Dispersed Market Power, Phillips Multiplier, and the Optimal Inflation Target

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Preliminary, please do not circulate
November 27, 2020

Abstract

The average markup of firms in the United States has increased due to the increase in the right tail of the markup distribution (De Loecker *et al.* 2020). We complement these empirical findings by showing that the left tail of the markup distribution has declined. We then study the implications of these findings based on a Multi-sector New Keynesian mode with heterogenous markups and nominal rigidities. First, more dispersed markups lead to higher (lower) money non-neutrality in an economy with decreasing (increasing) returns to scale. Second, changes in the markup distribution have minimal impact on the Phillips Multiplier in the U.S. due to the offsetting effects of the increase in the right tail and the decrease in the left tail of the markup distribution. Third, markups are negatively correlated with nominal rigidities across sectors, which has important implication for the design of the optimal inflation target. Particularly, our findings challenge the conventional wisdom that the central bank should always attach a higher weight to a sector with a higher degree of nominal rigidity. We construct the optimal inflation index, and show how it has evolved over time.

Keywords: Dispersed Markups, Phillips Curve, Money Non-neutrality, Inflation Targeting
JEL Classification: E31, E32, E52, E58

*The views expressed in this paper are the sole responsibility of the authors and do not necessarily reflect the views of the Bank of Italy.

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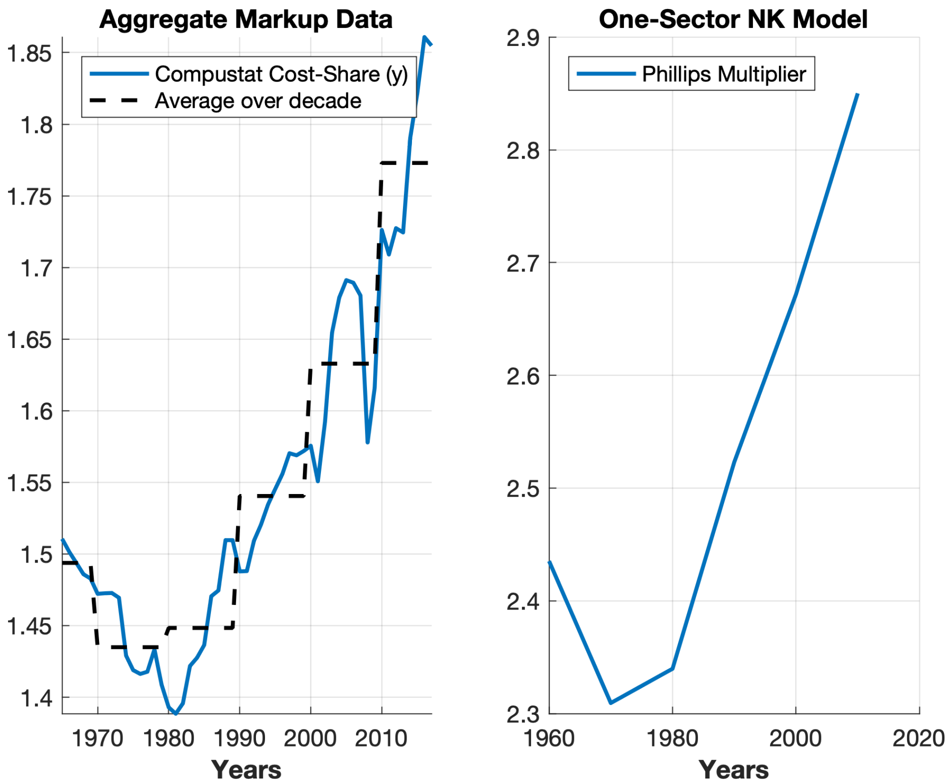
1 Introduction

> *Several indicators suggest that competition may be decreasing in many economic sectors, including the decades-long decline in new business formation and increases in industry-specific measures of concentration.* (Council of Economic Advisors, 2016)

Recent contributions in empirical macroeconomics have highlighted that the average markup/market power for firms in the U.S has increased over the past decades (De Loecker *et al.* 2020). Market power has an interesting interaction with nominal rigidities if firms have a non constant returns to scale production function. According to a basic one-sector New Keynesian model with decreasing returns to scale, this increase in markups implies an increasing Philips Multiplier and a decreasing monetary non-neutrality (see e.g., Coibion and Gorodnichenko 2015), see Figure 1. The intuition is the following. Facing a reduction in marginal cost, a firm resets it's price downward. Due to staggered prices, such a price cut then generates excess demand in the future. With decreasing (increasing) returns to scale, marginal cost would rise (decrease) in the future. As a result, the firm cuts it's price less (more) than it would otherwise do in the absence of this feedback effect. The degree of competition in the market amplifies this effect, altering the effects of real shocks to nominal prices. The previous analysis ignores the possibility that (i) the distribution of the average markups are dispersed and that (ii) the entire distribution might be evolving over time. How has the entire distribution of markups evolved over time in the data? What are the implications for the conduct of monetary policy?

In this paper, we first complement the recent empirical literature by showing that the distribution of steady-state markups spread out over time. This is driven by both the increase in the top quantiles but also a decrease in the bottom quantiles of the distribution.

Figure 1: The Evolution of the Aggregate steady-state Markup in the U.S. and its Implication



Note: Authors' own calculation. The solid blue line in the left panel plots the markup computed using the cost share approach using data from Compustat that covers publicly listed firms in the U.S. The dashed black line reports the average over the decade, which we interpret as the steady-state value of markup. The right panel reports the implied Phillips Multiplier for each decade based the simple New Keynesian model outlined in Galí (2008).

We then study the implications of those findings based on a New Keynesian model with heterogenous sectors. Particularly, we examine the implications of dispersed markups and the evolution of the distribution of markups over time for (i) monetary non-neutrality, (ii) the Phillips Multiplier, and (iii) the optimal inflation index (OII) stabilization policy. Understanding the degree of monetary non-neutrality, the size of the Phillips Multiplier, and the composite of the OII are important for the conduct of monetary policy. In fact, these three statistics are the foundations of the Federal Reserve's (Fed) dual mandate: foster economic conditions that achieve both stable prices and maximum sustainable employment. The first statistic, monetary non-neutrality, measures the central bank's ability to stimulate the economy to achieve the maximum sustainable employment. The second statistic, the Phillips Multiplier (Barnichon and Mesters, 2019)—defined as the ratio of the cumulative response of inflation to the cumulative response of real GDP after an exogenous monetary intervention—measures the trade-off between the stabilization of prices and the stimulation of employment. The third, the OII informs policy makers which inflation index they should target to achieve price stability and whether their current practice is close to the optimal. In the U.S., the Fed monitors the headline and the core of the personal consumption expenditures (PCE) inflation, which not necessarily coincide with the OII. Our contributions in showing how dispersed steady-state markups affect those three statistics both on average and over time are relevant for guiding policy discussions.

We derive the following results. First, dispersed steady-state markups lead to stronger (weaker) money non-neutrality and a smaller (bigger) Phillips Multiplier in the presence of decreasing (increasing) returns to scale. This is because monetary non-neutrality is concave (convex) in the steady-state level of markups. This result suggests that the central bank's ability in stimulating employment might be higher (lower) than previously thought, based on a model with homogenous market power and/or constant returns to scale.

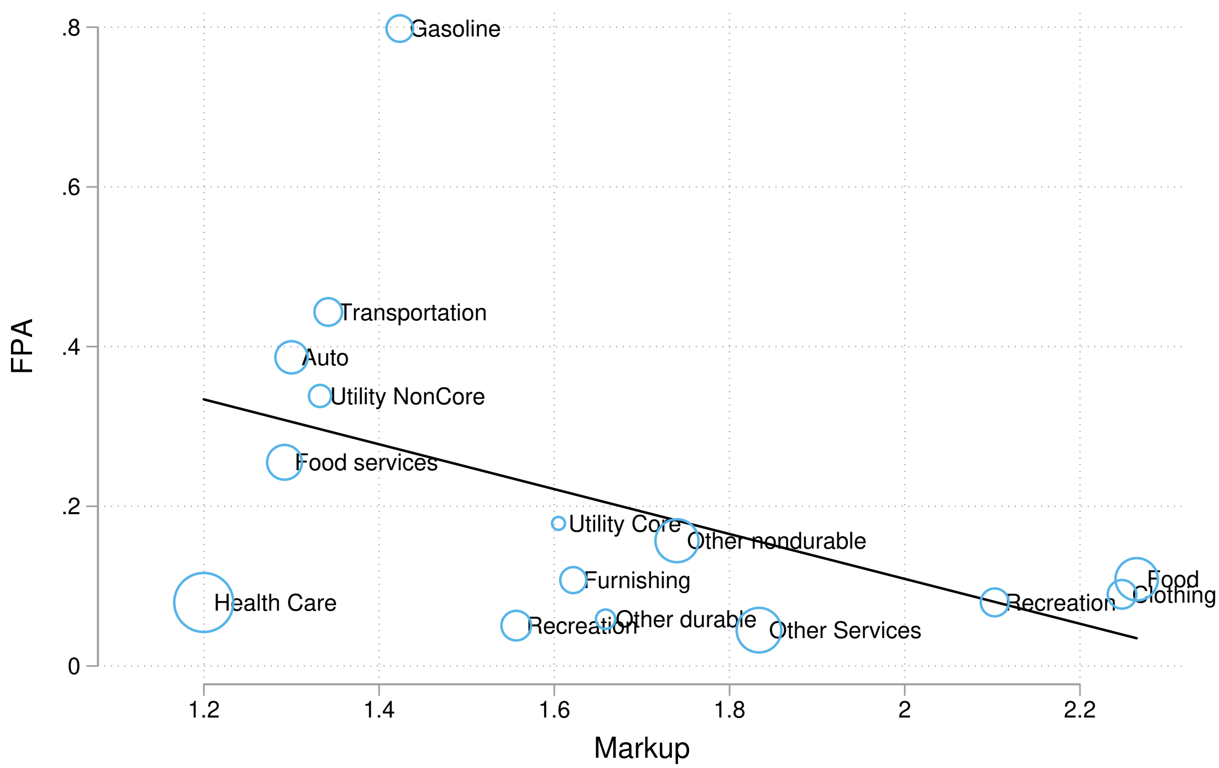
Second, we investigate how changes in steady-state markup distribution and sector sizes affect the size of the Phillips Multiplier based on calibrations of a seventeen-sectors model that consists of constitutes of the PCE index in the United States. We find that changes in steady-state markup distribution have minimal impact on the size of the Phillips Multiplier independent of the returns to scale. This result is driven by the spreading out of the markup distribution: effects arise through the changes in the right tail cancel out with the effects emerging from the movements in the left tail. However, our calibrated model predicts a 20% reduction in the Philips Multiplier due to the reallocation of resources to stickier price sectors.

We then study the policy implications. Specifically, we study the OII stabilization policy: the optimal weighted inflation index that the central bank should target to minimize the social welfare loss. One important mechanism that academics and policy makers focus on is the relative price stickiness channel. We label this as the *Stickiness Channel*, see e.g., Aoki (2001), Benigno (2004) and Mankiw and Reis (2003). In this paper, we address the heterogeneity in markups (the *Competition Channel*) in the design of the OII.

Empirically, we document that markups are negatively correlated with nominal rigidities across sectors (see Figure 2), which is consistent with costly price adjustment models developed by Barro (1972), Sheshinski and Weiss (1977) and Golosov and Lucas (2007). Therefore, analyzing the stickiness channel without considering the origin of the relative frequency of price adjustment might be misleading.

We show that a more competitive (lower market power) sector is associated with a higher weight in the OII. In the extreme case when a market is infinitely close to a perfect competition market (flat demand curve), the optimal inflation index is the one that only consists of inflation in that sector. The intuition as follows. In a more competitive market, firms face a flatter demand curve. Consequently, a given change in prices leads to a more significant movement in quantity. In the presence of price stickiness, this results in a more significant dispersion in output, which is welfare detrimental due to consumers' love of variety. In sum, inflation in a more competitive sector creates a bigger distortion. Therefore, stabilizing inflation in that sector is relatively more important, hence the higher weight. To illustrate the interaction between the competition channel and the stickiness channel, we calibrate a two-sector model with heterogenous degrees of nominal rigidities and market power to the manufacture and service sectors in the data. Interestingly, the competition channel offsets the stickiness channel. As a result, the PCE (weighted by the size of the market) stabilization performs similarly as compared to the stabilization of an

Figure 2: **Frequencies of Price Adjustment v.s. Markups**



Note: Authors' own calculation. This figure plots the frequencies of price adjustment against steady-state markups in seventeen sectors that are constitutes of the PCE index in the U.S. The size of a circle measures the size of the underlying industry. The black line is the fitted linear relationship according to the OLS.

inflation index that is merely based on the relative price stickiness. We label the latter as the sticky price index (SPI). This finding challenges the conventional wisdom that the central bank should always attach a higher weight to a sector with a higher degree of nominal rigidity.

We compute the OII for the seventeen-sectors models calibrated to the US data over time. In the 1960s, the competition channel played a minimal role: the welfare loss associated with the stabilization of the SPI is almost identical to the case of the stabilization of the OII. However, changes in the distribution of market powers that have occurred in the data affected this result. In the twenty-first century, ignoring the heterogeneity in market competition results into a welfare loss that is 6.1%, measured in terms of welfare loss under a PCE stabilization policy, higher than the outcome under the OII stabilization policy.

Lastly, we conduct a positive analysis by plotting the OII and compare it with the headline and the core PCE. A simple visual inspection suggests that during the Great Moderation periods, the OII was consistently higher than the two PCE measures that the Fed rely on in their policy analysis. This demonstrates that the OII stabilization cannot be achieved by monitoring a weighted average of the headline and the core PCE. During the periods following the Great Recession of the 2008, similar to other measures of inflation, the OII is blow the 2% target.

Literature review This paper is related to studies on multi-sector New Keynesian models. Those studies share the insight that heterogeneous price rigidity increases the effects and persistence of demand shocks, e.g. Carvalho (2006), Nakamura and Steinsson (2010) or Carvalho and Schwartzman (2015). In recent studies, the focus has shifted to the interaction of nominal rigidities with other sources of heterogeneities. Pasten, Schoenle and Weber (2020) or Hoynck (2020) show that production networks can magnify the importance of price rigidities through its effects on marginal costs. In a contemporaneous work, Meier and Reinholt (2020) show that firms with more rigid prices optimally set higher markups due to the precautionary price-setting motive. We study the implications of steady-state markup dispersions for the conduct of monetary policy, and we highlight the importance of monitoring the entire distribution.

