a) $\lambda = 852\,nm$

$$E = \frac{hc}{\lambda} = \underline{2.33 \times 10^{-19}\,J}$$

$$\sim \underline{1.45\,eV} \qquad = \frac{\quad}{e}\,eV$$

$$E = h\nu \rightarrow \nu = \frac{E}{h}$$

$$\rightarrow \nu = \underline{3.5 \times 10^{14}\,Hz}$$

$E = K_B T$

$$\rightarrow T = \frac{E}{K_B} \approx 1.688 \times 10^{4}\,K$$

b) $E = \frac{hc}{\lambda} = \frac{1}{2} m v^2$

$\longrightarrow \quad v = 4.28 \text{ km/s}$

c) $p = \frac{h}{\lambda}$, $\lambda_{Na} \sim 589 \text{ nm}$

$\longrightarrow \quad mv = \frac{h}{\lambda}$

$\longrightarrow \quad \underline{v_R = 0.029 \text{ m/s}}$

$E = \frac{1}{2} m v_R^2 = 1.65 \times 10^{-29} \text{ J}$

$\underline{E \sim 1 \mu K} \quad \sim \sqrt{m}$

II

$$[\bar{P}_x, \bar{A}(r,t)] \neq$$

$$A = \hat{x}(a + bx + cx^2 \cdots) + O(y)$$

$$[\bar{r}, \bar{A}(\bar{r}, t)] = 0$$

$$[\bar{r}, U(\bar{r}, t)] = 0$$

$$[\bar{P}, U(r, t)] \neq 0$$

$$\frac{d}{dt}\langle O\rangle = \frac{i}{\hbar}\langle [O, H]\rangle + \left\langle \frac{\partial}{\partial t} O\right\rangle$$

Ex.

$$H = \begin{pmatrix} E_0 & b \\ b^* & E_1 \end{pmatrix}$$

$[O, H] \propto b / b^*$

Recall Heisenberg eqn. of motion

$$\frac{d}{dt} O(t) = -\frac{i}{\hbar}[H, O(t)] + \left(\frac{\partial O}{\partial t}\right)$$

Ehrenfest theorem is basically exp. operator of Heis. eqn. of motion.

b)

$$H = \frac{1}{2m}\left[\bar{P} - q\,\bar{A}(r,t)\right]^2 + q\,U(r,t)$$

$$\langle v \rangle = \frac{d}{dt}\langle r \rangle$$

$$= \frac{1}{i\hbar}\left\langle [r, H] \right\rangle + \cancel{\left\langle \frac{\partial}{\partial t} r \right\rangle}$$

$$H = \frac{1}{2m}\left(\bar{P}^2 + q^2 \bar{A}^2(r,t) - q\,\bar{P}\cdot\bar{A}(r,t) - q\,\bar{A}\cdot\bar{P}\right) + qU(r,t)$$

what we need to compute

$$[\bar{r}, \bar{p}^2] \to [\underline{x}, \underline{P_x}^2] = P_x [x, P_x] + [x, P_x] P_x$$

$$= 2i\hbar P_x$$

in total

$$[\bar{r}, \bar{p}^2] = 2i\hbar(\hat{x} P_x + \hat{y} P_y + \hat{z} P_z) = 2i\hbar \bar{p}$$

$$[r_x, P_x - A_x(r,t)] = P_x \cancel{[r_x, A_x]}^{\,0} + \underbrace{[r_x, P_x]}_{i\hbar} A_x$$

$$= i\hbar A_x$$

$$\frac{1}{i\hbar}[\bar{r}, H] = \frac{2}{i\hbar m} i\hbar (\bar{p} - q\bar{A}(r,t))$$

$$\frac{d}{dt}\langle \bar{r} \rangle = \frac{1}{m} \left\{ \underbrace{\langle \bar{p} - q\bar{A}(r,t) \rangle}_{P_{eff}} \right\}$$

$$H = \frac{P_{eff}^2}{2m} + qU(r,t)$$

$$\bar{F} = m\frac{d}{dt}\langle v\rangle$$

(c) $$\frac{d}{dt}\langle\bar{v}\rangle = \frac{1}{i\hbar}\langle[\bar{v}, H]\rangle + \left\langle\frac{\partial v}{\partial t}\right\rangle$$

$$\langle v\rangle = \frac{1}{m}\langle\bar{P} - q\bar{A}(r,t)\rangle$$

don't ignore $\frac{\partial\langle\bar{v}\rangle}{\partial t}$

$$[\bar{r}, U(r,t)] \to 0$$

$$[\bar{v}, U(r,t)] \neq 0$$

$\to$ term like $[\hat{P}, U(r,t)]$

useful formula

$$[p, f(x)] = -i\hbar \frac{\partial}{\partial x} f(x)$$

to derive...

