.7
8	1951-08-01	-0.2	-1.0	0.2
9	1951-09-01	-1.1	-0.2	-1.0
10	1951-10-01	-1.0	-1.1	-0.2
11	1951-11-01	-0.8	-1.0	-1.1
12	1951-12-01	-0.7	-0.8	-1.0

0.10 Now applying regressions

Variable 1 as defined above

```
In [348]: # fitting the linear model
          lm.i.1 <- lm(df$T_i ~ df$T_i_1, data=df)

          df['pred_T_i'] <- predict(lm.i.1, df['T_i_1'])

          df['pred_T_i_residuals'] <- df['T_i'] - df['pred_T_i']
```

Variable 2 as defined above

```
In [349]: df.2.1 <- lm(df$T_i_2 ~ df$T_i_1, data=df)

          df['pred_T_i_2'] <- predict(df.2.1, df['T_i_1'])

          df['pred_T_i_2_residuals'] <- df['T_i_2'] - df['pred_T_i_2']
```

0.11 Calculating the correlation between the two sets of residuals

```
In [350]: cor(df$pred_T_i_residuals, df$pred_T_i_2_residuals, method = 'pearson')
```

0.296123035546276

0.12 Using R's built-in PACF function

```
In [351]: pacf(df$T_i, lag.max = 10, plot=FALSE)[2]
```

```
Partial autocorrelations of series df$T_i, by lag

    2
0.298
```

0.13 And now plotting the PACF

```
In [352]: pacf(df$T_i, lag.max = 100, plot=TRUE)
```