C228: Extragalactic Astronomy and Cosmology September 30 2022

C.–P. Ma

Problem Set 4

Due 6pm Friday October 7

1. **Age of the Universe**

The age of a universe, t_0, depends on cosmological parameters.

(a) For $\Omega_\Lambda = 0$ models, plot t_0 in units of h^{-1} Gyr versus the matter density parameter $\Omega_{0,m}$, for $0 \le \Omega_{0,m} \le 3$. What is the general trend of t_0 as the matter density increases?

(b) On the same plot, add a curve for t_0 vs. $\Omega_{0,m}$ for flat models (i.e. $\Omega_{0,m} + \Omega_{0,\Lambda} = 1$). For a given $\Omega_{0,m}$ (where $\Omega_{0,m} < 1$), is the age for a flat universe with non-zero $\Omega_{0,\Lambda}$ smaller or larger than an open universe without a cosmological constant?

(c) Consider the flat models in (b). Plot H_0, in units of km s^{-1} Mpc^{-1}, versus $\Omega_{0,m}$, for 3 values of t_0: 11, 13.8, 18 Gyr. Current observations find $H_0 \approx 70$ km s^{-1} Mpc^{-1}, and the oldest objects in the universe is *at least* 11 Gyr old. Given these, what is the constraint on $\Omega_{0,m}$ implied by your curves? For some years, $\Omega_{0,m} = 1$ was the favored value. What would be the constraint on H_0 implied by $t_0 > 11.5$ Gyr if $\Omega_{0,m} = 1$? How is it compared with the observed H_0?

2. **The Most Distant Galaxies**

A recent flurry of JWST preprints claims detections of galaxies at $z \approx 13$. The most distant photons we have detected are from the cosmic microwave background at $z \approx 1100$. Compute the ages of the universe when the light that we receive today was emitted from these two sources for three cosmological models: $(\Omega_{0,m}, \Omega_{0,\Lambda}) = (0.32, 0.68)$, (0.32, 0.0), and (1.0, 0.0). Assume $h = 0.70$ for the Hubble parameter.

3. **The Curious Behavior of the Angular Diameter**

(a) For an object of physical size L at redshift z, write down a general expression for its angular diameter as a function of L, z, $\Omega_{0,m}$, and H_0 (assume $\Omega_\Lambda = 0$ for simplicity). Rewrite the formula so that the angular diameter is in units of h arcsec (where $H_0 = 100\,h$ km/s/Mpc) and L in units of kpc.

(b) The physical size of the luminous part of a Milky-Way-like galaxy is about 20 kpc. On log-log scales, plot the angular diameter of such a galaxy versus redshift ($0.01 \le z \le 10$) for three values of $\Omega_{0,m}$: 0.3, 1.0, and 3.0. **Make sure you have three curves on a single figure and not three separate figures**. Comment on any interesting features in your curves. (Again assume $\Omega_\Lambda = 0$.)