Bread Price Diagnostics

April 18, 2020

1 Lecture 8: Diagnostic Checking & Introduction to Forecasting

From last time, fitted **AR(1)** model to bread price data

Let's load that bread price data up before looking at Overfitting as a diagnostic tool for model selection.

```
In [1]: price <- c(5.8, 6.1, 5.4, 6.2, 5.0, 4.6, 5.8, 5.1, 5.3, 5.1, 4.8, 5.3, 6.8, 9.0, 8.6,
        9.0, 7.4, 6.4, 4.8, 3.9, 3.9, 5.6, 5.7, 7.5, 7.3, 7.4, 7.5, 9.7, 6.1, 6.0, 5.7, 5.0,
        4.2, 4.6, 5.9, 5.4, 5.4, 5.4, 5.6, 7.6, 7.4, 5.4, 5.1, 6.9, 7.5, 5.9, 6.2, 5.6, 5.8,
        5.6, 6.6, 4.8, 5.2, 4.5, 4.4, 5.3, 5.0, 6.4, 7.8, 8.5, 5.6, 7.1, 7.1, 8.0, 7.3, 5.7,
        4.8, 4.3, 4.4, 5.7, 4.7, 4.1, 4.1, 4.7, 7.0, 8.7, 6.2, 5.9, 5.4, 6.3, 4.9, 5.5, 5.4,
        4.7, 4.1, 4.6, 4.8, 4.5, 4.7, 4.8, 5.4, 6.0, 5.1, 6.5, 6.2, 4.6, 4.5, 4.0, 4.1, 4.7,
        5.1, 5.2, 5.3, 4.8, 5.0, 6.2, 6.4, 4.7, 4.1, 3.9, 4.0, 4.9, 4.9, 4.8, 5.0, 4.9, 4.9,
        5.4, 5.6, 5.0, 4.5, 5.0, 7.2, 6.1)
```

```
In [99]: BP.ts <- ts(price, start=1634, frequency = 1)
```

```
In [106]: BP.train <- ts(price[1:104], start=1634, frequency = 1)
```

```
In [107]: BP.test <- ts(price[105:124], start=(1634+104), frequency = 1)
```

1.0.1 AR(1)

```
In [111]: # fitting an ARIMA(1,0,0) model
          ##ăwhich is  the same as fitting an AR(1) model
          BP.ar.1 <- arima(BP.train, order=c(1,0,0))
```

```
In [112]: #ăcan see we have a single coefficient - the phi_1 parameter - and it's standard err
          ##ănote also the similarity between the above estimate of phi_1 = 0.647 and R's MLE
          BP.ar.1
```

```
Call:
arima(x = BP.train, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
```

```
      0.6396     5.7389
s.e.  0.0744     0.2580

sigma^2 estimated as 0.9296:  log likelihood = -144.04,  aic = 294.08
```

2 Lecture 8

3 8.1 Overfitting

Overfitting is a method of diagnostic checking

Once you think you have an appropriate model (i.e. AR(1) in this case), fit a more general model with an extra parameter, and estimate whether the additional paramater value, taking the standard error of the additional parameter into account, differs significantly from zero.

If the extra parameter doesn't significantly add anything (i.e. the parameter's 95% confidence interval includes **zero**) then you can be more confident rejecting the model with the additional parameter and keeping with your initial model.

So we try and fit an AR(2) model...

```
In [8]: BP.ar.2 <- arima(BP.ts, order=c(2,0,0))
```

```
In [9]: BP.ar.2
```

```
Call:
arima(x = BP.ts, order = c(2, 0, 0))

Coefficients:
         ar1      ar2  intercept
      0.7231  -0.1235     5.6546
s.e.  0.0888   0.0892     0.2051

sigma^2 estimated as 0.8521:  log likelihood = -166.31,  aic = 340.62
```

And find that the second parameter has a value of -0.1235 with a standard error of 0.892

This gives us a t-statistic of -0.1235 / 0.0892

```
In [34]: t_stat <- -0.1235 / 0.0892
         t_stat <- t_stat
```

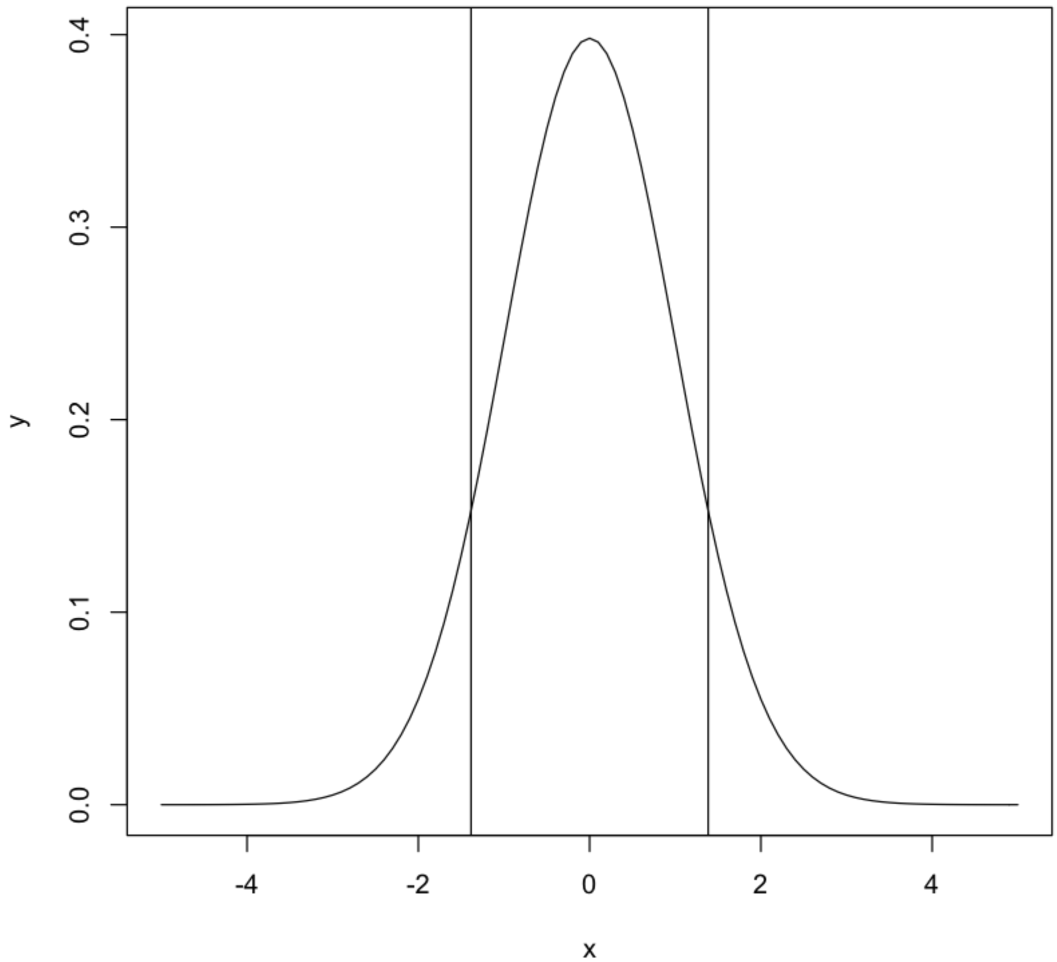
And we want to know, for that t-statistic, what's the probability of observing an equal or more extreme value by chance, if the paramater is really 0.

So you're after the probability distribution function, for P(X <= x), where x here is -1.38

In [35]:
```
# we say  that lower.tail = TRUE because we want to look at the extremes
##ăso you're looking at  the probability of the t-distribution having a value of less
##ăthan the t-stat you see, or more than the negative of the t-stat you see

##ăif you had a positive t-value, like 1.6, then you'd do P(X > x)
## you pick lower.tail because you're dealing  with a negative t-stat
2*pt(t_stat, 120, lower.tail = TRUE)
```

0.168765584886517

In [36]:
```
x <- seq(-5,5,0.1)
y <- dt(x, 120)

plot (x,y,type='l')
abline(v=t_stat)
abline(v=-t_stat)
```



Since the second parameter is not significant at the 5% level, we reject the usage of the second paramter, and stick with the AR(1) model, rather than going for the extra parameter with the AR(2) model.

This conclusion is inline with AIC + parsimony.

4 8.2 Diagnostic checking of the residuals

4.1 Checking the adequecy of the model

Examining the residials - the fitted data Vs the observed data

We would expect the residuals to have no trend - and for the residuals to appear to be from a white noise process

The resisuals should be uncorrelated with each other, with mean zero.

