CHM 696: Quantum Optics

Problem Set 4 (Bloch vector and Ramsey interferometery)

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I Bloch-vector and spontaneous emission

(a) Solve the optical Bloch equations for the case of a vanishing light field $\left(\Omega_0 = \frac{\mathcal{V}}{\hbar} = 0 \text{ and } \Delta = 0\right)$ and finite spontaneous emission ($\gamma > 0$).

(b) Assume that the atom is initially in the excited state. Calculate and draw the Bloch vector at the times $t = 0, t = \frac{\ln(2)}{\gamma}$ and $t = \infty$. Sketch its path on the Bloch sphere for the above case.

(c) Repeat the last exercise for the initial state given by $R_1 = 1, R_2 = 0, R_3 = 0$.

(d) Now assume the presence of a resonant light field $\Delta = 0$, $\Omega_0 > 0$. What is the steady state value of the Bloch vector? Calculate and draw the steady state Bloch vector for $\Omega_0 = 0$, $\Omega_0 = \gamma$, $\Omega_0 = \infty$.

II Ramsey-Sequence

In the lecture we discussed the photon echo technique. It is closely related to the Ramsey sequence, which consists of two $\pi/2$-pulses with a delay τ between them and is used e.g. in atomic clocks to precisely measure the transition frequency of an atom.

(a) After the second $\pi/2$-pulse, the population in the excited state is measured. How does the measured fraction depend on the time τ between the pulses and the detuning Δ of the light?

Note: Assume that the Rabi frequency during the pulses is much larger than the detuning ($\Omega_0 \gg \Delta$), such that the $\pi/2$ pulses are short compared to $1/\Delta$. *Hint: construct rotation matrices around the $\hat{x}$- and $\hat{z}$-axis.*

(b) The Ramsey-sequence is also widely used in nuclear magnetic resonance (NMR) techniques, where the two-level system of interest consists of two nuclear spin states in the presence of a magnetic field and the $\pi/2$-pulses are microwave pulses. In this case, the resonance frequency – and thereby the detuning – depends strongly on the magnetic field. In an ensemble, magnetic field inhomogeneities create different detunings within the ensemble and can lead to a fast dephasing of the signal, i.e. the Ramsey fringes decay as a function of the waiting time τ. Is this a homogeneous or an inhomogeneous broadening effect?

As discussed in the lecture, one common technique to avoid this is the so-called spin-echo sequence, where an additional π-pulse is inserted in the middle of the waiting time ($\tau/2$) between the two $\pi/2$-pulses. Explain graphically the effects of the above dephasing on the Bloch sphere and how the additional spin-echo pulse improves the contrast of the Ramsey fringes!

III Numerical exercise: driven two-level system

In the interaction picture the Hamiltonian $\hat{H}$ of a driven two-level system can be expressed as

$$\hat{H} = -\hbar\Omega_0 \cos(\omega t) \left(e^{-i\omega_{21}t}|1\rangle\langle 2| + e^{i\omega_{21}t}|2\rangle\langle 1|\right)$$

with $\omega_{21} = \omega_2 - \omega_1$.

In this exercise we want to investigate numerically the rotating-wave approximation (RWA).

(a) Determine the equations of motion from the Schrödinger equation, $i\hbar\partial_t|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$, and projection onto the components of the wave function.

(b) Take the Hamiltonian defined above and use the following values:

$$\begin{aligned}\omega_{21} &= 20\\ \omega &= 21\\ \Omega_0 &= 1\end{aligned}$$

Let the system be in state $|1\rangle$ at $t = 0$, and evolve the state up to $t = 5$. Plot, on the same axes the analytic solution for $P_1 = |\langle 1|\psi(t)\rangle|^2$ obtained using the rotating-wave approximation (detuned Rabi oscillations).

(c) You should notice some small "wiggles" in the exact solution, on top of the Rabi oscillation. Derive the amplitude and frequency of these wiggles (up to factors of proportionality) by considering the counter-rotating terms of the full Hamiltonian.

IV Journal club: Ramsey fringes in a Quantum Dot

The next paper we will discuss demonstrates a quantum optics effect realized in a semiconductor implementation: "Ramsey fringes in a single InGaAs/GaAs quantum dot", Physica Status Solidi B 243, No. 10, 2229-2232 (2006).

A quantum dot is a structure in a semiconductor which is so small that, similar to an atom, it has discrete energy levels for electrons. For our purposes, only two of these states are relevant and you can see the entire device as a generic two-level system - the exact nature of the two states does not matter here. The same applies for the "photocurrent": This is simply a method to measure the excited-state probability of the quantum dot.

Please read the paper, and pay special attention to the following questions. These should also be answered – but only in a few keywords each.

(a) What is the sequence of coupling pulses used here?

(b) Why is there a "detuning" – what is detuned from what? And how does that relate to the voltage the authors refer to?

(c) How do the authors argue that the spectral resolution is enhanced by the two-pulse method?

(d) What is the main result of the paper, and how does the approximate shape of the results shown in the graphs arise?