Pairs Trading

A Statistical Arbitrage Strategy

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Background

- Developed in the 1980's by a group of Quants at Morgan Stanley, who reportedly made over $50 million profit for the firm in 1987
- A contrarian strategy that tries to profit from the principles of mean-reversion processes
- In theory, one could expand the strategy to include a basket of more than a pair of related stocks

Main Idea

- Choose a pair of stocks that move together very closely, based on a certain criteria (i.e. Coke & Pepsi)
- Wait until the prices diverge beyond a certain threshold, then short the “winner” and buy the “loser”
- Reverse your positions when the two prices converge --> Profit from the reversal in trend

Example of a Pairs Trade

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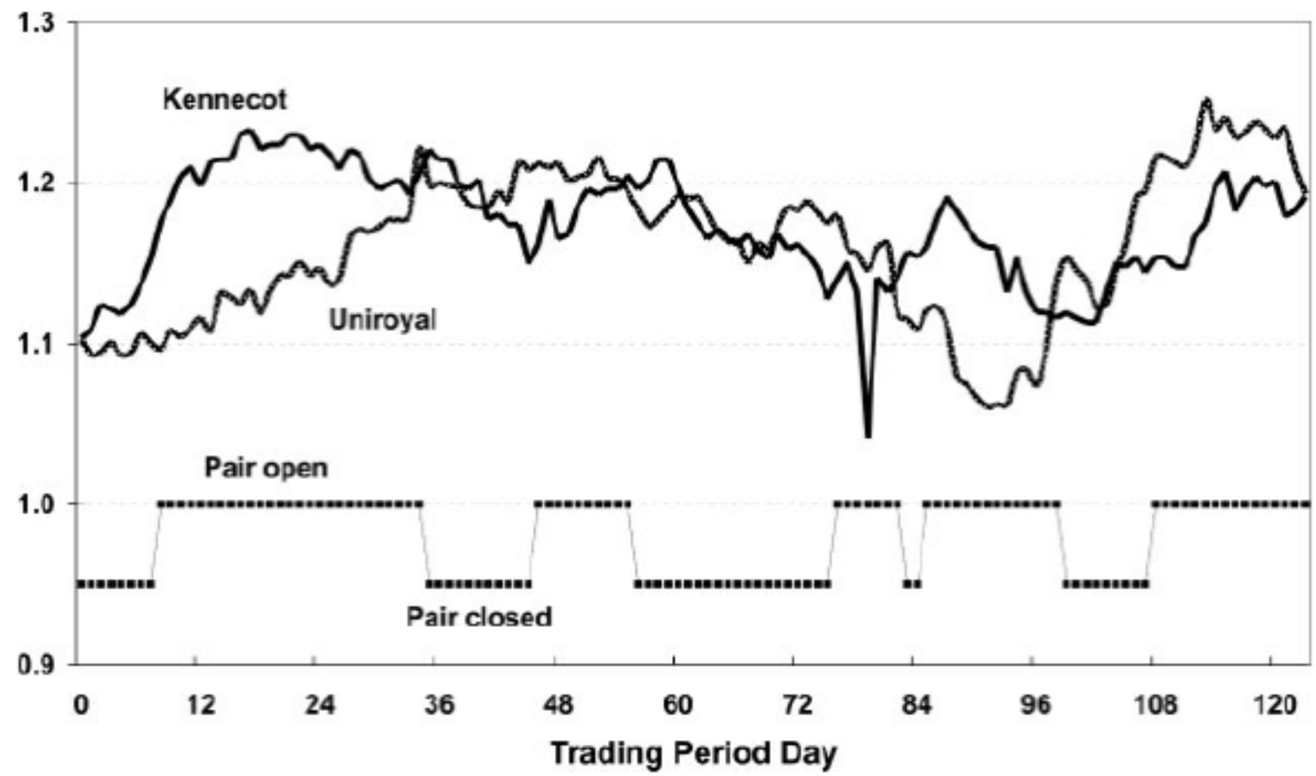


Figure 1
Daily normalized prices: Kennecott and Uniroyal (pair 5)
Trading period August 1963–January 1964.

