


Proble-1 (Ishman)

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} f(t) dt$$

$$\begin{array}{c} \bullet \\ \updownarrow \omega_1 \\ \hline \end{array} \begin{array}{l} |e\rangle \\ |g\rangle \end{array}$$

\updownarrow
 $E(t)$

$$\underline{E(t)} = E_0 \Theta(t) e^{-\frac{\Gamma}{2}t} e^{i\omega t}$$

Γ - decay term

$$\Theta(t) = \begin{cases} \Theta(t) = 1 & \text{for } t \geq 0 \\ \Theta(t) = 0 & \text{for } t < 0 \end{cases}$$

a) $\tau \rightarrow$ lifetime of $|e\rangle$

$$\Gamma \leftrightarrow \tau$$

$$\Gamma = \frac{1}{\tau}$$

$$b) E(t) \rightarrow E(\omega)$$

$$\Theta(t) = 1$$

$$\therefore, t \geq 0$$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} E_0 \Theta(t) e^{-\frac{\Gamma}{2} t} e^{i\omega_a t} dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-i\omega t} \cdot (1 \cdot e^{-\frac{\Gamma}{2} t} \cdot e^{i\omega_a t}) dt$$

$$e^{i\omega_a t} \cdot e^{-\frac{\Gamma}{2} t}$$

$$= e^{(i\omega_a - \frac{\Gamma}{2})t}$$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} E_0 \int_0^{\infty} e^{i\omega t} \cdot e^{(i\omega_a - \frac{\Gamma}{2})t} dt$$

$$\int_0^{\infty} e^{-(a+ib)t} \cdot e^{i\omega t} = \frac{1}{a+i(\omega-b)}$$

$$a = \frac{\Gamma}{2}; \quad b = \omega_a, \quad \omega = \omega$$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} E_0 \cdot \frac{1}{\frac{\Gamma}{2} + i(\omega - \omega_a)}$$

Ans.

c) Intensity

$$I(\omega) = |E(\omega)|^2$$

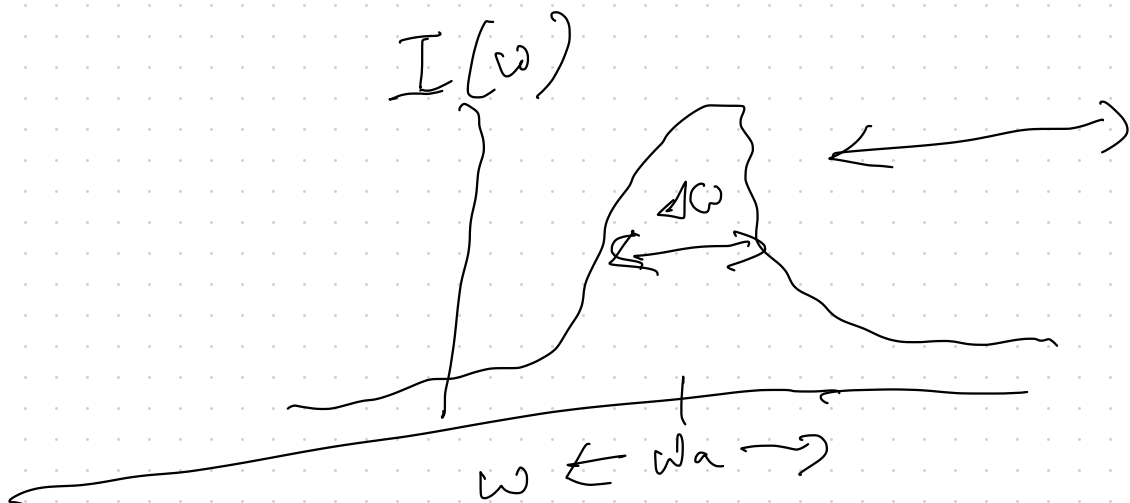
$$\left| \frac{1}{\sqrt{2\pi}} E_0 \cdot \frac{1}{\frac{\Gamma}{2} + i(\omega - \omega_a)} \right|^2$$

against ω

$$E_0 = 1$$

$$\Gamma = 2 \rightarrow 4 \rightarrow 10$$

$$\omega_a = 2$$



$$\Gamma \quad \Delta\omega = \frac{1}{\Gamma}$$

② (Shubalik)

$$\hat{H} = \hbar\omega_{21} \sigma_{21} \xrightarrow{\text{free atom}} + \hbar\omega_{\text{atom}} + \hbar g (\sigma_{21} + \sigma_{12})$$