Previous literature on the optimal inflation index is abundant, but most conclusions are drawn based on frameworks that introduce nominal rigidity into different markets, in the spirit of Aoki (2001), Benigno (2004) and Mankiw and Reis (2003). Erceg *et al.* (2000) show that in the presence of nominal wage rigidity, the optimal monetary policy index includes wage inflation. Huang and Liu (2005) demonstrate that with price stickiness in intermediate sectors, it is optimal for the central bank to respond to both PCE inflation and PPI inflation. By introducing nominal rigidity to the investment goods sector, Basu and Leo (2016) conclude that the optimal policy reacts to inflations in both consumption goods and investment goods. Anand *et al.* (2015) consider the optimal inflation targeting policy for developing countries. They show that with a significant fraction of hand-to-mouth workers in the food sector, stabilizing headline PCE is welfare improving as compared to maintaining core PCE. Eusepi et al. (2011) derive an optimal inflation index considering heterogeneity in nominal rigidity and the labor share and find that optimal weights mostly depend on price stickiness. We show that a sector with more rigid price is not necessarily associated with a higher weight in the OII due to the competition channel and its empirical correlation with the stickiness channel.[1]

2 Empirical Evidence

In this section, we document new empirical observations on the dispersion of markups and the empirical relationship between nominal rigidity and market power across sectors. In detail, we show additional facts to the increase in average market power over time: (i) the dispersion in markups increases over time, (ii) this is driven not only by increases in the markups of high markup firms but also by decreases for low markup firms and (iii) firms in more competitive sectors change prices more often.

[1]More broadly, this paper is related to the literature that studies the optimal monetary policy with a dynamic price elasticity originating from firm entry and exit. See, for example, Bilbiie *et al.* (2008), Bilbiie *et al.* (2014), Bergin and Corsetti (2008), Cooke (2016), Etro and Rossi (2015), Faia (2012) and Lewis (2013). In contrast to those studies, this paper focuses on the heterogeneity in the *steady-state* price elasticity across sectors. In another closely related paper, Andrés *et al.* (2008) rely on cross-country heterogeneity in competition to explain inflation differentials in the EMU.

2.1 Data

Before we turn to the evidence, we first outline the data we use for the empirical analysis and in order to calibrate our theoretical model. Specifically, we combine and match data from three different data sources.

Firm-Level Markups We use quarterly firm-level balance-sheet from 1967 - 2017 of publicly traded firms in Compustat to calculate firm-level markups. The data covers sales, employment, capital, and input factors of firms (cost of goods sold) over a long sample for a wide range of sectors covering not only manufacturing but also service sector firms. We estimate firm-level markups following the single-input approach of Hall (1986,88) and DeLoecker and Warzynski (2012). According to this approach, the markup $\mu_{i,t}$ of a firm i at time t can be computed from one flexible input, X_i, as the ratio of the output elasticity of the input, ϵ_{Q,X_i}, to the revenue share of that input, s_{R,X_i}

$$\mu_{i,t} = \frac{\epsilon_{Q,X_i}}{s_{R,X_i}}. \tag{1}$$

Compustat reports a composite input called Cost of Goods Sold (COGS), which consists of intermediate and labor input and that will be used as the (partially) flexible input, X_i. The authors use a variant of the technique introduced by Olley and Pakes (1996) and described in DeLoecker and Warzinsky (2012) to estimate a Cobb-Douglas function and obtain a time-independent estimate of output elasticity at the sector level. The markups are then derived by dividing the former by the share of COGS to revenue. We split our analysis in two parts given the critique on estiming markups using the production approach using revenue data, e.g. Bond et al. (2020) or Basu (2019). First, in the main part of the paper we follow the cost share approach and focus on revenue shares to learn about the variation in markups across firms and over time. Bond et al. (2020) outline that this approach can be used to study this variation under minimal restrictions without estimating an output elasticity. We check the robustness of these results to calculating markups as in DeLoecker, Eeckhout and Unger (2020) and report those results in the appendix.

When transforming the data, we drop all firms in the sectors goverment or FIRE. We consider only observations that are positive and linear interpolate observations that are missing for one period. Additionally, we perform outlier adjustments by trimming at 1% (5%) of calculated markups.

One concern with Compustat is that it covers only publicly traded firms and thus is not representative of the distribution of the universe of firms. We account for a representativeness bias by using the weights of each sector in the Compustat data from the PCE expenditure shares to account for sectoral composition (while we still calculate markups from publicly traded firms).

Frequency of Price Adjustment We use sector-level frequencies of price adjustment, FPA, from producer price data (PPI) averaged over the period 2005 - 2011 from Pasten, Schoenle and Weber (2020). The PPI measures selling prices of goods from the perspective of producers and covers goods-producing industries as well as services. The original confidential micro price data that underlies the PPI and collected by the BLS, covers about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The data we use is median frequency at the 6-digit NAICS level.

The aggregate price adjustment frequency, FPA, of matched industries is 0.64 which is close to the reported price frequencies in Nakamura and Steinsson (2005) or Bils and Klenow (2004). We are able to match 88 percent of firms with a FPA. Wherever, we use it, we define implied price duration following Nakamura and Steinsson (2008) as -1/ln(1-FPA). In contrast to Nakamura and Steinson (2008) or Bils and Klenow (2004), the source of the data are not consumer prices, but producer prices. This aims to account for the facts that first markups are set at the producer level, and second the Compustat data is defined on NAICS levels.[2]

Personal Consumption Expenditures and Sectoral Prices The use of personal consumption expenditure (PCE) data has two advantages. First, the main inflation target for monetary policy in the United States is the PCE deflator. Thus, we are going to use the PCE deflator as the reference point for our optimal policy analysis. Second, we can use PCE bridge tables to match NAICS sectors with the sectors that compromise the PCE deflator. For the baseline calibration we choose 17 sectors. This choice is driven by the 15 major types of products of the PCE plus a division of utilities into a core and non-core component. This allows us to compare the resulting index to core price PCE index. Moreover, we look at food services since they have a relatively large share and are characteristically very different to accommodations to which they usually count. We measure the size of sectors by their average expenditure share over each decade from NIPA Table 2.3.5.U. that reports personal consumption expenditures by Major Type of Product and by Major Function. We use PCE Bridge tables from the underlying detail estimates of the Industry Economic Accounts. These are annual tables that outline the commodity composition of the PCE categories from the National Income and Product Accounts (NIPAs) from 1997-2019.[3] In detail, they specify for each PCE category, the commodities it is composed of together with the purchasers' value, which we will use as weights. We will use this information to match each PCE sector into the different NAICS sectors for which we have generated estimates on markups and price rigidity.[4] Finally, we aggregate the matched

[2]Eusepi et al. (2011) aggregate FPA of entry level items (ELIs) in the nonshelter component of the consumer price index (PCE) from Nakamura and Steinsson (2008) into PCE sectors. As we show when we discuss our seventeen sectors economy, when we aggregate the PPI based FPAs into PCE sectors, we arrive at frequencies that are mostly similar.

[3]For the years prior to 1997, we do not have information on the weights of commodity composition. We deal with this by considering the average weights between 1997-2019 for all years.

[4]Results of this exercise can be seen in Table 2

data into three different levels: a one-sector economy, a two-sector economy – in which we distinguish between goods and services – and a 17-sector case composing of the major types of products of the PCE. In order to study the optimal price index, we use sectoral price indices data from the underlying detail table 2.4.4.U. for personal consumption expenditures by type of product.

2.2 Markup Dispersion and Correlation

We derive new empirical results on the evolution of markups over time and their relationship to price-setting behavior of firms. Our focus is twofold. First, we want to document the evolution of the distribution of markups over time. Since we later uncover consequences of different parts of the markup distribution on monetary policy, we can use those results to draw conclusions about changes to the transmission mechanism induced by these changes. Second, we will use the cross-sectional variation to calibrate a 17-sector version of our theoretical model.

Observation 1: Markup dispersion increases over time We estimate yearly markups at the firm-level from 1967-2017. We take 10-year moving averages of markups and receive a distribution of smooth markups for each year. Panel A in Figure 1 plots the resulting average markup over time. The increase in markups compares to other recent estimates in a literature that documents increasing market power in economies. DeLoecker et al. (2020) compares the distribution of markups in 1980 and 2016 and argues that a thicker right tail – more mass of firms with high markups – leads to the higher estimate of the average markup over the sample time.

Based on the distribution of smooth markups over time, we then calculate the dispersion of individual markups in each year via three measures: (i) the variance (ii) the interquartile range and (iii) the range between the 90th and 10th percentiles. The panel

A of Figure 3 and Figure 4 show that the resulting dispersion is increasing over time, independent on the considered measure. For all three measures this process began in the 1980s and quantitatively led for instance to more than a doubling of the standard deviation. As for the two measures of range, we can observe an additional acceleration in the end of the 1990s. Moreover, the interquantile range (right y-axis) is increasing less, reflecting the stronger increasing dispersion between very high markup firms and very low markup firms. Next, we want to investigate where the increase in dispersion is coming from.

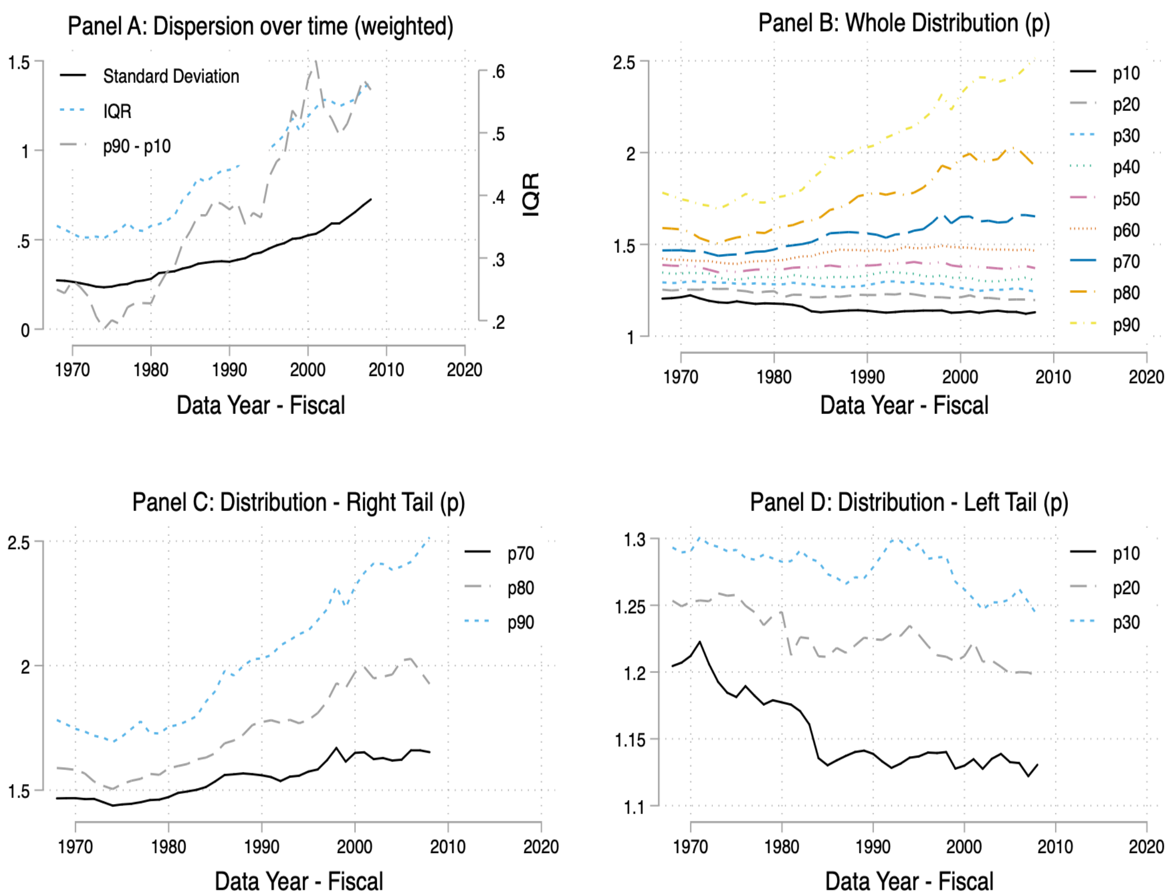
Observation 2: Gap between left and right tail widens by both sides Where does the dispersion come from? Conceptually, it could arise because markups of high markup firms increases, due to falling markups at the left tail of the markup distribution, or both. DeLoecker et al. (2020) reports that the increase in the mean markup is due to composition effects. Firms with larger markups increase in size. This leads to increases in the highest percentiles of the distribution.

Panels B and C of Figure 3 and Figure 4 confirm these results. For the highest percentiles (80 and 90), prices increase by more than 50 percent since the 1980s. In contrast, we observe decreases at the bottom of the markup distribution, however in smaller proportions. They are not sufficient to counteract the overall increase in average markups, but as we will argue, will become important later to due non-linearities in theoretical models even if their relative size appears to be irrelevant compared to the impressive increases at the top. We redo the same exercise for the seventeen sectors that constitute the PCE index. Figure 4 shows that we can find the same results on the dispersion but also on the gap between the left and the right tail in the less aggregated data.

In summary, we observe large increases in the dispersion of markups across firms in the U.S. since the 1980's. We find that these are not only because of increases at the right tail of the distribution but also due to decreasing markups for low markup firms.