Investor Decisions

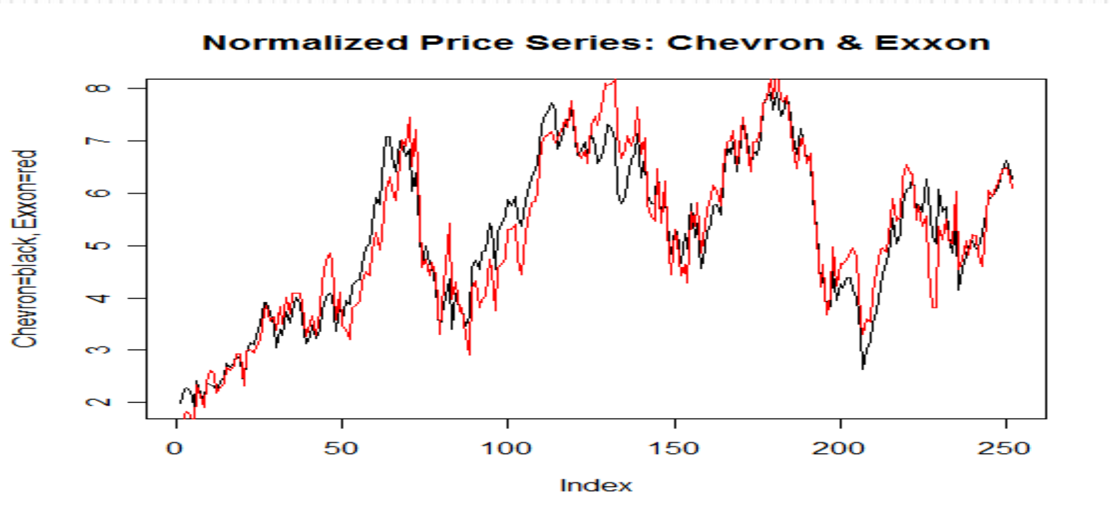
- Pair Selection Criteria
  - Correlation (Parametric & Non-Parametric Spearman's Rho)
  - Dickey-Fuller Test Statistic (Cointegration)

- Trading Threshold (areas of consideration)
  - Volatility of the Market
  - Historical returns
  - Cost of each transaction

Normalization of Stock Data

- METHOD:
  - Find pair that has maximal correlation
  - Normalize price series, plot spread over 1 year "formation period"
  - Generate optimal threshold non-parametrically: choose a threshold T_i=c*sd(spread), calculate profit for each T_i, choose T_i generating max profit
    - Calculate profit by going $1 short on winner, $1 long on loser; close position when prices converge, i.e. spread=0
  - Normalize price series in 6 month "trading period" using mean and sd from formation period
  - Plot spread using optimal threshold found from formation period, calculate profit
  - Lower thresholds→ More transactions→ Higher transaction costs → Lower Returns
  - Higher transaction costs→ Smaller Returns