May also wish to assume that the white noise residuals are normally distributed... so maybe we could also try out a qq plot

4.2 Residuals, e_t:

$e_t = y_t - \hat{y}_t$,
where $\hat{y}_t$ is the fitted value, given for the AR(1) model by:

$$\hat{y}_t - \hat{\mu} = \phi(y_{t-1} - \hat{\mu}),$$

because the ARIMA model you have produced has translated $Y_t \rightarrow Y_t - \mu$, but when we actually use real data we of course input y_t rather than $y_t - \hat{\mu}$, where we have to estimate the sample mean $\hat{\mu}$, as we don't know the population mean μ.
This equation involving $\hat{y}_t$ can be re-arranged to be:

$$\hat{y}_t = (1 - \hat{\phi})\hat{\mu} + \hat{\phi}y_{t-1}, 2 \leq t \leq T.$$

4.3 Standardised residuals:

These are the residuals normalised by the variance of the white noise, ϵ_t.

Standardised residuals, $d_t = e_t/\hat{\sigma}^2$.

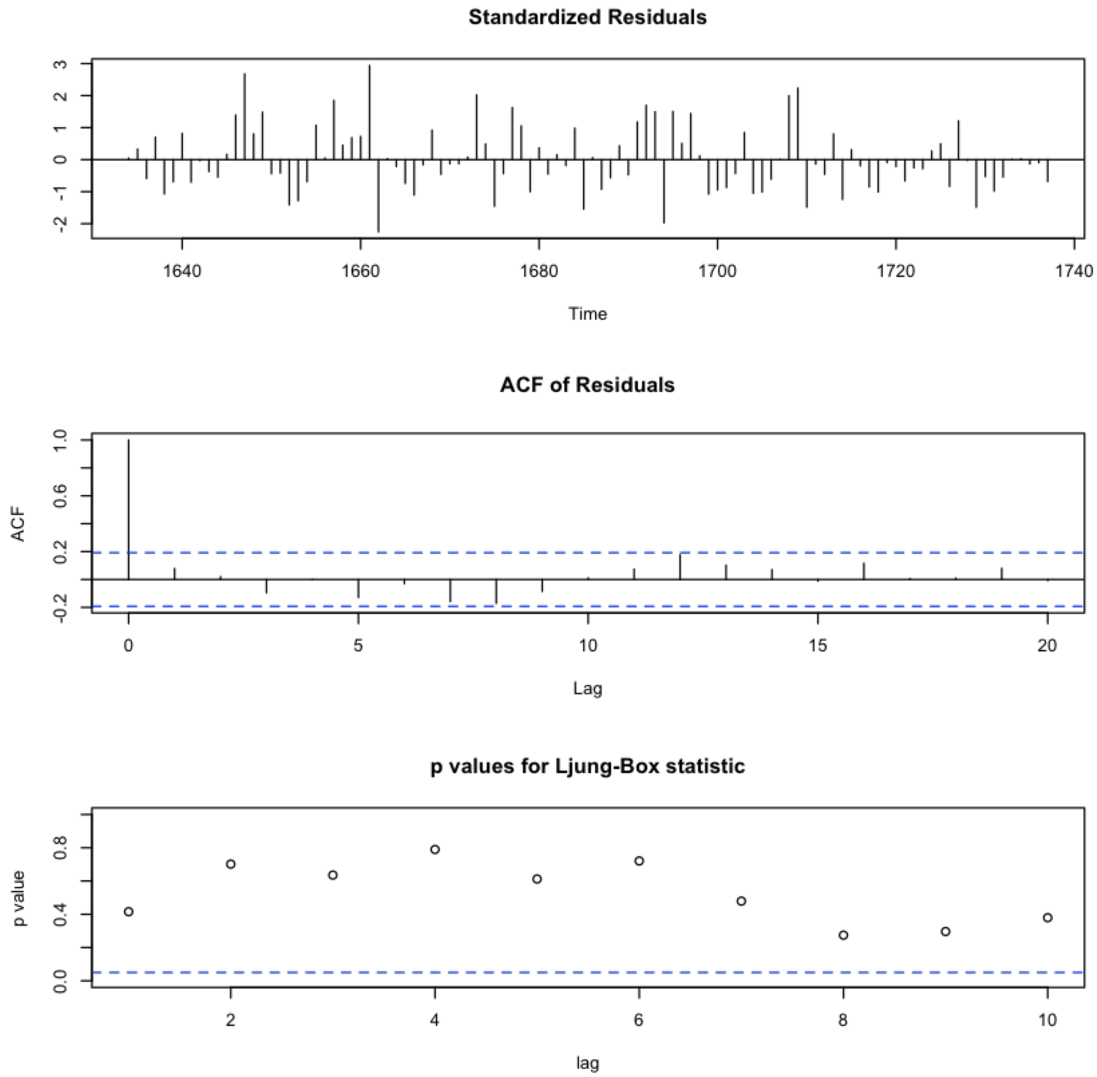
Can then examine the ACF plot of the standardised residuals, to see if there's autocorrelation between standardised residuals, and to deduce whether the residuals represent a white noise process.

We have the 95% confidence bands in the ACF, so after $\rho_0 = 1$, we expect most residuals to fall below this white noise 95% probability limit - and only see about 1/20 data points fall above the 95% probability bands.

the **tsdiag** function in R produces both a plot of the standardised residuals, as well as an ACF + 95% confidence band plot, as well as a third plot which we'll look at later...

5 8.3: Bread Price Example: Diagnostic Checking

In [139]:
```
#ăthe tsdiag function is specifically for ARIMA model diagnostics
tsdiag(BP.ar.1)
```



6 8.4 The portmanteau statistics

Given a lag K, the **Box-Pierce** test statistic based upon the first K autocorrections is:

$Q^*_K = T\sum_{\tau=1}^K r_\tau^2$

where r_τ here is the sample autocorrelations **of the residuals**.

If r_τ **ARE** the sample autocorrelations from the residuals from fitting an ARMA(p,d,q) model, and **the model is correct**, then the Q^*_K should follow a chi-squared distribution, χ^2_{K-p-q}.

An **improved** statistic is the **Ljung-Box** statistic. This is the statistic that's produced by R's *tsdiag* function.

The Ljung-Box statistic is:

$Q_K = T(T+2)\sum_{\tau=1}^{K} r_\tau^2/(T-\tau)$

Where K is the lag you're determining the statistic at, T is the size of the sample, r_τ is the autocorrelation at lag τ *of the residuals*.

This is sufficiently complicated that I don't think we'd ever get given a tough question on this in an exam, and I don't think we'd need to remember this equation...

tsdiag computes Q_K (but doesn't show it)

you get a p-value per lag

this is the p-value associated with each Q_K statistic on a chi-squared distribution with $K-p-q$ degrees of freedom.

if any of these Q_K values is significant, then we may reject the null hypothesis that the model is correct

So with **Ljung-Box** diagnostics we're running a hypothesis test, and we may reject the model if a lag is found to be significant at the 5% level.

As no Q_K falls within the 5% significance level, we choose not to reject the null hypothesis, and thus we accept the AR(1) model.

The AR(1) model is deemed adequete.

7 8.5 Airline passenger data

From lecture 3

```
In [142]: passengers<-c(112, 118, 132, 129, 121, 135, 148, 148, 136, 119, 104, 118, 115, 126,
                       +141, 135, 125, 149, 170, 170, 158, 133, 114, 140, 145, 150, 178, 163,
                       +199, 199, 184, 162, 146, 166, 171, 180, 193, 181, 183, 218, 230, 242,
                       +172, 194, 196, 196, 236, 235, 229, 243, 264, 272, 237, 211, 180, 201,
                       +235, 227, 234, 264, 302, 293, 259, 229, 203, 229, 242, 233, 267, 269,
                       +364, 347, 312, 274, 237, 278, 284, 277, 317, 313, 318, 374, 413, 405,
                       +271, 306, 315, 301, 356, 348, 355, 422, 465, 467, 404, 347, 305, 336,
                       +362, 348, 363, 435, 491, 505, 404, 359, 310, 337, 360, 342, 406, 396,
                       +548, 559, 463, 407, 362, 405, 417, 391, 419, 461, 472, 535, 622, 606,
                       +390, 432)
```

```
In [150]: air.ts <- ts(passengers, start=1950, frequency = 12)
```