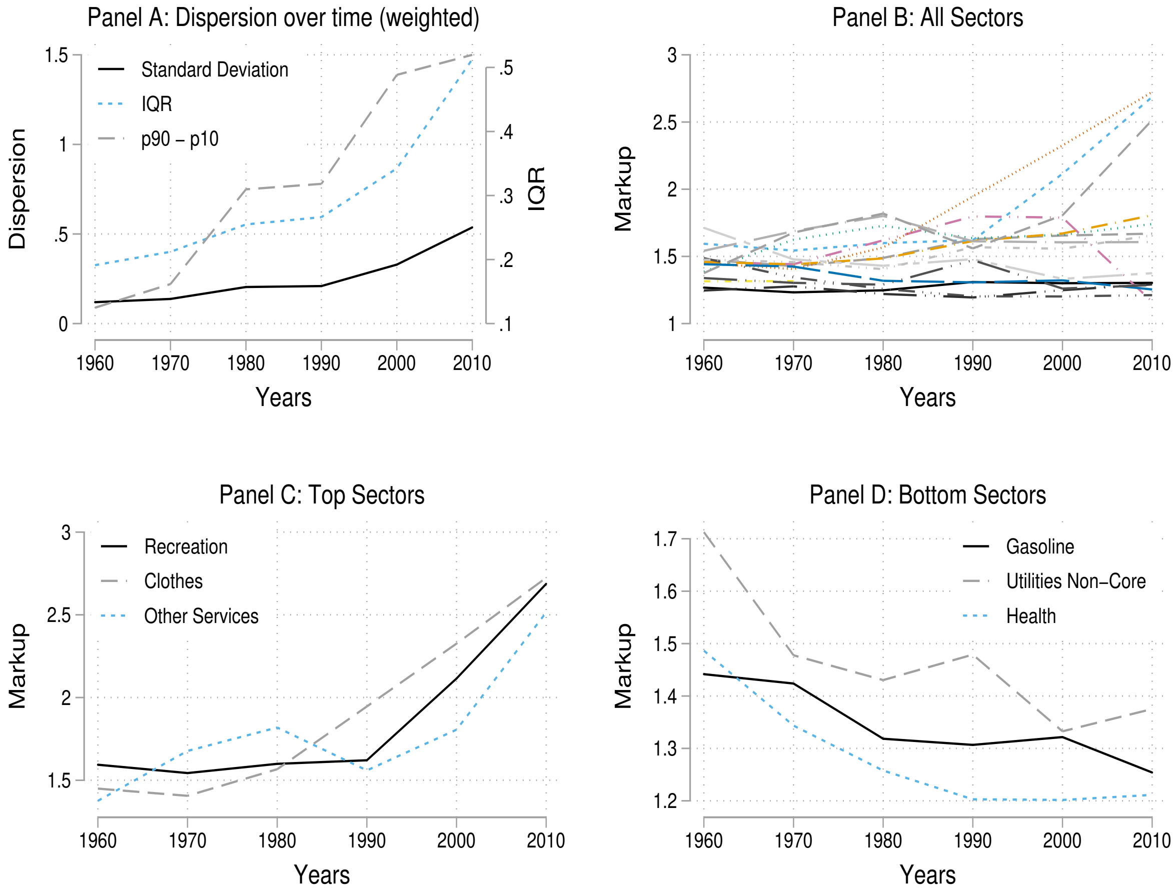
Observation 3: Firms with higher markups change prices less often We also compare markups and nominal rigidity. In the data, we observe that firms can keep prices unchanged for extended period of times. The degree of price rigidity is a leading explanation for the large effects of demand shocks (e.g. monetary policy) on output and is a central ingredient/building block in New Keynesian macroeconomics. To test the relationship, we observe matched markup-frequency pairs at the 3-digit industry level from our constructed dataset. Since frequencies have been calculated over the 2005-2011 period, we calculate sectoral markups over same timespan. In the first piece of evidence, we aggregate these 3-digit sectors, into seventeen sectors that composite the PCE price index. Figure 2 illustrates that there is a clear negative correlation between markups and frequency of price adjustment. The interpretation is that firms in sectors that are more competitive change prices more frequently. This is confirmed by a negative slope coefficient of an OLS regression on these seventeen sectors. We further verify this result by running OLS regressions with controls on a panel of 3-digit sectors. In all versions, we find a negative and significant relationship between markups and the frequency of price adjustment. Details of this exercise are in Appendix D.

Figure 3: steady-state Markups by Quantiles and their Dispersion



Note: Authors' own calculation. This figures plots different quantiles and dispersions of steady-state markups (measured as cost share) in Compustat from 1967-2017.

Figure 4: **steady-state Markups by Sectors and their Dispersion**



Note: Authors' own calculation. This figures plots steady-state markups (measured as cost share) in Compustat from 1967-2017 for seventeen sectors that are constitutes of the PCE, as well as their dispersions.

3 The Economic Mechanism

In the previous section, we documented that the steady-state markups are dispersed and evolving over time. We highlight both the positive trend in the right tail and the negative trend in the left tail of the markup distribution. We will now assess the implications of those facts for the conduct of monetary policy. Before moving to the full model, we illustrate the key mechanism based on a basic NK model borrowed from Galí (2015) Chapter 3, for details about the setup and meanings of parameters we refer readers to the original textbook. The following equations characterizes the equilibrium the economy:

$$\widetilde{y}_t = \mathbb{E}\widetilde{y}_{t+1} - \frac{1}{\sigma}[i_t - \mathbb{E}\pi_{t+1} - \rho)], \tag{2}$$

$$\pi_t = \beta\mathbb{E}\pi_{t+1} + \kappa\widetilde{y}_t, \tag{3}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t, \phi_\pi > 1 \tag{4}$$

where $\widetilde{y}_t, \pi_t, i_t$ denote the output gap, inflation and the nominal interest rate, respectively. Monetary shocks v_t, which follow a AR(1) process $v_t = \rho_v v_{t-1} + \epsilon_t^v$, are the only shocks that hit the economy. The slope of the Philips Curve κ is a composite of the deep parameters in the model:

$$\kappa \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right), \tag{5}$$

where the key parameter ϵ that we are interested in is the elasticity of substitution across goods. It is worth to emphasize that ϵ is the measure of degree of competition in the economy, i.e., it is negatively related to the degree of the market power. Specifically, the steady-state markup is $\frac{\epsilon}{\epsilon-1}$. The parameter α determines whether the production is decreasing ($\alpha > 0$) or increasing ($\alpha < 0$) returning to scale.

The basic model can be solved analytically to obtain:

$$y_t = \widetilde{y}_t = -\Omega v_t, \tag{6}$$

where $\Omega \equiv \frac{1-\beta\rho_v}{(1-\beta\rho_v)[\sigma(1-\rho_v)+\phi_y]+\kappa(\phi_\pi-\rho)}$ measures the size of the effect of a monetary policy shock on real GDP, which we denote as the degree of Money Non-Neutrality. It is straightforward to derive the following lemma.

Lemma 3.1. *With decreasing returns to scale, the slope of the Philipps Curve κ is a decreasing convex function of the elasticity of substitution across goods ϵ and the degree of monetary non-neutrality (Ω) is an increasing concave function of ϵ.*

Jensen's inequality implies that with decreasing (increasing) returns to scale the average of the money non-neutralities in different economies with heterogenous market powers is higher (lower) than the money non-neutrality of a representative economy featuring the average market power. Similarly, the average of the slopes of the Philipps Curve in different economies with heterogenous market powers is smaller (bigger) than an economy with the average market power.

The intuition for the slope of the Philipps Curve changing with ϵ is the following. In the presence of nominal rigidity, a firm's optimal price does not depend on the current marginal cost, but also the future ones:

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\psi_{t+k|t}\}, \tag{7}$$

where ψ_t denote the log marginal cost, and $\psi_{t+k|t}$ denotes the marginal cost in period $t+k$ for a firm that last reset its price in period t. Moreover, the following relationship holds:

$$\psi_{t+k|t} = \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}). \tag{8}$$

This equation states that the marginal cost in period $t+k$ for a firm that last reset its price in period t is decreasing (increasing) in p_t^*, as long as the marginal product of labor is decreasing (increasing) in output. Therefore, the firm sets a lower (higher) p_t^* than it would otherwise do in the absence of this endogenous feedback effect. The market power channel that we emphasize interacts with this endogenous feedback effect. In particular, the latter is amplified in a more competitive market (bigger ϵ). Because for the same amount of the price differential ($p_t^* - p_{t+k}$), the quantity differential is larger in a more competitive market. As a result, firms' prices respond less (more) to shifts in marginal costs. In other words, the slope of the Philipps Curve is flatter (steeper). It followed that the degree of monetary non-neutrality is increasing (decreasing) in ϵ.

Although a multi-sector version of the model is not a simple weighted average of the multiple basic models, the intuition provided in this subsection does carry over. The remaining of this section is to illustrate the quantitative importance in a multi-sector model.

4 Implications of Dispersed Markups

We assess the implications of the facts we documented in section (2) for the conduct of monetary policy based on a multi-sector NK model (Woodford (2011), Carvalho2006). In this section, we discuss the implications for the degree of monetary non-neutrality and the Phillips Multiplier. We analyze the inflation targeting policy in section 5.

4.1 The Multi-sector New Keynesian Model

In this section, we present a dynamic multi-sector model (Woodford (2011), Carvalho2006) with heterogenous degrees of market power, nominal rigidities, and sizes across sectors. The heterogeneity in market power is modeled as the following. We assume that within a sector k, firm-level goods are aggregated into the sectoral aggregate goods C_{kt} according to the following CES function:

$$C_{kt} \equiv \left[n_k^{-1/\epsilon_k} \int_0^{n_k} C_{kt}(i)^{(\epsilon_k - 1)/\epsilon_k} di \right]^{\epsilon_k/(\epsilon_k - 1)} \tag{9}$$

with an elasticity of substitution ϵ_k (hence the market power) that varies across sectors. The multi-sector economy is populated by a continuum of (0,1) of households, a fraction of n_k of monopolistic competitive firms in sector k for $k = 1, 2.., K$, a government and a central bank. A fraction of $1 - \theta_k$ of firms in sector k is allowed to reset their prices in each period. The degree of market power in each sector k is characterized by the elasticity of substitution across goods within the sector. Households consume the composite goods, buy a one-period risk-free government bond, supply labor to the sectoral competitive labor market, receive dividends (profits) from firms and pay taxes or receive transfers from the government. Firms demand labor to produce and sell goods to households. The government issues government bonds, collect lump-sum taxes (or pay transfers) from (to) households. The central bank follows a Taylor rule. We leave the detail description of the model to the Appendix F.

Sectoral Phillips Curves By solving the firms' optimization problem we obtain the New Keynesian Philipps Curve (NKPC) for each sector k:

$$\pi_{kt} = \kappa_k \widetilde{y}_t + \gamma_k \widetilde{y}_{R,kt} + \beta \mathbb{E}_t \pi_{k,t+1} \tag{10}$$

where $\widehat{y}_{R,kt} \equiv \widehat{y}_{kt} - \widehat{y}_t$, $\kappa_k \equiv \lambda_k(\sigma + \frac{\varphi+\alpha}{1-\alpha})$, $\gamma_k \equiv \lambda_k(\eta^{-1} + \frac{\varphi+\alpha}{1-\alpha})$, $\lambda_k \equiv \frac{(1-\beta\theta_k)(1-\theta_k)}{\theta_k}\Theta_k$, $\Theta_k \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon_k}$. Sectorial heterogeneities give birth to relative price (or quantity) dispersion across sectors, therefore a full stabilization of both inflation and output gap is no longer feasible. More specifically, the aggregate Phillips Curve can be obtained by summing up the Sectoral NKPCs:

$$\pi_t = \sum_{k=1}^{K} n_k \kappa_k \widetilde{y}_t + \sum_{k=1}^{K} n_k \gamma_k \widetilde{y}_{R,kt} + \beta \mathbb{E}_t \pi_{t+1}, \tag{11}$$

where $\pi_t \equiv \sum_{k=1}^{K} n_k \pi_{k,t}$ denotes the aggregate inflation index (PCE).

Monetary Policy The central bank sets the nominal interest rate according to a Taylor rule that targets an inflation index (π_t^{cb}) and the output gap:

$$i_t = \phi_\pi \pi_t^{cb} + \phi_y \widehat{y}_t + v_t, \tag{12}$$

where v_t denotes exogenous monetary policy shocks following an AR(1) process with persistence ρ_v. Before moving to the discussion of the optimal choice of π_t^{cb}, we assume that the central bank targets the PCE index π_t.

Calibration We use the evidence from the empirical section to calibrate the parameters of the Multi-sector model that govern sectoral heterogeneity. For the other parameters that characterize aggregate dynamics, we largely follow Galí (2015). The model is calibrated to match the main categories that underly the PCE price index in the NIPA tables. We additionally decomopose utilites into two categories to differentiate between core and non core components. The model is calibrated at quarterly basis.

Homogenous Parameters. Most parameters are calibrated to values that are frequently used in the literature. We set the discount factor to 0.99, implying a (annualized) steady-state interest rate of 4%. $\sigma = 2$ implies that inter-temporal elasticity of substitution equal to 0.5. The Frisch elasticity of labor supply ($1/\varphi$) is set to be $1/5$. The production function has decreasing returns to scale with $\alpha = 1/3$, a value that is commonly used in business cycle literature. We also study increasing returns to scales by chosing $\alpha = -0.05$, a value that is consistent with DeLoecker et al. (2020) and still guarantees determinacy. We adopt interest rule coefficients suggested by Taylor (1993) as $\phi_\pi = 1.5$ and $\phi_y = 0.125$. Shock persistence are set to 0.8 and variance following Billi and Gali at $\sigma_{ak} = 0.033$ and $\sigma_m = 0.044$. One important parameter is the across-sector elasticity of substitution, η. In one-sector models there is only one parameter that governs aggregate markups. Here, we differentiate between within sector elasticities, which finally govern sector specific markups, and across sector elasticities of substitution. We follow Atalay (2019) that calibrates the across sector elasticity using different approaches, finding that it is usually very small implying that goods are rather inelastic across sectors. We follow his medium estimate and use $\eta = 0.5$.

Utility function	$\sigma = 2$, $\varphi = 5$
Discount factor	$\beta = 0.99$
Production function	$\alpha = 1/3$ or $\alpha = -0.05$
Technology shocks	$\rho_k = 0.8$, $\sigma^2_{ak} = 0.033$
Demand shock	$\rho_m = 0.8$, $\sigma^2_m = 0.044$
Elasticity of substitution	$\eta = 0.5$

Table 1: Calibration Homogenous Parameters

Heterogenous Parameters. Table 2 shows the calibrated values for all heterogenous parameters.

First heterogeneity in the size of a sector, n_k. This corresponds to the steady-state share of expenditure the consumer assigns to this sector. Accordingly, we use average personal consumption expenditures of households that is attributed to this sector over the different decades.

Second, frequency of price adjustment. In the Calvo model, the frequency of price adjustment directly matches into price rigidity, since every quarter only a fraction $(1 - FPA_k)$ of goods within sector can adjust prices. We calculate the sales-weighted median of the frequency of price adjustment within each category as the respective measure of price stickiness. We convert monthly frequencies from the table into quarterly frequency. Here, two possible approaches: from the consumer side or the producer side. While the former uses Nakamura and Steinsson's (2006) estimates aggregated by Eusepi et al. (2011), the latter represents our aggregations based on the data on FPAs from Weber et al. (2019).[5] The aggregate calibrated rigidity in the 17-sector model is 0.6, which is in line with the aggregate rigity (0.63) found in Gorodnichenko and Weber (2016). One important observation is the similarity of many frequencies across this highly different approaches.[6] This is the more suprising given that they do not share the same data base as origin. Instead, one is aggregated from ELI good prices and the other from PPI prices. This gives us confidence also for the aggregation of the markups that follows the same methodology as for the PPI frequencies.

