



Atividade de Fixacao 2 - Algoritmos Numericos II
Resolucao de EDO's pelos Metodos de Diferencas Finitas e Runge Kutta
Aluno: Christian Jonas Oliveira

```
clear
clc
format(17);
//          OBSERVACAO: DEVE-SE ALTERAR NO CODIGO O BLOCO          //
//          REFERENTE AO CALCULO PELO METODO DAS DIFERENCAS FINITAS          //

// Bloco de alteracao pelo Programador/Usuario ///////////////////
x0 = 0;
xn = 3;
y0 = 2;
xk = x0;
yk = y0;

h(1) = 0.5;
h(2) = 0.2;
h(3) = 0.1;
h(4) = 0.05;

function output = funcao(x, y)
    output = x - y
endfunction

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function output = RK(h)
    c = 2;
    count = round(xn/h)+2;
    A(1) = y0;
    while(xk < xn)
        K1 = h * funcao(xk, yk); //A(c)(1) = K1;
        K2 = h * funcao(xk + (h/2), yk + (K1/2)); //A(c)(2) = K2;
        K3 = h * funcao(xk + (h/2), yk + (K2/2)); //A(c)(3) = K3;
        K4 = h * funcao(xk + h, yk + K3); //A(c)(4) = K4;
        yk = yk + (1/6) * (K1 + 2*K2 + 2*K3 + K4);

        A(c) = yk;

        xk = xk+h;
        c = c+1;

        if(c==count) c = 1; end
    end
end
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    xk = x0;
    yk = y0;
    output = A;
endfunction

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function imprimeMatriz(matriz, h);
    ordem = round(xn/h)+1;
    for i=1:ordem
        printf("y%d = %.5f\n", i-1, matriz(i));
    end
endfunction

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function output = montaX(h)
    X(1)=x0;
    count = x0+h;
    i = 2;
    for count=count:h:xn;
        X(i) = count;
        count = count + h;
        i = i+1;
    end

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    output = X;
endfunction

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function output = solExata(X, ordem)
    A(1) = y0;
    i = 2;
    while i <= ordem+1;
        A(i) = (X(i)-1) + 3 * %e^(-X(i));
        i = i+1;
    end
    output = A;
endfunction

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function output = funcaoA(h, lim, o)
    for i=1:o
        x(i)=(h*i)+lim);

        // Digite aqui o primeiro termo (que acompanha yi-1)
        primeiraParcela = -1;

        // Digite aqui o segundo termo (que acompanha yi)

        parcelaDoMeio = h+1;
    end
endfunction

```

```

// Digite aqui o terceiro termo (que acompanha yi+1)

ultimaParcela = 0;
for j=1:o
    if (i-1>0)
        A(i)(i-1)= primeiraParcela;
    end

    A(i)(i)= parcelaDoMeio;

    if (i+1<=o)
        A(i)(i+1)= ultimaParcela;
    end
end
end
output = A;
endfunction

```

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function output = funcaoB(h, vi, o)
    for i=1:o
        x(i) = (h*i) + x0;
        // Digite aqui o primeiro termo (que acompanha yi-1)
        primeiraParcela = -1;
        // Digite aqui a funcao que vem apos a igualdade
        funcB = h*x(i);
        if(i==1) then
            B(i)(1)= ((-1)* vi * primeiraParcela + funcB);
        else
            B(i)(1) = funcB;
        end
    end
    output = B;
endfunction

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function output = reorganiza(A)
    i = 1;
    B(1) = y0;
    for i=1:length(A)
        B(i+1) = A(i);
    end
    output = B;
endfunction

```

```

function output = Erro (sExata, difFinitas)
    tam = length(sExata);
    for i=1:tam
        e(i) = abs(difFinitas(i) - sExata(i));
    end
endfunction

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```

end
output = e;
endfunction

function main()

    for i=1:4
        printf("\n\n \t SOLUCAO \n")
        ordem = round((xn-x0)/h(i));
        x = montaX(h(i));
        tit = "\n\n\nPara H = " + string(h(i));
        printf(tit);

