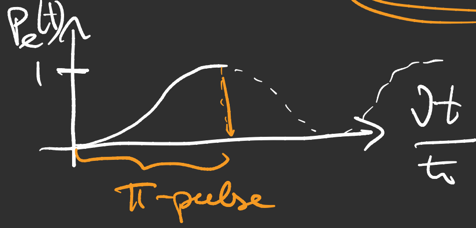
π - pulse

→ pulse that results in a full population inversion on resonance

$|g\rangle \xrightarrow{\pi} |e\rangle$



→ the laser should be applied for a duration

$$T_\pi = \frac{1}{2} \frac{2\pi}{\Omega_R}$$

$\pi/2$ pulse : pulse of duration $T_{\pi/2} = \frac{1}{4} \cdot \frac{2\pi}{\Omega_R}$

$$|g\rangle \xrightarrow{\pi/2} \frac{1}{\sqrt{2}}\left(|g\rangle + |e\rangle\right)$$

2π pulse : 2-level system (TLS) restores its initial state

$$|g\rangle \xrightarrow{2\pi} |g\rangle$$

6.4 Ensembles of two-level systems: density matrix formalism

→ advantage: integrate decay into description

general mixed state

statistical weights (time indep.)

$$\hat{\rho} = \sum_{|\psi\rangle} P_\psi |\psi\rangle\langle\psi|$$

$$\partial_t |\psi\rangle = -\frac{i}{\hbar}\hat{H}|\psi\rangle \longleftrightarrow \partial_t \langle\psi| = \frac{i}{\hbar}\langle\psi|\hat{H}$$

$$\dot{\hat{\rho}} = \sum_{|\psi\rangle} P_\psi \left[\left(-\frac{i}{\hbar}\hat{H}|\psi\rangle\right)\langle\psi| + |\psi\rangle\left(\frac{i}{\hbar}\langle\psi|\hat{H}\right)\right]$$

$$\Rightarrow \boxed{\dot{\hat{\rho}} = \frac{i}{\hbar}[\hat{\rho}, \hat{H}]}$$

von Neumann equation

opposite sign compared to Heisenberg eq. of motion

side remark:

$$\boxed{\langle 1|1\rangle\langle 2|1\rangle = \rho_{21}}$$

Reminder:

populations

coherences

$$\rho = \begin{pmatrix} \rho_{gg} & \rho_{ge} \\ \rho_{eg} & \rho_{ee} \end{pmatrix}$$

phenomenological description of decay:

$\mathcal{M} = |e\rangle\langle e|$

$\mathcal{M} = |g\rangle\langle e|$

$-\gamma\rho_{ee}$	for ρ_{ee}	$-\frac{\Gamma}{2}\rho_{ge}$	for ρ_{ge}
$+\gamma\rho_{ee}$	for ρ_{gg}	$-\frac{\Gamma}{2}\rho_{eg}$	for ρ_{eg}

6.5 Optical Bloch equation

$$\hat{H} = \hat{H}_0 + \hat{H}', \quad H_0 = \begin{pmatrix} E_g & 0 \\ 0 & E_e \end{pmatrix}, \quad \hat{H}' = \mathcal{V}\cos\omega t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\dot{\rho}_{gg} = \gamma\rho_{ee} + \frac{i}{\hbar}\mathcal{V}\cos(\omega t)(\rho_{ge} - \rho_{eg})$$

$$\dot{\rho}_{ee} = -\gamma\rho_{ee} - \frac{i}{\hbar}\mathcal{V}\cos(\omega t)(\rho_{ge} - \rho_{eg})$$

$$\dot{\rho}_{ge} = \left(i\omega_0 - \frac{\Gamma}{2}\right)\rho_{ge} - \frac{i}{\hbar}\mathcal{V}\cos(\omega t)(\rho_{ee} - \rho_{gg})$$

$\omega_0 = \frac{E_e - E_g}{\hbar}$

$\frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$

* As $\hat{\rho}$ is hermitian, $\rho_{eg} = \rho_{ge}^*$
* Time dependence of ρ-entries when $\mathcal{V} \to 0$ (no coupling)

$$\rho_{ge}(t) \sim e^{i\omega_0 t - \frac{\Gamma}{2}t}, \quad \rho_{eg}(t) = e^{-i\omega_0 t - \Gamma/2\, t}, \quad \rho_{ee}(t) \sim e^{-\Gamma t}$$

RWA (rotating wave approximation)

→ with coupling ($\nu \neq 0$), we drop from $\cos(\omega t) = \frac{1}{2}(e^{-i\omega t} + e^{i\omega t})$ the rapidly oscillating terms

e.g.: $\rho_{ge} \cdot e^{i\omega t}$

$$\dot{\rho}_{gg} = \gamma \rho_{ee} + \frac{i}{2\hbar}\nu\Big(\overbrace{e^{-i\omega t}\rho_{ge}}^{=\tilde{\rho}_{eg}^{*}} - \overbrace{e^{i\omega t}\rho_{eg}}^{\tilde{\rho}_{eg}}\Big)$$

$$\dot{\rho}_{ee} = -\gamma \rho_{ee} - \frac{i}{2\hbar}\nu\Big(\underbrace{e^{-i\omega t}\rho_{ge}} - \underbrace{e^{i\omega t}\rho_{eg}}\Big)$$

$$\circledast\ \dot{\rho}_{ge} = \left(i\omega_0 - \frac{\gamma}{2}\right)\rho_{ge} - \frac{i}{2\hbar}\nu e^{i\omega t}(\rho_{ee} - \rho_{gg})$$

Optical Bloch eq. under RWA

multiply $\circledast$ by $e^{-i\omega t}$

$$\dot{\rho}_{ge} \cdot e^{-i\omega t} = \left(i\omega_0 - \frac{\gamma}{2}\right)\underline{\rho_{ge} e^{-i\omega t}} - \frac{i}{2\hbar}\nu(\rho_{ee} - \rho_{gg})$$

$$= \partial_t\left(\underline{\rho_{ge} e^{-i\omega t}}\right) + i\omega \underline{\rho_{ge} e^{-i\omega t}}$$

$$\circledast\ \partial_t \tilde{\rho}_{ge} = i\left([\omega_0 - \omega] - \frac{\gamma}{2}\right)\tilde{\rho}_{ge} - \frac{i\nu}{2\hbar}(\rho_{ee} - \rho_{gg})$$