Third, markups across sectors. We calculate markups using either costshare or De-Loecker et al. (2020) approach at the firm level. In a first step, we aggregate them at the 3 digit NAICS level using Compustat declarations and take average values decades. We then use PCE bridge tables to assign the NAICS sectors from which personal consumption in PCE originates. This provides us with a matching from NAICS into PCE categories. We use weights based on the producer value of goods. The markups we derive following this approach are reported in Table 2 and can be matched into sector specific

[5]We are grateful to Michael Weber for sharing this data with us.

[6]Particularly interesting is the health care sector. The frequency based on the health care ELIs included in the CPI research database studied by Nakamura and Steinsson (2006) is 3.4 percent, implying an average duration of prices of 29 months. Given this high rigidity, Eusepi et al (2011) deviate and chose an ad-hoc value of 8.3 to have an implied duration of 12 months to match spikes in October and January of the underlying data. Our estimates based on the PPI data generate an rigidity of 7.96, close to the implied duration of 12 months supporting the view that prices in this sector change at least once a year.

elasticities of substitution, ϵ_k.

4.2 Results: Money Non-Neutrality

In this section, we report first theoretical results on the relationship between non-neutrality and the Phillips multiplier with different components of the markup distribution – (i) mean and (ii) dispersion. Note that we consider mean and dispersion as stand-ins for the whole distribution of markups. In this sense, it is likely that higher moments like kurtosis will also effect non-neutrality. While the analysis of those is out of the scope of this paper, the aim is to motivate to look at the whole distribution instead of solely the average markup to study the relationship between non-neutrality and markups. We do so in a simple three-sector version of the multi-sector model, where we will control the moments of the markup distribution. The homogenous parameters follow the exposition in the last section and sector sizes as well frequency of price adjustment are calibrated to their average levels. In the second part, we will then look at predictions concerning the Phillips multiplier in the different decades of the seventeen-sector calibration of the model as outlined in Table 2.

Non-Neutrality decreases (increases) in Average Markup First, we study the effect of increases to the average markup on non-neutrality in the multi-sector model, while keeping other moments of the distribution constant. This resembles the exercise performed in the motivational example with the difference that there we showed the effect of an increase in a one-sector model (as e.g. in Galí 2008). In particular, we calibrate different versions of the model that feature heterogeneity in sectoral markups and differ in the average markup across versions.

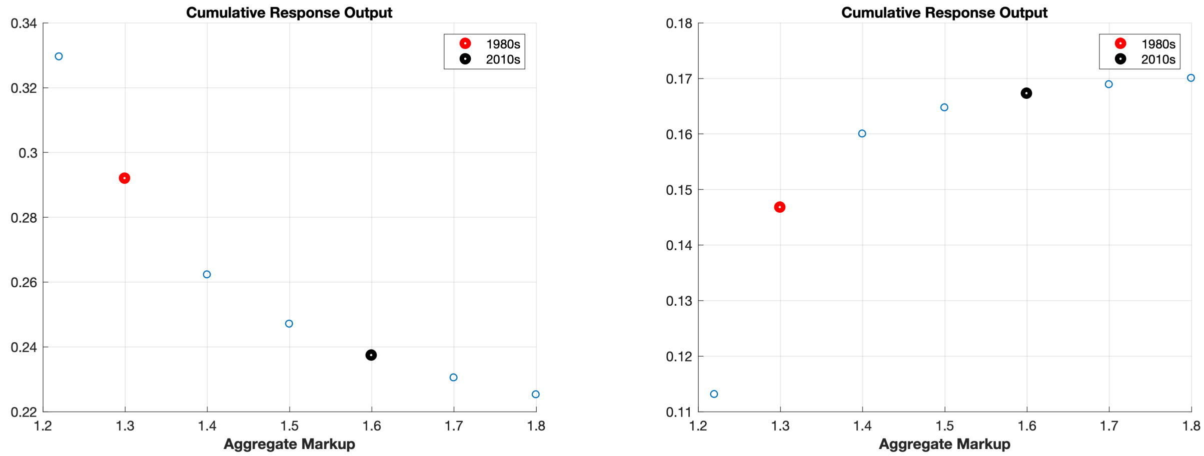
Figure 5 shows the cumulative response of output to a 25bp expansionary monetary policy shock in different calibrations of the model with the average markup of those cal-

Table 2: Calibrated Parameter Values

Sector name	Core	PCE share 1960's	PCE share 2000's	FPA PPI	FPA CPI	Markup 1960's	Markup 2000's
i) One sector							
PCE		100	100	14.17	12.93	1.3	1.6
ii) 2 sector							
Manufacturing		51.67	34.78	11,57		1.2	1.58
Services		48.33	65.22	8,47		1.6	1.81
iii) 17 sectors							
Motor vehicles	X	6.51	5.03	38.66	36.6	1.27	1.30
Furnishings and household	X	4.93	3.26	10.76	9.2	1.50	1.94
Recreational goods	X	2.51	3.75	9.48		1.64	3.03
Other durable goods	X	1.45	1.80	6.55	10.9	1.39	1.65
Food (off-premises)		17.98	8.62	16.96	13	1.42	1.93
Clothing and footwear	X	7.97	3.94	6.92	32.2	1.41	2.41
Gasoline & energy goods		4.62	3.40	79.83	87.6	1.50	1.32
Other nondurable goods	X	8.90	8.90	15.59	10.4	1.41	1.73
Housing	X	15.17	17.09	29.79	10.3	1.52	1.90
HH Utilities Core	X	0.48	0.81	17.86	11.4	1.35	1.60
HH Utilities Non Core		2.62	2.37	33.82	38.5	1.72	1.33
Health care	X	6.31	16.78	7.96	8.3	1.49	1.20
Transportation	X	2.98	3.68	10.55	71.5	1.28	1.27
Recreation services	X	2.19	4.23	5.06	10	1.47	1.91
Food services	X	6.10	5.83	25.51		1.34	1.27
Accommodations	X	0.43	0.95	21.28		1.25	1.25
Other services	X	8.84	9.56	4.49	7.5	1.38	1.95

This table shows the calibrated parameters of the different sector economies. Share and frequencies are in percentage points. FPA PPI is from Weber et al. 2020 and FPA CPI is from Nakamura and Steinsson 2006. Markups are calculated using Compustat at the firm level and aggregated to PCE sectors using PCE bridge tables.

Figure 5: **Monetary Non-neutrality and Aggregate Markup**



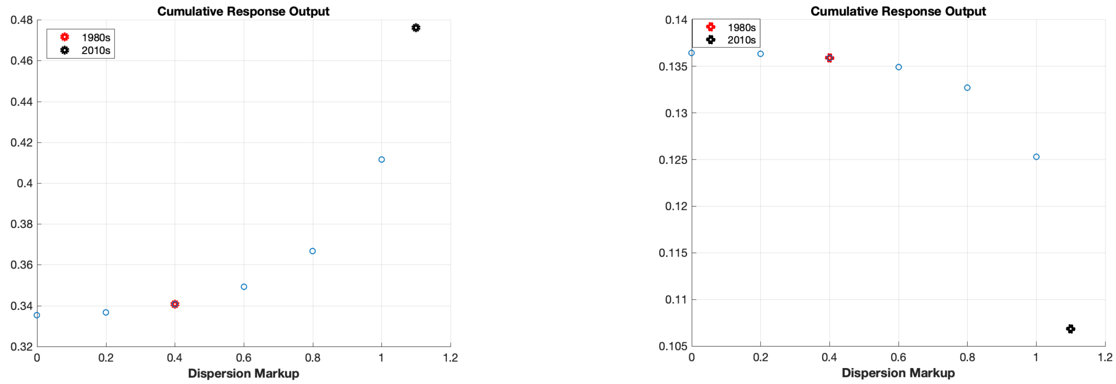
(a) Decreasing returns to scale ($\alpha = 1/3$)

(b) Increasing returns to scale ($\alpha = -0.05$)

Note: This figure shows the cumulative output response to a 25bp expansionary monetary policy shock in different three sector calibrations of the multi-sector model that differ in the aggregate degree of market power.

ibrations on the horizontal dimension. Panel a) of Figure 5 shows the results in the case of decreasing returns to scale. We find a clear negative relationship between markups and non-neutrality as in the text-book one-sector model. With less competition in an economy, monetary policy becomes less effective. We also highlight the size of average markups in the economy in the 1980s and 2000s. The increase in markups of 30 percentage points decreased non-neutrality by over 20 percent according to the model. This is a direct application of Lemma 3.1 since an increase in the average markup is related to a decrease in the aggregate elasticity of substitution, ϵ_k. Intuitively, if demand is less elastic to changes in prices, increases in demand will lead to changes in prices with smaller output adjustments. Thus, money non-neutrality becomes smaller for all sectors. This implies also that the Phillips curve multiplier becomes steeper for all sectors and thus also on the aggregate. In conclusion, the results in the multi-sector model are consistent with the one-sector model. In Panel b) we report the same results for the case of increasing returns to scale. We document that with increasing returns to scale, larger markups result in larger

Figure 6: **Monetary Non-neutrality and Markup Dispersion**



(a) Decreasing returns to scale ($\alpha = 1/3$) **(b)** Increasing returns to scale ($\alpha = -0.05$)

Note: This figure shows the cumulative output response to a 25bp expansionary monetary policy shock in different three sector calibrations of the multi-sector model that differ in the dispersion of markups while having the same aggregate degree of market power.

monetary non-neutrality.

Non-Neutrality increases (decreases) in Markup Dispersion As we have seen in the empirical evidence, dispersion as measured by standard deviation of idiosyncratic markups more than doubled over time. Therefore, we are interested to measure the effect of increases in the dispersion of sectoral markups on non-neutrality in the multi-sector model. For this, we consider again the three sector model, but this time we keep the average markup constant. Instead, we increase step-wise markup dispersion, by decreasing (increasing) the lower (higher) markup in the same proportions. In the one-sector model, a change in the composition of sectoral markups does not not change any predictions as long as the aggregate markup stays the same.

Figure 6 shows the cumulative response of output to a 25bp expansionary monetary policy shock when the dispersion of markups across sectors increases (horizontal axis). Panel a) of Figure 6 shows a clear positive relationship between markup dispersion and non-neutrality in the case of decreasing returns to scale. With more dispersed markups in an economy, monetary policy becomes more effective. Specifically, the increase in dispersion in the U.S. economy from the 1960s until today, could have increased non-neutrality by more than 40 percent. The reason is the non-linearity in the size of output response to the level of markup exposed in Lemma 3.1. Intutively, decreasing the markup of the low markup sector increases it's non-neutrality by more than can be off-set by the equivalent increase in the markup of the high markup sector. The evidence in the case of increasing returns to scale are again inverse as documented in Panel b). It shows that, larger dispersion decreases monetary non-neutrality again because of the effects at both ends of the markup distribution. In consequence, the increase in dispersion over time could potentially off-set the predictions of increasing markups in the one-sector and multi-sector models. If this is indeed the case, we will study in the next section, where we calibrate the multi-sector model to the actual moments we observed for the U.S. in between 1967 - 2017.

Phillips Multiplier in the U.S. Now we consider the seventeen-sector version of the multi-sector model and calibrate it to the data described in the previous section and summarized in Table 2. To identify the effects of changes in markup distribution, we fix sectoral sizes at their 2000 level. Figure 7 shows the Phillips multiplier for each decade from the 1960s until the 2010s for both the case with decreasing returns to scale (Panel A) and the case with increasing returns to scale (Panel A). In our analysis, we define the Phillips multiplier as the ratio of the cumulative responses of inflation vis-a-vis output to a 25bp expansionary monetary policy shock. We find a relatively stable Phillips multiplier (blue lines in Panel A and C) and monetary non-neutrality (blue lines in Panel B and D) over time.[7] The reason for these stable statistics is the spreading-out of the markup distribu-

[7]Due to aggregation effects, the Phillips multiplier and slope of the Phillips curve tend to be larger in multi-sector models than in an one-sector model. The reason is that κ_k in equation (11) is non-linear in θ_k and ϵ_k.

tion. The effects of a decline in the left tail offset the effects an increase in the right tail of the markup distribution. This is verified by the red lines in Panel A and C: had the left tail remained constant over time,[8] the Phillips multiplier would have increased (decreased) due to decreasing (increasing) returns to scale.

Next, we calibrate our seventeen-sector over time to take into account the changes in sector sizes. 8 plots the results. Due to changes in sector sizes, the degree of monetary non-neutrality in the U.S. has increased overtime and the inflation-output tradeoff summarized by the Phillips multiplier has declined by roughly 20%. This is true for increasing and decreasing returns to scale. Moreover, Panel A of Figure 8 shows another important result. The graph compares the calibrated 17-sectors model with a counterfactual that keeps the lowest markups constant in the case of decreasing returns to scale. The prediction on the change in the Phillips multiplier would be of opposite sign and about 50% higher. This confirms the importance of monitoring the whole markup distribution.

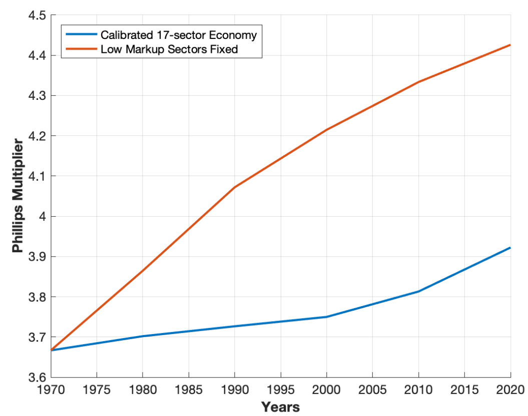
Next, we want to understand where the decreasing Phillips multiplier in Panel A of Figure 8 comes from. Therefore, we focus on the case of increasing returns to scale. To understand these results we look at the aggregate New Keynesian Phillips curve, equation (11). The slope of the Phillips curve is the elasticity of inflation to changes in the in output gap. In the present case, it is represented by the coefficient in front of the output gap, $\bar{\kappa} = \sum n_k \kappa_k$.[9]

Figure 9 shows that the decrease in the Phillips multiplier (blue solid line) is almost completely explained by the decreasing aggregate slope coefficient (red solid line) in the model. The slope coefficient, $\bar{\kappa}$ depends non-linearly on the sectoral elasticity of substitution. As a consequence, changes to smaller ϵ_k have a smaller impact on the aggregate

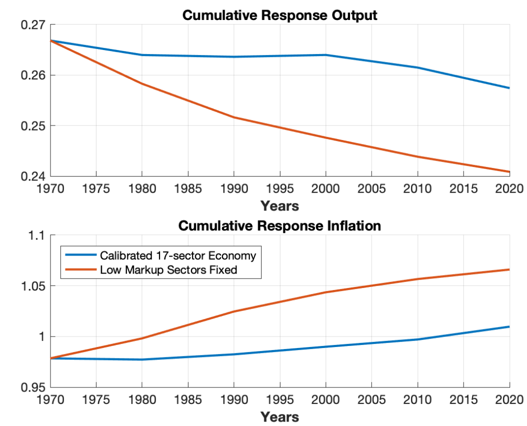
[8] In the counterfactual we fix the markups of the four sectors with the lowest markups at their 1960's value.

