(Thanks to Peter Berman.) The explanation of the identification of derivatives from Euler's homogeneity theorem is a little brief. After all, writing a homogeneous function as $f(x,y) = xg(x,y) + yh(x,y)$ does not identify g and h as the partial derivatives. (Just take $f(x,y) = 0$ and write $f(x,y) = x \times (-y/x) + y \times 1 = x \times (-y^2/x) + y \times y$.) However, it's easy to identify "the term that multiplies K" in the Black-Scholes formula separately as the K-derivative: With ξ denoting the (lognormal) density of $S(T)$ we get

$$\begin{aligned}
\frac{\partial}{\partial K} e^{-r\tau} \mathbf{E}^Q((S(T) - K)^+) &= e^{-r\tau} \frac{\partial}{\partial K} \int_K^\infty (s - K)\xi(s) ds \\
&= -e^{-r\tau} \int_K^\infty \xi(s) ds \\
&= -e^{-r\tau} N(d_2),
\end{aligned}$$

where the second equality follows by using the fundamental theorem of calculus (twice - or Leibniz' rule once), and the third from the standard calculation.