Class expectations

- light-matter interaction
- EIT + Autler-Townes
- nonlinear optics
- Raman spectroscopy
- beam splitters
- mirror effect on temporal compression
- BBO crystals
- photon counting
- Generation of entangled photons
- in-depth understanding of quantum optics
- Excitons (Frenkel + Wannier)
- General quantum optics
- "get to work properly"
- relation to quantum information + computation

- coherence in photon feild
- generation + understanding of nonclassical light
- $g^{(n)}(\tau)$ correlations
- Wigner / Husimi functions
- entanglement theory
- Cavities + QED
- bosonic behavior of photons
- QFT
- research-oriented course
- coherent dynamics + ET
- relation of QO to experiment
- few- and many-body physics
- cavities

TOPICS

(QUANTUM) LIGHT FIELDS

- beam splitters
- photon counting
- coherence in photon field
- $g^{(n)}(\tau)$ correlations
- Wigner / Husimi functions
- entanglement theory

FIELD-MATTER INTERACTION

- light-matter interaction
- EIT + Autler-Townes
- Raman spectroscopy
- Generation of entangled photons
- generation + understanding of nonclassical light
- coherent dynamics + EIT

OTHER TOPICS

- nonlinear optics
- mirror effect on temporal compression
- BBO crystals
- Excitons (Frenkel + Wannier)
- relation to quantum information + computation
- QFT
- few- and many-body physics
- cavities
- Cavities + QED
- bosonic behavior of photons

COURSE STYLE

- in-depth understanding of quantum optics
- General quantum optics
- "get to work properly"
- research-oriented course
- relation of QO to experiment

will be covered

will be covered if there is time

will likely not be covered

Aug. 29

→ acceptable that $|n\rangle$ is defined while $\hat{E}$ is uncertain

$$[\hat{n}, \hat{E}_x] = \varepsilon_0 \sin(kz)(\hat{a}^+ - \hat{a})$$

If two operator $\hat{A}, \hat{B}$ do not commute, they satisfy an uncertainty relation

$$[\hat{A}, \hat{B}] = \hat{C} \Rightarrow (\Delta\hat{A})(\Delta\hat{B}) \geq \frac{1}{2}|\langle\hat{C}\rangle|$$

$$\underbrace{(\Delta n)}_{=0}(\Delta E) \geq \frac{1}{2}\varepsilon\ |\sin kz|\ \underbrace{|\langle\hat{a}^+\rangle - \langle\hat{a}\rangle|}_{=0}$$

2.2 Quadrature operators (single-mode field)

Define: $\hat{X}_1 \equiv \frac{1}{2}(\hat{a} + \hat{a}^+)$; $\hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^+)$

$\sim$ "q" $\quad\quad \sim$ "p"

$$\hat{E}_x(z,t) = \varepsilon(\hat{a}e^{-i\omega t} + \hat{a}^+ e^{i\omega t})\sin(kz)$$

$$= 2\varepsilon \sin(kz)(\hat{X}_1\cos(\omega t) + \hat{X}_2\sin(\omega t))$$

$$[\hat{X}_1, \hat{X}_2] = \frac{1}{4i}[\hat{a} + \hat{a}^+, \hat{a} - \hat{a}^+] = \dots = \frac{i}{2}$$

$$\boxed{(\Delta X_1)(\Delta X_2) \geq \frac{1}{4}}$$ (in general)

In number state: $\langle n|\hat{X}_1|n\rangle = \langle n|\hat{X}_2|n\rangle = 0$

$$\langle n|\hat{X}_1^2|n\rangle = \frac{1}{4}\langle n|\hat{a}^2 + \hat{a}\hat{a}^+ + \hat{a}^+\hat{a} + \hat{a}^{+2}|n\rangle$$
$$= \frac{1}{4}(2n+1)$$
$$\langle n|\hat{X}_2^2|n\rangle = \frac{1}{4}(2n+1)$$
$$(\Delta X_1) = (\Delta X_2) = \frac{1}{2}\sqrt{2n+1}$$

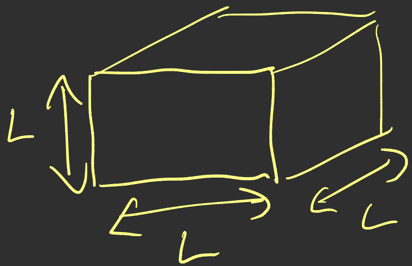
$$\Rightarrow \boxed{(\Delta X_1)(\Delta X_2) = \frac{1}{4}(2n+1)}$$ (for number states)

→ Quadratures share uncertainty for number states

→ minimal at $n=0$

$$(\Delta X_1)(\Delta X_2) = \frac{1}{4}$$

2.3 Generalization to multi-mode fields



$$\vec{E}(0,y,z) = \vec{E}(L,y,z)$$
$$\vec{E}(x,0,z) = \vec{E}(x,L,z)$$
$$\vec{E}(x,y,0) = \vec{E}(x,y,L)$$
$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0$$

$\Rightarrow$ PBC give running wave solution

$$e^{i\vec{k}\vec{r}}, \quad k = \frac{2\pi}{L}(m_x, m_y, m_z)$$

$$m_{x,y,z} = 0, \pm 1, \pm 2, \ldots \quad \Bigg| \quad \omega_{\vec{k}} = c|\vec{k}| = \frac{2\pi}{L} c \left(m_x^2 + m_y^2 + m_z^2\right)^{1/2}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \vec{k} \cdot \vec{e} = 0 \Leftrightarrow \text{transverse wave}$$