\downarrow
 photon

$$(a) \quad \sigma^+ \quad , \quad \sigma^-$$

$$|g\rangle \quad , \quad |e\rangle$$

$$\sigma^+ = |e\rangle\langle g| \rightarrow |e\rangle$$

$$\sigma^- = |g\rangle\langle e| \leftarrow$$

$$\sigma^+ |g\rangle = |e\rangle\langle g|g\rangle$$

$$\sigma^+ |e\rangle = 0$$

$$= |e\rangle$$

$$\sigma |g\rangle = 0$$

$$\sigma^\dagger |e\rangle = |g\rangle$$

(b) uncoupled systems $g=0$

$$\hat{H}_{\text{free}} = \hbar \omega_{21} \hat{\sigma}_z + \hbar \omega_a \hat{a}^\dagger \hat{a}$$

$$\hat{H}_{\text{free}} |\psi\rangle = E |\psi\rangle$$

$$|\psi\rangle = \sum_i |e, i\rangle + \sum_j |g, j\rangle$$

$$\eta \hbar \omega_{21} + \hbar \omega$$

$$= |e, n\rangle$$

$$\eta \hbar \omega_{21}$$

$$= |g, n\rangle$$

$$② \quad |e, n\rangle \quad \& \quad |g, n+1\rangle$$

$$\hat{H} = \underbrace{\hbar \omega_{21} \sigma_c^x}_{\text{acts atomic space}} + \hbar \omega_{at} a^\dagger a + \hbar g (a^\dagger \sigma_c + \sigma_c^\dagger a)$$

$$\langle \psi_2 | \hat{H} | \psi_1 \rangle \neq 0$$

$$\hookrightarrow \text{coupling}$$

$$|\psi\rangle = |e, i\rangle + |g, j\rangle$$

\uparrow atomic \uparrow photon

$$\langle \psi_2 | \hat{H} | \psi_1 \rangle \rightarrow \neq 2 \text{ coupled}$$

$$= \underbrace{(\langle e, i_2 | + \langle g, j_2 |)}_{\text{II}} \hbar \omega_{21} \sigma_c^x \underbrace{(|e, i_1\rangle + |g, j_1\rangle)}_{\text{I}} + \hbar \omega_{at} a^\dagger a (|e, i_1\rangle + |g, j_1\rangle)$$

$\xrightarrow{\text{I}} \text{I}$

$$(\langle e, i_2 | + \langle g, f_2 | + \dots) (a^\dagger + a) (|e, i_1\rangle + |g, f_1\rangle)$$

$$\textcircled{I} = \hbar\omega \left[\langle e, i_2 | a^\dagger a | e, i_1 \rangle \checkmark \right. \\ + \langle e, i_2 | a^\dagger a | g, f_1 \rangle \\ + \langle g, f_2 | a^\dagger a | g, f_1 \rangle \checkmark \\ + \left. \langle \underline{g, i_2} | a^\dagger a | e, i_1 \rangle \right]_{\text{atom}}$$

$$\hbar\omega a^\dagger a = \overset{\text{atom}}{(a^\dagger a)}_{\text{atomic}} \textcircled{X} \underline{\underline{\text{Photon}}}$$

→ Side note

$$\rightarrow a^\dagger |g\rangle, |e\rangle \dots$$

$$\begin{aligned}
 \textcircled{I} &\Rightarrow \hbar \omega_{21} \langle j_1 | j_2 \rangle \langle g | g \rangle \xrightarrow{g} \mathbb{I} \\
 &+ \hbar \omega_{21} \langle i_1 | i_2 \rangle \langle e | e \rangle \xrightarrow{g} \mathbb{I} \\
 j_1 = j_2 &= \uparrow \delta_{i_1 i_2}
 \end{aligned}$$