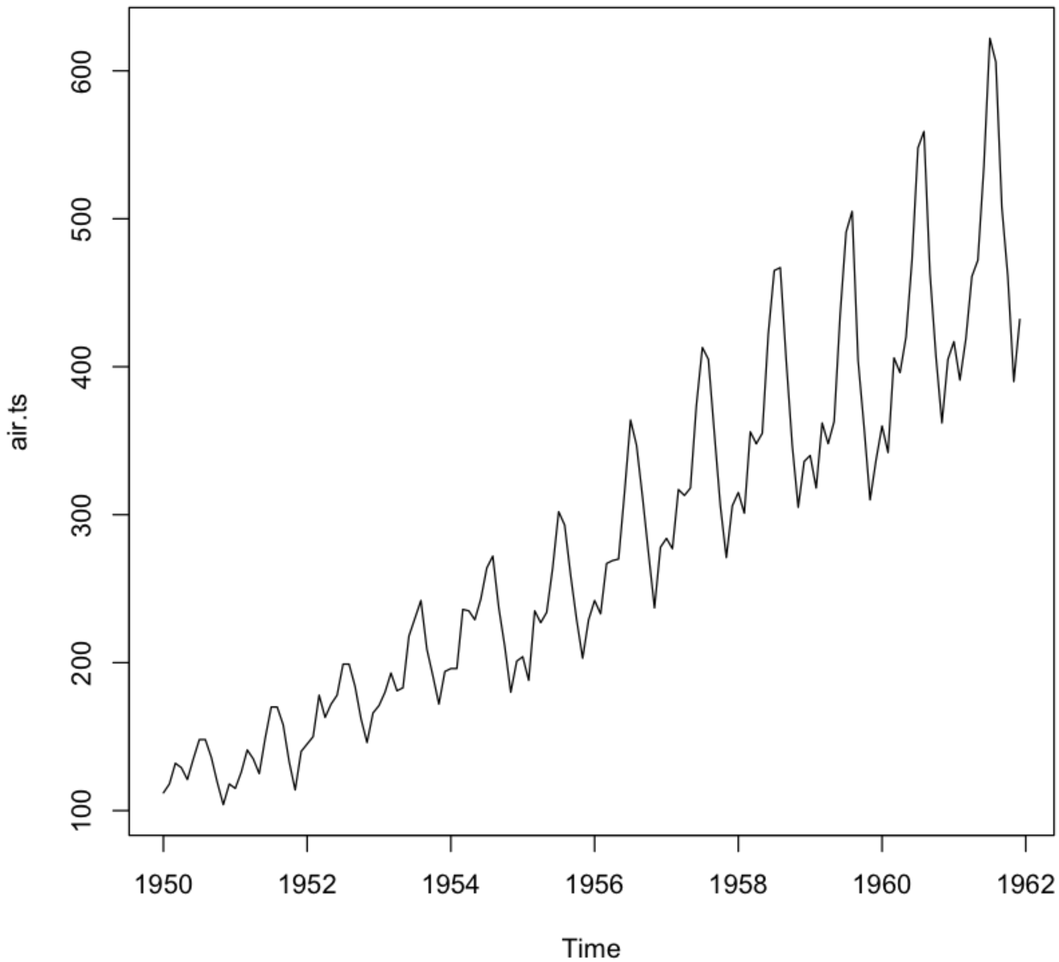
```
In [151]: air.ts
```

```
ERROR while rich displaying an object: Error in repr_matrix_generic(obj, "\n%s%s\n", sprintf("

Traceback:
1. FUN(X[[i]], ...)
2. tryCatch(withCallingHandlers({
 .     if (!mime %in% names(repr::mime2repr))
 .         stop("No repr_* for mimetype ", mime, " in repr::mime2repr")
 .     rpr <- repr::mime2repr[[mime]](obj)
 .     if (is.null(rpr))
 .         return(NULL)
 .     prepare_content(is.raw(rpr), rpr)
 . }, error = error_handler), error = outer_handler)
3. tryCatchList(expr, classes, parentenv, handlers)
4. tryCatchOne(expr, names, parentenv, handlers[[1L]])
5. doTryCatch(return(expr), name, parentenv, handler)
6. withCallingHandlers({
 .     if (!mime %in% names(repr::mime2repr))
 .         stop("No repr_* for mimetype ", mime, " in repr::mime2repr")
 .     rpr <- repr::mime2repr[[mime]](obj)
 .     if (is.null(rpr))
 .         return(NULL)
 .     prepare_content(is.raw(rpr), rpr)
 . }, error = error_handler)
7. repr::mime2repr[[mime]](obj)
8. repr_markdown.ts(obj)
9. repr_ts_generic(obj, repr_markdown.matrix, ...)
10. repr_func(m, ..., rows = nrow(m), cols = ncol(m))
```

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1950	112	118	132	129	121	135	148	148	136	119	104	118
1951	115	126	141	135	125	149	170	170	158	133	114	140
1952	145	150	178	163	172	178	199	199	184	162	146	166
1953	171	180	193	181	183	218	230	242	209	191	172	194
1954	196	196	236	235	229	243	264	272	237	211	180	201
1955	204	188	235	227	234	264	302	293	259	229	203	229
1956	242	233	267	269	270	315	364	347	312	274	237	278
1957	284	277	317	313	318	374	413	405	355	306	271	306
1958	315	301	356	348	355	422	465	467	404	347	305	336
1959	340	318	362	348	363	435	491	505	404	359	310	337
1960	360	342	406	396	420	472	548	559	463	407	362	405
1961	417	391	419	461	472	535	622	606	508	461	390	432

7.1 Will now start a sequence of plotting, logging, and differencing to produce a stationary process to model using ARIMA.

```
In [180]: plot(air.ts)
```

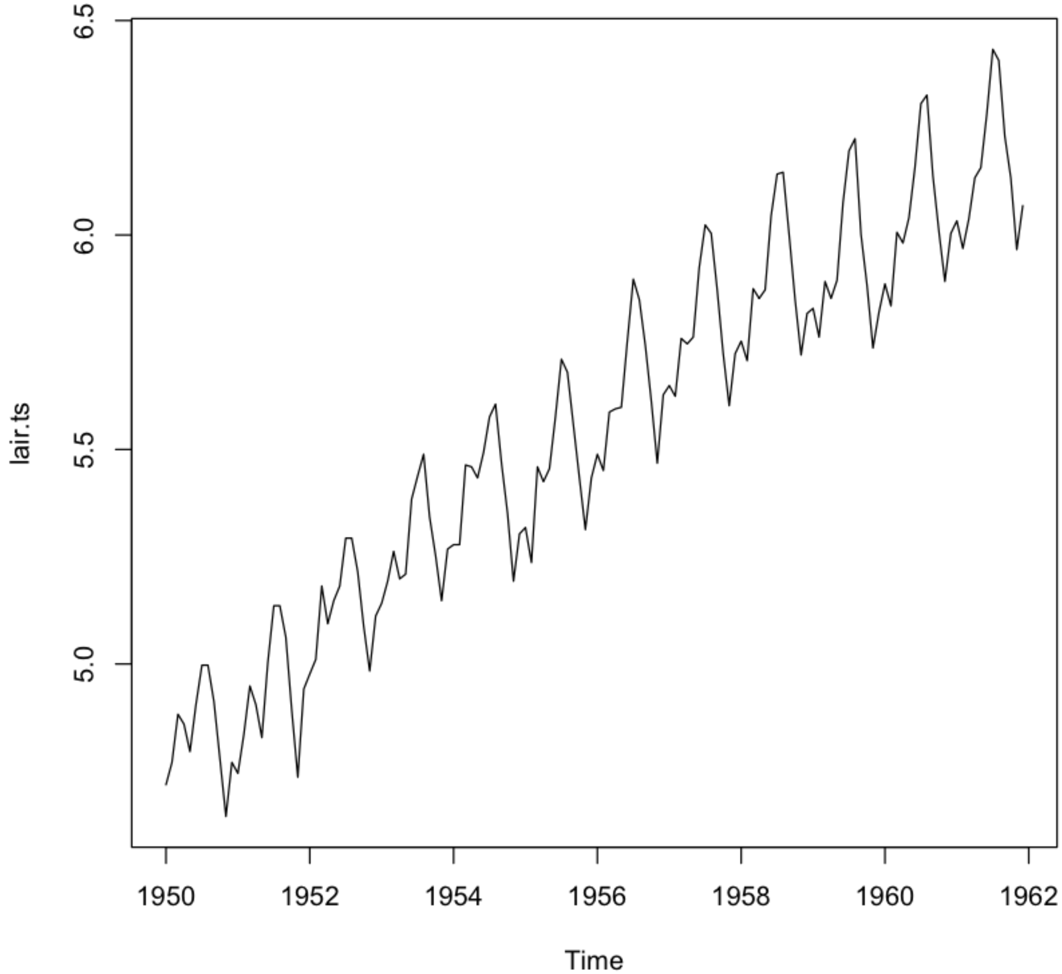


Can see that the variance is increasing with time.

