

**B.Sc./Grad. Dip.: Probability Models and Time Series**  
**MAS programmes: Stochastic Models and Time Series**

**Examples 2**

1. Consider a network consisting of five nodes  $A, B, C, D$  and  $E$ . The network has six arcs joining  $A$  with  $B$ ,  $B$  with  $C$ ,  $B$  with  $E$ ,  $B$  with  $D$ ,  $C$  with  $D$ , and  $D$  with  $E$ .

A particle moves round the nodes of this network by selecting with equal probability one of the arcs attached to the presently occupied node and moving along it to the next node, where the process is then repeated.

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Note: do **not** confuse the network structure with the 1-step transition diagram!

Thus, for e.g.,  $p_{BA} = \frac{1}{4}$  but  $p_{AB} = 1$ .

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Let  $X_n$  be the node that is occupied by the particle just after all transitions at the  $n$ -th stage of the process have taken place.

- (a) Find the transition matrix,  $\mathbf{P}$ , for the movement of the particle.  
(b) By calculating the appropriate entries of  $\mathbf{P}^2$ , find

$$\mathbb{P}(X_2 = D | X_0 = B), \quad \mathbb{P}(X_2 = B | X_0 = B).$$

- (c) Does the stationary distribution of the chain exist? Justify your answer. If it does, calculate it.

2. Supplemental to Qu.1.

Suppose that nodes  $A$  and  $E$  become absorbing. What would the revised transition matrix,  $\mathbf{P}_{\text{new}}$ , look like?