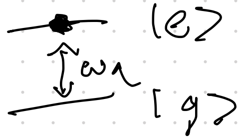
Problem-1 (Ishman)

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\, dt$$

$$f(t) \updownarrow E(t)$$



$$\underline{E(t)} = E_0\, \Theta(t)\, e^{-\frac{\Gamma}{2}t} e^{i\omega t}$$

Γ - decay term

$\Theta(t) = \quad \Theta(t) = 1 \quad$ for $t \geq 0$

$\Theta(t) = 0 \quad$ for $t < 0$

a) $\tau \rightarrow$ lifetime of $|e\rangle$

$$\Gamma \Longleftrightarrow \tau$$

$$\Gamma = \frac{1}{\tau}$$

b) $E(t) \rightarrow E(\omega)$

$\Theta(t) = 1$
$t \geq 0$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} E_0 \Theta(t) e^{-\frac{\Gamma}{2}} \cdot e^{i\omega_a t}$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-i\omega t} \cdot 1 \cdot e^{-\frac{\Gamma}{2}} \cdot e^{i\omega_a t} dt$$

$$e^{i\omega_a t} \cdot e^{-\frac{\Gamma}{2}t}$$

$$= e^{(i\omega_a - \frac{\Gamma}{2})t}$$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} E_0 \int_0^{\infty} e^{-i\omega t} \cdot e^{(i\omega_a - \frac{\Gamma}{2})t} dt$$

$$\int_0^{\infty} e^{-(a+ib)t} \cdot e^{-i\omega t} = \frac{1}{a + i(\omega - b)}$$

$a = \frac{\Gamma}{2}$; $b = \omega_a$, $\omega = \omega$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} E_0 \cdot \frac{1}{\frac{\Gamma}{2} + i(\omega - \omega_a)}$$

Ans.

c) Intensify

$$I(\omega) = |E(\omega)|^2$$

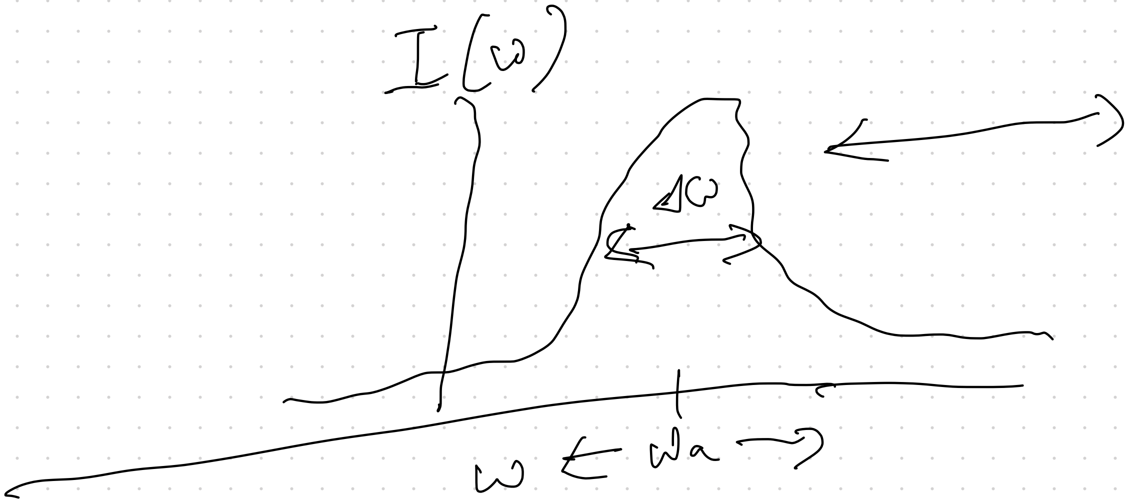
$$\left| \frac{1}{\sqrt{2\pi}} E_0 \cdot \frac{1}{\frac{\Gamma}{2} + i(\omega - \omega_a)} \right|^2$$

against ω

$E_0 = 1$

$\Gamma = 2 \longrightarrow 4 \longrightarrow 10$

$\omega_a = 2$



Γ

$$\Delta\omega = \frac{1}{\Gamma}$$

② (Shubalik)