        // Runge Kutta
        ya = RK(h(i));
        printf("\n Solucao Por Runge Kutta\n");
        imprimeMatriz(ya, h(i));
        subplot(2, 2, i);
        plot(x, ya, 'r');

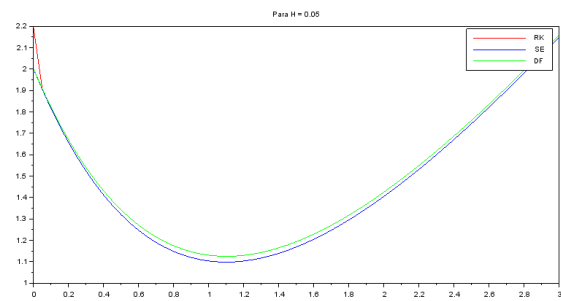
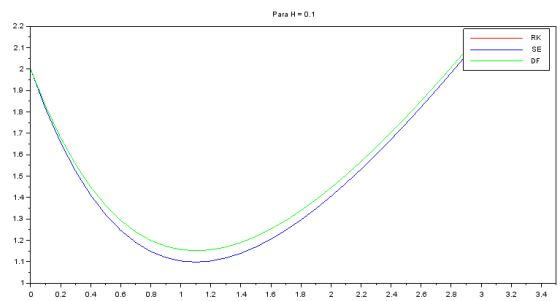
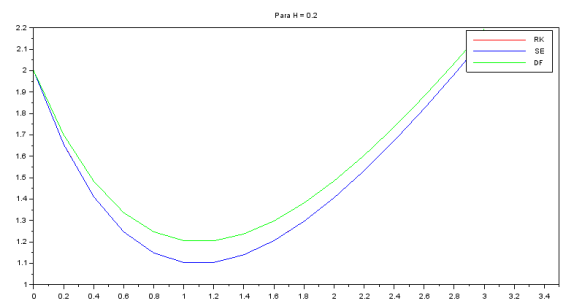
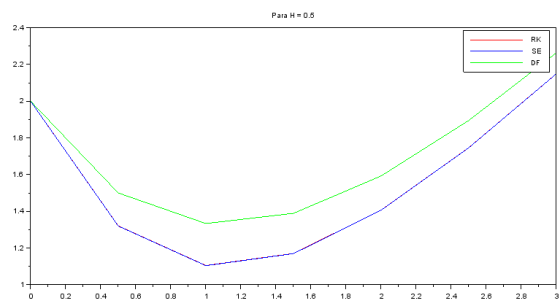
        // Solucao exata
        yb = solExata(x, ordem);
        printf("\n Solucao Exata\n");
        imprimeMatriz(yb, h(i));
        subplot(2, 2, i);
        plot(x, yb, 'b');

        // Diferencas Finitas
        printf("\n Solucao Por Diferencas Finitas\n");
        A = funcaoA(h(i), x0, ordem);
        B = funcaoB(h(i), y0, ordem);
        S = linsolve(A, -B);
        S = reorganiza(S);
        imprimeMatriz(S, h(i));
        subplot(2, 2, i);
        plot(x, S, 'g');
        legend(['RK'; 'SE'; 'DF']);
        title(tit);

        printf("\n\n \t ERRO \n")
        printf("\n Por Diferencas Finitas\n")
        E1 = Erro(yb, S);
        imprimeMatriz(E1, h(i));
        printf("\n Por Runge Kutta\n")
        E2 = Erro(yb, ya);
        imprimeMatriz(E2, h(i));
    end
end

```

