Electric Field of Squeezed Coherent States

Single-mode field:

$$\hat{E}_x(z,t) = \mathcal{E}_0 \sin kz \left(\hat{a}\, e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}\right)$$

recall $\langle \hat{a} \rangle_{\alpha,\xi} = \alpha$, $\langle \hat{a}^\dagger \rangle_{\alpha,\xi} = \alpha^*$

$\alpha = |\alpha| e^{i\theta}$

$$\Rightarrow \langle \hat{E}_x(z,t) \rangle_{\alpha,\xi} = \mathcal{E}_0 \sin kz \left(|\alpha| e^{i\theta} e^{-i\omega t} + |\alpha| e^{-i\theta} e^{i\omega t} \right)$$

$$= 2\mathcal{E}_0 |\alpha| \sin kz \cos(\omega t - \theta)$$

$\Rightarrow$ no sign of squeezing here!

$$\langle \hat{E}_x^2(z,t) \rangle_{\alpha,\xi} = \mathcal{E}_0^2 \sin^2 kz \langle \hat{a}^2 e^{-i2\omega t} + \hat{a}^{\dagger 2} e^{i2\omega t} + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger \hat{a} \rangle_{\alpha,\xi}$$

$$= 2\mathcal{E}_0^2 \sin^2 kz \left[|\alpha|^2 \cos(2\omega t - 2\theta) - \cos(2\omega t - \phi) \sinh r \cosh r + |\alpha|^2 + \sinh^2 r + \frac{1}{2} \right]$$

$$\boxed{(\Delta \hat{E}_x)_{\alpha,\xi} = \sqrt{2}\, \mathcal{E}_0 |\sin kz| \left[\cosh(2r) - \cos(2\omega t - \phi) \cdot \sinh(2r) \right]^{1/2}}$$

Special cases:

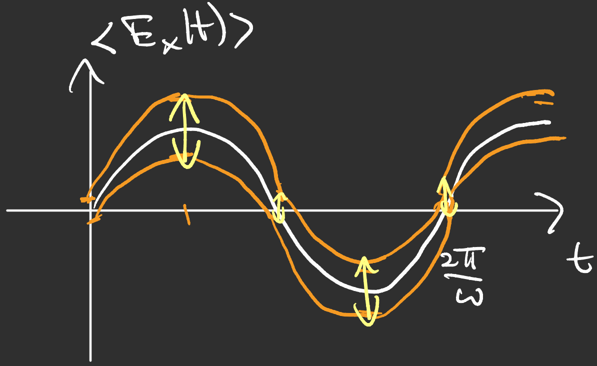
1) $r=0$ (no squeezing): $(\Delta\hat{E}_x)_{\alpha,\xi} = \sqrt{2}\,\mathcal{E}_0|\sin kz|$

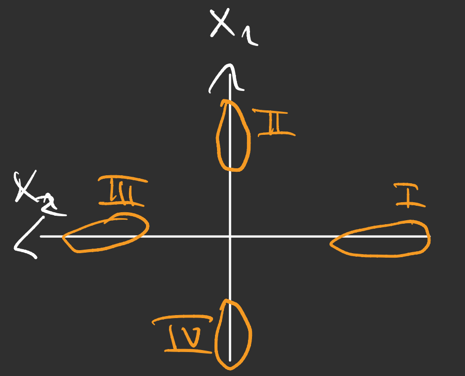
(as in coherent state)

2) $r\neq 0,\ \phi=0$: fluctuations change over time!

at $\omega t = 0$: $(\Delta\hat{E}_x)_{\xi,\alpha}(z,0) = \sqrt{2}\,\mathcal{E}_0|\sin kz|\,e^{-r}$

at $\omega t = \frac{\pi}{2}$: $(\Delta\hat{E}_x)_{\xi,\alpha}\left(z,\frac{\pi}{2\omega}\right) = \sqrt{2}\,\mathcal{E}_0|\sin kz|\,e^{r}$



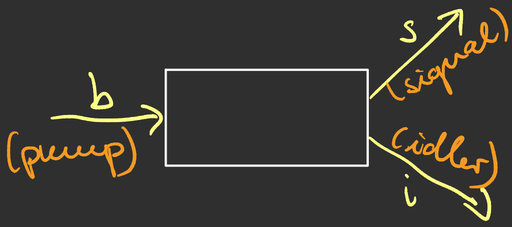


8.3 Generation of squeezed light

Typical mechanisms:

