B.Sc./Grad. Dip.: Probability Models and Time Series
MAS programmes: Stochastic Models and Time Series

Examples 4

1. Using the Cauchy-Schwarz inequality that

$$\left(\sum_{t=1}^{n} a_t b_t\right)^2 \leq \sum_{t=1}^{n} a_t^2 \sum_{t=1}^{n} b_t^2,$$

show that the sample autocovariances c_τ satisfy

$$c_\tau^2 \leq c_0^2, \qquad \tau = 0, 1, \ldots, T-1.$$

Deduce that the sample autocorrelations r_τ satisfy $|r_\tau| \leq 1$.

2. For both the following cases, identify a time series model that will produce the specified autocorrelation function, including the values of the model parameters. In both cases only the first four autocorrelations are listed, correct to three decimal places.

(a)

τ	1	2	3	4
ρ_τ	-0.400	0.160	-0.064	0.026

(b)

τ	1	2	3	4
ρ_τ	0.400	0.000	0.000	0.000

3. Using the Cauchy-Schwarz inequality - introduced in Qu.1. - (with $n = \infty$), show that $\gamma_\tau \to 0$ as $\tau \to \infty$, for any linear process, $Y_t = \sum_{i=0}^{\infty} \psi_i \epsilon_{t-i}$. [Recall that one of the properties of a linear process is that $\sum_{i=0}^{\infty} \psi_i^2 < \infty$].

4. For the MA(1) model, $Y_t = \epsilon_t + \theta\epsilon_{t-1}$, prove that, as the value of the parameter θ varies, the range of values of the autocorrelation ρ_1 is the closed interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and that if the process $\{Y_t\}$ is constrained to be invertible then the range of the ρ_1 values is restricted to the open interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$.