Int. terms (g)

$$\left[\langle e, i_1 | + \langle g, i_2 | \right] \hbar g \left(\sigma^+ a + \underline{\underline{\sigma^- a^\dagger}} \right) \left[\begin{array}{c} |e, i_1\rangle \\ + |g, i_1\rangle \end{array} \right]$$

$$\sim \langle e | \underline{\underline{\sigma^+}} | e \rangle \langle i_2 | a | i_1 \rangle$$

$$+ \underbrace{\langle e | \sigma^+ | g \rangle}_{=1} \langle i_2 | a | j_1 \rangle$$

$$+ \underbrace{\langle g | \sigma^- | e \rangle}_{=1} \langle j_2 | a^\dagger | i_1 \rangle$$

$$+ \langle g | \sigma^- | g \rangle \langle j_2 | a^\dagger | j_1 \rangle$$

$$\rightarrow = 0$$

$$\sim \underline{\langle e | g \rangle} \langle i_2 | i_1 \rangle$$

$$= 0$$

$$\langle i_2 | a | s_1 \rangle + \langle g_2 | a^\dagger | i_1 \rangle$$

$$\sqrt{j_1} \langle i_2 | s_1 - 1 \rangle + \sqrt{i_1 + 1} \langle j_2 | i_1 + 1 \rangle$$

$$\downarrow$$

$$i_2 = j_1 - 1$$

coupling

$$\downarrow$$

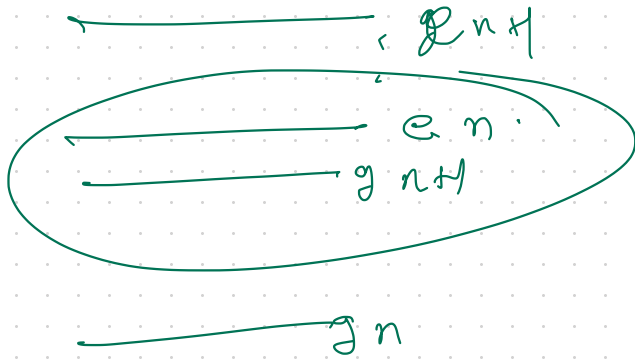
$$j_2 = i_1 + 1$$

$$(|e, i_2\rangle, |g, i_2 + 1\rangle)$$

$$(|e, j_2\rangle, |g, j_2\rangle)$$

$$|e, n\rangle \quad (|g, n+1\rangle)$$

$$|e, n-1\rangle \quad (|g, n\rangle)$$



(d, e)

\hat{H}

In basis of $\left\{ \begin{array}{l} |e, n\rangle \\ g_{n+1} \end{array}, \begin{array}{l} |g, n+1\rangle \\ e_n \end{array} \right\}$

$$\hat{H} = \begin{array}{c} g_{n+1} \\ e_n \end{array} \begin{bmatrix} \hbar \omega(n+1) & \hbar g \sqrt{n+1} \\ \hbar g \sqrt{n+1} & \hbar \omega n + \hbar \omega_2 \end{bmatrix}$$

Eigen values

$|g, n+1\rangle$

$|e, n\rangle$

$$E_{\pm} = \frac{\hbar\omega}{2} + \hbar\omega \left(n + \frac{1}{2}\right) \pm \frac{\hbar\omega}{2} \frac{\Delta}{\omega_n}$$

The diagram illustrates energy levels and transitions. On the left, two horizontal lines represent the ground state $|g\rangle$ and the excited state $|e\rangle$. To the right, a series of horizontal lines represent the harmonic oscillator energy levels. A dashed arrow indicates a transition from $|g\rangle$ to the first excited oscillator level. A double-headed vertical arrow indicates a transition between the $|e\rangle$ and $|g\rangle$ states, with a circled label $\frac{\hbar\omega}{2} \frac{\Delta}{\omega_n}$ indicating the energy shift. Above the oscillator levels, a double-headed vertical arrow is labeled $\frac{\hbar\omega}{2}$, and another double-headed vertical arrow is labeled $\hbar\omega \left(n + \frac{1}{2}\right)$.

