Problem 1 (Patrizia parts a+b)

$\Omega_0 = \frac{\nu}{\hbar} = 0 \qquad \gamma > 0 \qquad \Delta = 0$

① $\partial_t \rho_{gg} = \gamma \rho_{ee} - \cancel{\frac{i}{2\hbar}\nu} 2i \, \mathrm{Im}(\tilde{\rho}_{eg})$

② $\partial_t \rho_{ee} = -\gamma \rho_{ee} + \cancel{\frac{i}{2\hbar}\nu} 2i \, \mathrm{Im}(\tilde{\rho}_{eg})$

③ $\partial_t \tilde{\rho}_{ge} = (\cancel{i\Delta} - \gamma/2]\tilde{\rho}_{ge} - \cancel{\frac{i}{2\hbar}\nu}(\rho_{ee} - \rho_{gg})$

② $\partial_t \rho_{ee} = \underbrace{-\gamma \rho_{ee}}$ $\qquad e^x$

$\int \partial_t \rho_{ee} = -\gamma \rho_{ee}$ then $\rho_{ee}(t) = e^{-\gamma t}$

$dt(e^{-\gamma t}) = -\gamma e^{-\gamma t}$

$t=0 \rightarrow e^0 = 1$

③ $\partial_t \rho_{ge} = -\frac{1}{2}\gamma \rho_{ge}$

$\int \partial_t \rho_{ge} = -\frac{1}{2}\gamma \rho_{ge}$

$$\rho_{ge} = e^{-1/2\gamma} \qquad \partial t\left(e^{-1/2\gamma}\right) = -\frac{1}{2}e^{-1/2\gamma}$$

$$t=0 \rightarrow \rho_{ge} = e^{0} = 1$$
$$\downarrow$$
$$0$$

(since all atoms are in excited state $\rho_{ge}=0$)

① $\partial t \rho_{gg} = \gamma \rho_{ee} = \gamma e^{-\gamma t}$

then $\int \partial t \rho_{gg} = \gamma \rho_{ee} = \gamma e^{-\gamma t}$

$$\rho_{gg} = - \quad e^{-\gamma t} \qquad \partial t\left(e^{-\gamma t}\right) = -\gamma e^{-\gamma t}$$

$\rho_{gg} = 1 - e^{-\gamma t}$

$\Delta = 0 \quad \frac{\nu}{\hbar} = 0$

① $\partial t R_1 = -\frac{\gamma}{2} R_1 + \cancel{\Delta R_2}$

② $\partial t R_2 = -\cancel{\Delta R_1} - \gamma/2 R_2 + \cancel{\frac{\nu}{\hbar} R_3}$

③ $\partial t R_3 = -\gamma(R_3+1) - \cancel{\frac{\nu}{\hbar} R_2}$

① $\partial t R_1 = -\frac{1}{2}\gamma R_1 \qquad R_1(t) = R_1 e^{-1/2\gamma t}$

② $\partial t R_2 = -\frac{1}{2}\gamma R_2 \qquad R_2(t) = e^{-1/2\gamma t} R_2(0)$

③ $\partial t R_3 = -\gamma R_3 - \gamma \qquad R_3(t) = e^{-\gamma t} \cdot C - 1$

$\hookrightarrow R_3(0)$

$$c = 2e^{-\gamma t} - 1$$

$$\boxed{R_3(t) = -1 + e^{-\gamma t}\left(R_3(0) + 2\right)}$$

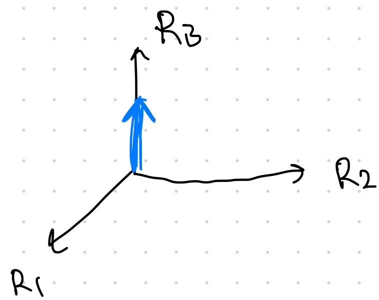
part b

$$\left.\begin{aligned} R_1(t) &= 2\,\mathrm{Re}\left(\rho_{ge}^{*}(t)\right) \\ R_2(t) &= 2\,\mathrm{Im}\left(\rho_{ge}^{*}(t)\right) \\ R_3(t) &= \rho_{ee}(t) - \rho_{gg}(t) \end{aligned}\right\}$$