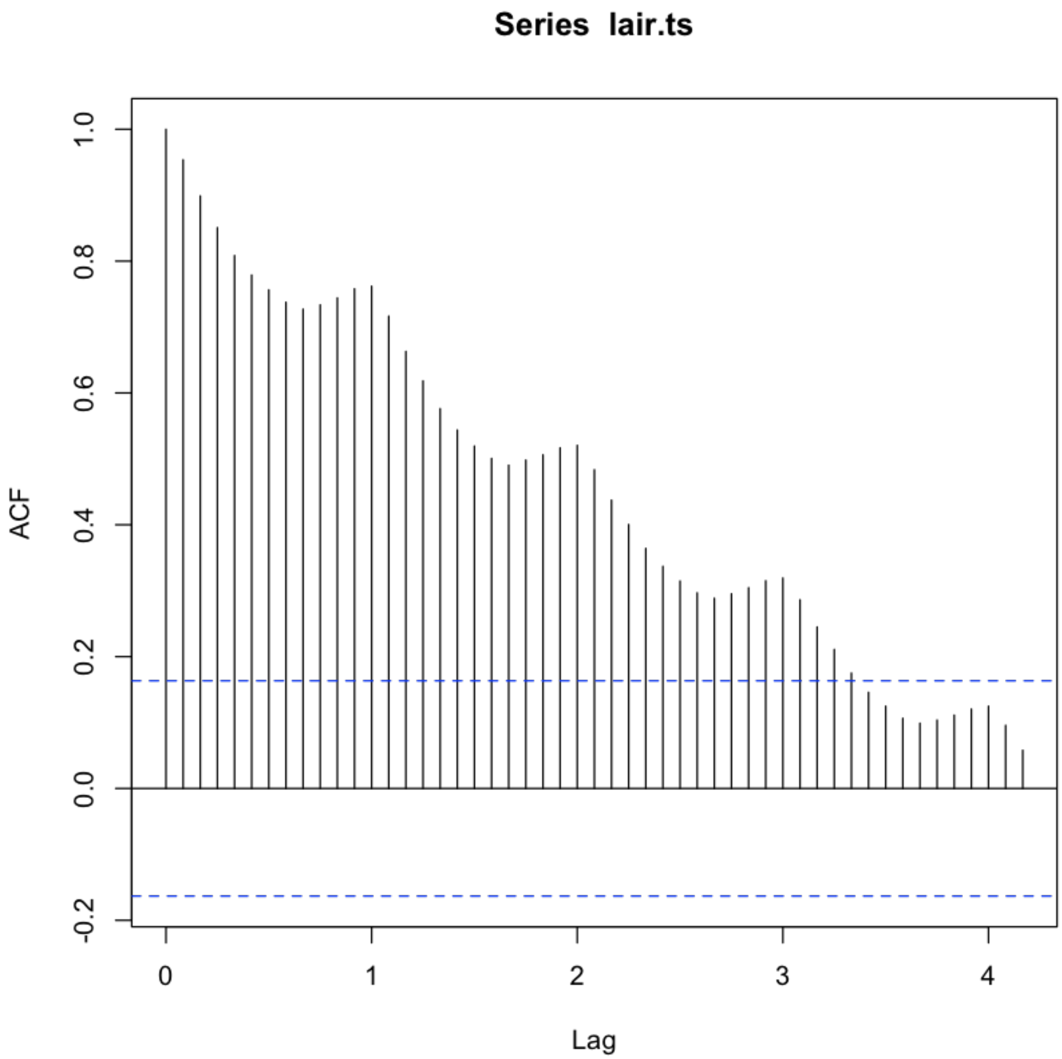
7.2 Logging the time series to reduce that increasing variance with time.

```
In [152]: lair.ts <- log(air.ts)
```

```
In [153]: plot(lair.ts)
```




```
In [156]: lair.acf <- acf(lair.ts, 50)
```



Now we see that the variance is relatively constant with increasing time.

There's still obvious seasonalilty and general trend though in the time series.

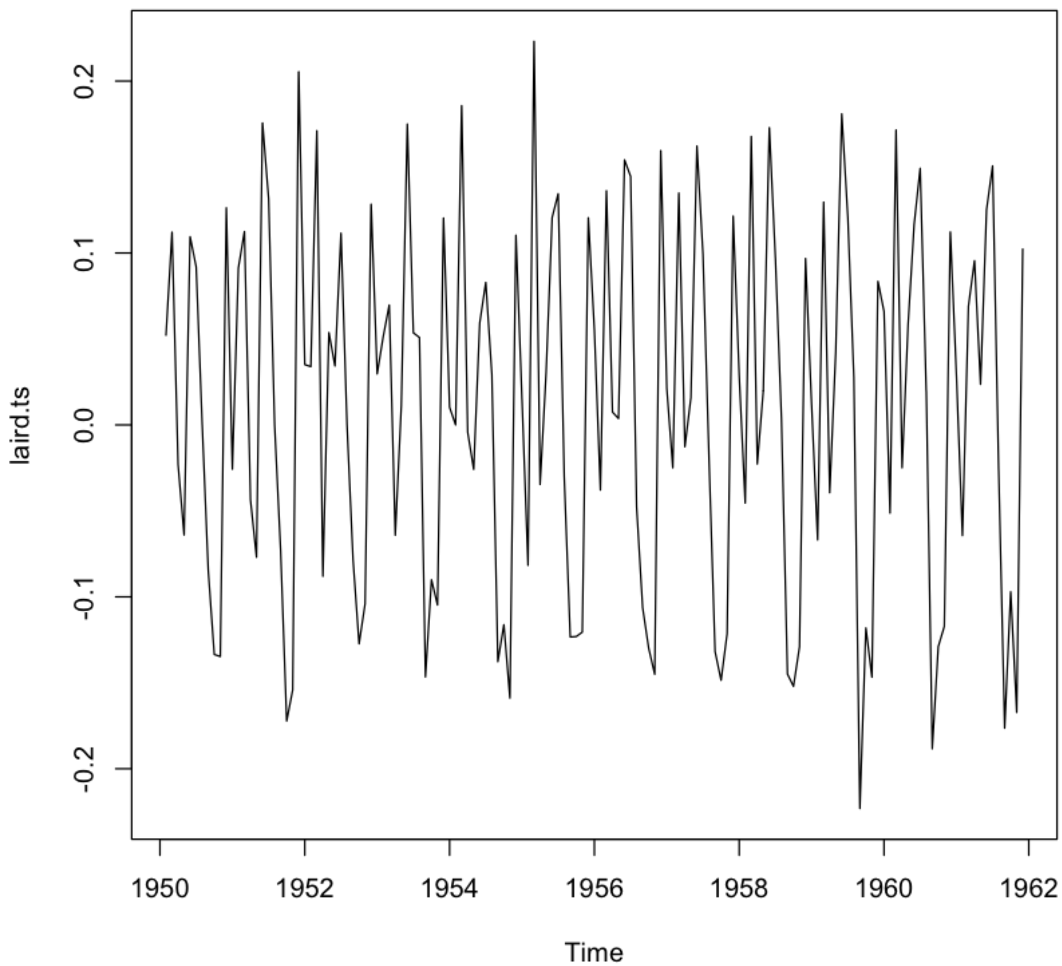
And the auto correlation function has a very slow decrease towards zero.

