Generalized spin echo

Review and introduction

Let's start with reviewing photon echo and Ramsey sequence. Consider classical light field, the Hamiltonian of atom coupled to field is:

$$H = \begin{pmatrix} E_g & \\ & E_e \end{pmatrix} + \Omega \cos\omega t \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

solving optical Bloch equation, we find out in strong coupling case, this Hamiltonian leads to rotation about x axis. Dependent on coupling Ω, we can chose the how long the field is applied to rotate Bloch vector of atom by certain angle (e.g., $\frac{\pi}{2}$ and pulse, see animation). Now let's generalize the pulses to more atoms. In simple case: ① no interatomic interaction

② no decoherence

We are interested in following questions:

1. What happens if we vary time seperation of $\frac{\pi}{2}$ pulses in a Ramsey sequence?
2. If atoms have different frequencies, applying photon echo preserves the population.

To address these questions, we simulate Ramsey and photon echo sequences using qutip. Plot excited state population as a function of pulse seperation time and compare different sequences.

Spin - Spin interaction

We have seen free atoms dynamics in Ramsey and "echo" pulses. What if atoms have interactions?

Since we got the idea from Ising model, let's call the qubits as "spins" in stead of "atoms". We'll check the effect out in a simple case:

① two spins have the same frequency and coupling

② interaction depends on whether spins direction orientations are the same.

③ classical field

④ decoherence is ignored

Therefore, we write the Hamiltonian as:

$$H = \sum_{i=1}^{2}\left(\frac{1}{2}\omega_0 \sigma_z^i + \Omega^i \cos\omega t\, \sigma_x^i\right) + J\,\sigma_z^1 \sigma_z^2$$

We are interested in following questions:

1. What can we find in a Ramsey sequence?
2. Is anything preserved in a spin echo sequence?
3. Can we still measure frequence of a spin despite its interaction with other spins?

1. To understand what's happening in a Ramsey sequence under interaction case, we can analytically calculate the expectation value.

$|\downarrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad |\uparrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad \sigma_z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$

$\frac{\pi}{2}$ pulse about $x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$

π pulse about $x = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

Do tensor product for all these vectors and operatos to be in the multi-spin basis.

Then solve Schrodinger equation and plot the expectation values in terms of free evolving time between two $\frac{\pi}{2}$ pulses.

Fortunately, the Hamiltonian is diagonal if we take the field part out, so use pulse operators instead of including field term in Schrodinger equation.

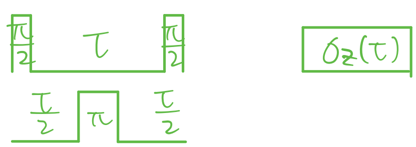
2. Use qutip to simulate. First benchmark the code without interaction. Then run spin echo and find out what's preserved.

3. Since the interaction is state dependent, a sequence that seperates Ramsey pulses and π pulse into two spins may remove the effect of inter-spin interaction on the spin running Ramsey sequence.

sequence:

spin 1: $|\uparrow\rangle$

spin 2: $|\uparrow\rangle$

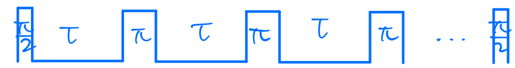


Dynamic decoupling

Although in the previous simulations, we solved master equation but we haven't take decoherence into aceount. Now if we consider a multi-atom system coupled to a bath, can we generate a sequence to prevent the system from decay?

Try this pulse:

atom: $|e\rangle$



measure atom state after sequence for different τ and compare with Ramsey sequence.

The idea is that once the atom evolves, do a π pulse to "reset" its state. As we increase the frequency of the π pulse, the state is kept unchanged.

Consider the system is coupled to a bath via H_B, evolution during time τ is $U_B(\tau) = e^{-i\tau H_B}$.

Then apply a strong unitary pulse P only to the system, and let it evolve for time τ again followed by another unitary pulse $P^\dagger$. Overall evolution is:

$$
\begin{aligned}
U_B(2\tau) &= P^\dagger e^{-i\tau H_B} P \, e^{-i\tau H_B} \\
&= e^{-i\tau P^\dagger H_B P} e^{-i\tau H_B} \\
&= e^{i\tau H_B} e^{-i\tau H_B} \\
&= \mathbb{1}
\end{aligned}
$$