

B.Sc./Grad. Dip.: Probability Models and Time Series
MAS programmes: Stochastic Models and Time Series

Examples 7

1. For the ARIMA(0,1,1) model specified by the equation

$$Y_t = c + Y_{t-1} + \epsilon_t + \theta\epsilon_{t-1} \quad t \in \mathbb{Z},$$

prove by induction on h that

$$Y_{T+h} = ch + Y_T + \theta\epsilon_T + \epsilon_{T+h} + (1 + \theta) \sum_{i=1}^{h-1} \epsilon_{T+h-i} \quad h \geq 1.$$

Deduce expressions for the forecast $\hat{y}_T(h)$ and for the corresponding forecast error, $e_T(h)$.

2. Using R and the data on sheep population, from Examples 6, obtain forecasts of the next five population values, for the years 1940 to 1944, based upon fitting the ARIMA(3,1,0) with no constant (or no process mean for the differenced series).
 - (a) Explain why it is not so simple to write down the forecast function.
 - (b) By examining the forecasts produced by R for a large enough lead time, write down the *eventual forecast function* for this model. Justify your answer.