Let's difference to try and remove some of that seasonality and trend

$\Delta y_t = (1 - L)y_t = y_t - y_{t-1}$

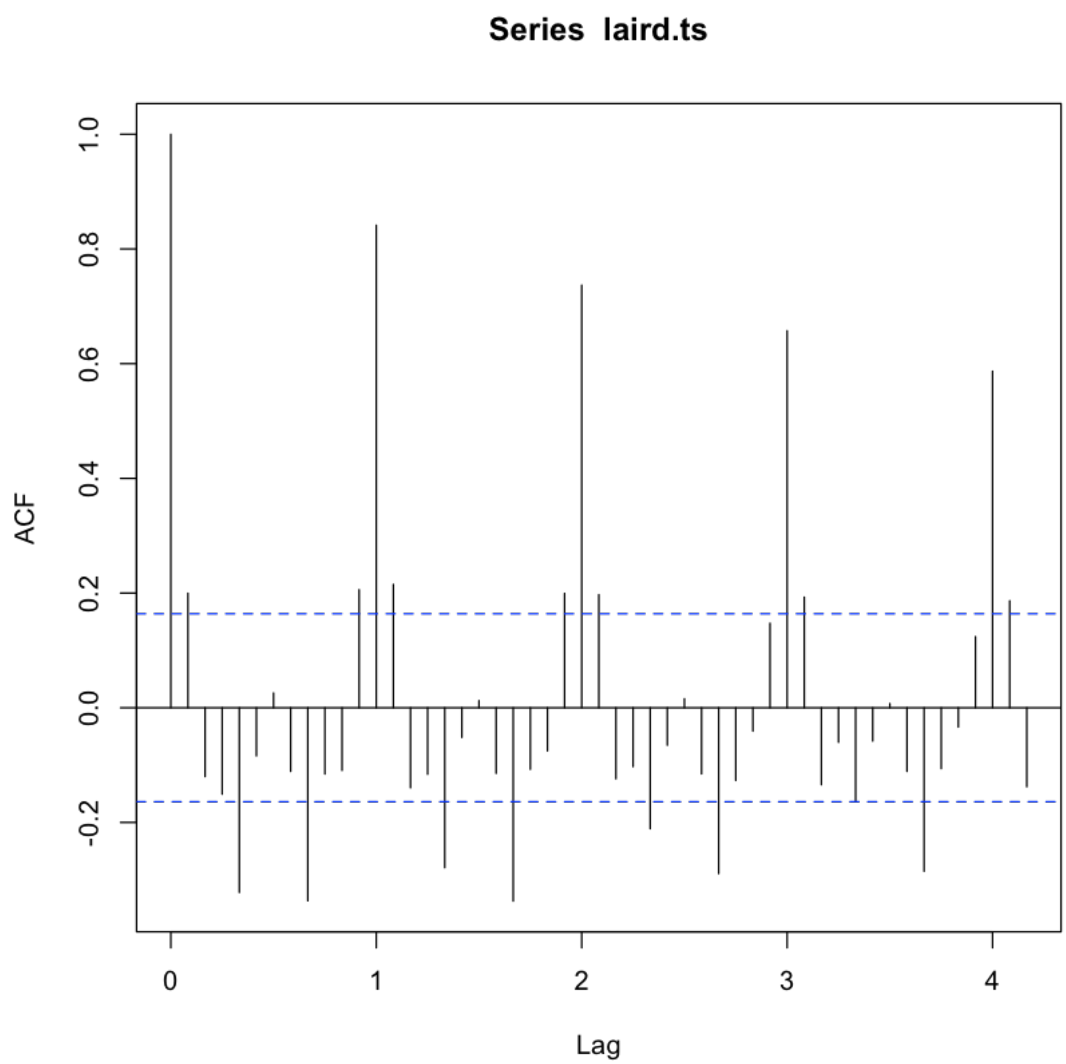
```
In [181]: laird.ts <- diff(lair.ts)
```

```
In [182]: plot(laird.ts)
```



That's cut out a lot of the general trend, but there's still some seasonality that you can clearly see in the ACF:

```
In [184]: laird.acf <- acf(laird.ts, 50)
```

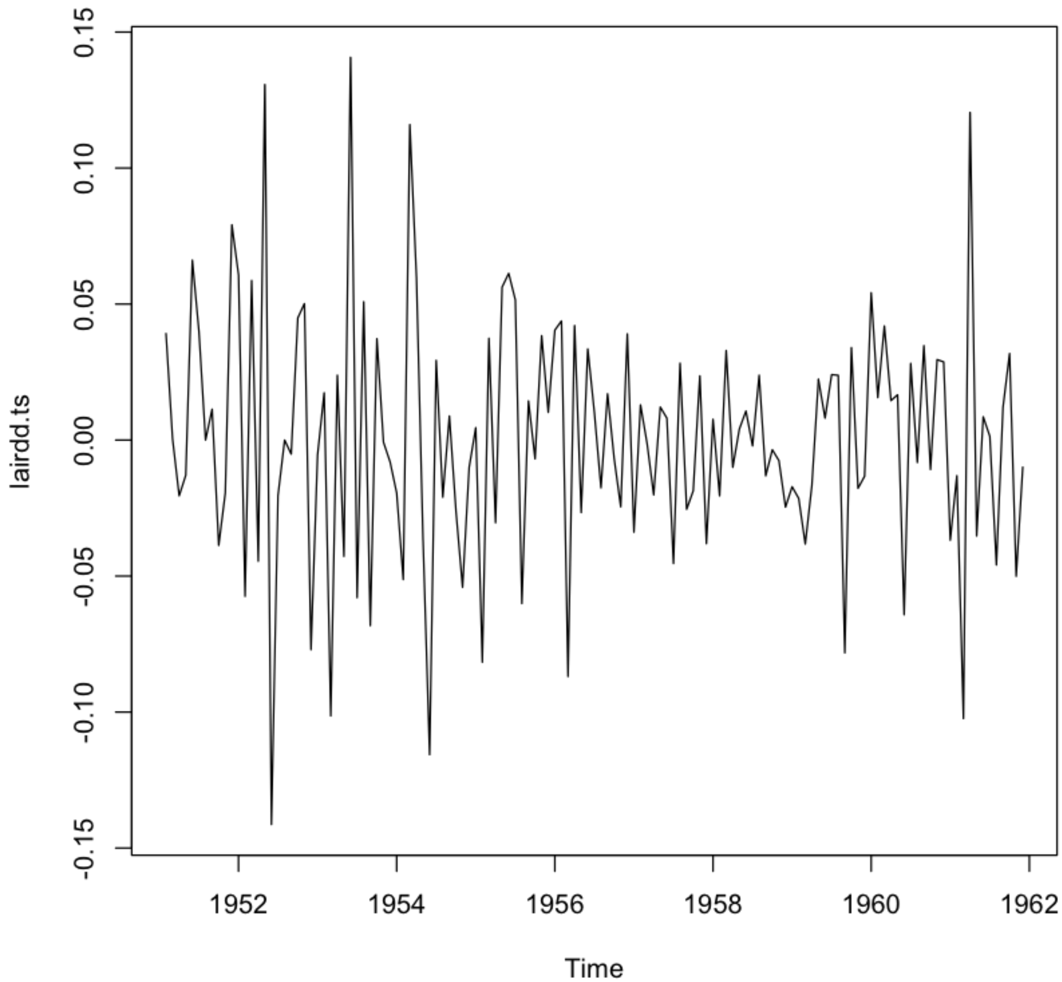


7.3 So differencing using Δ_s, where s is the frequency of the seasonality that we're trying to remove from the non-stationary process before we can model it.