[9] Hoynck (2020) showed that in multi-sector models, the slope of the Phillips curve is measured with a bias, since relative output gaps are possibly correlated with the aggregate output gap. In the present case, the bias appears not to be driving the results.

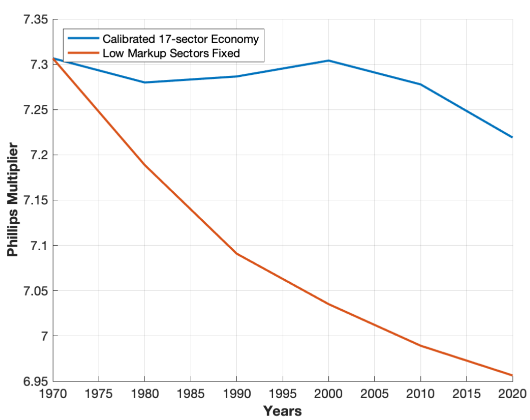
Figure 7: The Implied Evolution of the Phillips Multiplier in the U.S.



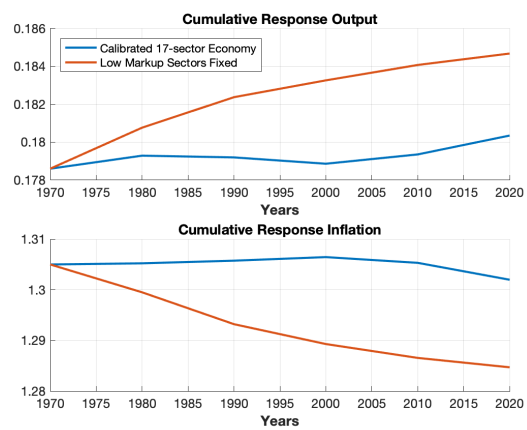
(a) Decreasing returns to scale ($\alpha = 1/3$)



(b) Decreasing returns to scale ($\alpha = 1/3$)



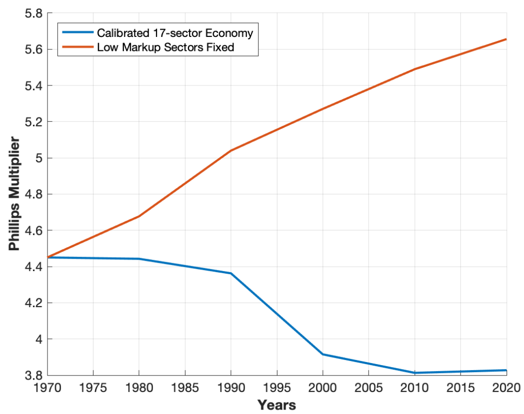
(c) Increasing returns to scale ($\alpha = -0.05$)



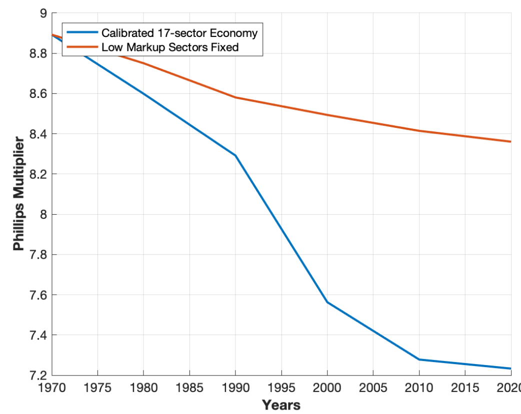
(d) Increasing returns to scale ($\alpha = -0.05$)

Note: This figure shows the dynamic multiplier as the ratio of the cumulative responses of inflation and output to a 25bp expansionary monetary policy shock in seventeen sectors calibrations of the multi-sector model over time.

Figure 8: **The Implied Evolution of the Phillips Multiplier in the U.S.**



(a) Decreasing returns to scale ($\alpha = 1/3$)

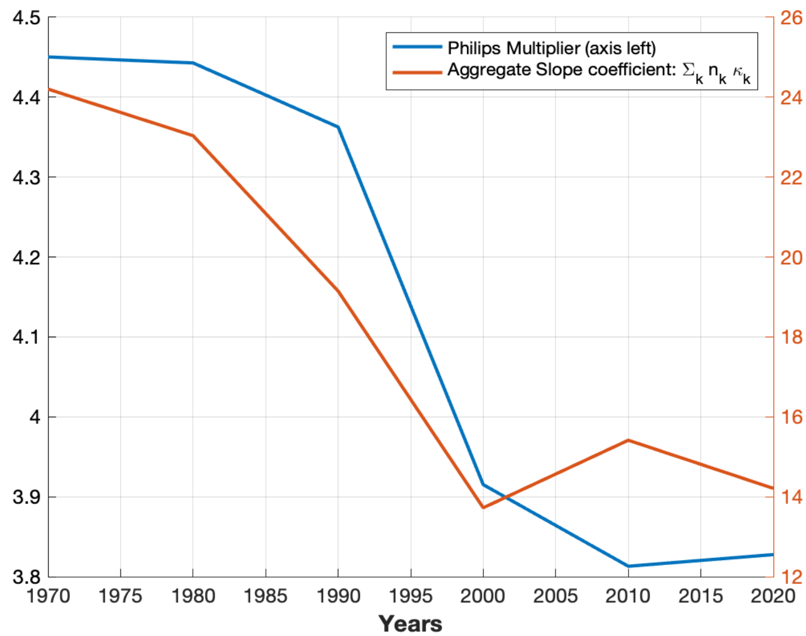


(b) Decreasing returns to scale ($\alpha = 1/3$)

Note: This figure shows the dynamic multiplier as the ratio of the cumulative responses of inflation and output to a 25bp expansionary monetary policy shock in seventeen sectors calibrations of the multi-sector model over time.

coefficient those to than larger ϵ_k. This is confirmed if we look at the cumulative responses of output and inflation in Panel B of Figure 9. The Phillips multiplier is smaller because the slope coefficient decreases; larger fluctuations in output require smaller deviations of inflation (blue line).

Figure 9: Phillips Multiplier v.s. Phillips Curve



Note: This Figure shows the aggregate slope coefficient together with the estimated Phillips multiplier over time.

5 The Optimal Inflation Target Policy

Welfare Loss Function Before moving to the central bank's problem, we will derive the welfare loss function, which is the objective of the central bank. Following Rotemberg and Woodford (1997, 1999) and Woodford (2002), we derive the welfare loss function as the second order approximation of the representative consumer's period welfare loss expressed in consumption equivalent variation (CEV):

$$L = \sum_{k=1}^{K} \frac{\epsilon_k}{\lambda_k} n_k var(\pi_{kt}) + (\sigma + \frac{\varphi + \alpha}{1 - \alpha}) var(\widetilde{y}_t) + (\eta^{-1} + \frac{\varphi + \alpha}{1 - \alpha}) \sum_{k=1}^{K} n_k var(\widetilde{y}_{R,kt}) \tag{13}$$

where $\lambda_k \equiv \frac{(1-\beta\theta_k)(1-\theta_k)}{\theta_k}\Theta_k$ defined as above. Normalize the weights on π_{kt} such that $\sum \phi_k = 1$:

$$L = \sum_{k=1}^{K} \phi_k var(\pi_{kt}) + \lambda_y var(\widetilde{y}_t) + \lambda_{R_y} \sum_{k=1}^{K} n_k var(\widetilde{y}_{R,kt}) \tag{14}$$

where

$$\phi_k = \frac{n_k \epsilon_k \lambda}{\lambda_k}, \qquad \lambda_y = (\sigma + \frac{\varphi + \alpha}{1-\alpha})\lambda, \qquad \lambda_{Ry} = (\eta^{-1} + \frac{\varphi + \alpha}{1-\alpha})\lambda$$

and λ is defined as:

$$\lambda \equiv (\sum_{0}^{K} n_k \epsilon_k \lambda_k^{-1})^{-1}$$

See Appendix (G) for details of the derivations.

Interestingly, by allowing for sectorial heterogeneity in market power, inflation of a sector with a higher elasticity of demand enters in the welfare loss function with a bigger relative weight, i.e., $\frac{\partial \phi_k}{\partial \epsilon_k} > 0$.

The Optimal Inflation Index Stabilization Policy The central bank adopts inflation targeting as the means of conducting monetary policy. This is the case for many central banks around the world. We assume that the target rate is zero (the steady-state inflation rate) and the goal is always achieved. This is equivalent to a Taylor rule with strict inflation index targeting. The monetary instrument is the ex-ante choice of an inflation index that the central bank stabilizes ex-post. This question can be formulated as the following:

$$\min_{\{\omega_k\}} L = \min_{\{\omega_k\}} \sum_{k=1}^{K} \phi_k var(\pi_{kt}) + \lambda_y var(\widetilde{y}_t) + \lambda_{R_y} \sum_{k=1}^{K} n_k var(\widetilde{y}_{R,kt}) \tag{15}$$

subject to equilibrium conditions, resources constraints, and $\sum_{k=1}^{K} \omega_k \pi_{kt} = 0$.

Previous studies have uncovered two main results. First, if sectors share the same degrees of nominal rigidities and market competition, the stabilization of PCE is optimal. Second, it is optimal to give higher weight to the sector with a higher degree of nominal rigidity. The remaining of this paper is to investigate the role of market power and in particular how it might interact with the stickiness channel.

5.1 Special Cases

We begin with analyzing a liming case in which one sector is infinitely close to perfect competition[10], i.e., $\epsilon_k \to \infty$. In this case, the loss function collapses to:

$$L \to var(\pi_k).$$

It follows immediately that:

Proposition 1. *In the limiting case $\epsilon_k \to \infty$ and $\theta_k \neq 0$, the optimal monetary policy is to set $\pi_k = 0$.*

This does not mean that the welfare loss under the optimal monetary policy is zero. In fact, due to asymmetric shocks, the aggregate and the relative output gap, and inflation in the remaining sectors fluctuate, which give rise welfare loss. It means that if goods in sector k are almost perfect substitutes then, in terms of welfare loss, stabilizing inflation in this sector is infinitely more valuable than stabilizing any other variables. This is the case because, with a flat demand curve and nominal rigidity, price dispersion that arises from inflation leads to an infinitely big dispersion in output.

[10]It only makes sense to talk about the infinitely close case, because in the limiting case with perfect competition firms are price takers. Therefore the firm's problem discussed in the previous section, price setter firms, would not carry over.

Next, we investigate whether the competition channel affects the optimality of core inflation stabilization suggested by Aoki (2001) and Benigno (2004).

Proposition 2. *If price is flexible in sector k, independent of the relative market power, the optimal weight for this sector is zero.*

Proof: see Benigno (2004).

If the price is flexible, inflation does not lead to price dispersion. Therefore welfare loss originating from inflation is trivial no matter how competitive the market is.

A more interesting interaction between market power and nominal rigidity arises in the general case.

5.2 Inflation Targeting Policies in a More Aggregated Two-sector Model

We begin with illustrating the mechanism based on a two-sector model calibrated to the manufacture and service sector in the U.S.. Those two sectors represent a big fraction of aggregate production in the U.S. Therefore, the results we find below represent the findings for a relatively (compared to a 17-sector model) more aggregated model.

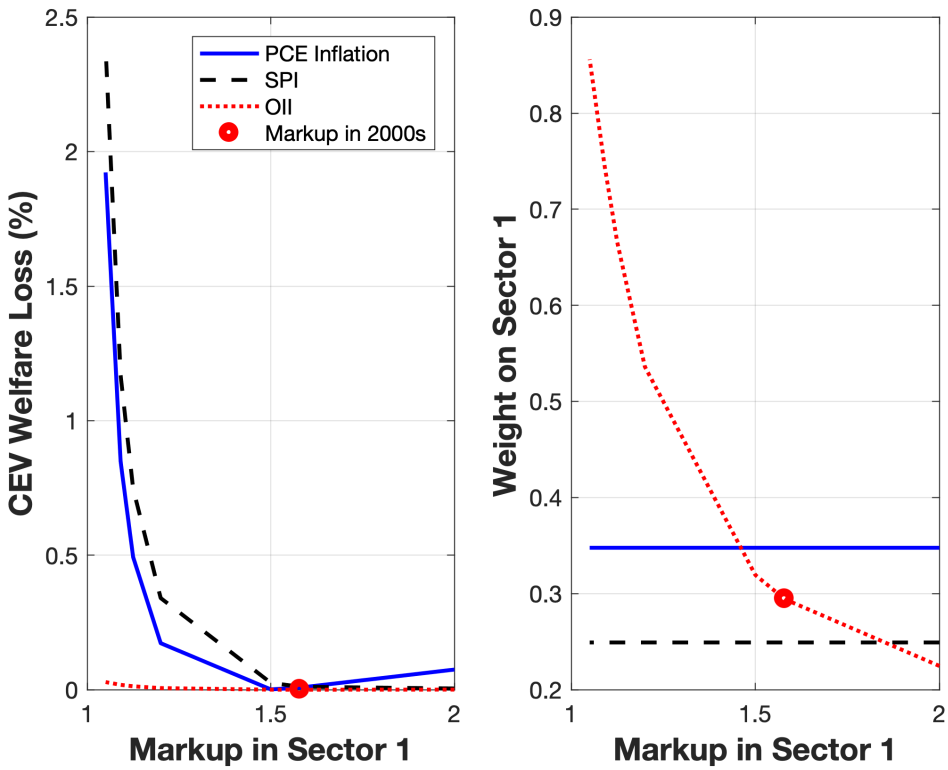
Calibration Unless otherwise specified, the model's parameters are calibrated to be those reported in Table (1) and (2) in the 2000's. The heterogenous parameters are calibrated to match their counterparts in the manufacturing (sector 1) and service (sector 2) sectors in the U.S. from 2000-2010. The sectoral degrees of nominal rigidity are $\theta_1 = 0.69$ and $\theta_2 = 0.77$ for the manufacturing and service sectors. They are chosen to match the monthly frequency of price adjustments based on the PPI and are similar to those reported in Gorodnichenko and Weber (2016). The sectorial elasticities of substitution are calibrated to be $\epsilon_1 = 2.72$ and $\epsilon_2 = 2.25$ in order to match markups in manufacturing (1.58) and service sectors (1.81) estimated by Loecker and Eeckhout (2017). Those markups are higher than the values that are typically assumed in the literature: 1.1 or 1.2. We provide a robustness check using those commonly used values, and qualitatively results are unchanged. What matters is the markup in the service sector is higher than manufacturing, which is confirmed by Christopoulou and Vermeulen (2012) in their estimates of markups for both the U.S. and the Euro Area.