Chevron & Exxon



Formation Period Corr=0.93

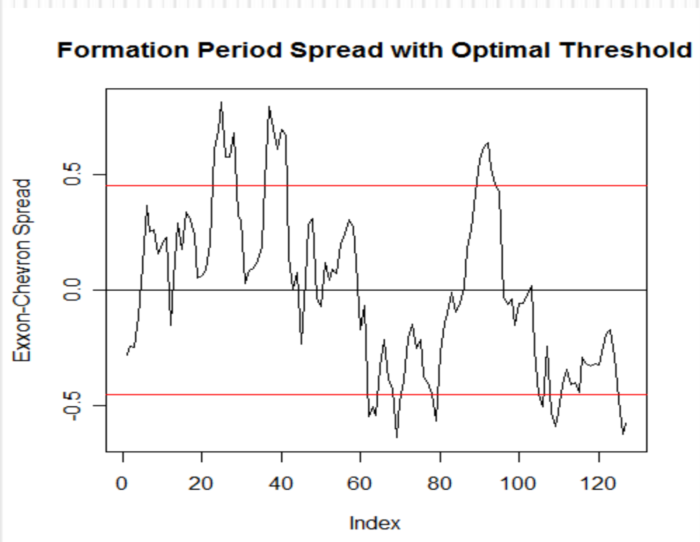
Trading Period Corr=0.96

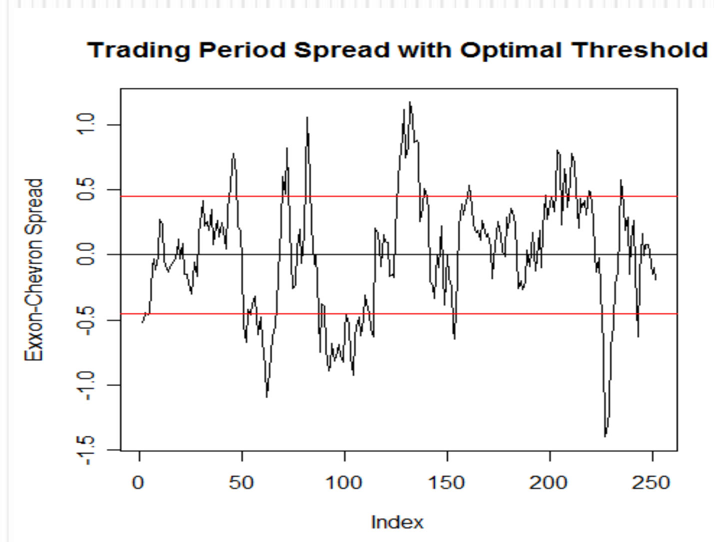
Optimal Threshold=1.25*sd's

Transactions=10

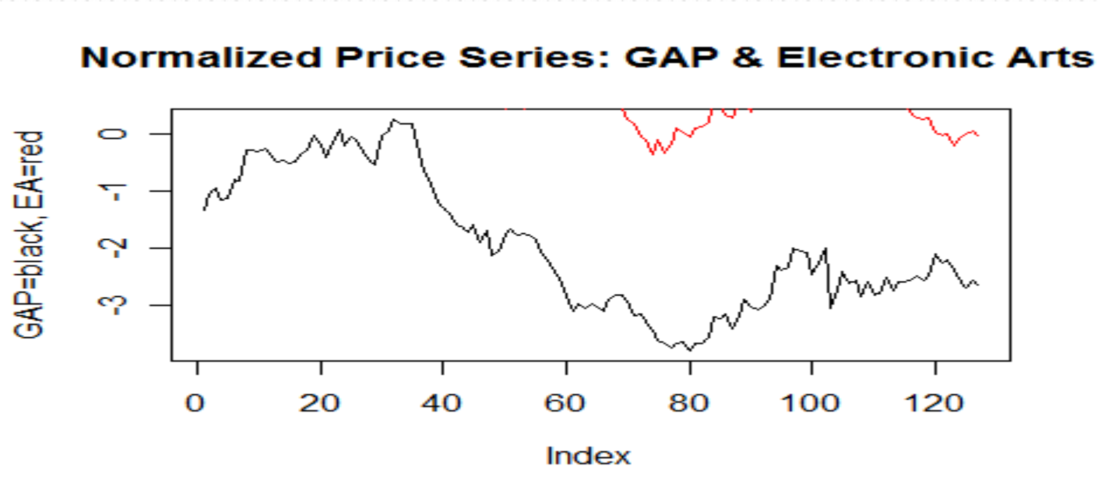
Returns=15%

Win.