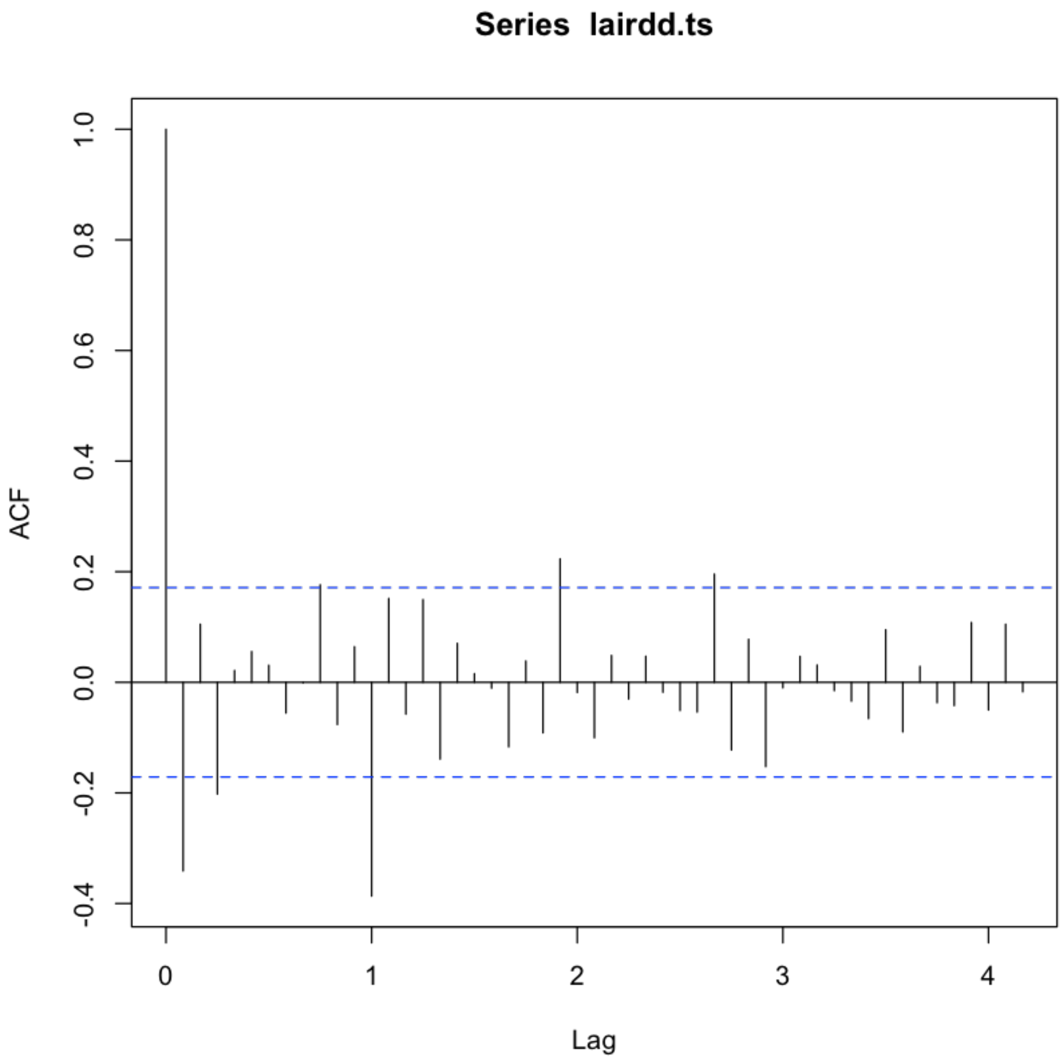
We notice from the ACF that there's correlation occuring at the 12 month lags (i.e. there's some annual seasonality in the data)

So now differencing by applying Δ_{12} such that our process is now $\{\Delta_{12}\Delta y_t\}$

```
In [191]: lairdd.ts <- diff(laird.ts, lag=12)

In [192]: plot(lairdd.ts)
```




```
In [193]: lairdd.acf <- acf(lairdd.ts, 50)
```




```
In [199]: lag <- 0:50

In [200]: lair <- lair.acf$acf
          laird <- laird.acf$acf
          lairdd <- lairdd.acf$acf

In [201]: airacf <- data.frame(lag, lair, laird, lairdd)

In [202]: airacf
```

lag	lair	laird	lairdd
0	1.00000000	1.00000000	1.0000000000
1	0.95370337	0.19975134	-0.3411237983
2	0.89891595	-0.12010433	0.1050467496
3	0.85080249	-0.15077204	-0.2021386642
4	0.80842517	-0.32207432	0.0213592288
5	0.77889939	-0.08397453	0.0556543435
6	0.75644222	0.02577843	0.0308036696
7	0.73760171	-0.11096075	-0.0555785695
8	0.72713135	-0.33672146	-0.0007606578
9	0.73364870	-0.11558631	0.1763686815
10	0.74425525	-0.10926704	-0.0763581912
11	0.75802665	0.20585223	0.0643839399
12	0.76194292	0.84142998	-0.3866128596
13	0.71650448	0.21508704	0.1516020121
14	0.66304279	-0.13955394	-0.0576067980
15	0.61836286	-0.11599576	0.1495652202
16	0.57620873	-0.27894284	-0.1389421819
17	0.54380132	-0.05170646	0.0704823385
18	0.51945611	0.01245814	0.0156307241
19	0.50070292	-0.11435760	-0.0106106130
20	0.49040280	-0.33717439	-0.1167285978
21	0.49818190	-0.10738490	0.0385542023
22	0.50616664	-0.07521120	-0.0913645276
23	0.51674339	0.19947518	0.2232689055
24	0.52048973	0.73692070	-0.0184181674
25	0.48352367	0.19726236	-0.1002881161
26	0.43739831	-0.12388430	0.0485657567
27	0.40040669	-0.10269904	-0.0302396339
28	0.36413092	-0.21099219	0.0471343505
29	0.33698229	-0.06535684	-0.0180304684
30	0.31472272	0.01572846	-0.0510696473
31	0.29677522	-0.11537038	-0.0537672361
32	0.28861644	-0.28925562	0.1957284827
33	0.29535468	-0.12688236	-0.1224193885
34	0.30454726	-0.04070684	0.0777498102
35	0.31509613	0.14741061	-0.1524548378
36	0.31929315	0.65743810	-0.0099950101
37	0.28621139	0.19290864	0.0469202805
38	0.24501605	-0.13431247	0.0312375443
39	0.21089584	-0.06023711	-0.0150867473
40	0.17509464	-0.16270560	-0.0341315285
41	0.14584968	-0.05802668	-0.0655933853
42	0.12482768	0.00736649	0.0950573679
43	0.10645564	-0.11095442	-0.0896620926
44	0.09900334	-0.28526755	0.0288258081
45	0.10378457	-0.10617644	-0.0368860804
46	0.11126270	-0.03364527	-0.0421346581
47	0.12042286	0.12402117	0.1081915971
48	0.12479240	0.58689883	-0.0501473118
49	0.09575361	0.18653823	0.1050148119
50	0.05795009	-0.13775391	-0.0171246719

7.4 Fitting an ARIMA model to the logged $\{y_t\}$ process

So we're happy with the differencing applied, so we've got the **d** and **s** covered, but we're wondering what **p** / **q** will be.

We're not seeing a geometrically decreasing series in the ACF - we're actually seeing a few autocorrelations that are significantly different than zero, so let's introduce a few MA terms.

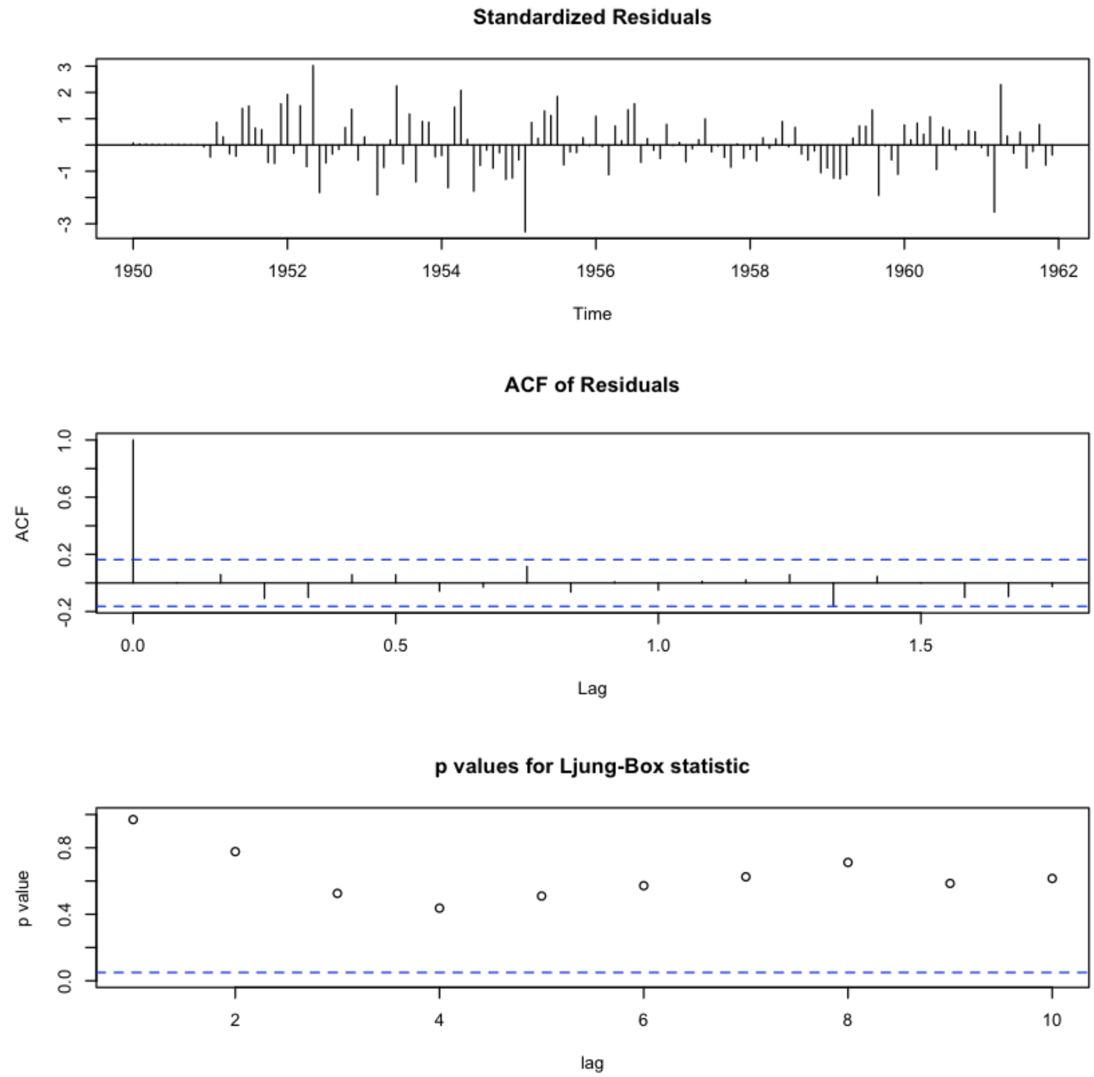
Let's try and fit an ARIMA(0,1,1)x$(0,1,1)_{12}$ model:

```
In [234]: arima.111.111.12 <- arima(lair.ts, order=c(0,1,1),seasonal=list(order = c(0,1,1), pe
```

```
In [235]: arima.111.111.12
```

```
Call:
arima(x = lair.ts, order = c(0, 1, 1), seasonal = list(order = c(0, 1, 1), period = 12))

Coefficients:
          ma1     sma1
      -0.4018  -0.5569
s.e.   0.0896   0.0731

sigma^2 estimated as 0.001348:  log likelihood = 244.7,  aic = -483.4
```

```
In [229]: tsdiag(arima.111.111.12)
```