Free atom

$$\hat{H} = \underline{\hbar\omega_{21}\sigma^{\dagger}\sigma} + \hbar\omega a^{\dagger}a + \hbar g(a^{\dagger}\sigma + a\sigma^{\dagger})$$

photons

(a) $\sigma^+ \; , \; \sigma$

$|g\rangle \; , \; |e\rangle$

$\sigma^+ = |e\rangle\langle g| \rightarrow |e\rangle$

$\sigma = |g\rangle\langle e| \quad e$

$\sigma^+|g\rangle = |e\rangle \underbrace{\langle g|g\rangle}_{1}$

$= |e\rangle$

$\sigma^+|e\rangle = 0$

$\sigma|g\rangle = 0$

$\sigma^{\cancel{+}}|e\rangle = |g\rangle$

(b) uncoupled systems $g = 0$

$$\hat{H}_{free} = \hbar \omega_{21} \sigma^{+} \sigma + \hbar \omega a^{+} a$$

$$\hat{H}_{free} |\psi\rangle = E |\psi\rangle$$

$$|\psi\rangle = \sum_{i} |e, \cancel{x} i\rangle + \sum_{j} |g, j\rangle$$

$$n \hbar \omega_{21} + \hbar \omega = |e, n\rangle$$

$$n \hbar \omega_{21} = |g, n\rangle$$

(c) $|e, n\rangle \leftrightarrow |g, n+1\rangle$

$$\hat{H} = \hbar\omega_{21}\sigma^{\dagger}\sigma + \hbar\omega a^{\dagger}a + \hbar g(a^{\dagger}\sigma + \sigma^{\dagger}a)$$

acts atomic space

$$\langle\Psi_2|\hat{H}|\Psi_1\rangle \neq 0$$

coupling

$$|\Psi\rangle = |e, i\rangle + |g, j\rangle$$

atomic platform

$$\langle\Psi_2|\hat{H}|\Psi_1\rangle \neq 2$$

coupled

$$= (\langle e, i_2| + \langle g, j_2|)\, \hbar\omega_{21}\sigma^{\dagger}\sigma(|e, i_1\rangle + |g, j_1\rangle) \rightarrow \mathrm{I}$$

$$\hbar\omega a^{\dagger}a(|e, i_1\rangle + |g, j_2\rangle) \rightarrow \mathrm{II}$$

||

$$\left(\langle e, i_2| + \langle g, f_2|\right) \hbar g \left(a^\dagger \sigma + \sigma^\dagger a\right) \left(|e, i_1\rangle + |g, f_1\rangle\right)$$

$$\text{I} = \hbar\omega_{12} \Big[\langle e, i_2| \sigma^\dagger \sigma | e, i_1\rangle$$

$$+ \langle g, i_2| \sigma^\dagger \sigma | g, f_1\rangle$$

$$+ \langle g, f_2| \sigma^\dagger \sigma | g, f_1\rangle$$

$$+ \langle g, i \cancel{f}_2| \sigma^\dagger \sigma | e, i_1\rangle \Big]$$

photon

$$\hbar\omega\, \sigma^\dagger \sigma \overset{\text{atm}}{=} (\sigma^\dagger \sigma)_{\text{atomic}} \otimes \text{II}$$

