$$\dot{\rho}_{gg} = \gamma \rho_{ee} + \frac{i}{2\hbar}\Omega\Big(\underbrace{e^{-i\omega t}\rho_{ge}}_{=\tilde{\rho}_{eg}^*} - \underbrace{e^{i\omega t}\rho_{eg}}_{=\tilde{\rho}_{eg}}\Big)$$

$$\dot{\rho}_{ee} = -\gamma \rho_{ee} - \frac{i}{2\hbar}\Omega\left(e^{-i\omega t}\rho_{ge} - e^{i\omega t}\rho_{eg}\right)$$

$$\dot{\tilde{\rho}}_{ge} = i\left(\Delta - \frac{\gamma}{2}\right)\tilde{\rho}_{ge} - \frac{i}{2\hbar}\Omega\left(\rho_{ee} - \rho_{gg}\right)$$

$$\partial_t \rho_{gg} = \gamma \rho_{ee} - \frac{i}{2\hbar}\Omega \cdot 2i \,\mathrm{Im}(\tilde{\rho}_{eg})$$

$$\partial_t \rho_{ee} = -\gamma \rho_{ee} + \frac{i}{2\hbar}\Omega \cdot 2i \,\mathrm{Im}(\tilde{\rho}_{eg})$$

$$\partial_t \tilde{\rho}_{ge} = \left(i\Delta - \frac{\gamma}{2}\right)\tilde{\rho}_{ge} - \frac{i}{2\hbar}\Omega\left(\rho_{ee} - \rho_{gg}\right)$$

$$\partial_t \tilde{\rho}_{ge}^* = \left(-i\Delta - \frac{\gamma}{2}\right)\tilde{\rho}_{ge}^* + \frac{i}{2\hbar}\Omega\left(\rho_{ee} - \rho_{gg}\right)$$

→ get equations for $\mathrm{Re}(\tilde{\rho}_{eg})$ and $\mathrm{Im}(\tilde{\rho}_{eg})$

$$\partial_t \underbrace{\left(\tilde{\rho}_{ge} + \tilde{\rho}_{ge}^*\right)}_{} = -\frac{\gamma}{2}\left(\tilde{\rho}_{ge} + \tilde{\rho}_{ge}^*\right) + i\Delta\left(\tilde{\rho}_{ge} - \tilde{\rho}_{ge}^*\right)$$

$$\partial_t \underbrace{\left(\tilde{\rho}_{ge}^* - \tilde{\rho}_{ge}\right)}_{} = -i\Delta\left(\tilde{\rho}_{ge}^* + \tilde{\rho}_{ge}\right) - \frac{\gamma}{2}\left(\tilde{\rho}_{ge}^* - \tilde{\rho}_{ge}\right) + \frac{i}{2\hbar}\Omega \cdot 2 \cdot \left(\rho_{ee} - \rho_{gg}\right)$$

$$\partial_t \underbrace{(\rho_{gg} - \rho_{ee})} = -\underbrace{2\gamma\rho_{ee}}_{=-\gamma(R_3+1)} - \frac{\Omega}{\hbar} \cdot 2\,\mathrm{Im}(\tilde{\rho}_{eg})$$

6.6 Vector model for the OBE (optical Bloch equations)

Introduce $\vec{R} = \hat{e}_1 R_1 + \hat{e}_2 R_2 + \hat{e}_3 R_3 \leftarrow$ Bloch vector

$R_1(t) = 2\,\mathrm{Re}(\tilde{\rho}_{eg}(t)) \leftarrow$ "dispersive part of Bloch vector"

$R_2(t) = 2\,\mathrm{Im}(\tilde{\rho}_{eg}(t)) \leftarrow$ "absorptive part of Bloch vector"

$R_3(t) = \rho_{ee}(t) - \rho_{gg}(t) \leftarrow$ "population inversion"

→ Convert equations of motion of $\rho_{gg}, \tilde{\rho}_{eg} \ldots$ into equations of motion for Bloch vector

$$\partial_t R_1 = -\frac{\gamma}{2} R_1 + \Delta R_2$$

$$\partial_t R_2 = -\Delta R_1 - \frac{\gamma}{2} R_2 + \frac{\Omega}{\hbar} R_3$$

$$\partial_t R_3 = -\gamma(R_3+1) - \frac{\Omega}{\hbar} R_2$$

$$\boxed{\partial_t \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} -\vartheta/t_0 \\ 0 \\ -\Delta \end{pmatrix} \times \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} - \gamma \begin{pmatrix} R_1/2 \\ R_2/2 \\ R_3+1 \end{pmatrix}}$$

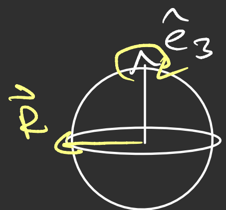
→ Bloch vector rotates around $\begin{pmatrix} -\vartheta/t_0 \\ 0 \\ -\Delta \end{pmatrix}$ and is damped by γ.

without decay: $\partial_t \vec{R} = \vec{\Omega} \times \vec{R}$

To visualize the dynamics of the Bloch vector

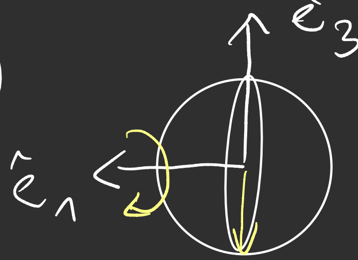
a) no decay: $\gamma = 0$

i)