7.5 Let's take a quick look at the parameter values with their standard errors:

```
In [214]: p1 <- -0.4018

In [215]: se1 <- 0.0896

In [216]: p2 <- -0.5569

In [217]: se2 <- 0.0731

In [218]: t1 <- p1 / se1

In [219]: t2 <- p2 / se2
```

```
In [220]: c(t1, t2)
```

1. -4.484375 2. -7.61833105335157

```
In [221]: length(lair.ts)
```

144

```
In [222]: df <- length(lair.ts) - 4
```

7.6 P-values associated with the t-statistics (parameter / se-parameter) from the ARIMA model

```
In [236]: #ăp-value of first parameter
          2*pt(t1, df, lower.tail=TRUE)
```

1.51065810452479e-05

```
In [237]: # p-value of second parameter
          2*pt(t2, df, lower.tail=TRUE)
```

3.49897514509632e-12

7.7 Model fitted:

Recall: ARMA(p,q) model can be written as:

$Y_t = \mu + \sum_{k=1}^{p} \phi_k (Y_{t-k} - \mu) + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$,

with finite μ. It can be written as:

$Y_t - \mu - \sum_{k=1}^{p} \phi_k (Y_{t-k} - \mu) = \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$

let $X_t = Y_t - \mu$

$(1 - \sum_{k=1}^{p} \phi_k L^k) X_t = (1 + \sum_{i=1}^{q} \theta_i L^i) \epsilon_t$

$\phi(L) X_t = \theta(L) \epsilon_t$,

where $\phi(L)$ and ϵ_t are the AR and MA characteristic polynomials, respectively.

7.8 Applying the differencing:

7.8.1 1) Simple difference operator $\Delta = (1 - L)$ to Y_t:

$W_t = \Delta Y_t = (1 - L) Y_t$

WE NOW FIT THE ARMA(p,q) MODEL TO W_t, AND THEN WORK BACKWARDS

$\phi(L)(W_t - \mu) = \theta(L) \epsilon_t$

Fitting ARMA(0,1) to W_t

$\phi(z) = 1 - \phi_1 z - \phi_2 z^2 - ... - \phi_p z^p$

$\theta(z) = 1 + \theta_1 z + \theta_2 z^2 + ... + \theta_p z^p$

ARMA(0,1) will have $\phi(L) = 1$

ARMA(0,1) will have $\theta(L) = (1 + \theta L)$

AMRA(0,1) model equation: $(W_t - \mu) = (1 + \theta L)\epsilon_t = \epsilon_t + \theta\epsilon_{t-1}$

Subbing $W_t = \Delta Y_t = (1 - L)Y_t$ into the ARMA(0,1) produces the ARIMA(0,1,1) model:

$Y_t = \mu + Y_{t-1} + \epsilon_t + \theta\epsilon_{t-1}$

7.8.2 2) The general ARIMA model with seasonal differencing is called a multiplicative model:

$\Delta_s = 1 - L^s$

$\Delta_s Y_t = (1 - L^s)Y_t = Y_t - Y_{t-s}$

For seasonal differencing, we again define a W_t ARMA equation, but now W_t includes both normal and seasonal differencing:

$W_t = \Delta^d \Delta_s^D Y_t = (1 - L)^d (1 - L^s)^D Y_t$

You need to add seasonal versions of the AR and MA characteristic polynomials: these are called the *seasonal AR characteristic polynomial*, $\Phi(z^s)$ and the *seasonal MA characteristic polynomial*, $\Theta(z^s)$

The seasonal W_t ARMA:

$\phi(L)\Phi(L^s)(W_t - \mu) = \theta(L)\Theta(L^s)\epsilon_t$

Where $\Phi(z^s) = 1 - \Phi_1 z^s - \Phi_2 z^{2s} - ... - \Phi_P z^{2P}$

And $\Theta(z^s) = 1 + \Theta_1 z^s + \Theta_2 z^{2s} + ... + \Theta_Q z^{Qs}$

When you sub $W_t = (1 - L)^d (1 - L^s)^D Y_t$ back into the above, then you have the multiplicative seasonal ARIMA(p,d,q) x (P,D,Q)$_s$ model.

7.9 And so finally, the form of the ARIMA(0,1,1) x (0,1,1)$_s$ model that we fitted to the airline data:

$(W_t - \mu) = \theta(L)\Theta(L^{12})\epsilon_t$

$(1 - L)(1 - L^{12})Y_t = \mu + (1 + \theta L)(1 + \Theta L^{12})\epsilon_t$

We can multiple this out like usual:

$(1 - L^{12} - L + L^{13})Y_t = \mu + (1 + \Theta L^{12} + \theta L + \theta\Theta L^{13})\epsilon_t$

$$Y_t - Y_{t-12} - Y_{t-1} + Y_{t-13} = \mu + \epsilon_t + \Theta\epsilon_{t-12} + \theta\epsilon_{t-1} + \theta\Theta\epsilon_{t-13}$$

```
In [238]: arima.111.111.12


Call:
arima(x = lair.ts, order = c(0, 1, 1), seasonal = list(order = c(0, 1, 1), period = 12))

Coefficients:
          ma1     sma1
      -0.4018  -0.5569
s.e.   0.0896   0.0731

sigma^2 estimated as 0.001348:  log likelihood = 244.7,  aic = -483.4
```

$\theta = -0.4018$

$\Theta = -0.5569$

and therefore $\theta\Theta = -0.4018 \times -0.5569 = 0.2238$

And so the fitted model is:

$$Y_t = Y_{t-12} + Y_{t-1} - Y_{t-13} + \epsilon_t - 0.5569\epsilon_{t-12} - 0.4018\epsilon_{t-1} + 0.2238\epsilon_{t-13}$$