Photon

↳ Side note

$$\rightarrow \sigma^\dagger (|g\rangle, |e\rangle \ldots$$

$$\textcircled{I} \rightarrow \hbar\omega_{21} \langle f_1 | f_2 \rangle \underbrace{\langle g | g \rangle}_{\to 1}$$

$$+ \hbar\omega_{21} \langle i_1 | i_2 \rangle \underbrace{\langle e | e \rangle}_{\to 1}$$

$$f_1 = f_2 \qquad = \qquad \delta_{i_1 i_2}$$

Int. terms (g)

$$\left[\langle e, i_2| + \langle g, f_2|\right] \hbar g_c \left(\sigma^+ a + a^+ \sigma\right) \left[|e, i_1\rangle + |g, f_1\rangle\right]$$

$$\sim \langle e | \sigma^+ | e \rangle \langle i_2 | a | i_1 \rangle$$

$$+ \underbrace{\langle e | \sigma^+ | g \rangle}_{=1} \langle i_2 | a | f_1 \rangle$$

$$+ \underbrace{\langle g | \sigma | e \rangle}_{=1} \langle f_2 | a^+ | i_1 \rangle$$

$$+ \langle g | \sigma | g \rangle \langle f_2 | a^+ | f_1 \rangle$$

$$= 0$$

$$\sim \underline{\langle e | g\rangle} \langle i_2 | a | i_1\rangle$$

$$= 0$$

$$\langle i_2 | a | j_1\rangle + \langle j_2 | a^\dagger | i_1\rangle$$

$$\sqrt{j_1}\,\langle i_2 | j_1 - 1\rangle + \sqrt{i_1+1}\,\langle j_2 | i_1+1\rangle$$

$$i_2 = j_1 - 1 \qquad j_2 = i_1 + 1$$

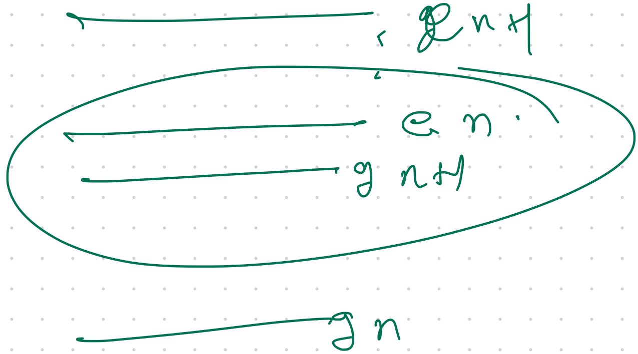
coupling

$$(|e, i_2\rangle,\ |g\ i_2+1\rangle)$$

$$(|e, j_2 - 1\rangle,\ |g, j_2\rangle)$$

$$|e, n\rangle \qquad (|g\ n+1\rangle)$$

$$|e, n-1\rangle \qquad (|g, n\rangle)$$



(d, e) $\hat{H}$

In basis of $\{ |e, n\rangle , |g, n+1\rangle \}$

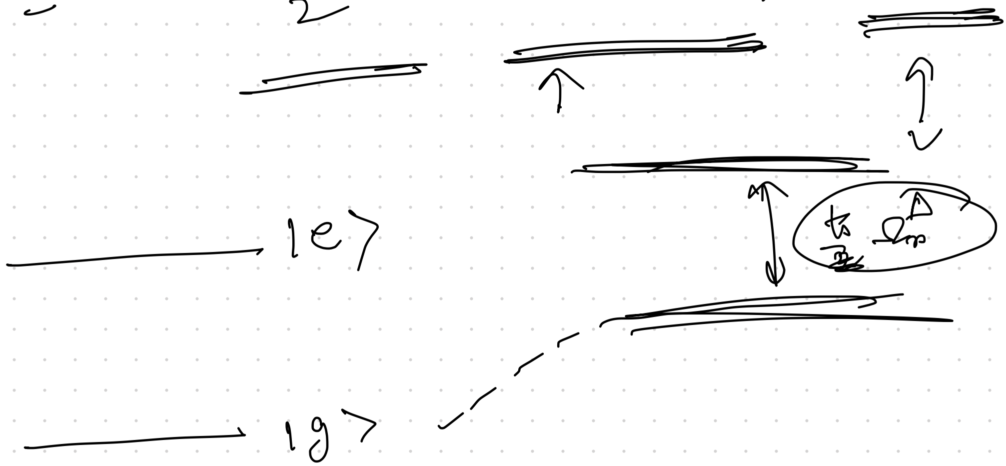
$$\underline{H} = \begin{array}{c|c} & \begin{array}{cc} g, n+1 & e, n \end{array} \\ \begin{array}{c} g\,n+1 \\ \\ e\,n \end{array} & \begin{bmatrix} \hbar\omega(n+1) & \hbar g\sqrt{n+1} \\ \\ \hbar g\sqrt{n+1} & \hbar\omega n + \hbar\omega_{21} \end{bmatrix} \end{array}$$