The Competition Channel offsets the Stickiness Channel We conduct welfare analysis under alternative inflation index stabilization policies: the OII, the PCE and the SPI. The welfare comparison is done for different values of steady-state markup in sector 1. Figure (10) reports the results. The reported welfare loss is the CEV defined above in deviation from the CEV under the optimal monetary policy. The left panel shows the welfare loss under alternative policy rules, and the right panel plots the associated weights.

Interesting results arise when comparing the performance of PCE stabilization with the stabilization of SPI. When markup in sector 1 is big enough. stabilizing the inflation index based on stickiness as recommended by Benigno (2004) and Mankiw and Reis (2003) is welfare improving as compared to the stabilization PCE. However, if sector 1 is competitive with a small markup, stabilizing PCE is superior. Hence, stabilizing SPI is a sensible policy advise (as compared to the stabilization of PCE) if the sector with relatively more flexible price (sector 1) is associated with a bigger markup. This is not the case in the data: Figure(2) shows a negative relationship between price flexibilities and steady-state markups.[11] This suggests that the competition channel works against the stickiness channel. The red dot in Figure (10) points out the steady-state markup in sector 1 in the data. Interestingly, in the calibrated two-sector model that we consider here, the competition channel fully offsets the stickiness channel. As a result, the stabilization of PCE or SPI leads to welfare losses of the similar amount.

[11]Costly price adjustment models developed by Barro (1972), Sheshinski and Weiss (1977) and Golosov and Lucas (2007) predict more flexible price in a sector with higher competition.

Figure 10: **Inflation Targeting Policy in a Two-sector model**



Note: Welfare loss as a function of markup in sector 1 under alternative policies. The welfare loss is the corresponding CEV in deviation from the CEV under optimal monetary policy. The red dot corresponds to the point where markup in sector 1 equals to 1.58 — the markup for the manufacturing sector in the data.

5.3 Inflation Targeting Policies in a Calibrated Seventeen-Sector Model

We now consider the inflation targeting policies in a seventeen-sector model. The model's parameters are calibrated to be those reported in Table (1) and (2) for the 1960's and the 2000's subsamples. Sectors are heterogenous in their degree of nominal rigidities, market powers, and sector sizes. We consider four inflation index weights: (i) the OII weights that takes into account of all three heterogeneities; (ii) the SPI weights that are computed without the consideration of heterogeneity in market power i.e. sectoral markups are set to the economy-wide average level; (iii) the Markup weights that are inferred from an economy where there is no heterogeneity in nominal rigidities and all sectoral frequencies of price adjustment are set to the economy-wide average level; (iv) the PCE weights are simply given by the size of an sector which is inferred by it's expenditure share in the data, n_k.

The Importance of the Markup Channel Increased Over Time Table (3) reports the percentage gain in the welfare loss under alternative inflation targeting policies relative to that under the PCE stabilization policy. That is, we report $100 * (L_{PCE} - L_i)/(L_{PCE})$, where $i = \{OII, SPI, Markup\}$ and L_i denotes the welfare loss under an inflation targeting policy i. The role of the market channel played a minor role in the 1960's. However, in the twenty-first century the market channel has gained its importance in the design of inflation targeting policy. The stabilization of SPI leads to a welfare loss that is 11.3% smaller than the loss under the stabilization of the PCE, whereas the OII that takes market power heterogeneity into consideration leads to a welfare gain of 17.4%.

Table 3: Welfare Gain Relative to the Stabilization of PCE

Inflation Targeting Policies	The 1960's Calibration	The 2000's Calibration
OII	7.5%	17.4%
SPI	7.4%	11.3%
Markup	0.3%	8.2%

The pattern on the importance of the markup channel over time is confirmed in Figures (11), where we plot the OII weights, SPI weights, and the markup weights for each sector in the two subsamples. A clear pattern stands out: in the 1960's the SPI weights almost coincides with the OII weights, whereas the difference among the two weights are more pronounced in the 2000's. The service sectors are plotted towards the end of the x-axis. The optimal weights for those sectors in the 2000's (red bar) lay in between the

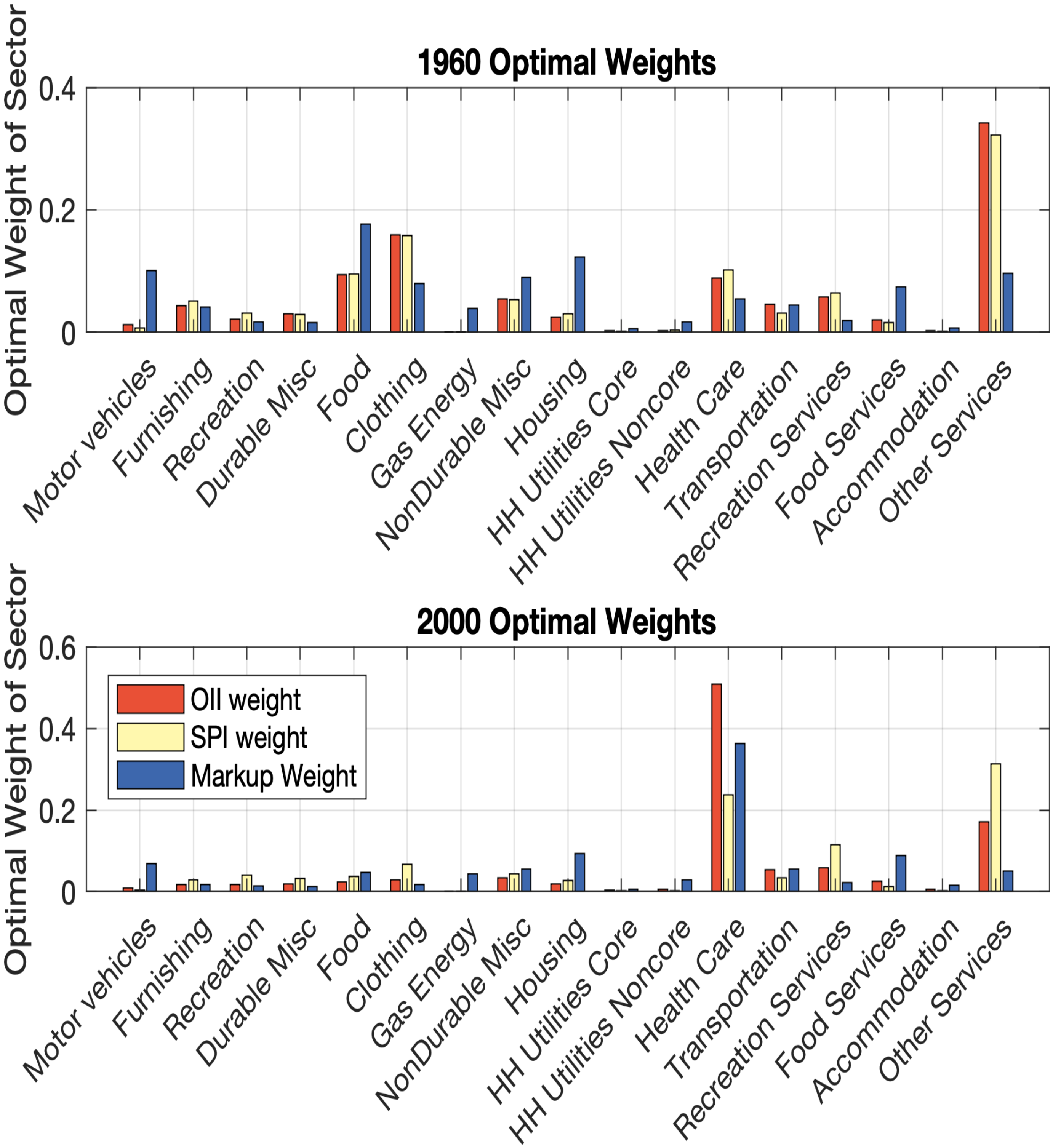


Figure 11: Optimal weights 1960's vis-a-vis 2000's.

SPI weights (yellow bar) and the markup weights (blue bar), indicating that the markup channel offsets the stickiness channel as we explained in the two-sector calibration. However, the major change between the 1960's and the 2000's is the weights on the health care sector. Our OII weight on the health care sector is of the similar magnitude as in Eusepi et al. The health care sector deserves a big weight in the 2000' because it is a sector with big size, low markup, and low frequency of price adjustment. As it is shown in Figure (2), the health care sector is an outlier in the markup and price flexibility relationship.

The Evolution of the OII We construct the time series of realized OII over time, by combining the OII weights with realized sectoral price dynamics. We build the OII as

$$\pi_t^{OII} = \sum_{k=1}^{K} \omega_k^{OII} \pi_{k,t}$$

where the weights ω_k^{OII} are the optimal weights derived in the 17-sectors calibration in the multi-sector model with heterogeneity in markups and nominal rigidity. The sectoral inflation rates, $\pi_{k,t}$, are the historical time series of realized inflation rates in the 17 PCE categories. Replacing the optimal weights by the expenditure shares of sectors, n_k, one can derive the time series of realized PCE inflation.

Figure (12) plots the OII together with the headline and the core PCE. During the Great Moderation periods, the OII was consistently higher than the two PCE measures that the Fed rely on in their policy analysis. This demonstrates that the OII stabilization cannot be achieved by monitoring a weighted average of the headline and the core PCE. During the periods following the Great Recession of the 2008, similar to other measures of inflation, the OII is blow the 2% target.

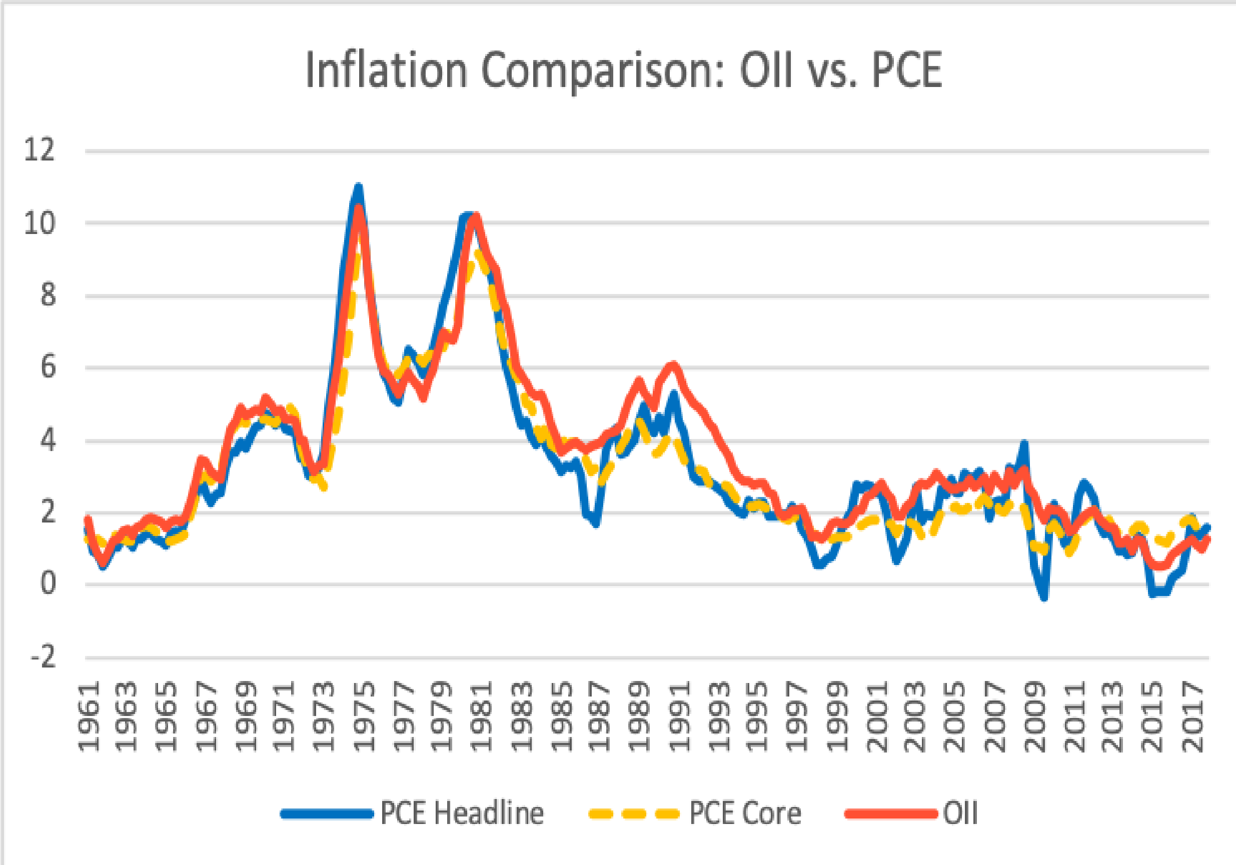


Figure 12: The Optimal Inflation Index

6 Conclusion

We have witnessed a substantial change in the average markups in the U.S. in the last decades. In this paper, we document that those changes are heterogenous across sectors. Particularly, while there is a persistent increase in the right tail of the markup distribution, firms in the left tail suffered a persistent decline in the steady-state markups. This finding has important implications for the conduct of monetary policy.

First, the degree of monetary non-neutrality is higher (lower) in a model that features heterogeneity in steady-state markups and decreasing (increasing) returns to scale as compared to a model with homogenous market powers

Second, changes in the markup distribution have minimal impact on the Phillips Multiplier in the U.S. due to the offsetting effects of the increase in the right tail and the decrease in the left tail of markup distribution.

Third, addressing markup dispersions is relevant for the optimal inflation targeting policy. Based on a two-sector model calibrated to the manufacture and service sectors in the U.S, we show that the heterogeneity in markups offsets the stickiness channel. This finding challenges the conventional wisdom in academic and policy circle that a sector's optimal weight in the OII is not necessarily proportional to its relative price rigidity, heterogeneity in market powers matter too. Crucial to this finding is the negative relationship sectoral steady-state markups and frequency of price adjustment that we document in this paper. Moreover, we show, based on the model calibrated to seventeen sectors in the U.S., the importance of the markup channel became more important in the 2000's as compared to the 1960's.