```
endfunction
main();
```



Para $H = 0.5$

Solução por Runge-Kutta	Solução Exata	Solucao por Diferenças Finitas
y0 = 0.00000 y1 = 0.18041 y2 = 0.22970 y3 = 0.21950 y4 = 0.18659 y5 = 0.14881 y6 = 0.11401	y0 = 2.00000 y1 = 1.31959 y2 = 1.10364 y3 = 1.16939 y4 = 1.40601 y5 = 1.74625 y6 = 2.14936	y0 = 2.00000 y1 = 1.50000 y2 = 1.33333 y3 = 1.38889 y4 = 1.59259 y5 = 1.89506 y6 = 2.26337

Erro

Por Diferencas Finitas	Por Runge Kutta
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y0 = 0.00000 y1 = 0.18041 y2 = 0.22970 y3 = 0.21950 y4 = 0.18659 y5 = 0.14881 y6 = 0.11401	y0 = 0.00000 y1 = 0.00072 y2 = 0.00087 y3 = 0.00080 y4 = 0.00064 y5 = 0.00049 y6 = 0.00036
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Para $H = 0.2$

Solucao Por Runge Kutta	Solucao Exata	Solucao Por Diferencas Finitas
y0 = 2.00000 y1 = 1.65620 y2 = 1.41097 y3 = 1.24645 y4 = 1.14800 y5 = 1.10366 y6 = 1.10360 y7 = 1.13981 y8 = 1.20570 y9 = 1.29591 y10 = 1.40602 y11 = 1.53242 y12 = 1.67216 y13 = 1.82283 y14 = 1.98244 y15 = 2.14937	y0 = 2.00000 y1 = 1.65619 y2 = 1.41096 y3 = 1.24643 y4 = 1.14799 y5 = 1.10364 y6 = 1.10358 y7 = 1.13979 y8 = 1.20569 y9 = 1.29590 y10 = 1.40601 y11 = 1.53241 y12 = 1.67215 y13 = 1.82282 y14 = 1.98243 y15 = 2.14936	y0 = 2.00000 y1 = 1.70000 y2 = 1.48333 y3 = 1.33611 y4 = 1.24676 y5 = 1.20563 y6 = 1.20469 y7 = 1.23724 y8 = 1.29770 y9 = 1.38142 y10 = 1.48452 y11 = 1.60376 y12 = 1.73647 y13 = 1.88039 y14 = 2.03366 y15 = 2.19472

Erro

Por Diferencas Finitas	Por Runge Kutta
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y0 = 0.00000	y0 = 0.00000
y1 = 0.04381	y1 = 0.00001
y2 = 0.07237	y2 = 0.00001
y3 = 0.08968	y3 = 0.00002
y4 = 0.09877	y4 = 0.00002
y5 = 0.10199	y5 = 0.00002
y6 = 0.10111	y6 = 0.00002
y7 = 0.09745	y7 = 0.00002
y8 = 0.09201	y8 = 0.00002
y9 = 0.08552	y9 = 0.00001
y10 = 0.07851	y10 = 0.00001
y11 = 0.07135	y11 = 0.00001
y12 = 0.06432	y12 = 0.00001
y13 = 0.05757	y13 = 0.00001
y14 = 0.05123	y14 = 0.00001
y15 = 0.04536	y15 = 0.00001

Para $H = 0.1$

Solucao Por Runge Kutta	Solucao Exata	Solucao Por Diferencas Finitas
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y0 = 2.00000	y0 = 2.00000	y0 = 2.00000
y1 = 1.81451	y1 = 1.81451	y1 = 1.82727
y2 = 1.65619	y2 = 1.65619	y2 = 1.67934
y3 = 1.52246	y3 = 1.52245	y3 = 1.55394
y4 = 1.41096	y4 = 1.41096	y4 = 1.44904