→ i) Degenerate parametric downconversion (PDC) - Kerr NL ($\chi^{(2)}$)

ii) Four-wave mixing ("$\chi^{(3)}$")



process such that:

$$\omega_p = \omega_s + \omega_i$$

$$\vec{k}_p = \vec{k}_s + \vec{k}_i$$

Effective Hamiltonian:

$$H_{PDC} = \hbar k \left(\overbrace{\hat{a}_s^{\dagger} \hat{a}_i^{\dagger} \hat{b}}^{\mathrm{I}} + \overbrace{\hat{a}_s \hat{a}_i \hat{b}^{\dagger}}^{\mathrm{II}} \right)$$

$\chi^{(2)}$ coupling constant

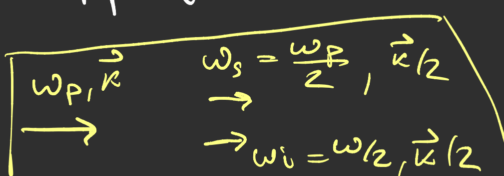
pump

Strong beam: I dominant and

$$\hat{b} \xrightarrow{\text{classical}} \beta_p \cdot e^{i\phi_p}$$

- degenerate case: $\hat{a}_s = \hat{a}_i = \hat{a}$

$$\Rightarrow \hat{H}_{PDC} = \hbar k \beta_p \left(\hat{a}^{\dagger 2} e^{-i\phi_p} + \hat{a}^2 e^{i\phi_p} \right)$$



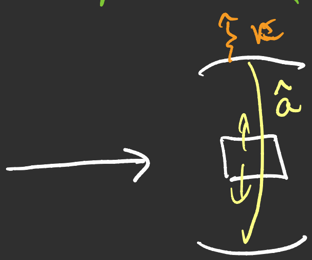
Time evolution operator:

$$U(t,t_0) = e^{-i\hat H (t-t_0)/\hbar} \overset{t_0=0}{=} e^{-i\frac{\cancel{\hbar}\kappa\beta_p}{\cancel{\hbar}}\cdot t\left(\hat a^{\dagger 2} e^{-i\phi_p} + \hat a^2 e^{i\phi}\right)}$$

with $\boxed{\alpha = 2\kappa\beta_p e^{i\phi_p}\cdot t}$ $U(t)$ corresponds to $\hat S(\alpha)$

→ time evolution acts like squeezing
(limit $t \to \infty$ makes squeezing very strong)

ASIDE ↓



$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \dots$$

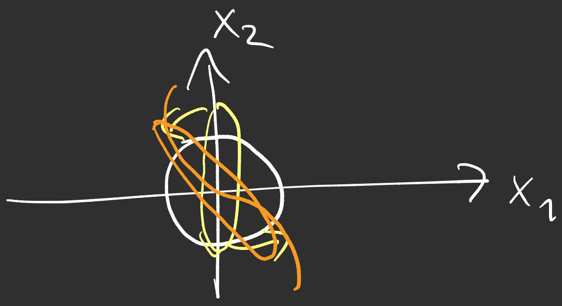
$$H = \hbar\omega \hat a^\dagger \hat a + \hbar\kappa\left(\hat a^{\dagger 2} e^{-i\phi} + \hat a^2 e^{i\phi}\right)$$

$$\partial_t \hat a = i[\hat H, \hat a] = -i\omega \hat a - 2i\kappa\, \hat a^\dagger e^{-i\phi}$$

$$\tilde{\hat a} = a e^{+i\omega t} \longleftrightarrow \tilde{\hat a}^\dagger = \hat a^\dagger e^{-i\omega t} = \tilde{\hat a}^\dagger$$

$$e^{-i\omega t}\partial_t \tilde{\hat a} = -2i\kappa e^{i\omega t} \tilde{\hat a}^\dagger e^{-i\phi}$$

$$\partial_t \hat{\tilde{a}} = -2i\kappa\, e^{2i\omega t}\, \hat{\tilde{a}}^{+}\, e^{-i\phi}$$
$$\partial_t \hat{\tilde{a}}^{+} = 2i\kappa\, e^{-2i\omega t}\, \hat{\tilde{a}}\, e^{i\phi}$$