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A Figures

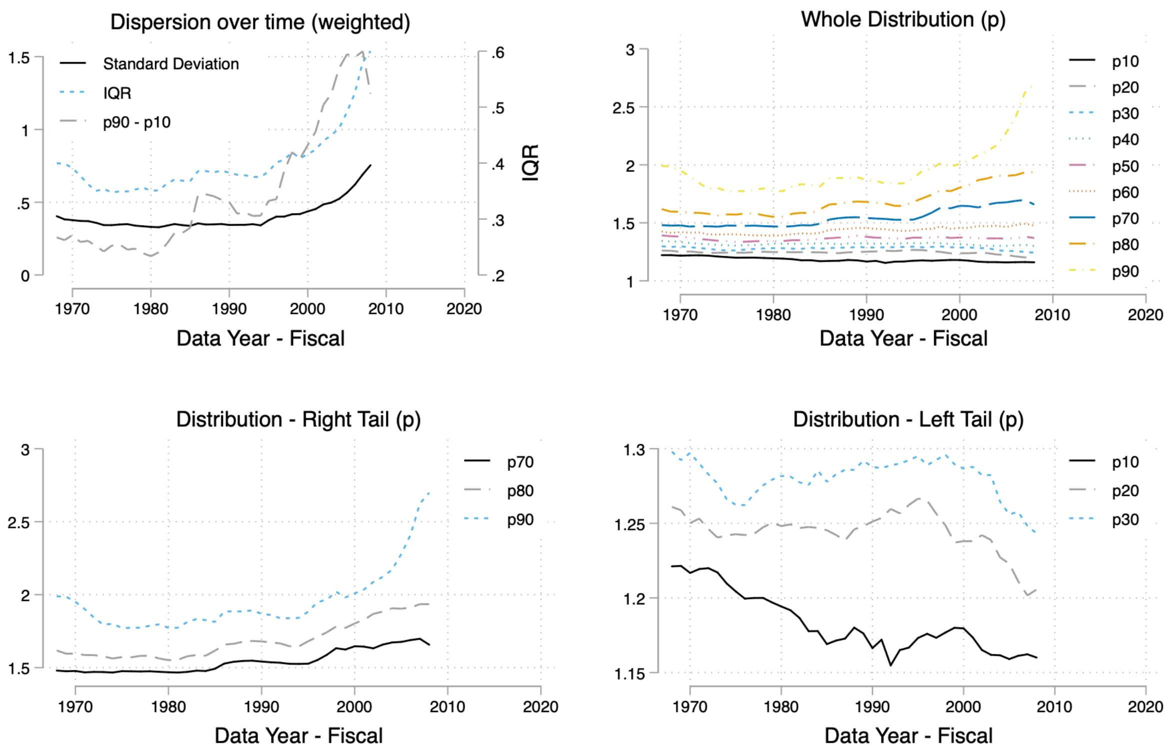

Figure 13: Robustness check μ_0: Dispersion in markups in Compustat from 1967-2017.

B Figures

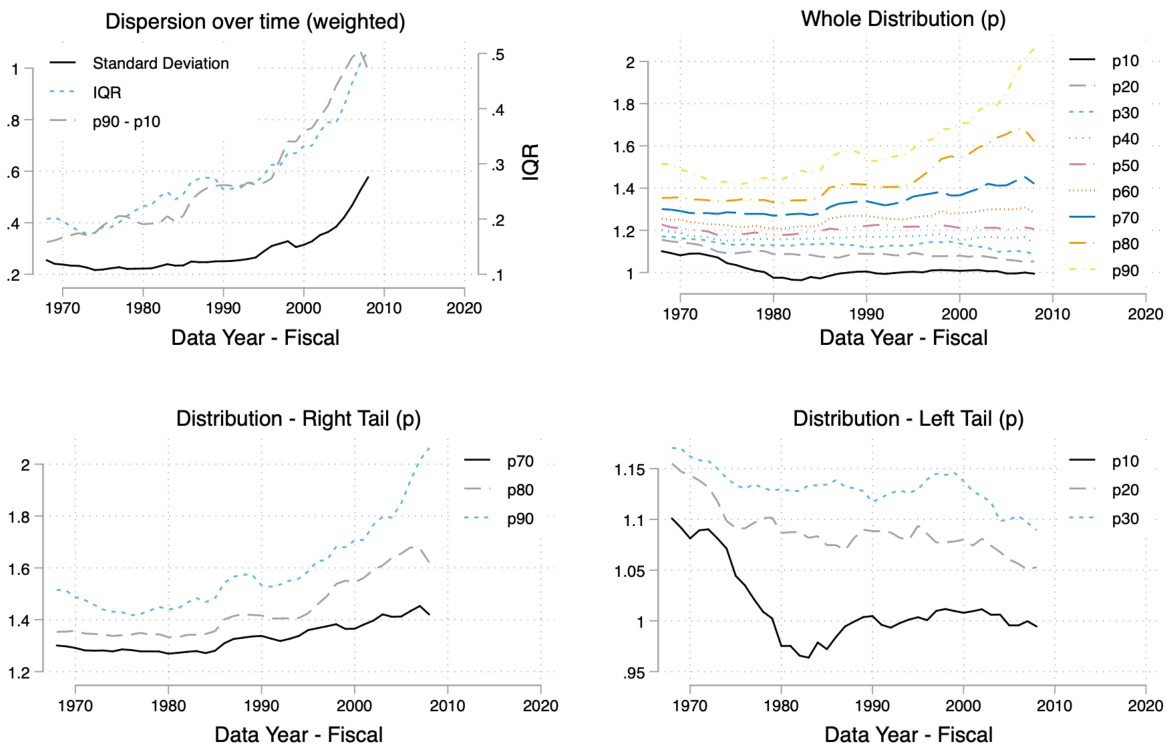

Figure 14: Robustness check μ_1: Dispersion in markups in Compustat from 1967-2017.

C Figures

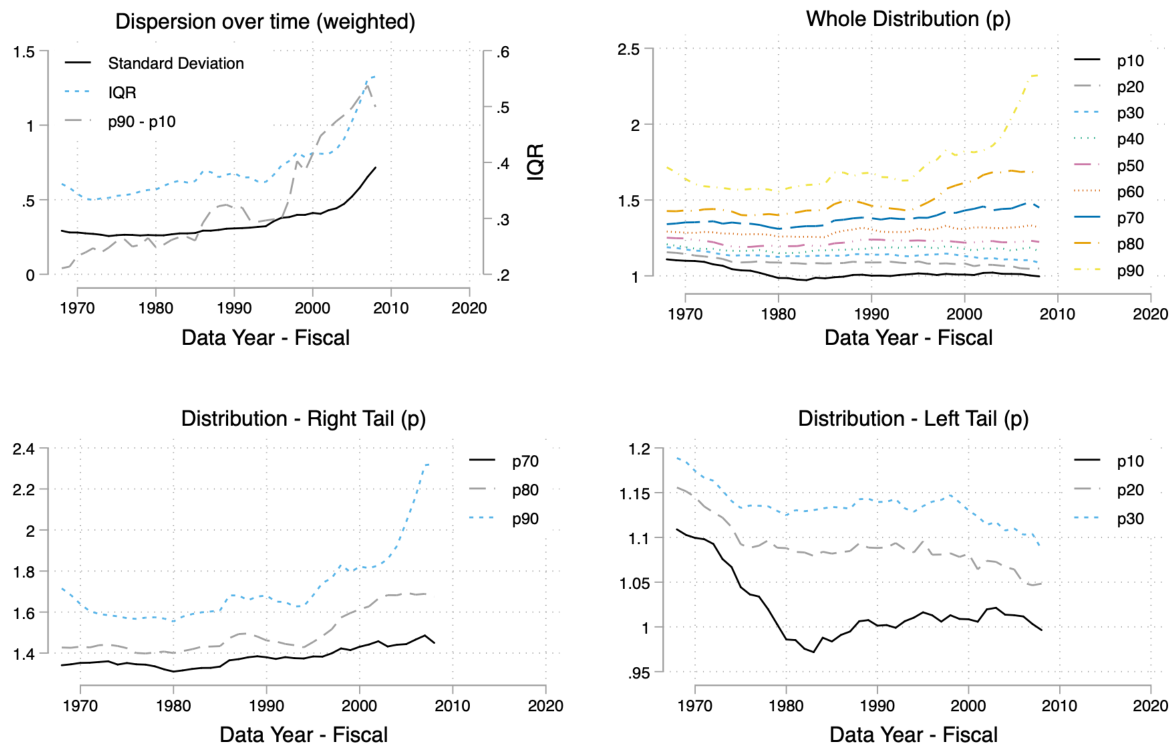

Figure 15: Robustness check μ_2: Dispersion in markups in Compustat from 1967-2017.

D Estimation of Negative Correlation between markups and frequency

aWe want to estimate the size and sign of the relationship between markups and nominal rigidity. To test the relationship, we observe matched markup-frequency pairs at the 3-digit industry level from our constructed dataset. Since frequencies have been calculated over the 2005-2011 period, we calculate sectoral markups over same timespan.

Columns (1) and (4) of Table 4 document that firms with higher markups change prices less often. In order to control for possible excluded variables / omitted variable bias, we add two sets of control variables: (i) firm characteristics such as size, output or fixed capital stock and (ii) firm- and time-specific volatilities. Together with the former fixed effects at the 2-digit industry level intend to control for other sector-specific characteristics we did not consider. Columns (2) and (5) show that the estimated relationship is of the same sign, size and significance, but the adjusted R-squared has increased. Firms could change prices more often when they are subject to more volatile shocks. In columns (3) and (6) we control for those effects via sector- and time-specific volatilities. Note that the number of observations decreases in this case because there were not sufficient observations to calculate volatilities for each sector. With these controls the estimated correlation becomes stronger and remains of the same sign.

Table 4: Regression Markups and Nominal Rigdity 2005-2011

	FPA			Duration		
	(1)	(2)	(3)	(5)	(6)	(7)
Markup	-0.057***	-0.066***	-0.079***	2.278***	2.475***	2.836***
	(0.016)	(0.016)	(0.019)	(0.54)	(0.54)	(0.56)
Sales		0.0376*	-0.0732		-0.0687	5.07***
		(0.0157)	(0.045)		(0.541)	(1.35)
Employment		-0.134	0.999		-24.3	-76.9*
		(1.01)	(1.21)		(34.9)	(36.2)
Output		-0.374	-0.144		-4.38	-12.6
		(0.252)	(0.261)		(8.69)	(78.1)
Capital		0.954**	0.679*		-6.08	3.51
		(0.333)	(0.337)		(1.15)	(1.01)
$Volatility_i$			-0.0254			0.102
			(0.102)			(3.06)
$Volatility_t$			3.59*			-16.6***
			(1.39)			(4.16)
FE	NO	YES	YES	NO	YES	YES
N	501	501	377	501	501	377
adj. R^2	0.022	0.253	0.244	0.032	0.202	0.294

Note: Table shows results of regressing FPA (duration) on markups. Controls include sales, output (in million), employment and capital (per 100.000) and volatitility of sales across time or within sector. Fixed effects are at the 2-digit sector level. Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

E Description of the model

E.1 Households

A representative household seeks to maximize the following utility function:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\sigma}}{1-\sigma}-\sum_{k=1}^{K}\frac{N_{k,t}^{1+\varphi}}{1+\varphi}\right],$$

subject to budget constraint:

$$P_tC_t + Q_tB_{t+1} \leq B_t + \sum_{k=1}^{K} W_{kt}N_{kt} + \sum_{k=1}^{K} T_{kt}$$

where P_t denotes the aggregate price defined below, Q_t denotes the price at time t of a one period bond that pays B_{t+1} at time $t+1$, W_{kt} the sectorial wage and T_{kt} the lump-sum transfer including profit from firms. There are K sectors in the economy, each of those sectors requires a sector-specific labor N_k. The aggregate consumption that enters utility function is a CES aggregate of K subindices:

$$C_t \equiv \left[\sum_{k=1}^{K} n_k^{1/\eta} C_{kt}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{1}$$

with the elasticity of substitution across sectors $\eta > 0$ and n_k denotes the size of the sector k with $\sum_{k=1}^{K} n_k = 1$. Each subindices C_{kt} is a CES aggregate of the following form:

$$C_{kt} \equiv \left[n_k^{-1/\epsilon_k}\int_0^{n_k} C_{kt}(i)^{(\epsilon_k-1)/\epsilon_k} di\right]^{\epsilon_k/(\epsilon_k-1)} \tag{2}$$

with an elasticity of substitution ϵ_k that varieties across sectors.

The implied sectorial prices index are:

$$P_{kt} \equiv \left[n_k^{-1} \int_0^{n_k} p_{kt}(i)^{1-\epsilon_k} di \right]^{1/(1-\epsilon_k)}, \tag{3}$$

The implied aggregate price index is:

$$P \equiv \left[\sum_{k=1}^{K} n_k P_{kt}^{1-\eta} \right]^{1/(1-\eta)} \tag{4}$$

Solving the consumers' problem regarding the optimal allocation of demand across varieties yields the following demand functions:

$$C_{kt}(i) = \frac{1}{n_k} C_{kt} \left(\frac{P_{kt}(i)}{P_{kt}} \right)^{-\epsilon_k}, \qquad C_{kt} = n_k C_t. \left(\frac{P_{kt}}{P_t} \right)^{-\eta} \tag{5}$$

The former is the demand function faced by an individual firm i in sector k and the one on the right is the sectorial demand faced by sector k. It is worth to emphasize that the price elasticity of demand faced by firm i in sector k is $-\epsilon_k$, the same magnitude as the elasticity of substitution with the opposite sign (downward sloping). This is intuitive: the higher is the elasticity of substitution the easier it is for a consumer to substitute goods i by another goods j in the same sector. Hence, the more elastic is the demand hence more competitive this sector is. In the limiting case of $\epsilon_k \to \infty$, the market is perfectly competitive.

E.2 Firms

There are K sectors in the economy, with a continuous of monopolistic competitive firms operate in each of those sectors. All sectors share the production function of the same functional form but are subject to different shocks:

$$Y_{kt} = e^{a_{kt}} N_{kt}^{1-\alpha}, \tag{6}$$

Firms are subject to nominal rigidity *à la* Calvo (1983): each firm may reset its price with probability $1-\theta_k$. Hence, the log level price at sector k, p_{kt}, evolves as the following:

$$p_{kt} = \theta_k p_{k,t-1} + (1-\theta_k) p_{kt}^*$$

where p_{kt}^* is the optimal price that a reoptimizing firm at sector k would set, which is the solution to the following problem:

$$\max_{P_{kt}^*} \sum_{h=0}^{\infty} \theta_k^h \mathbb{E}_t \Big\{ Q_{t,t+h} \big(P_{kt}^* Y_{k,t+h|t} - \Psi_{k,t+h}(Y_{k,t+h|t}) \big) \Big\} \tag{7}$$

subject to its demand constraints specified in (7). Where $Q_{t,t+h} \equiv \beta^k (C_{t+h}/C_t)^{-\sigma}(P_t/P_{t+h})$ denotes the stochastic discount factor, $\Psi_{k,t+h}$ denotes the cost function and $Y_{k,t+h|t}$ is the output for a firm in sector k that last reset its price in period t.

