L9 Revision Part 1

June 2, 2020

1 L9 Revision: Part 1 Forecasting AR(1)

1.1 Practical aspects of making forecasts

Will look back at our Bread Price example where we found an AR(1) model an adequete fit to the data.

1.2 Bread Price Diagnostics

```
In [1]: price <- c(5.8, 6.1, 5.4, 6.2, 5.0, 4.6, 5.8, 5.1, 5.3, 5.1, 4.8, 5.3, 6.8, 9.0, 8.6,
        9.0, 7.4, 6.4, 4.8, 3.9, 3.9, 5.6, 5.7, 7.5, 7.3, 7.4, 7.5, 9.7, 6.1, 6.0, 5.7, 5.0,
        4.2, 4.6, 5.9, 5.4, 5.4, 5.4, 5.6, 7.6, 7.4, 5.4, 5.1, 6.9, 7.5, 5.9, 6.2, 5.6, 5.8,
        5.6, 6.6, 4.8, 5.2, 4.5, 4.4, 5.3, 5.0, 6.4, 7.8, 8.5, 5.6, 7.1, 7.1, 8.0, 7.3, 5.7,
        4.8, 4.3, 4.4, 5.7, 4.7, 4.1, 4.1, 4.7, 7.0, 8.7, 6.2, 5.9, 5.4, 6.3, 4.9, 5.5, 5.4,
        4.7, 4.1, 4.6, 4.8, 4.5, 4.7, 4.8, 5.4, 6.0, 5.1, 6.5, 6.2, 4.6, 4.5, 4.0, 4.1, 4.7,
        5.1, 5.2, 5.3, 4.8, 5.0, 6.2, 6.4, 4.7, 4.1, 3.9, 4.0, 4.9, 4.9, 4.8, 5.0, 4.9, 4.9,
        5.4, 5.6, 5.0, 4.5, 5.0, 7.2, 6.1)
```

```
In [2]: # use time series function to transform it into time series format (auto indexes the t
        BP.ts <- ts(price, start=1634, frequency = 1)
```

