1. Q-Representation

$$Q(\alpha) = \frac{1}{\pi} \langle\alpha|\hat{\rho}|\alpha\rangle$$

i) $\hat{\rho} = |\Psi\rangle\langle\Psi|$

$$\Rightarrow Q(\alpha) = \frac{1}{\pi}\langle\alpha|\Psi\rangle\langle\Psi|\alpha\rangle$$

$$= \frac{1}{\pi}\langle\alpha|\Psi\rangle(\langle\alpha|\Psi\rangle)^\dagger = \frac{1}{\pi}|\langle\alpha|\Psi\rangle|^2$$

ii) $|\alpha'\rangle$

$$\langle\alpha|\alpha'\rangle = e^{\alpha'\alpha^* - \frac{|\alpha'|^2}{2} - \frac{|\alpha|^2}{2}}$$

$$|\alpha'\rangle = e^{-\frac{|\alpha'|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha'^n}{\sqrt{n!}}|n\rangle$$

$$* \langle\alpha| = (|\alpha\rangle)^\dagger = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(\alpha^*)^n}{\sqrt{n!}}\langle n|$$

$$\Rightarrow \langle\alpha|\alpha'\rangle = e^{-\frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}} \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} \frac{(\alpha')^n}{\sqrt{n!}} \frac{(\alpha^*)^{n'}}{\sqrt{n'!}} \langle n|n'\rangle$$

$$= e^{-\frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}} \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty} \frac{(\alpha')^n}{\sqrt{n!}} \frac{(\alpha^*)^{n'}}{\sqrt{n'!}} \delta_{nn'}$$

$$= e^{-\frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}} \sum_{n=0}^{\infty} \frac{(\alpha^*\alpha')^n}{n!}$$

$$= \underline{e^{\alpha'\alpha^* - \frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}}}$$

$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle$$

$$= \frac{1}{\pi}|\langle\alpha|\alpha'\rangle|^2 = \frac{1}{\pi}\left(e^{\alpha'\alpha^* - \frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}}\right) *$$

$$* \left(e^{(\alpha')^*\alpha - \frac{|\alpha|^2}{2} - \frac{|\alpha'|^2}{2}}\right)$$

$$= \frac{1}{\pi}\exp\left[\alpha'\alpha^* + (\alpha')^*\alpha - |\alpha|^2 - |\alpha'|^2\right]$$

$$= \frac{1}{\pi}\exp\left[-\left(\underline{|\alpha|^2 - (\alpha')\alpha^* - (\alpha')^*\alpha + |\alpha'|^2}\right)\right]$$

$$(\alpha - \alpha')^*(\alpha - \alpha') = |\alpha - \alpha'|^2$$

$$= \alpha^*\alpha - (\alpha')^*\alpha - \alpha'\alpha^* + (\alpha')^*\alpha'$$

$$= |\alpha|^2 - (\alpha')^*\alpha - \alpha'\alpha^* + |\alpha'|^2$$

$$= \frac{1}{\pi}e^{-|\alpha - \alpha'|^2}$$

$|m\rangle$ number state

$$\hat{\rho} = |m\rangle\langle m| \Rightarrow Q(\alpha) = \frac{1}{\pi}\langle\alpha|m\rangle\langle m|\alpha\rangle$$

$$= \frac{1}{\pi}|\underline{\langle\alpha|m\rangle}|^2$$

$$\langle\alpha|m\rangle = \left(e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{(\alpha^*)^n}{\sqrt{n!}}\langle n|\right)|m\rangle$$

$$= e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{(\alpha^*)^n}{\sqrt{n!}}\delta_{nm} = e^{-\frac{|\alpha|^2}{2}}\frac{(\alpha^*)^m}{\sqrt{m!}}$$

$$\Rightarrow |\langle\alpha|m\rangle|^2 = e^{-|\alpha|^2}\frac{|\alpha|^{2m}}{m!}$$

$$\Rightarrow Q(\alpha) = \frac{1}{\pi}|\langle\alpha|m\rangle|^2 = \frac{1}{\pi}e^{-|\alpha|^2}\frac{|\alpha|^{2m}}{m!}$$