$$[p, f(x)]\, \psi(x) = p\, f(x)\, \psi(x) - f(x)\, p\, \psi(x)$$

we know

$$p\, \psi(x) \longrightarrow -i\hbar \frac{\partial}{\partial x} \psi(x)$$

$$\hookrightarrow \langle x | p | \psi \rangle \longrightarrow -i\hbar \frac{\partial}{\partial x} \langle x | \psi \rangle$$

more precise

$$\boxed{\begin{aligned} &-i\hbar\, \partial_x \left(f(x)\, \psi(x) \right) \\ &- f(x) \left(-i\hbar\, \partial_x\, \psi(x) \right) \end{aligned}}$$

The result...

$$\frac{d}{dt}\langle \bar{v} \rangle = -\frac{q}{m}\langle \nabla U(r,t) \rangle$$

$$+\frac{q}{m}\left\langle -\frac{\partial}{\partial t}\bar{A}(\bar{r},t) \right\rangle$$

$$+\frac{q}{2m}\left\langle (\hat{v}\times\vec{B})_m - (\vec{B}\times\hat{v})_m \right\rangle$$

$$-\nabla V - \partial_t \hat{A} = \vec{\mathcal{E}}$$

$$\underset{\smile}{v} = \frac{1}{m}(\bar{p} - q\bar{A}) = \frac{P_{eff}}{m}$$

$$H = \frac{1}{2}mv^2 + qU$$

$$[\tilde{v}, H] \neq [\bar{v}, qU]$$

$$\hat{\vec{v}} = \left(\vec{P} - \frac{q\vec{A}}{2m}\right)$$

$$\hat{v}^2 = \frac{1}{2m}\left[\vec{P}^2 + \frac{q^2\vec{A}^2}{2m} - \frac{q\hat{\vec{P}}\cdot\hat{\vec{A}}}{2m} + \frac{q\vec{A}\cdot\hat{P}}{2m}\right]$$

$$[\hat{v}, H] \rightarrow [\hat{v}_x, H]$$

$$\rightarrow [\hat{v}_y, H]$$

$$\rightarrow [\hat{v}_z, H]$$

$$[\hat{V}_x, H]$$

$$= \frac{1}{m}\Big[\hat{P}_x - \hat{A}_x(x, y, z, t), \; \frac{1}{m}\big(\hat{P}_x^2 + \hat{P}_y^2 + \hat{P}_z^2\big)$$

$$+ \frac{q^2 A^2}{2m} - \frac{q}{m}\sum_i \hat{P}_i \hat{A}_i$$

$$- \frac{q}{m}\sum_i \hat{A}_i \hat{P}_i\Big]$$

$$[A_x(x,y,z), \hat{P}_y^2]$$

$$A_x = \sum A_m^y y^m$$

$$[A_x, \hat{P}_y^2] = \sum A_m^y [y^m, \hat{P}_y^2]$$

$$[y^m, \hat{P}_y^2]$$

$$= [\hat{y}^m, P_y]P_y + P_y[\hat{y}^m, \hat{P}_y]$$

$$m\hat{y}^{m-1}P_y + mP_y\hat{y}^{m-1}$$

$$= m\hat{y}^{m-1}P_y + mP_y\hat{y}^{m-1}$$

$$\sum A_m^y \left[m y^{m-1} P_y + m P_y y^{m-1} \right]$$

$$\left(\sum m A_m^y y^{m-1} \right) P_y$$

$$\frac{\partial A}{\partial y} \cdot P_y + P_y \cdot \frac{\partial A}{\partial y}$$

$$\left[P_i - A_i(\vec{r}, t) \right.$$

$$\left. \sum_i \frac{P_i^2}{2m} - P_i A_i - A_i P_i \right]$$

$\hat{\rho} = |\psi\rangle\langle\psi|$ pure state