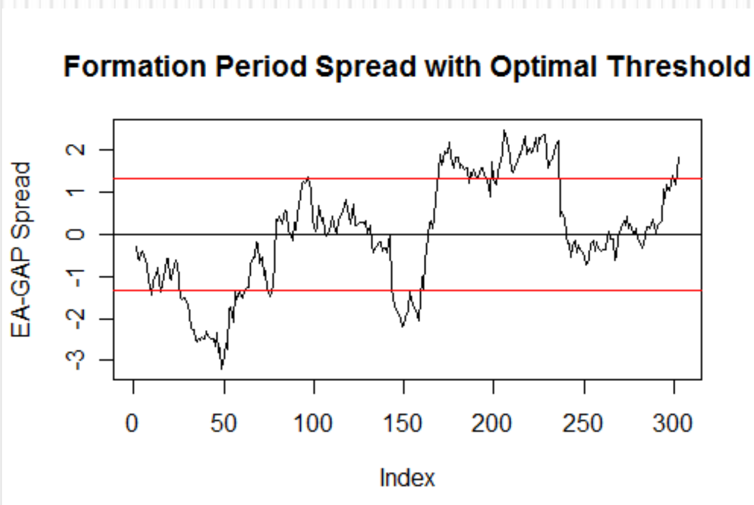


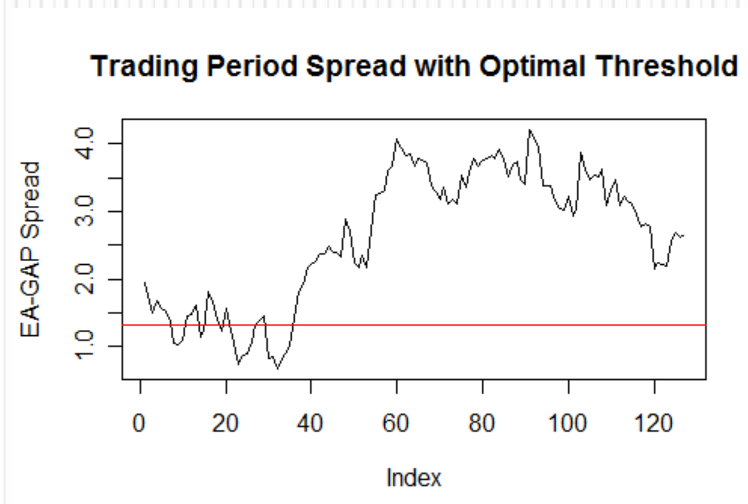


Electronic Arts & GAP

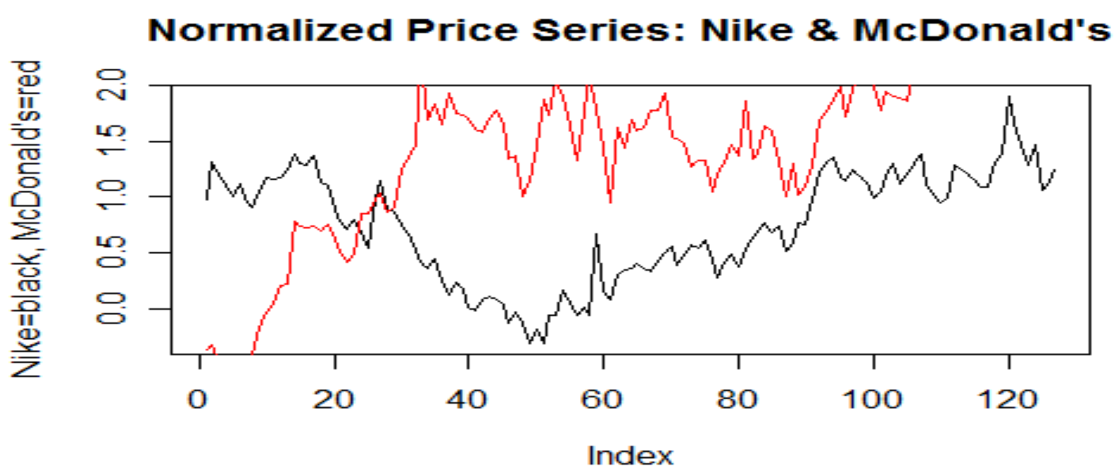


- Formation Corr=0.12
- Trading Corr=0.56
- Optimal Threshold=1 sd
- #Transactions=0 (Open a position, but spread never returns to 0)
- Return= -0.04
- **Lose.**