The optimality condition implied by the firm's problem is:

$$\sum_{h=0}^{\infty} \theta_k^h \mathbb{E}_t \Big\{ Q_{t,t+h} Y_{k,t+h|t} \big(P_{kt}^* - \frac{\epsilon_k}{\epsilon_k - 1} \Psi'_{k,t+h}(Y_{k,t+h|t}) \big) \Big\} = 0$$

Thus, the desired markup, defined as the markup under flexible price, is equal to $\frac{\epsilon_k}{\epsilon_k - 1}$. The frictionless markup is decreasing in ϵ_k: the monopolistic competitive firm charges a lower markup in a more competitive market.

E.3 Equilibrium

Solve the household's problem and log-linearize to obtain the dynamic IS equation:

$$\widetilde{y}_t = \mathbb{E}\widetilde{y}_{t+1} - \frac{1}{\sigma}[i_t - \mathbb{E}\pi_{t+1} - r_t^N)], \tag{8}$$

where

$$\widetilde{y}_t \equiv y_t - y_t^N, \qquad y_t^N = \psi^a \sum_{k=1}^{K} n_k a_{kt}, \qquad r_t^N \equiv \sigma\psi^a \sum_{k=1}^{K} n_k \mathbb{E}_t \triangle a_{k,t+1}$$

with $\psi^a \equiv \frac{(1+\varphi)}{\sigma(1-\alpha)+\varphi+\alpha}$. Throughout this paper, a variable with tilde denotes this variable in deviation from its natural level. And a variable with hat denotes this variable in deviation from its steady-state. Solve the firms' optimization problem and log-linearize, we obtain the New Keynesian Philipps Curve (NKPC) for each sector k:

$$\pi_{kt} = \lambda_k(\widehat{mc}_{kt} - \widehat{p}_{R,kt}) + \beta\mathbb{E}_t\pi_{k,t+1} \tag{9}$$

where $\lambda_k \equiv \frac{(1-\beta\theta_k)(1-\theta_k)}{\theta_k}\Theta_k$, $\Theta_k \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon_k}$, $p_{R,kt}$ is the sector k's relative price (relative to aggregate price), defined as $p_{kt} - p_t$. And $\widehat{mc}_{kt}$ is the real marginal cost in sector k, which is defined as:

$$\widehat{mc}_{kt} = \sigma(\widehat{y}_t - \widehat{y}_t^N) + \frac{\alpha+\varphi}{1-\alpha}(\widehat{y}_{kt} - \widehat{y}_{kt}^N) + \eta^{-1}(\widehat{y}_t^N - \widehat{y}_{kt}^N). \tag{10}$$

In the derivations of $\widehat{mc}_{kt}$, we have used household's labor supply equations and the fact that $\widehat{mc}_{kt}^N = -\eta^{-1}(\widehat{y}_{kt}^N - \widehat{y}_t^N)$ as it is implied by the sectorial demand function together with firms' frictionless desired prices. Plug (12) into (11) and replace $p_{R,kt}$ by $-\eta^{-1}\widehat{y}_{R,kt}$, where $\widehat{y}_{R,kt} \equiv \widehat{y}_{kt} - \widehat{y}_t$, we obtain the following sectorial NKPC:

$$\pi_{kt} = \kappa_k \widetilde{y}_t + \gamma_k \widetilde{y}_{R,kt} + \beta \mathbb{E}_t \pi_{k,t+1} \tag{11}$$

where $\kappa_k \equiv \lambda_k(\sigma + \frac{\varphi+\alpha}{1-\alpha})$ and $\gamma_k \equiv \lambda_k(\eta^{-1} + \frac{\varphi+\alpha}{1-\alpha})$. Alternatively, the NKPC can be rewritten as:

$$\pi_{kt} = \kappa_k \widetilde{y}_t - \eta \gamma_k \widetilde{p}_{R,kt} + \beta \mathbb{E}_t \pi_{k,t+1} \tag{12}$$

As it is the case in standard multi-sector NK models, sectorial heterogeneities give birth to relative price (or quantity) dispersion across sectors, therefore a full stabilization of both inflation and output gap is no longer feasible. Moreover, while a positive aggregate output gap arises inflation in all sectors, an increase in relative price (or quantity) in one sector has a disinflationary impact in that sector and increases inflation pressure in other sectors.

F Derivation of the Welfare Loss Function

The second order Taylor expansion of the representative household's utility U_t around a steady-state (C, L) in terms of log deviations can be written as:

$$U_t - U \approx U_c C\Big(\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2\Big) + \sum_{k=1}^{K} U_{L_k} L_k \Big(\widehat{l}_{kt} + \frac{1+\varphi}{2}\widehat{l}_{kt}^2\Big) di.$$

Note that

$$(1-\alpha)\widehat{l}_{kt} = \widehat{y}_{kt} - a_{kt} + d_{kt}$$

where $d_{kt} \equiv (1-\alpha)log(\frac{P_{kt}(i)}{P_{kt}})^{-\frac{\epsilon_k}{1-\alpha}}$.

Lemma F.1. $d_{kt} = \frac{\epsilon_k}{2\Theta} var_i\{p_{kt}(i)\}$, *with* $\Theta_k \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon_k}$

Proof: Gali (2008, chapter 4)

Therefore,

$$U_t - U \approx U_c C\Big(\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2\Big) + \sum_{k=1}^{K} \frac{U_{L_k} L_k}{1-\alpha}\Big(\widehat{y}_{kt} + \frac{\epsilon_k}{2\Theta_k} var_i\{p_{kt}(i)\} + \frac{1+\varphi}{2(1-\alpha)}(\widehat{y}_{kt} - a_{kt})^2\Big) + t.i.p.$$

where $t.i.p$ denotes the terms independent of policy. Under the assumption that cost of employment is subsidized optimally at sectorial level to eliminate distortions originate from monopolistic competition, the steady-state is efficient and $-\frac{U_{L_k}}{U_c} = MPN$.

Approximate the CES aggregate C_t defined in (3) around $c_k = c + log(n_k)$:

$$\sum_{k=1}^{K} n_k \widehat{y}_{kt} \approx \widehat{y}_t - \frac{1-\eta^{-1}}{2} \sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2$$

with $\sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 \equiv \sum_{k=1}^{K} n_k (\widehat{y}_{kt} - \widehat{y}_t)^2$. Using the fact that $MPN = (1-\alpha)(Y_k/L_k), Y =$

C, it follows that:

$$\begin{aligned}
\frac{U_t - U}{U_c C} &\approx -\frac{1}{2}\Big[\sum_{k=1}^{K}\Big(\frac{\epsilon_k n_k}{\Theta_k} var_i\{p_{kt}(i)\}\Big) - (1-\sigma)\widehat{y}_t^2 - (1-\eta^{-1})\sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 \\
&+ \frac{1+\varphi}{1-\alpha}\sum_{k=1}^{K} n_k(\widehat{y}_{kt} - a_{kt})^2\Big] + t.i.p \\
&= -\frac{1}{2}\Big[\sum_{k=1}^{K}\Big(\frac{\epsilon_k n_k}{\Theta_k} var_i\{p_{kt}(i)\}\Big) - (1-\sigma)\widehat{y}_t^2 - (1-\eta^{-1})\sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 \\
&+ \frac{1+\varphi}{1-\alpha}\sum_{k=1}^{K} n_k(\widehat{y}_{kt}^2 - 2\widehat{y}_{kt} a_{kt})\Big] + t.i.p \\
&= -\frac{1}{2}\Big[\sum_{k=1}^{K}\Big(\frac{\epsilon_k n_k}{\Theta_k} var_i\{p_{kt}(i)\}\Big) + (\sigma + \frac{\varphi+\alpha}{1-\alpha})\widehat{y}_t^2 + (\eta^{-1} + \frac{\varphi+\alpha}{1-\alpha})\sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 \\
&- 2\frac{1+\varphi}{1-\alpha}\sum_{k=1}^{K} n_k \widehat{y}_{kt} a_{kt}\Big] + t.i.p \\
&= -\frac{1}{2}\Big[\sum_{k=1}^{K}\Big(\frac{\epsilon_k n_k}{\Theta_k} var_i\{p_{kt}(i)\}\Big) + (\sigma + \frac{\varphi+\alpha}{1-\alpha})\widehat{y}_t^2 + (\eta^{-1} + \frac{\varphi+\alpha}{1-\alpha})\sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 \\
&- 2\frac{1+\varphi}{1-\alpha}\sum_{k=1}^{K} n_k(\widehat{y}_t + \widehat{y}_{kt} - \widehat{y}_t) a_{kt})\Big] \\
&= -\frac{1}{2}\Big[\sum_{k=1}^{K}\Big(\frac{\epsilon_k n_k}{\Theta_k} var_i\{p_{kt}(i)\}\Big) + (\sigma + \frac{\varphi+\alpha}{1-\alpha})\widehat{y}_t^2 + (\eta^{-1} + \frac{\varphi+\alpha}{1-\alpha})\sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 \\
&- 2(\sigma + \frac{\varphi+\alpha}{1-\alpha})\widehat{y}_t y^N - 2(\eta^{-1} + \frac{\varphi+\alpha}{1-\alpha})\sum_{k=1}^{K} n_k(\widehat{y}_{kt} - y_t)(\widehat{y}_{kt}^N - y_t^N)\Big] + t.i.p \\
&= -\frac{1}{2}\Big[\sum_{k=1}^{K}\Big(\frac{\epsilon_k n_k}{\Theta_k} var_i\{p_{kt}(i)\}\Big) + (\sigma + \frac{\varphi+\alpha}{1-\alpha})\widetilde{y}_t^2 + (\eta^{-1} + \frac{\varphi+\alpha}{1-\alpha})\sum_{k=1}^{K} n_k \widetilde{y}_{R,kt}^2\Big] + t.i.p
\end{aligned}$$

where $\widetilde{y}_t \equiv y_t - y_t^N$. From line 2 to line 3, we have used the fact that$\sum_{k=1}^{K} n_k \widehat{y}_{kt}^2 = \sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 + (\sum_{k=1}^{K} n_k \widehat{y}_{kt})^2 \approx \sum_{k=1}^{K} n_k \widehat{y}_{R,kt}^2 + \widehat{y}_t^2$. From line 4 to line 5, where the fact was used that $a_{kt} = \frac{\sigma(1-\alpha)+\alpha+\varphi}{1+\varphi} y_t^N$ and $a_{kt} - \sum_{k=1}^{K} a_{kt} = \frac{-\eta^{-1}(1-\alpha)+\alpha+\varphi}{1+\varphi}\widehat{(}y_{kt}^N - y_t^N)$.

To summarize, the second order approximation of the representative consumer's welfare loss as a fraction of steady-state consumption is:

$$
\begin{aligned}
W &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{U_t - U}{U_c C} \Big) \\
&= -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\sum_{k=1}^{K} \Big(\frac{\epsilon_k n_k}{\Theta_k} var_i\{p_{kt}(i)\} \Big) + (\sigma + \frac{\varphi + \alpha}{1-\alpha}) \widetilde{y}_t^2 + (\eta^{-1} + \frac{\varphi + \alpha}{1-\alpha}) \sum_{k=1}^{K} n_k \widetilde{y}_{R,kt}^2 \Big] + t.i.p.
\end{aligned}
$$

Lemma F.2. $\sum_{t=0}^{\infty} \beta^t var_i\{p_{kt}(i)\} = \frac{\theta_k}{(1-\beta\theta_k)(1-\theta_k)} \sum_{t=0}^{\infty} \beta^t \pi_{kt}^2$

Proof: Woodford (2003, chapter 6)

Thus we obtain the following welfare loss function:

$$
W = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\sum_{k=1}^{K} \frac{\epsilon_k}{\lambda_k} n_k \pi_{kt}^2 + (\sigma + \frac{\varphi + \alpha}{1-\alpha}) \widetilde{y}_t^2 + (\eta^{-1} + \frac{\varphi + \alpha}{1-\alpha}) \sum_{k=1}^{K} n_k \widetilde{y}_{R,kt}^2 \Big] + t.i.p.
$$

where $\lambda_k \equiv \frac{(1-\beta\theta_k)(1-\theta_k)}{\theta_k} \Theta_k$ defined as above. Normalize the weights on π_{kt} such that $\sum \omega_k = 1$:

$$
W = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\sum_{k=1}^{K} \phi_k \pi_{kt}^2 + \lambda_y \widetilde{y}_t^2 + \lambda_{R_y} \sum_{k=1}^{K} n_k \widetilde{y}_{R,kt}^2 \Big] + t.i.p. \tag{13}
$$

where

$$
\phi_k = \frac{n_k \epsilon_k \lambda}{\lambda_k}, \qquad \lambda_y = (\sigma + \frac{\varphi + \alpha}{1-\alpha})\lambda, \qquad \lambda_{Ry} = (\eta^{-1} + \frac{\varphi + \alpha}{1-\alpha})\lambda
$$

and λ is defined as:

$$
\lambda \equiv (\sum_{0}^{K} n_k \epsilon_k \lambda_k^{-1})^{-1}
$$

From the sectorial demand equation, one can rewrite sectorial output dispersion as a function of sectorial price dispersion:

$$W = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\sum_{k=1}^{K} \frac{\epsilon_k}{\lambda_k} n_k \pi_{kt}^2 + (\sigma + \frac{\varphi + \alpha}{1-\alpha}) \widetilde{y}_t^2 + \eta (1 + \frac{\varphi + \alpha}{1-\alpha} \eta) var_k(\widetilde{p}_{kt}) \Big] + t.i.p.$$

Normalize the weights on π_{kt}:

$$W = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\sum_{k=1}^{K} \phi_k \pi_{kt}^2 + \lambda_y \widetilde{y}_t^2 + \lambda_{R_p} var_k(\widetilde{p}_{kt}) \Big] + t.i.p. \tag{14}$$

where

$$\lambda_{Rp} = \eta (1 + \frac{\varphi + \alpha}{1-\alpha} \eta) \lambda$$