At $t=0$

$$R_1(0) = 2\,\mathrm{Re}\left(e^{-1/2\gamma(0)}\right) = 2(1) = 2$$

$$R_2(0) =$$



$$R_1(t) = R_1(0)\, e^{-\gamma t/2}$$

$$R_2(t) = R_2(0)\, e^{-\gamma t/2}$$

$$R_3(t) = -1 + e^{-\gamma t}\left(R_3(0) + 1\right)$$

$$t=0 \Rightarrow \begin{pmatrix} R_1 = 0 \\ R_2 = 0 \\ R_3 = 1 \end{pmatrix} \qquad \vec{R} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$t = \frac{\ln(2)}{\gamma} = T_1$$

$$R_1(T_1) = 0 \qquad R_1(\infty) = 0$$

$$R_2(T_1) = 0 \qquad R_2(\infty) = 0$$

$$R_3(T_1) = 0 \qquad R_3(\infty) = -1$$

4 $\Rightarrow$

$$R_1^S = 0$$

$$R_2^S = -\frac{2\Omega}{\gamma\left(1 + \frac{2\Omega^2}{\gamma^2}\right)}$$

$$R_3^S = \left(-\frac{1}{1 + \frac{2\Omega^2}{\gamma^2}} \right)$$

at $\Omega = 0$

$$R_1^S = 0 \,, \quad R_2^S = 0 \,, \quad R_3^S = -1$$

at $\Omega = \gamma$

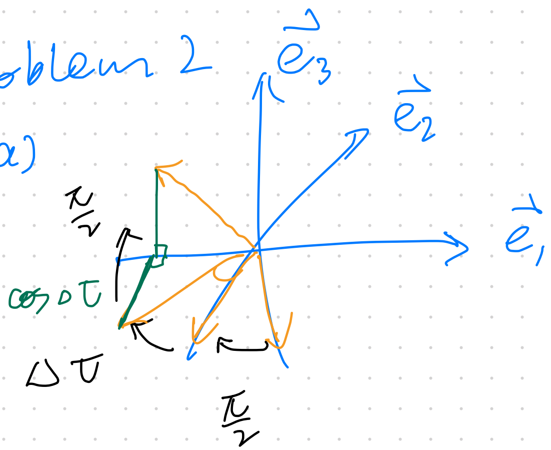
$$R_1^S = 0 \,, \quad R_2^S = -\frac{2}{3} \,, \quad R_3^S = -\frac{1}{3}$$

at $\Omega \to \infty$

$$R_1^S = 0 \,, \quad R_2^S = 0 \,, \quad R_3^S = 0$$

Problem 2

(a)

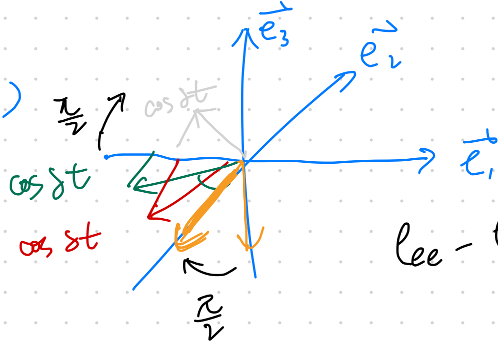


$$R_3 = \rho_{ee} - \rho_{gg} = \cos \Delta\tau$$

$$\Downarrow \quad \rho_{ee} + \rho_{gg} = 1$$

$$\rho_{ee} = \frac{1}{2}\cos\Delta\tau + \frac{1}{2}$$

(b)



$$\frac{1}{C}\int_0^{\delta_0}\cos\delta t\; d\delta$$

$$\rho_{ee}-\rho_{gg}=\frac{\sin\delta_0 t}{C\,t} \to 0$$

$t \to \infty$