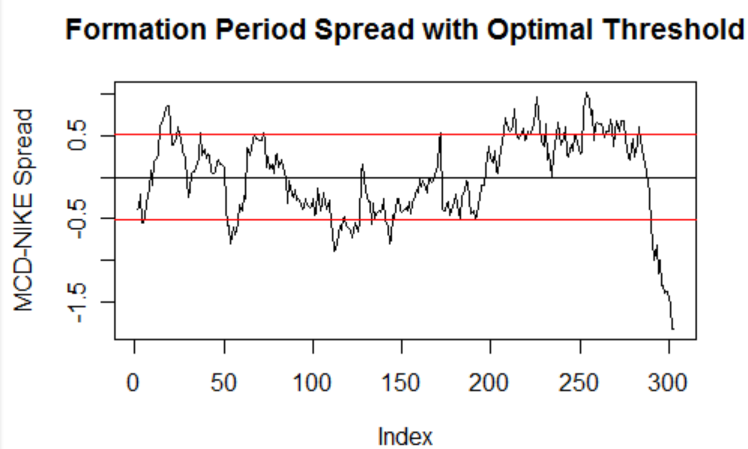


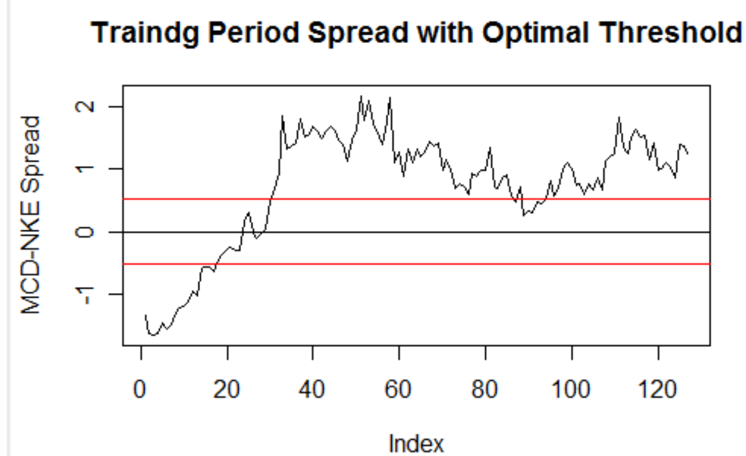


Nike & McDonald's



- Formation Corr=0.87
- Trading Corr=0.02
- #Transactions=1
- Return= -0.05
- Lose.
- Correlation is imperfect criteria for selecting pairs.





Interesting result involving market volatility

		Formation Period		Trading Period		
Pair	Dates	Corr.	Optimal Threshold*	#Trans	Returns	Corr.
Exxon, Chevron	Period 1	0.93	1	6	0.11	0.85
	Period 2	0.85	1.75	6	0.05	0.69
	Period 3	**0.93**	**1.25**	**10**	**0.15**	**0.96**
Nike, McDonald's	Period 1	0.87	1.5	2	-0.05	0.02
	Period 2	0.10	1	6	-0.02	0.29
	Period 3	**0.87**	**2**	**4**	**0.04**	**0.87**
Electronic Arts, GAP	Period 1	0.12	1	0	-0.04	0.56
	Period 2	0.19	2	4	-0.03	-0.09
	Period 3	**0.31**	**1.75**	**4**	**0.06**	**0.10**

Positive profits for all measured pairs in period 3.

Period 3 includes January 2008, a very volatile month for the stock market.

It seems that high market volatility allows the possibility for positive profits for uncorrelated pairs which would not generate such profits in low volatility periods, although this can surely work either way.

