

B.Sc./Grad. Dip.: Probability Models and Time Series
MAS programmes: Stochastic Models and Time Series

Examples 5

1. Consider the AR(2) model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t$$

in the two cases, (i) $\phi_1 = \frac{3}{10}$, $\phi_2 = \frac{1}{25}$ and (ii) $\phi_1 = \frac{2}{5}$, $\phi_2 = -\frac{1}{16}$.

In both cases check that the stationarity condition is satisfied and find

- (a) the infinite moving average representation of the stationary process $\{Y_t\}$,
- (b) numerical values, correct to 4 decimal places, for the first four autocorrelations, ρ_1, ρ_2, ρ_3 and ρ_4 ,
- (c) a general expression for the autocorrelation function, $\{\rho_\tau : \tau \geq 0\}$.

2. Consider a process $\{Y_t\}$ that satisfies the equation

$$Y_t = \frac{5}{4}Y_{t-1} + \frac{1}{8}Y_{t-2} - \frac{3}{8}Y_{t-3} + \epsilon_t.$$

- (a) Find the equation satisfied by the process $\{W_t\}$ of first differences, i.e., where $W_t = Y_t - Y_{t-1}$.
- (b) Show that $\{W_t\}$ is a stationary process and deduce to which of the family of ARIMA processes $\{Y_t\}$ belongs.