---

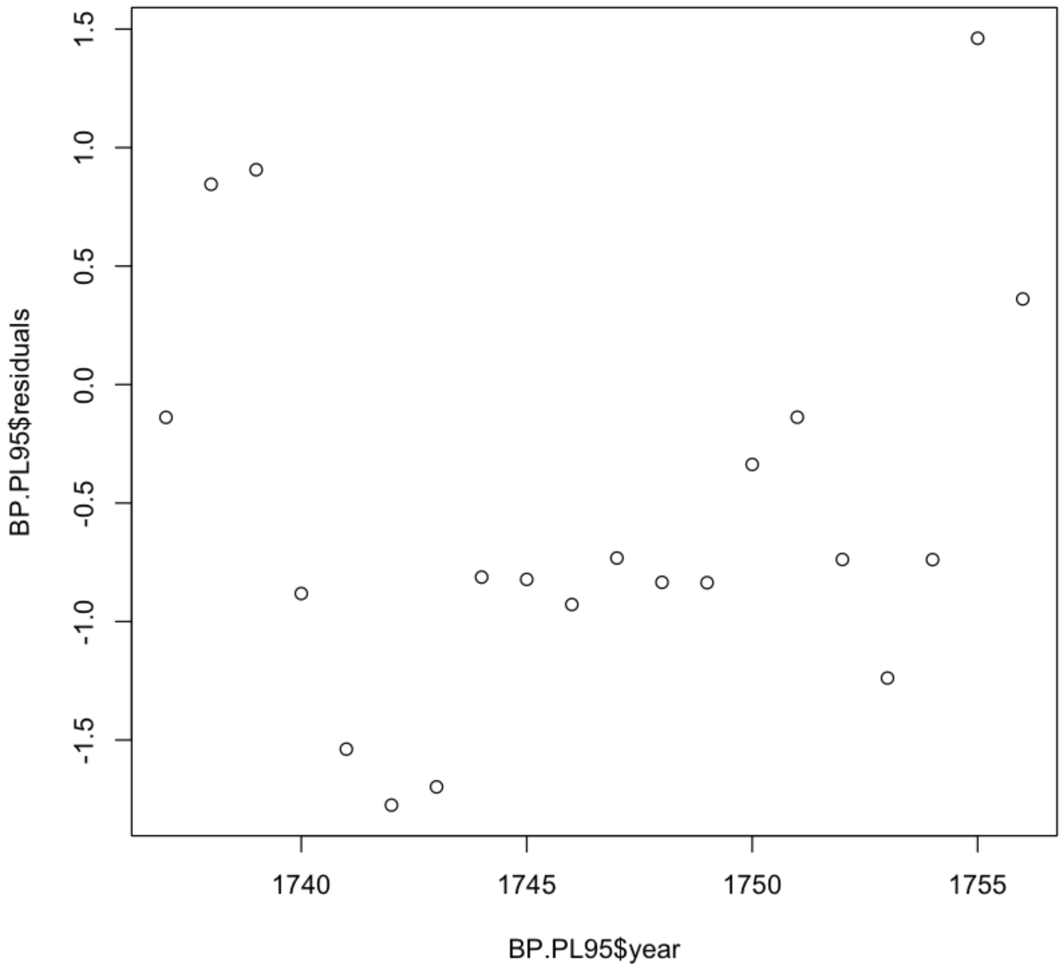
8 Forecasting using R ARIMA model

```
In [162]: X <- data.frame(BP.train, 1634:(1634+length(BP.train)-1))
          colnames(X) <- c('price','year')

In [163]: num_years_pred <- 20

          BP.fore <-  predict(BP.ar.1, num_years_pred)

In [116]: pred <- BP.fore$pred

          pred.se <- BP.fore$se

In [117]: num_se <- qt(0.025, 120, lower.tail=FALSE)

In [118]: L95 <- pred - num_se*pred.se
          U95 <- pred + num_se*pred.se

          year <- max(X['year']):(max(X['year'])+num_years_pred-1)
```

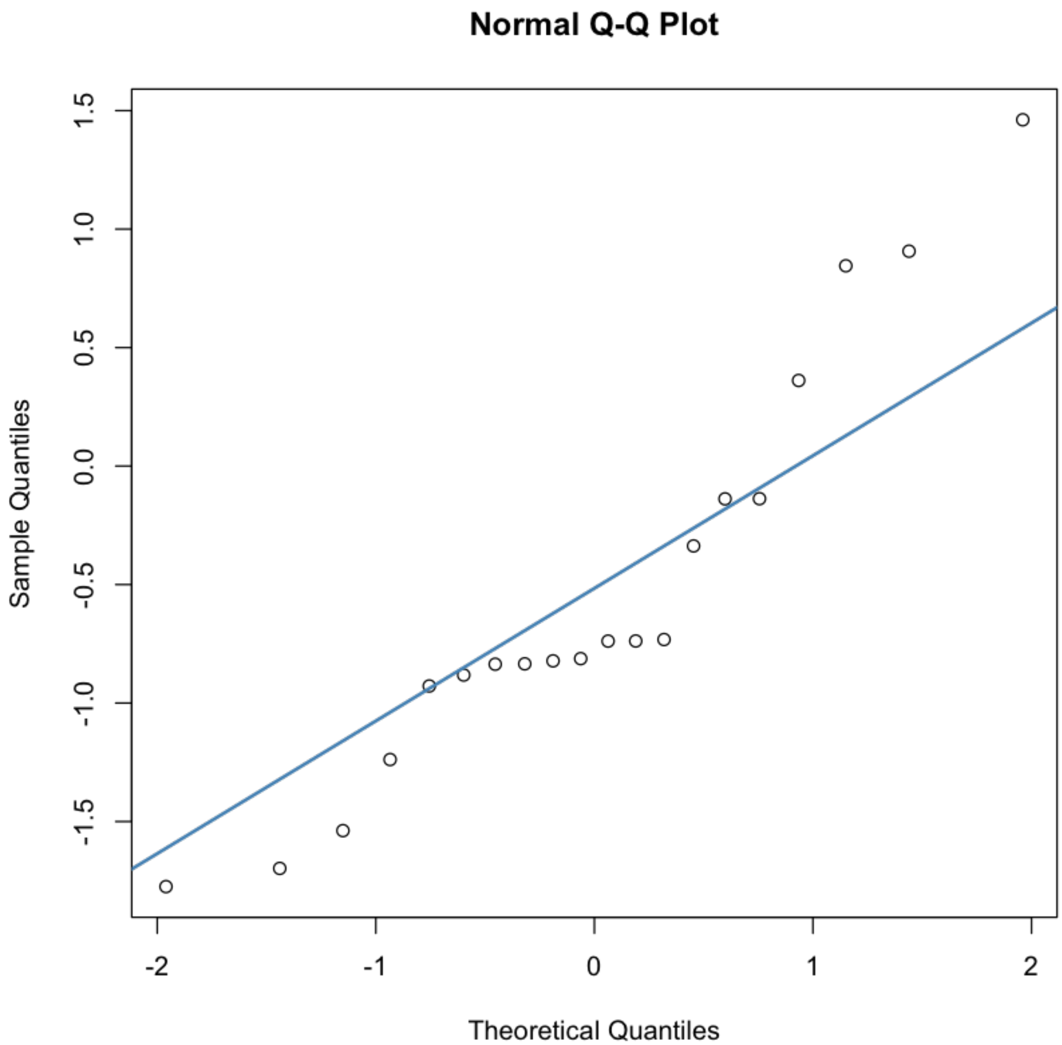
```
In [122]: BP.PL95 <- data.frame(year, L95, pred, U95, BP.test)

          BP.PL95['residuals'] <- BP.PL95['BP.test'] - BP.PL95['pred']

          BP.PL95
```

year	L95	pred	U95	BP.test	residuals
1737	3.229417	5.138416	7.047414	5.0	-0.1384157
1738	3.088823	5.354850	7.620876	6.2	0.8451504
1739	3.096487	5.493270	7.890053	6.4	0.9067300
1740	3.133543	5.581797	8.030051	4.7	-0.8817969
1741	3.169417	5.638414	8.107412	4.1	-1.5384143
1742	3.197192	5.674624	8.152056	3.9	-1.7746240
1743	3.216908	5.697782	8.178656	4.0	-1.6977819
1744	3.230312	5.712593	8.194873	4.9	-0.8125926
1745	3.239209	5.722065	8.204920	4.9	-0.8220647
1746	3.245032	5.728123	8.211213	4.8	-0.9281227
1747	3.248810	5.731997	8.215184	5.0	-0.7319970
1748	3.251249	5.734475	8.217701	4.9	-0.8344749
1749	3.252818	5.736060	8.219302	4.9	-0.8360596
1750	3.253824	5.737073	8.220322	5.4	-0.3370731
1751	3.254470	5.737721	8.220973	5.6	-0.1377213
1752	3.254883	5.738136	8.221388	5.0	-0.7381358
1753	3.255148	5.738401	8.221654	4.5	-1.2384009
1754	3.255317	5.738570	8.221824	5.0	-0.7385705
1755	3.255426	5.738679	8.221932	7.2	1.4613211
1756	3.255495	5.738748	8.222001	6.1	0.3612517

```
In [131]: plot(BP.PL95$year, BP.PL95$residuals)
```




```
In [137]: qqnorm(BP.PL95$residuals)
          qqline(BP.PL95$residuals, col = "steelblue", lwd = 2)
```


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2
-1
0
1
2

```
In [ ]:
```