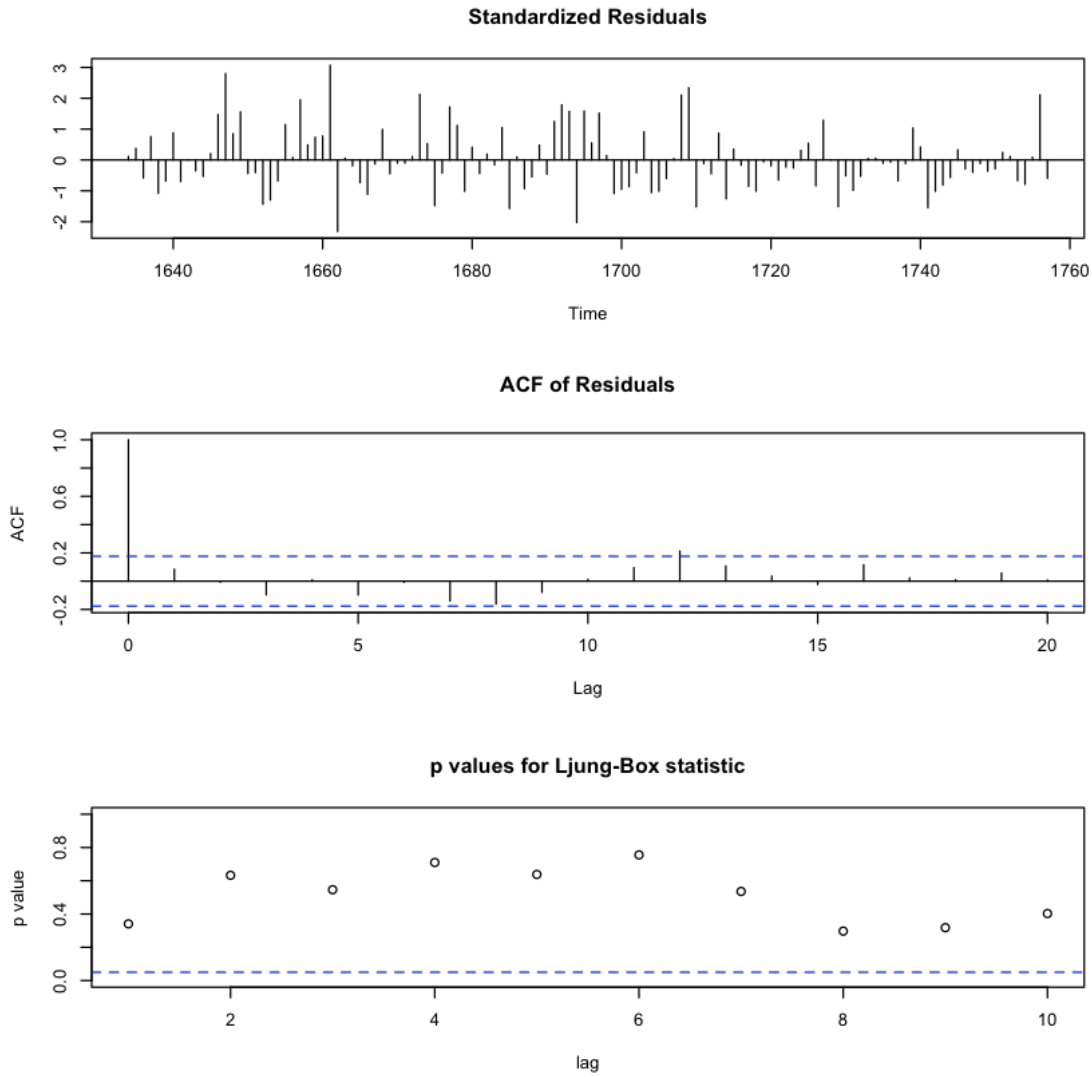
```
In [3]: ar.1 <- arima(BP.ts, order=c(1,0,0))
        ar.1
```

```
Call:
arima(x = BP.ts, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.6429     5.6608
s.e.  0.0678     0.2307

sigma^2 estimated as 0.8655:  log likelihood = -167.26,  aic = 340.52
```

```
In [4]: tsdiag(ar.1)
```



1.3 This was where we'd got up to last time.

1.4 Time to make some forecasts.

```
In [8]: print(BP.ts)
```

```
Time Series:
Start = 1634
End = 1757
Frequency = 1
  [1] 5.8 6.1 5.4 6.2 5.0 4.6 5.8 5.1 5.3 5.1 4.8 5.3 6.8 9.0 8.6 9.0 7.4 6.4
 [19] 4.8 3.9 3.9 5.6 5.7 7.5 7.3 7.4 7.5 9.7 6.1 6.0 5.7 5.0 4.2 4.6 5.9 5.4
 [37] 5.4 5.4 5.6 7.6 7.4 5.4 5.1 6.9 7.5 5.9 6.2 5.6 5.8 5.6 6.6 4.8 5.2 4.5
```

```
 [55] 4.4 5.3 5.0 6.4 7.8 8.5 5.6 7.1 7.1 8.0 7.3 5.7 4.8 4.3 4.4 5.7 4.7 4.1
 [73] 4.1 4.7 7.0 8.7 6.2 5.9 5.4 6.3 4.9 5.5 5.4 4.7 4.1 4.6 4.8 4.5 4.7 4.8
 [91] 5.4 6.0 5.1 6.5 6.2 4.6 4.5 4.0 4.1 4.7 5.1 5.2 5.3 4.8 5.0 6.2 6.4 4.7
[109] 4.1 3.9 4.0 4.9 4.9 4.8 5.0 4.9 4.9 5.4 5.6 5.0 4.5 5.0 7.2 6.1
```

```
In [24]: BP.fore <- predict(ar.1, 10)

         print(BP.fore)
```

```
$pred
Time Series:
Start = 1758
End = 1767
Frequency = 1
 [1] 5.943184 5.842361 5.777539 5.735862 5.709067 5.691839 5.680762 5.673641
 [9] 5.669062 5.666119

$se
Time Series:
Start = 1758
End = 1767
Frequency = 1
 [1] 0.9303292 1.1060230 1.1709735 1.1967927 1.2073042 1.2116227 1.2134034
 [8] 1.2141386 1.2144425 1.2145680
```