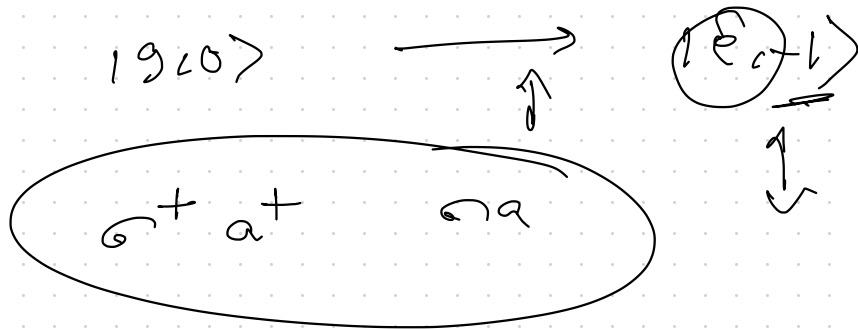
(f) $|g, 0\rangle$

$$\hat{H} |g, 0\rangle = 0 |g, 0\rangle$$

$$\frac{\hbar\omega}{2}$$

$$|g, n+1\rangle \longleftrightarrow |e, n\rangle$$

$$|g, n\rangle \longrightarrow |e, n-1\rangle$$



$$\begin{aligned}
 \textcircled{g} \quad |+\rangle &= \cos\left(\frac{\theta}{2}\right) |e, n\rangle \\
 &\quad + \sin\left(\frac{\theta}{2}\right) |g, n+1\rangle
 \end{aligned}$$

$$|-\rangle = \cos\frac{\theta}{2} |e, n\rangle - \sin\frac{\theta}{2} |g, n+1\rangle$$

$$\begin{aligned}
 \langle n+1 | \hat{H} | + \rangle &= E_+ \langle n+1 | + \rangle \\
 &= E_+ \cdot 1 = E_+
 \end{aligned}$$

$$\langle n-1 | \hat{H} | - \rangle = E_-$$

$$\begin{aligned}
 &\hbar\omega_1 \cos\frac{\theta}{2} + n\hbar\omega \cos\frac{\theta}{2} \\
 &\quad + \hbar g \cos\frac{\theta}{2} \sin\frac{\theta}{2} \\
 &= E_+
 \end{aligned}$$

$$\hbar \omega_{21} \cos \frac{\sigma}{2} + \hbar \omega_{21} \cos \frac{\sigma}{2}$$

$$- \hbar g \cos \frac{\sigma}{2} \sin \sigma$$

$$= 0$$

solve for θ

$$H = \begin{pmatrix} \omega_1 & \omega_2 \\ \omega_2 & \omega_1 \end{pmatrix}$$

$$\rightarrow H' = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$$

$$H' = U^\dagger H U$$

$$U^\dagger = \begin{pmatrix} \cos \frac{\sigma}{2} & \sin \frac{\sigma}{2} \\ -\sin \frac{\sigma}{2} & \cos \frac{\sigma}{2} \end{pmatrix}$$

Scattering Problem I

$$3. a) E_n^{\pm} = \frac{\hbar}{2} \omega_2 + \hbar \omega \left(n + \frac{1}{2} \right) \pm \frac{\hbar}{2} \Omega_n^{\Delta}$$

$$\Omega_n^{\Delta} = \sqrt{\Delta^2 + 4g^2(n+1)} \rightarrow 0$$

$$c) \text{ if } g=0, \quad \underline{\underline{\Omega_n^{\Delta} = \Delta}}$$

$$\checkmark E_n^{\pm} = \frac{\hbar}{2} \omega_2 + \hbar \omega \left(n + \frac{1}{2} \right) \pm \frac{\hbar}{2} \Delta$$

$$n=0 \Rightarrow E_0^{\pm} \Rightarrow \begin{aligned} E_0^{+} &= \hbar \omega & \underline{\Delta < 0} \\ E_0^{-} &= \hbar \omega_2 \end{aligned}$$

$$n=1 \Rightarrow E_1^{\pm} \Rightarrow \begin{aligned} E_1^{+} &= 5\hbar\omega/4 \\ E_1^{-} &= \frac{\hbar}{2} \left[2\omega_2 - \frac{1}{2}\omega \right] \end{aligned}$$

$$\searrow \quad \swarrow \quad \underline{\underline{\Delta > 0}}$$

$$\frac{5}{4}\hbar\omega \quad \hbar\omega_2 \quad \frac{\hbar}{2} \left[2\omega_2 - \frac{1}{2}\omega \right]$$