$\vartheta \to 0$ (weak coupling)

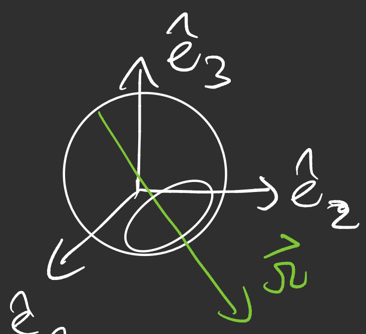
ii)



$\Delta \to 0$ (resonant field)

$\omega = \omega_0$

iii)



drive + detuning

+ depth

⇒ Mathematica script

→ Nielsen + Chuang: Quantum Information

6.7 Homogeneous and inhomogeneous broadening

environment

$|e\rangle$

$|g\rangle$

$\Rightarrow$ There are 2 types of broadening mechanisms

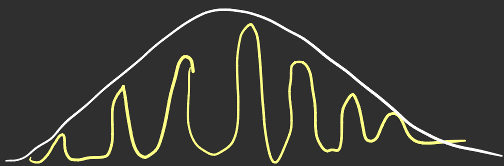
Homogeneous broadening: all 2LSs are affected in the same way

$\rightarrow$ examples: - spont. rad. decay
- thermal broadening

(difficult to combat)

Inhomogeneous broadening: $\rightarrow$ each 2LS experiences its own due to local strain/impurities (QDs) or Doppler shifts (atoms)

$\Rightarrow$ result: emission line is a superposition of a large collection of homogeneously broadened lines



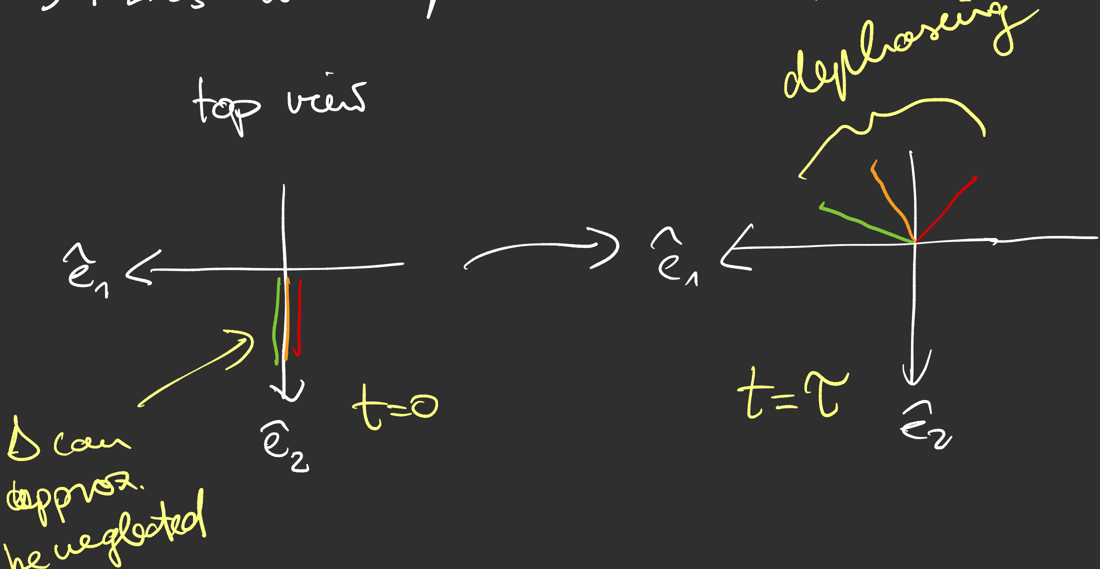
6.8 Photon echo

→ initialize a system of inhomogeneously broadened 2LSs in the ground state

→ drive a $\pi/2$ pulse

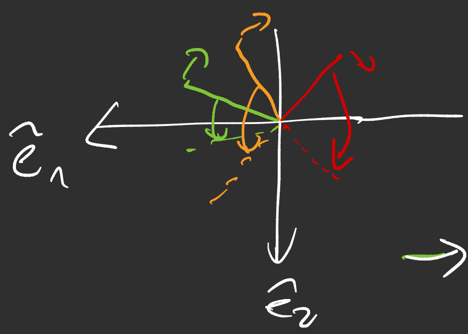
→ TLSs will dephase in xy plane

top view

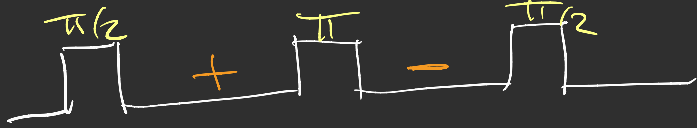


→ At $t=\tau$, apply π-pulse → $|g\rangle \leftrightarrow |e\rangle$

$$\begin{pmatrix} R_1 \to R_1 \\ R_2 \to -R_2 \\ R_3 \to -R_3 \end{pmatrix}$$



→ At $t = 2\tau$, all spins re-align



If no π-pulse is applied, the second $\pi/2$ rotation would yield no polarization (no echo)

$\Rightarrow$ sequence can be repeated

$$\pi/2 - \pi - \pi - \pi - \dots \pi - \pi/2$$

$\Rightarrow$ dynamical decoupling of 2LS from bath

$\rightarrow$ Comment: works because inhomogeneous broadening is a reversible process.