Eigen values

$|g, n+1\rangle$

$|e, n\rangle$

$$E_{\pm} = \frac{\hbar\omega_{21}}{2} + \hbar\omega\left(n+\frac{1}{2}\right) \pm \frac{\hbar\Omega_n^{\Delta}}{2}$$



(f) $|g,0\rangle$

$$\hat{H}\,|g,0\rangle = 0\,|g,0\rangle$$

$$\frac{\hbar\omega}{2}$$

$$|g,n+1\rangle \longleftrightarrow |e,n\rangle$$

$$|g,n\rangle \longrightarrow |e,n-1\rangle$$

$|g,0\rangle \longrightarrow |e,-1\rangle$

g $\updownarrow$

$\sigma^+ a^+ \quad \sigma a$

(g)

$$|+n\rangle = \cos\left(\frac{\theta}{2}\right)|e,n\rangle + \sin\left(\frac{\theta}{2}\right)|g,n+1\rangle$$

$$|-n\rangle = \cos\frac{\theta}{2}|e,n\rangle - \sin\frac{\theta}{2}|g,n+1\rangle$$

$$\langle n+|\hat{H}|+n\rangle = E_+\langle n+|n+\rangle = E_+\cdot 1 = E_+$$

$$\langle n-|\hat{H}|-n\rangle = E_-$$

$$\hbar\omega_{21}\cos\frac{\theta}{2} + n\hbar\omega\cos\frac{\theta}{2} + \hbar g\cos\frac{\theta}{2}\sin\frac{\theta}{2}$$

$$= E_+ \cancel{\cos\frac{\theta}{2}}$$

$$\hbar\omega_{21}\cos\frac{\theta}{2} + \hbar\omega\cos\frac{\theta}{2}$$

$$-\hbar g\cos\frac{\theta}{2}\sin\frac{\theta}{2}$$

$$= 0$$

solve for θ

$$H = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

$$H' = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$$

$$H' = U^\dagger H U$$

$$U^\dagger = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

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3. a) $E_n^{\pm} = \frac{\hbar}{2}\omega_{21} + \hbar\omega\left(n+\frac{1}{2}\right) \pm \frac{\hbar}{2}\Omega_n^{\Delta}$

$$\Omega_n^{\Delta} = \sqrt{\Delta^2 + 4g^2(n+1)} \longrightarrow 0$$

b) if $g=0$, $\underline{\Omega_n^{\Delta} = \Delta}$

$$E_n^{\pm} = \frac{\hbar}{2}\omega_{21} + \hbar\omega\left(n+\frac{1}{2}\right) \pm \frac{\hbar}{2}\Delta$$

$\underline{\Delta < 0}$

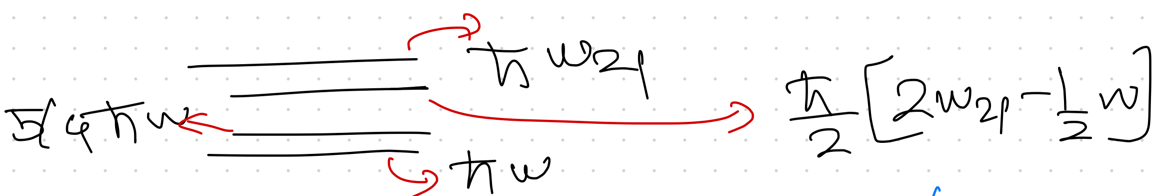
$n=0 \Rightarrow E_0^{\pm} \Rightarrow E_0^{+} = \hbar\omega$

