B.Sc./Grad. Dip.: Probability Models and Time Series
MAS programmes: Stochastic Models and Time Series

Examples 1

1. For each of the following Markov chains, determine the set of transient states and/or the irreducible closed set(s) of recurrent states, where they exist. Determine the periodicity of each of the irreducible closed sets. Label the states $\{a, b, c, \ldots\}$ etc.

   (a)

$$P_1 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

   (b)

$$P_2 = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

   (c)

$$P_3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

2. B blue and B green balls are distributed into two buckets in such a way that each contains B balls.

   At each stage n, draw a single ball at random from the first bucket and a single ball at random from the second bucket simultaneously and then place each ball in the bucket from which they were **not** drawn. After the switch is completed, the system state will then be taken to be equal to i, i.e. $X_n = i$ at stage n, if the first bucket contains i blue balls, for $i \in \{0, 1, 2, \ldots, B\}$.

   Write down a formula for the one-step transition probabilities $\{p_{ij} : i, j = 0, 1, 2, \ldots, B\}$.