y5 = 1.31959	y5 = 1.31959	y5 = 1.36276
y6 = 1.24644	y6 = 1.24643	y6 = 1.29342
y7 = 1.18976	y7 = 1.18976	y7 = 1.23947
y8 = 1.14799	y8 = 1.14799	y8 = 1.19952
y9 = 1.11971	y9 = 1.11971	y9 = 1.17229
y10 = 1.10364	y10 = 1.10364	y10 = 1.15663
y11 = 1.09861	y11 = 1.09861	y11 = 1.15148
y12 = 1.10358	y12 = 1.10358	y12 = 1.15589
y13 = 1.11760	y13 = 1.11760	y13 = 1.16899
y14 = 1.13979	y14 = 1.13979	y14 = 1.18999
y15 = 1.16939	y15 = 1.16939	y15 = 1.21818
y16 = 1.20569	y16 = 1.20569	y16 = 1.25289
y17 = 1.24805	y17 = 1.24805	y17 = 1.29353
y18 = 1.29590	y18 = 1.29590	y18 = 1.33958
y19 = 1.34871	y19 = 1.34871	y19 = 1.39052
y20 = 1.40601	y20 = 1.40601	y20 = 1.44593
y21 = 1.46737	y21 = 1.46737	y21 = 1.50539
y22 = 1.53241	y22 = 1.53241	y22 = 1.56854
y23 = 1.60078	y23 = 1.60078	y23 = 1.63503
y24 = 1.67215	y24 = 1.67215	y24 = 1.70458
y25 = 1.74626	y25 = 1.74625	y25 = 1.77689
y26 = 1.82282	y26 = 1.82282	y26 = 1.85172
y27 = 1.90162	y27 = 1.90162	y27 = 1.92883
y28 = 1.98243	y28 = 1.98243	y28 = 2.00803
y29 = 2.06507	y29 = 2.06507	y29 = 2.08912
y30 = 2.14936	y30 = 2.14936	y30 = 2.17193

Erro

Por Diferencas Finitas	Por Runge Kutta
------------------------	-----------------

y0 = 0.00000	y0 = 0.00000
y1 = 0.01276	y1 = 0.00000
y2 = 0.02315	y2 = 0.00000
y3 = 0.03149	y3 = 0.00000
y4 = 0.03808	y4 = 0.00000
y5 = 0.04317	y5 = 0.00000
y6 = 0.04699	y6 = 0.00000
y7 = 0.04972	y7 = 0.00000
y8 = 0.05154	y8 = 0.00000
y9 = 0.05258	y9 = 0.00000
y10 = 0.05299	y10 = 0.00000
y11 = 0.05287	y11 = 0.00000
y12 = 0.05231	y12 = 0.00000
y13 = 0.05140	y13 = 0.00000
y14 = 0.05020	y14 = 0.00000
y15 = 0.04879	y15 = 0.00000
y16 = 0.04720	y16 = 0.00000
y17 = 0.04548	y17 = 0.00000
y18 = 0.04368	y18 = 0.00000
y19 = 0.04182	y19 = 0.00000
y20 = 0.03993	y20 = 0.00000
y21 = 0.03802	y21 = 0.00000
y22 = 0.03613	y22 = 0.00000
y23 = 0.03426	y23 = 0.00000
y24 = 0.03242	y24 = 0.00000
y25 = 0.03063	y25 = 0.00000
y26 = 0.02890	y26 = 0.00000
y27 = 0.02722	y27 = 0.00000
y28 = 0.02560	y28 = 0.00000
y29 = 0.02405	y29 = 0.00000
y30 = 0.02256	y30 = 0.00000

Para $H = 0.05$

Solucao Por Runge Kutta	Solucao Exata	Solucao Por Diferencas Finitas
y0 = 2.19208	y0 = 2.00000	y0 = 2.00000
y1 = 1.90369	y1 = 1.90369	y1 = 1.90714
y2 = 1.81451	y2 = 1.81451	y2 = 1.82109
y3 = 1.73212	y3 = 1.73212	y3 = 1.74151
y4 = 1.65619	y4 = 1.65619	y4 = 1.66811
y5 = 1.58640	y5 = 1.58640	y5 = 1.60058