1.5 Summary

- Each prediction has a standard error associated with it.
- The standard errors are the square root of the forecast error variance, $V(h)$.
- We assume that our white noise is $\text{NID}(0, \sigma^2)$
- This means our Y_{T+h} is also normally distributed
- And this so too is the forecast error $e_T(h)$
- We know that $E[e_T(h)|H_T] = E[e_T(h)] = 0$
- And we know $\text{Var}(e_T(h)) = V(h)$

Therefore our error forecast:

$e_T(h) \sim \text{NID}(0, V(h))$

$\hat{y}_T(h)$ is our prediction for Y_{T+h}.

$e_T(h) = Y_{T+h} - \hat{y}_T(h)$

Therefore:

$$Y_{T+h} = e_T(h) + \hat{y}_T(h)$$

And the variance in Y_{T+h} will be:

$$\text{Var}(Y_{T+h}) = \text{Var}\left(e_T(h) + \hat{y}_T(h)\right) = V(h)$$

Since $\hat{y}_T(h)$ is just a number.
So to make prediction intervals for our predictions of Y_{T+h}, we use:

$$\hat{y}_T(h) \pm z_{\alpha/2} \times \sqrt{(}V(h))$$

```
In [25]: num.se <- qnorm(0.025, lower.tail=FALSE)
         num.se
```

1.95996398454005

```
In [26]: pred <- BP.fore$pred
         se <- BP.fore$se

         print(pred)
```

```
Time Series:
Start = 1758
End = 1767
Frequency = 1
 [1] 5.943184 5.842361 5.777539 5.735862 5.709067 5.691839 5.680762 5.673641
 [9] 5.669062 5.666119
```

```
In [27]: print(se)
```

```
Time Series:
Start = 1758
End = 1767
Frequency = 1
 [1] 0.9303292 1.1060230 1.1709735 1.1967927 1.2073042 1.2116227 1.2134034
 [8] 1.2141386 1.2144425 1.2145680
```

```
In [28]: L95 <- pred - num.se*se
         U95 <- pred + num.se*se

         year <-  1958:1967

In [29]: BP.PL95 <- data.frame(year, pred, L95, U95)

         BP.PL95
```

year	pred	L95	U95
1958	5.943184	4.119772	7.766596
1959	5.842361	3.674596	8.010127
1960	5.777539	3.482473	8.072605
1961	5.735862	3.390191	8.081533
1962	5.709067	3.342794	8.075339
1963	5.691839	3.317102	8.066576
1964	5.680762	3.302535	8.058989
1965	5.673641	3.293973	8.053309
1966	5.669062	3.288799	8.049326
1967	5.666119	3.285609	8.046628

1.6 Linking these figures to our theory.

Our estimate, $\hat{\sigma}^2$,for the white noise variance, σ^2 from R was: 0.8655

Our estimate $\hat{\phi}$ for ϕ was: 0.6429

And our estimate for $\hat{\mu}$ for μ was: 5.6608

Recall, we estimate $V(1)$ to be $\hat{\sigma}^2 = 0.8655$

Estimate for $\hat{y}_{124}(1)$:

$$\hat{y}_{124}(1) = \hat{\mu} + \hat{\phi}(\hat{y}_{124}(0) - \hat{\mu})$$

$$\hat{y}_{124}(1) = \hat{\mu} + \hat{\phi}(y_{124} - \hat{\mu})$$

Making prediction for $h = 1$:

$$\hat{y}_{124}(1) = 5.6608 + 0.6429(6.1 - 5.6608) = 5.94318$$

95% probability interval for $h = 1$:

$$\hat{y}_{124}(1) \pm 1.96 \times \sqrt{(}V(1))$$

$$\hat{y}_{124}(1) \pm 1.96 \times \sqrt{(}0.8655)$$

```
In [ ]:
```