$E_0^{-} = \hbar\omega_{21}$

$n=1 \Rightarrow E_1^{\pm} \Rightarrow E_1^{+} = 5\hbar\omega/4$

$E_1^{-} = \frac{\hbar}{2}\left[2\omega_{21} - \frac{1}{2}\omega\right]$

$\underline{\Delta > 0}$



correction Δ

b) AC starks shift :

$$\Delta E_n^{\pm} = E_n^{\pm}(g) - E_n^{\pm}(g=0)$$

$$\Omega_n^{\Delta} = \Delta \sqrt{1 + \frac{4g^2(n+1)}{\Delta^2}}$$

$$f(x) = \sqrt{1+x} \leftarrow \text{taylor Expand}$$

$$\Omega = \Delta \left[1 + \frac{1}{2}\left(\frac{4g^2 n+1}{\Delta^2}\right) \cdots \right] \rightarrow 4$$

$\Omega = \Delta =$ at $g = 0$

$$\Delta E_n^{\pm} = \underset{1}{E_n^{\pm}(g)} - \underset{2}{E_n^{\pm}(0)}$$

1st two terms = identical

$$\Delta E_n^{\pm} = \frac{\hbar}{2}(\Omega - \Delta) \qquad \text{from } 4$$

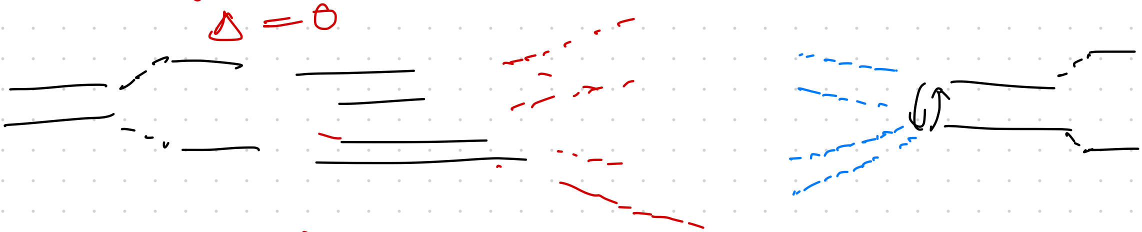
$$= \frac{\hbar}{2}\left[\frac{\cancel{\Delta}}{2} \frac{4g^2(n+1)}{\cancel{\Delta^2}} \right]$$

$$\boxed{\Delta E_n^{\pm} = \pm \frac{\hbar}{\Delta} g^2 (n+1)} \Rightarrow n=0$$

$$\Delta E_n \neq 0$$

c) $g=0$ $\quad \Delta > 0 \quad \Delta < 0$

$\Delta = 0$



Energy diagram

d) $$\Delta E_n^{\pm} = \frac{\hbar n}{\Delta} [g^2]$$

Lecture Notes

$$\rightarrow \quad g^2 = \frac{\hbar \omega_c}{\epsilon_0 V} (\sin^2 k_z) \frac{d^2}{\hbar^2}$$

$$\propto \frac{\text{Energy}}{\text{Volume}} \times \text{velocity}$$

$$c = \sqrt{\epsilon_0 \mu_0} \Rightarrow c^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$g^2 \propto \frac{\text{Energy}}{\text{Volume}} \frac{d^2}{\hbar^2} \left(c^2/ka\right) \sin^2 k_2$$

$$g^2 \propto \frac{\text{Energy}\ c^2}{\text{Volume}}$$