y6 = 1.52245	y6 = 1.52245	y6 = 1.53865
y7 = 1.46406	y7 = 1.46406	y7 = 1.48204
y8 = 1.41096	y8 = 1.41096	y8 = 1.43052
y9 = 1.36288	y9 = 1.36288	y9 = 1.38383
y10 = 1.31959	y10 = 1.31959	y10 = 1.34174

y11 = 1.28085	y11 = 1.28085	y11 = 1.30404	
y12 = 1.24643	y12 = 1.24643	y12 = 1.27051	
y13 = 1.21614	y13 = 1.21614	y13 = 1.24096	
y14 = 1.18976	y14 = 1.18976	y14 = 1.21520	
y15 = 1.16710	y15 = 1.16710	y15 = 1.19305	
y16 = 1.14799	y16 = 1.14799	y16 = 1.17433	
y17 = 1.13224	y17 = 1.13224	y17 = 1.15889	
y18 = 1.11971	y18 = 1.11971	y18 = 1.14656	
y19 = 1.11022	y19 = 1.11022	y19 = 1.13720	
y20 = 1.10364	y20 = 1.10364	y20 = 1.13067	
y21 = 1.09981	y21 = 1.09981	y21 = 1.12683	
y22 = 1.09861	y22 = 1.09861	y22 = 1.12555	
y23 = 1.09991	y23 = 1.09991	y23 = 1.12671	
y24 = 1.10358	y24 = 1.10358	y24 = 1.13020	
y25 = 1.10951	y25 = 1.10951	y25 = 1.13591	
y26 = 1.11760	y26 = 1.11760	y26 = 1.14372	
y27 = 1.12772	y27 = 1.12772	y27 = 1.15354	
y28 = 1.13979	y28 = 1.13979	y28 = 1.16528	
y29 = 1.15371	y29 = 1.15371	y29 = 1.17884	
y30 = 1.16939	y30 = 1.16939	y30 = 1.19413	
y31 = 1.18674	y31 = 1.18674	y31 = 1.21108	
y32 = 1.20569	y32 = 1.20569	y32 = 1.22960	
y33 = 1.22615	y33 = 1.22615	y33 = 1.24962	
y34 = 1.24805	y34 = 1.24805	y34 = 1.27106	
y35 = 1.27132	y35 = 1.27132	y35 = 1.29387	
y36 = 1.29590	y36 = 1.29590	y36 = 1.31797	
y37 = 1.32171	y37 = 1.32171	y37 = 1.34331	
y38 = 1.34871	y38 = 1.34871	y38 = 1.36982	
y39 = 1.37682	y39 = 1.37682	y39 = 1.39744	
y40 = 1.40601	y40 = 1.40601	y40 = 1.42614	
y41 = 1.43620	y41 = 1.43620	y41 = 1.45584	
y42 = 1.46737	y42 = 1.46737	y42 = 1.48652	
y43 = 1.49945	y43 = 1.49945	y43 = 1.51811	
y44 = 1.53241	y44 = 1.53241	y44 = 1.55058	
y45 = 1.56620	y45 = 1.56620	y45 = 1.58389	
y46 = 1.60078	y46 = 1.60078	y46 = 1.61799	
y47 = 1.63611	y47 = 1.63611	y47 = 1.65285	
y48 = 1.67215	y48 = 1.67215	y48 = 1.68843	
y49 = 1.70888	y49 = 1.70888	y49 = 1.72469	
y50 = 1.74626	y50 = 1.74625	y50 = 1.76161	
y51 = 1.78425	y51 = 1.78424	y51 = 1.79915	
y52 = 1.82282	y52 = 1.82282	y52 = 1.83729	
y53 = 1.86195	y53 = 1.86195	y53 = 1.87599	
y54 = 1.90162	y54 = 1.90162	y54 = 1.91523	
y55 = 1.94178	y55 = 1.94178	y55 = 1.95498	
y56 = 1.98243	y56 = 1.98243	y56 = 1.99522	
y57 = 2.02353	y57 = 2.02353	y57 = 2.03592	
y58 = 2.06507	y58 = 2.06507	y58 = 2.07707	
y59 = 2.10702	y59 = 2.10702	y59 = 2.11864	