For vacuum state let $m=0$ above

$$\Rightarrow Q(\alpha) = \frac{1}{\pi}e^{-|\alpha|^2}\frac{|\alpha|^{2\cdot 0}}{0!} = \frac{1}{\pi}e^{-|\alpha|^2}\left(\frac{1}{1}\right)$$

$$= \frac{1}{\pi}e^{-|\alpha|^2}$$

$$|\Psi\rangle = \frac{N}{\sqrt{2}}(|\alpha'\rangle + |-\alpha'\rangle) \qquad \text{Cat - State}$$

$$Q(\alpha) = \frac{1}{\pi}|\langle\alpha|\Psi\rangle|^2$$

$$= \frac{1}{\pi}\left|\frac{N}{\sqrt{2}}(\langle\alpha|\,(|\alpha'\rangle + |-\alpha'\rangle))\right|^2$$

$$= \frac{1}{\pi}\frac{N^2}{2}\left|\underline{\langle\alpha|\alpha'\rangle} + \underline{\langle\alpha|-\alpha'\rangle}\right|^2$$

$$= \frac{N^2}{2\pi}\left|e^{\alpha'\alpha^* - \frac{|\alpha'|^2}{2} - \frac{|\alpha|^2}{2}} + e^{-\alpha'\alpha^* - \frac{|\alpha'|^2}{2} - \frac{|\alpha|^2}{2}}\right|^2$$

$$= \frac{N^2}{2\pi} e^{-|\alpha'|^2 - |\alpha|^2} |e^{\alpha'\alpha^*} + e^{-\alpha'\alpha^*}|^2$$

$$= \frac{2N^2}{\pi} e^{-|\alpha'|^2 - |\alpha|^2} |\cosh(\alpha'\alpha^*)|^2$$

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3) $$H = -\frac{\hbar\delta}{2}\hat{\sigma}_z - \frac{\hbar\Omega_0}{2}\hat{\sigma}_x$$

$$\hat{=} \begin{pmatrix} -\frac{\hbar\delta}{2} & -\frac{\hbar\Omega_0}{2} \\ -\frac{\hbar\Omega}{2} & \frac{\hbar\delta}{2} \end{pmatrix}$$

$$i\partial_t \begin{pmatrix} c_g \\ c_e \end{pmatrix} = \begin{pmatrix} -\frac{\hbar\delta}{2} & -\frac{\hbar\Omega_0}{2} \\ -\frac{\hbar\Omega}{2} & \frac{\hbar\delta}{2} \end{pmatrix} \begin{pmatrix} c_g \\ c_e \end{pmatrix}$$

NDSolve , $c_g(0) = 1$

$c_e(0) = 0$

2)

I) $W(q,p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \langle q+\frac{x}{2} | \hat{\rho} | q-\frac{x}{2} \rangle e^{ipx/\hbar} dx$

II) $W(\alpha) = \frac{1}{\pi^2} \int e^{\lambda^*\alpha - \lambda\alpha^*} \mathrm{tr}[\hat{\rho}\, e^{\lambda\hat{a}^+ - \lambda^*\hat{a}}] d^2\lambda$

$\omega = \hbar = 1$

$$\lambda = \frac{1}{\sqrt{2}}(x+iy)$$

$$\int d^2\lambda = \frac{1}{2}\int dx \int dy$$

$$\lambda^*\alpha - \lambda\alpha^* = \frac{1}{2}\Big[(x-iy)(q+ip) - (x+iy)(q-ip)\Big] = i[-yq+px]$$

$$\lambda\hat{a}^+ - \lambda^*\hat{a} = \dots = i[\hat{q}y - x\hat{p}]$$

$$W(q,p) = \frac{1}{2\pi^2}\int dx \int dy\, e^{-i(-xp+yq)} \cdot \mathrm{Tr}\Big[\underbrace{e^{-i(x\hat{p}-y\hat{q})}}\hat{\rho}\Big]$$

BCH: $e^{-ix\hat{p}}\, e^{iy\hat{q}}\, e^{-i\frac{xy}{2}}$!

$$e^{\left(-i\frac{x\hat{p}}{2}\right.} e^{-\frac{ix\hat{p}}{2}}$$

$$= \frac{1}{2\pi^2} \int dx \int dy \, e^{-i(-xp+yq)} e^{-i\frac{xy}{2}}$$

$$\underbrace{\mathrm{Tr}\left[e^{-ix\hat{p}/2} e^{iy\hat{q}} \hat{\rho} e^{-ix\hat{p}/2} \right]}_{\int dq' \langle q' | \dots | q' \rangle}$$

$$= \frac{1}{2\pi^2} \int dx \int dy \int dq'$$

$$e^{-i(-xp+yq)} \cancel{e^{-i\frac{xy}{2}}}$$

$$\langle q' + \tfrac{x}{2} | e^{iy\hat{q}} \hat{\rho} | q' - \tfrac{x}{2} \rangle$$

$$\langle q' + \tfrac{x}{2} | e^{iy(q' + \cancel{\frac{x}{2}})}$$

$$= \frac{1}{2\pi^2} \int dx \int dy \int dq \, e^{ixp} e^{iy(q'-q)}$$

$$2\pi\delta(q-q') \langle q' + \tfrac{x}{2} | \hat{\rho} | q' - \tfrac{x}{2} \rangle$$

$$= \frac{1}{\pi} \int dx \, e^{ixp} \langle q' + \frac{x}{2} | \hat{\rho} | q' - \frac{x}{2} \rangle$$

Factor of 2?