↓
correction Δ

b) AC Stark shift:

$$\Delta E_n^{\pm} = E_n^{\pm}(g) - E_n^{\pm}(g=0)$$

$$\Omega = \Delta \sqrt{1 + \frac{4g^2(n+1)}{\Delta^2}}$$

$$f(x) = \sqrt{1+x} \quad \leftarrow \text{Taylor Expand}$$

$$\Omega = \Delta \left[1 + \frac{1}{2} \left(\frac{4g^2(n+1)}{\Delta^2} \right) \dots \right] \rightarrow \textcircled{4}$$

$$\underline{\Omega = \Delta} \text{ at } g=0$$

$$\Delta E_n^{\pm} = E_n^{\pm}(g) - E_n^{\pm}(0)$$

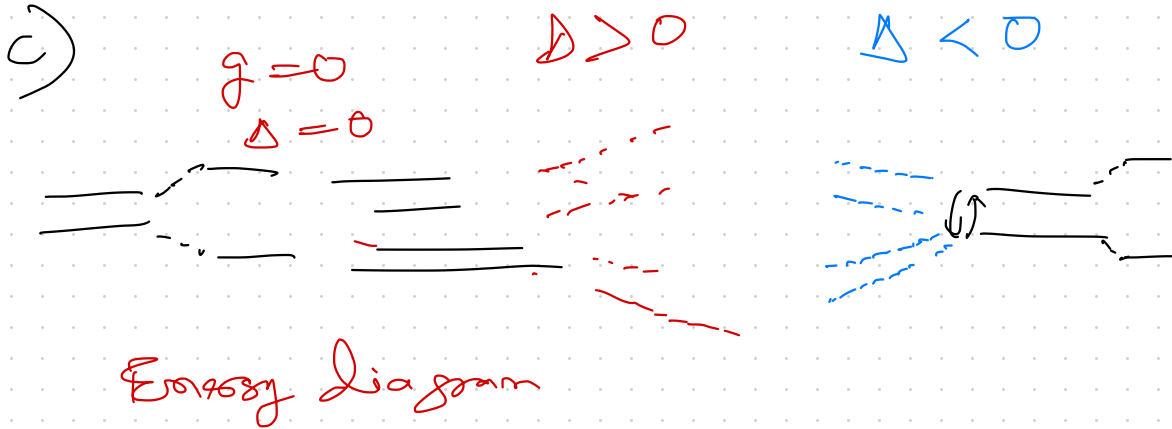
① ②

4th two terms = identical

$$\Delta E_n^{\pm} = \frac{\hbar}{2} (\Omega - \Delta) \quad \text{from } \textcircled{4}$$
$$= \frac{\hbar}{2} \left[\frac{\Delta}{2} + \frac{4g^2(n+1)}{\Delta^2} \right]$$

$$\Delta E_n^\pm = \pm \frac{\hbar}{\Delta} g^2 (n+1)$$

$\rightarrow n=0$
 $\Delta E_n \neq 0$



d)

$$\Delta E_n^\pm = \frac{\hbar n}{\Delta} [g^2]$$

lecture notes

$$\rightarrow g^2 = \frac{\hbar \omega_c}{E_c \sqrt{}} (\sin^2 k_z) \frac{d^2}{\hbar^2}$$

$$\star \frac{\text{Energy}}{\text{Volume}} \times \text{velocity}$$

$$c = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \Rightarrow c^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$g^2 \propto \frac{\text{Energy} \int (c^2 k^2) \sin^2 k_z}{\text{Volume } \hbar^2}$$

$$g^2 \propto \frac{\text{Energy } c^2}{\text{Volume}}$$