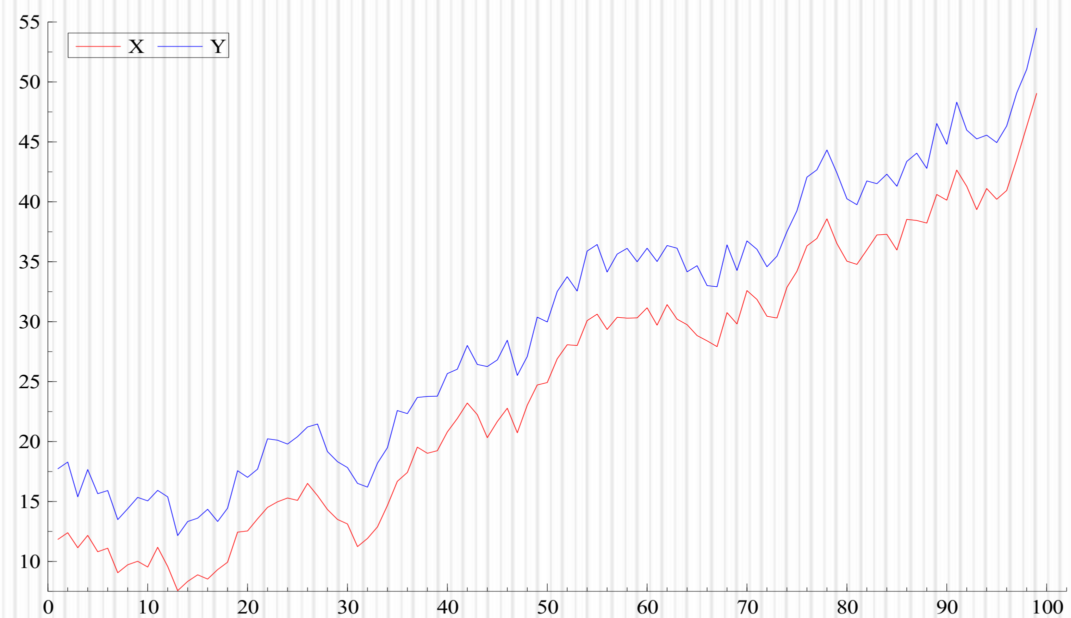
Cointegration

- If there exists a relationship between two non-stationary I(1) series, Y and X , such that the residuals of the regression

$$Y_t = \beta_0 + \beta_1 X_t + u_t$$

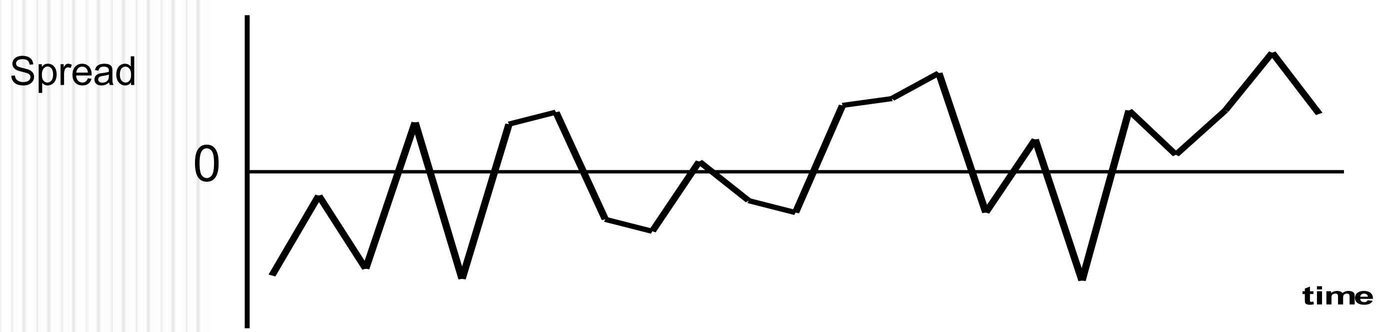
are stationary, then the variables in question are said to be cointegrated

Note: X and Y here are clearly not stationary, but they seem to move together. In fact, they are cointegrated --> (Y- β_1X-β_0)should be stationary



Application to Pairs Trading

- If we have two stocks, X & Y, that are cointegrated in their price movements, then any divergence in the spread from 0 should be temporary and mean-reverting.



- The important issues here are: 1) how to test for cointegration between prices and 2) estimating the constant

Testing For Cointegration

- Many Methods – most of them focus on testing whether the residuals of $Y_t = \beta_0 + \beta_1 X_t + u_t$ are stationary processes
- We use the Cointegrating Regression Dickey-Fuller Test, which essentially operates the following regression:

$$\boldsymbol{\Delta u_t = \varphi\, u_{t-1} + e_t}$$

- H_0: $\boldsymbol{\varphi} = 0$ => no cointegration*
- H_a: $\boldsymbol{\varphi} < 0$ => cointegration*
- To obtain the cointegration factor estimates, we must regress the de-trended Y_t on the de-trended X_t

* We must use critical values different from Gaussian ones due to non-symmetric properties of the Dickey-Fuller distribution

Results of Test

- NO PAIR OF PRICES ARE COINTEGRATED!
- No surprise there
- Alternative: take the "most cointegrated" pair & optimize thresholds as we did with normalized data
- Compare the results against normalized thresholds in the same time period