$\rightarrow$ label each mode by $\vec{k}, s$ where s refers to two ortogonal field polarizations

$H \Leftrightarrow V$

(horizontal) (vertical)

$LCP \Leftrightarrow RCP$

(left circular polarization) (right c. p.)

generalize Hamiltonian

$$H = \sum_{\vec{k}, s} \frac{1}{2} \left(\hat{p}^2_{\vec{k},s} + \omega^2_{\vec{k}} \hat{q}^2_{\vec{k},s} \right)$$

$\rightarrow$ independent harmonic oscillators

Impose:

$$[\hat{q}_{\vec{k}s}, \hat{q}_{\vec{k}'s'}] = [\hat{p}_{\vec{k}s}, \hat{p}_{\vec{k}'s'}] = 0$$

$$[\hat{q}_{\vec{k}s}, \hat{p}_{\vec{k}'s'}] = i\hbar\, \delta_{\vec{k},\vec{k}'}\, \delta_{ss'}$$

In terms of $\hat{a}, \hat{a}^\dagger$

$$\hat{H} = \sum_{\vec{k}s} \hbar \omega_{\vec{k}} \Big(\underbrace{\hat{a}^\dagger_{\vec{k}s} \hat{a}_{\vec{k}s}}_{\hat{n}_{\vec{k}s}} + \frac{1}{2} \Big)$$

→ standard comm. relations

→ A multimode photon number can be written as a direct product

$$|n_1\rangle \otimes |n_2\rangle \otimes \dots = |n_1, n_2, \dots\rangle$$

$$= |\{n_i\}\rangle$$

Completeness

$$\boxed{\sum_{n_j=0}^{\infty} |n_j\rangle\langle n_j| = \mathbb{1}_j}$$

Example: $|\{0\}\rangle = |0, 0 \dots\rangle$

multi-mode vacuum

A general number state can be generated as

$$|\{n_j\}\rangle = \prod_j \frac{(\hat{a}_j^{+})^{n_j}}{\sqrt{n_j!}} |0\rangle$$

where $\hat{a}_j |n_1 \ldots n_j \ldots\rangle = \sqrt{n_j}\, |n_1 \ldots n_j - 1 \ldots\rangle$

$\hat{a}_j^{+} |n_1 \ldots n_j \ldots\rangle = \sqrt{n_j + 1}\, |n_1 \ldots n_j + 1 \ldots\rangle$

Electric field operator

$$\hat{\vec{E}}(\vec{r},t) = \hat{\vec{E}}^{(+)}(\vec{r},t) + \hat{\vec{E}}^{(-)}(\vec{r},t)$$

$$\hat{\vec{E}}^{(+)}(\vec{r},t) = i \sum_{\vec{k}s} \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 V}}\, \hat{e}_{\vec{k}s}\, \underline{\hat{a}_{\vec{k}s}(t)}\, e^{ikr}$$

$$\hat{\vec{E}}^{(-)}(\vec{r},t) = \left[\hat{\vec{E}}^{(+)}(\vec{r},t)\right]^{+}$$

Similarly:

$$\vec{B}(\vec{r},t) = \hat{\vec{B}}^{(+)}(\vec{r},t) + \hat{\vec{B}}^{(-)}(\vec{r},t)$$

$$\uparrow$$

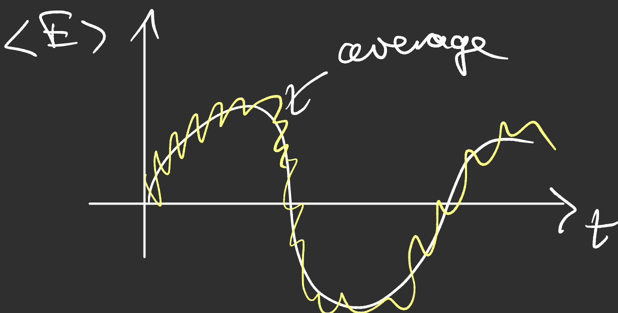
$$\frac{i}{c} \sum_{\vec{k}s} (\vec{k} \times \hat{e}_{ks}) \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 V}}\, \hat{a}_{\vec{k}s}(t)\, e^{i\vec{k}\vec{r}}$$

3. Coherent states

→ Number state $|n\rangle$ showed $\langle n|\hat{E}|n\rangle = 0$

→ Quantum states $|\alpha\rangle$ with

$$\langle\alpha|\hat{E}(\vec{r},t)|\alpha\rangle \sim \sin(\omega t - \vec{k}\vec{r} + \theta) \, ?$$



$$E_x \sim i(\hat{a} - \hat{a}^+)$$

$$\langle\alpha|\hat{a} - \hat{a}^+|\alpha\rangle \neq 0$$

→ look for for right (left) eigenstates of $\hat{a}$ $(\hat{a}^+)$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \longleftrightarrow \langle\alpha|\hat{a}^+ = \alpha^*\langle\alpha|$$

$$\Rightarrow \langle\alpha|\hat{E}_x|\alpha\rangle = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\langle\alpha|\hat{a}e^{i\vec{k}\vec{r}}e^{-i\omega t} - \hat{a}^+e^{-i\vec{k}\vec{r}}e^{i\omega t}|\alpha\rangle$$

$\alpha|\alpha\rangle$ $\quad$ $\alpha^*\langle\alpha|$

$$= 2|\alpha|\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\sin(\omega t - \vec{k}\vec{r} + \theta)$$

phase of α