↑ ASIDE

9. Optical Coherence Measures

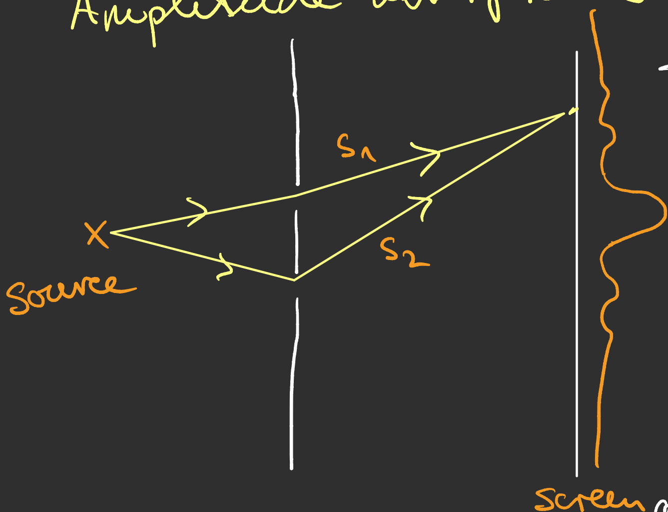
Complete coherence $\Rightarrow$ phase relationship among various component preserved over time, no phase-randomizing noise sources.

Distinction: 1^{st}-order coherence : Young's double slit

2^{nd}-order coherence : Hanbury-Brown Twiss (HBT)

} exp. realization

Amplitude Interference



$\to$ Detector:

Photographic film

$Ag^+X^- + h\nu \to Ag^+ + X + e^-$

halide

CCD / APD

charge coupled device / avalanche photodetector

absorbed photon gives rise to a photocurrent

In either case: detection means photon absorption $\hat{a}$

$\Rightarrow$ (as opposed to a quantum nondemolition measurement)
QND

Recall: $\hat{E}(\vec{r},t) = \hat{E}^{(+)}(\vec{r},t) + \hat{E}^{(-)}(\vec{r},t)$ ← drop brackets in the following

$$= i \sum_{\vec{k},s} \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 V}}\, \hat{e}_{\vec{k},s} \left[\hat{a}_{\vec{k},s} e^{i\vec{k}\vec{r}-i\omega t} - \hat{a}^{+}_{\vec{k},s} e^{-i\vec{k}\vec{r}+i\omega t} \right]$$

→ i.e. positive frequency part of electric field corresponds to photon annihilation

→ destructive detection $\hat{E}^{(+)} \sim e^{-i\omega t}$, $\omega > 0$

↳ $\langle F | \hat{H}^{(I)} | I \rangle = -\langle e | \vec{d} | g \rangle \langle f | \hat{E}^{(+)} | i \rangle$

Double slit:

→ $\hat{E}^{+}(\vec{r},t) = \hat{E}^{+}_{1}(\vec{r},t_1) + \hat{E}^{+}_{2}(\vec{r},t_2)$; $t_{1,2} = t - \frac{s_{1,2}}{c}$

(retarded times)

→ the part of the E-field intensity which is responsible for absorption

Fermi's → Golden Rule $\sum_f |\langle f | \hat{E}^{(+)} | i \rangle|^2 = \sum_f \langle f | \hat{E}^{(+)} | i \rangle \langle i | \hat{E}^{(-)} | f \rangle$

$$= \langle i | \hat{E}^{(-)} \hat{E}^{(+)} | i \rangle$$

$$|\hat{E}^{+}(\vec{r},t)|^2 = \underbrace{\hat{E}^{(-)}}_{\text{destroy phot.}}(\vec{r},t)\, \hat{E}^{(+)}\underbrace{(\vec{r},t)}_{\text{destroy phot.}}$$

$$= |\hat{E}^{+}_{1}(\vec{r},t_1)|^2 + |\hat{E}^{+}_{2}(\vec{r},t)|^2 + \underbrace{2\,\mathrm{Re}\{\hat{E}^{-}_{1}(\vec{r},t_1)\hat{E}^{+}_{2}(\vec{r},t_2)\}}_{\text{interference term}}$$