v60 = 2 14936	v60 = 2 14936	v60 = 2 16061	

y00 = 2.10000	y00 = 2.10000	y00 = 2.10000
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Erro	
Por Diferencas Finitas	Por Runge Kutta
y0 = 0.00000	y0 = 0.19208
y1 = 0.00345	y1 = 0.00000
y2 = 0.00658	y2 = 0.00000
y3 = 0.00939	y3 = 0.00000
y4 = 0.01192	y4 = 0.00000
y5 = 0.01418	y5 = 0.00000
y6 = 0.01619	y6 = 0.00000
y7 = 0.01798	y7 = 0.00000
y8 = 0.01956	y8 = 0.00000
y9 = 0.02094	y9 = 0.00000
y10 = 0.02215	y10 = 0.00000
y11 = 0.02319	y11 = 0.00000
y12 = 0.02408	y12 = 0.00000
y13 = 0.02483	y13 = 0.00000
y14 = 0.02545	y14 = 0.00000
y15 = 0.02595	y15 = 0.00000
y16 = 0.02635	y16 = 0.00000
y17 = 0.02665	y17 = 0.00000
y18 = 0.02685	y18 = 0.00000
y19 = 0.02698	y19 = 0.00000
y20 = 0.02703	y20 = 0.00000
y21 = 0.02701	y21 = 0.00000
y22 = 0.02694	y22 = 0.00000
y23 = 0.02680	y23 = 0.00000
y24 = 0.02662	y24 = 0.00000
y25 = 0.02639	y25 = 0.00000
y26 = 0.02613	y26 = 0.00000
y27 = 0.02582	y27 = 0.00000
y28 = 0.02549	y28 = 0.00000
y29 = 0.02513	y29 = 0.00000
y30 = 0.02474	y30 = 0.00000
y31 = 0.02433	y31 = 0.00000
y32 = 0.02391	y32 = 0.00000
y33 = 0.02347	y33 = 0.00000
y34 = 0.02301	y34 = 0.00000
y35 = 0.02255	y35 = 0.00000
y36 = 0.02208	y36 = 0.00000
y37 = 0.02160	y37 = 0.00000
y38 = 0.02111	y38 = 0.00000
y39 = 0.02062	y39 = 0.00000
y40 = 0.02013	y40 = 0.00000
y41 = 0.01964	y41 = 0.00000
y42 = 0.01915	y42 = 0.00000

y43 = 0.01866	y43 = 0.00000
y44 = 0.01817	y44 = 0.00000
y45 = 0.01769	y45 = 0.00000
y46 = 0.01721	y46 = 0.00000
y47 = 0.01674	y47 = 0.00000
y48 = 0.01627	y48 = 0.00000
y49 = 0.01581	y49 = 0.00000
y50 = 0.01536	y50 = 0.00000
y51 = 0.01491	y51 = 0.00000
y52 = 0.01447	y52 = 0.00000
y53 = 0.01404	y53 = 0.00000
y54 = 0.01361	y54 = 0.00000
y55 = 0.01320	y55 = 0.00000
y56 = 0.01279	y56 = 0.00000
y57 = 0.01239	y57 = 0.00000
y58 = 0.01200	y58 = 0.00000
y59 = 0.01162	y59 = 0.00000
y60 = 0.01125	y60 = 0.00000

Conclusões

Diante das respostas obtidas, percebe-se que o método Runge-Kutta se mostra melhor comparado ao método das Diferenças Finitas. Por RK o erro tende a zero mais rapidamente, enquanto pelo método das DF houve a necessidade de intervalos cada vez menores para que se tenha erros consideravelmente mais baixos. Analisando os gráficos isto é mais perceptível, visto que as duas curvas se sobrepõem (Resultado Exato e pelo método de Runge-Kutta).