Normalization Vs Cointegration

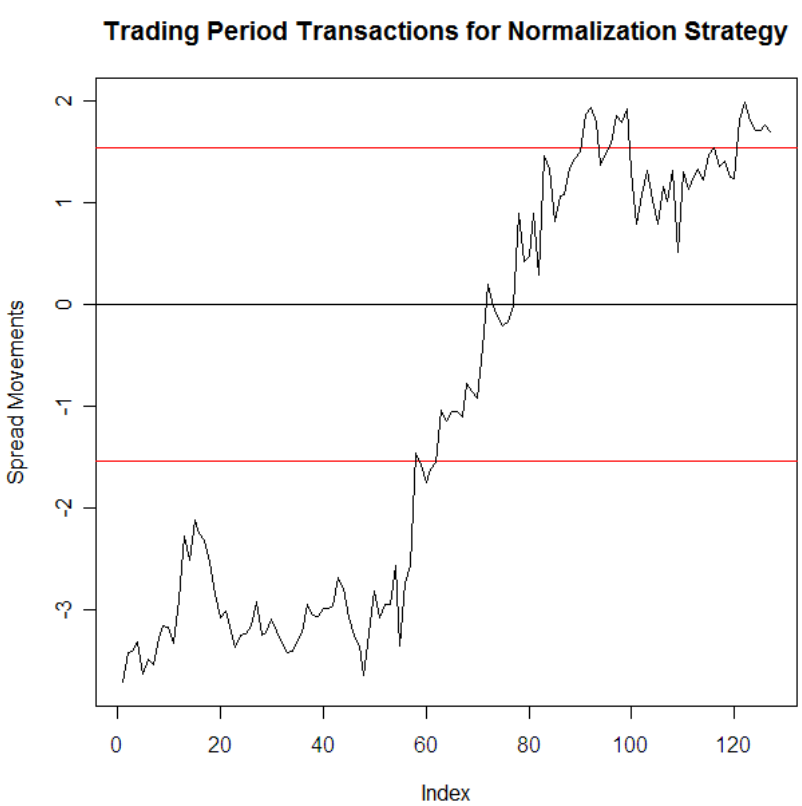
Figure 5: Normalized strategy VS Cointegrated strategy

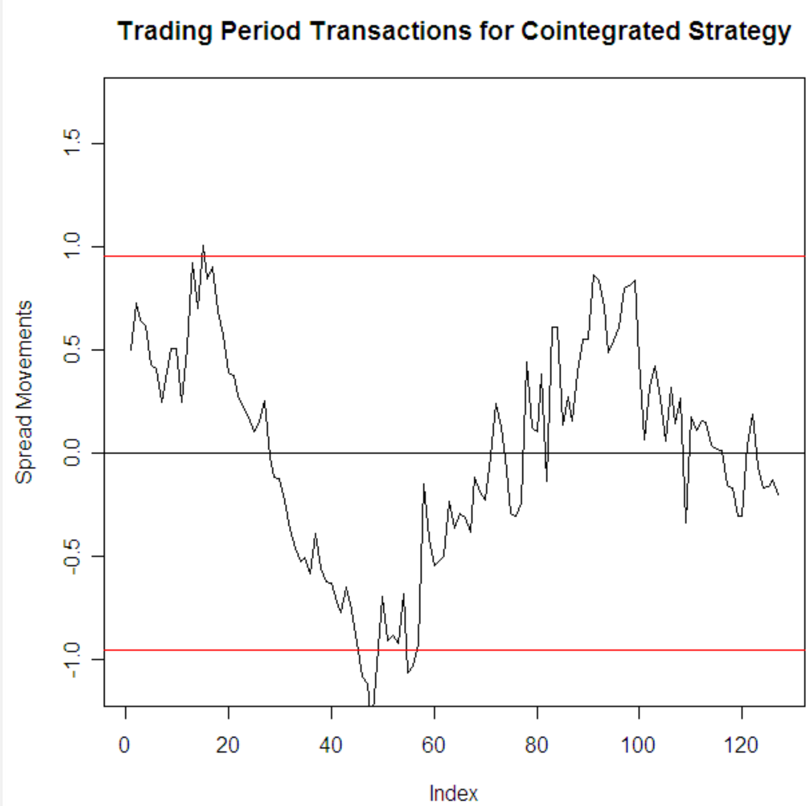
LUV(Southwest Airlines) & PLL (Pall Corporation)	Normalization	Cointegration
Correlation coef. /CRDF stat. over Formation Period	0.24	-0.52*
Cointegration Factor	N/A	0.43
Optimal SD Threshold over Formation Period	1.25 SDs	1.75 SDs
Optimal Returns over Formation Period	~0%	~2%
Number of Transactions over Trading Period	4	4
Returns over Trading Period	~5%	~13%

