→ Define the following total angular momentum operators

$$\hat{J}_{\pm} = \sum_i \sigma_{\pm}^i \; , \quad \hat{J}_1 \equiv \hat{J}_+ + \hat{J}_- \; , \quad \hat{J}_2 \equiv -i(\hat{J}_+ - \hat{J}_-)$$

$$\hat{J}_3 \equiv \frac{1}{2} \sum_i \hat{\sigma}_3^i$$

Levi-Levita symbol

$$\rightarrow \quad [\hat{J}_i, \hat{J}_j] = i\varepsilon_{ijk} \hat{J}_k \; , \quad \hat{J}^2 = \vec{J} \cdot \vec{J}$$

⇒ obtain: spin-$N/2$ system from N spin-$1/2$ systems

→ use $\hat{J}^2$ and J_3 as commuting set of observables to label states: $|J, M\rangle$ with

"σ_3"

$$\boxed{-J \leq M \leq J \qquad \hat{J}_3 |J,M\rangle = M \cdot |J,M\rangle \qquad \hat{J}^2 |J,M\rangle = J(J+1)\, |J,M\rangle}$$

The Hamiltonian we consider is

$$\boxed{\hat{H}_D = \hbar\omega\hat{a}^{\dagger}\hat{a} + \hbar\omega_0\hat{J}_3 + \hbar\lambda(\hat{a}+\hat{a}^{\dagger})\cdot\hat{J}_1}$$

Dicke model

Basic assumptions for superradiance

*) no center-of-mass motion (Doppler shifts), no atom-atom interactions (collisions, van der Waals forces)

*) e-m interactions only with one mode
[for a more general treatment → Gross + Haroche '82]

*) no re-absorption

*) initial state: $|\psi(t=0)\rangle = |e \ldots e\rangle$ ← symmetric!
(Note that $\hat{H}_D$ is also symmetric)

Symmetric superradiant states

→ under these assumptions, time evolution is confined to subspace invariant under atom permutation

→ N+1 fully symmetric states

$$|J,M\rangle = \sqrt{\frac{(J+M)!}{(2J)!\,(J-M)!}}\,\left(\hat{J}_-\right)^{J-M}\,\underbrace{|e \dots e\rangle}_{|J,\,M=J\rangle}$$

→ N spins have 2^N states → # symmetric states $= N+1 = 2J+1$

remaining: subradiant states / subspace

Dicke ladder

$$|J, M=J\rangle = |e \dots e\rangle$$

$\hat{J}_-$

$$|J, M=J-1\rangle = S\,|ge \dots e\rangle$$

$\vdots$

$$|J, M=0\rangle = S\,|\underbrace{g \cdots g}_{N/2}\underbrace{e \cdots e}_{N/2}\rangle$$

$\vdots$

$$|J, 1-J\rangle = S\,|g \dots ge\rangle$$

$$|J, -J\rangle = |g \dots g\rangle$$

How does spontaneous emission proceed down the "ladder"?

emission prob. $\sim \Gamma \cdot \langle \psi_N | \hat{J}_+ \hat{J}_- | \psi_N \rangle$

(compare 2LS: $\langle \psi | \sigma^+ \sigma^- | \psi \rangle = p_e$)

From theory of angular momenta

$$\boxed{\begin{aligned} \hat{J}_+ |J, M\rangle &= \sqrt{(J-M)(J+M+1)}\ |J, M+1\rangle \\ \hat{J}_- |J, M\rangle &= \sqrt{(J+M)(J-M+1)}\ |J, M-1\rangle \end{aligned}} \quad (*)$$

→ Apply to symmetric ladder:

$$W_N = \Gamma \overbrace{\underbrace{\langle J, M | \hat{J}_+}_{\text{adjoints}} \underbrace{\hat{J}_- | J, M\rangle}}^{\#\text{ excitations in the system}}$$

$$\overset{(*)}{=} \Gamma\ (J+M)(J-M+1)$$

↑

$|J,J\rangle$ $\quad W_N \overset{M=J}{=} 2J\Gamma = N\cdot\Gamma$

$N\cdot\Gamma \downarrow$

$|J,J-1\rangle$

$\vdots$

$\Gamma(\frac{N}{2}+1)\frac{N}{2}$ $\quad |J,1\rangle$ $\quad W_N \overset{M=1}{=} \Gamma\cdot(\frac{N}{2}+1)\ \frac{N}{2}$

$\vdots$

$|J,1-J\rangle$

$N\cdot\Gamma \downarrow$

$|J,-J\rangle$

emission is maximal $\propto N^2$ around $M=0$

Simple $N=2$

$|ee\rangle$

$|+\rangle = \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)$

$|gg\rangle$

superradiant subspace

$J=1$

no radiative transitions!

$|-\rangle = \frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$

subradiant state (anti-symmetric)

Dicke Hamiltonian $N=1$: $\langle e|\hat{H}_D|g\rangle = \lambda$

$N=2$: $\langle ee|\hat{H}_D|+\rangle = \sqrt{2}\,\lambda$

but $\langle ee|\hat{H}_D|-\rangle = 0$

Comments: *) superradiance has been observed, creation of subradiant states is more difficult

*) simplification : Tavis-Cummings model

$$\hat{H}_D = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_0\hat{J}_3 + \hbar\lambda(\hat{a}+\hat{a}^\dagger)\underbrace{\hat{J}_1}_{=\hat{J}_+ + \hat{J}_-}$$

$$\downarrow \text{RWA}$$

$$\Rightarrow \hat{H}_{TC} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_0\hat{J}_3 + \hbar\lambda(\hat{J}_+\hat{a} + \hat{J}_-\hat{a}^\dagger)$$

	$N=1$		$N\geq 1$
no RWA	Quantum Rabi model	$\longleftrightarrow$	Dicke model
	$\updownarrow$		$\updownarrow$
RWA	Jaynes-Cummings model	$\longleftrightarrow$	Tavis-Cummings model

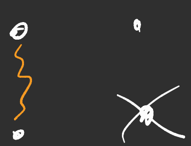
ASIDE

$$\hat{S}\,|gg\ldots ge\ldots e\rangle$$

$$\hat{S}\,|ge\rangle = \frac{1}{\sqrt{2}}\left(|ge\rangle + |eg\rangle\right)$$

$$\hat{S}\,|gee\rangle = \frac{1}{\sqrt{3}}\left(|gee\rangle + |ege\rangle + |eeg\rangle\right)$$

$$\hat{S}\,|ggee\rangle = \left(|ggee\rangle + |eegg\rangle + |geeg\rangle + |egge\rangle + |gege\rangle + |egeg\rangle\right)\frac{1}{\sqrt{6}}$$



* Office hour (Friday 3pm)