*CRDF statistic insignificant against the H_0: The Time Series is not cointegrated (5% critical value = -3.39)
**Fixed transaction costs implicit in both models

Trading Period Comparison

Figure 6: Normalized LUV & PLL spread VS Cointegrated LUV & PLL spread





Auto-Regressive Time Series

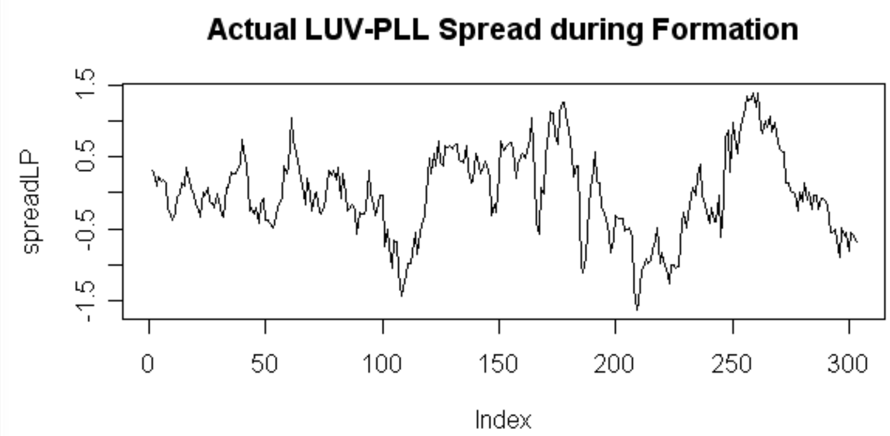
- Cointegration is an ideal construct for pairs trading
- But Dickey-Fuller Hypothesis Test is inconclusive
- Instead we can fit a time series to the spread data
  - AR(1): $Y_t = \beta Y_{t-1} + \varepsilon_t$
- Looking for a spread that produces an AR(1) with $|\beta| < 1$, so that will be stationary.

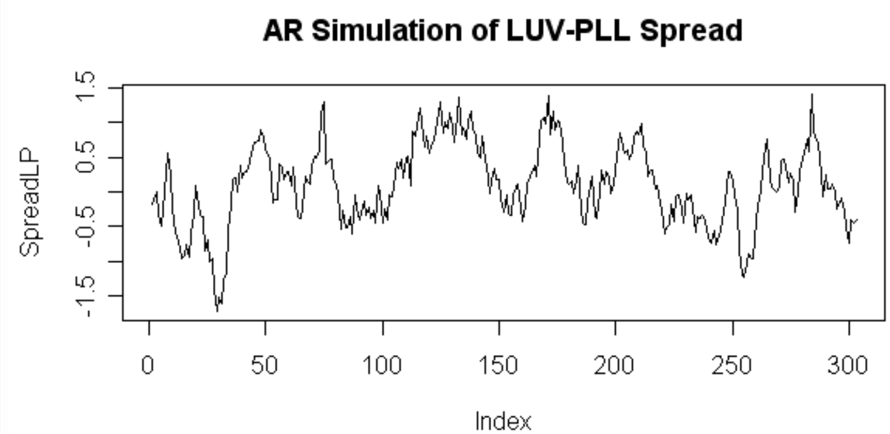
Choosing thresholds with AR(1)

- For the interest of time, we are only going to focus our most cointegrated pair: LUV and PLL.
- We will fit an AR(1) to the data by estimating β and the standard deviation of each iid white noise $\varepsilon_{t.}$
- Then we will run one thousand simulations of this AR(1) model and estimate each of their optimal benchmarks
- The average of the optimal benchmarks from each simulation will serve as our estimate for the optimal benchmark in the formation period.

Results of AR(1) Thresholds

AR(1) Coefficient estimate (μ-hat)	0.8605
Optimal Threshold estimate	1.046
SD of Optimal Threshold	0.2597
Number of Transactions	12
Returns over Trading Period	17.7%





Alternative Strategies

- Conditional correlation or some other measure of "relatedness", such as Copulas

- Modeling the spread as GARCH processes

- Optimize profits w.r.t. certain global indicators (i.e. market volatility, industry growth, etc.)

- Factor Analysis on the spread

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