Q1. $\hat{E}(\chi) = E_0(\hat{a}e^{-i\chi} + a^\dagger e^{i\chi})$

$$\hat{E}^2(\chi) = E_0^2(a^2 e^{-2i\chi} + aa^\dagger + a^\dagger a + a^{\dagger 2} e^{2i\chi})$$

(a) Vac. state $|0\rangle$.

$$\langle 0|\hat{E}(\chi)|0\rangle = E_0(\langle 0|\cancel{a}|0\rangle^{0} e^{-i\chi} + \langle 0|\cancel{a^\dagger}|0\rangle^{0} e^{i\chi}) = 0$$

$$\langle 0|\hat{E}^2(\chi)|0\rangle = E_0^2(\langle 0|\cancel{a^2}|0\rangle^{0} e^{-2i\chi} + \langle 0|a^\dagger a + aa^\dagger|0\rangle + \langle 0|\cancel{a^{\dagger 2}}|0\rangle^{0} e^{2i\chi})$$

$$= E_0^2$$

Foch st. $|n\rangle$.

$$\langle n|\hat{E}(\chi)|n\rangle = E_0(\langle n|a|n\rangle e^{-i\chi} + \langle n|a^\dagger|n\rangle e^{i\chi}) = 0$$

$$\langle n|\hat{E}^2(x)|n\rangle = E_0^2(\langle n|\cancel{a^2}|n\rangle^{0} e^{-2i\chi} + \langle n|a^\dagger a + aa^\dagger|n\rangle + \langle n|\cancel{a^{\dagger 2}}|n\rangle^{0} e^{2i\chi})$$

$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$$= E_0^2(\langle n|a^\dagger a|n\rangle + \langle n|aa^\dagger|n\rangle)$$

$a|n\rangle = \sqrt{n}\,|n-1\rangle$

$$= E_0^2(n\cancel{\langle n-1|n-1\rangle}^{1} + n+1\cancel{\langle n+1|n+1\rangle}^{1})$$

$$= E_0^2(n+n+1) = E_0^2(2n+1)$$

Coherent states: $|\alpha\rangle$.

$$\langle\alpha|E|\alpha\rangle = E_0\left(\langle\alpha|a|\alpha\rangle e^{-i\chi} + \langle\alpha|a^\dagger|\alpha\rangle e^{i\chi}\right)$$

$\alpha = |\alpha|e^{-i\phi}$

$$= E_0\left(\alpha \langle\alpha|\alpha\rangle e^{-i\chi} + \alpha^* \langle\alpha|\alpha\rangle e^{i\chi}\right) \quad \langle\alpha|\alpha\rangle = 1$$

$$= E_0 |\alpha|\left(e^{i(\phi+\chi)} + e^{-i(\phi+\chi)}\right)$$

$$= E_0 |\alpha|\, 2\cos(\chi+\phi).$$

$$\langle\alpha|E^2(\chi)|\alpha\rangle$$

$$= E_0^2\left(\langle\alpha|a^2|\alpha\rangle e^{-2i\chi} + \underline{\langle\alpha|aa^\dagger|\alpha\rangle} + \underline{\langle\alpha|a^\dagger a|\alpha\rangle}\right.$$

$$\left. + \langle\alpha|a^{\dagger 2}|\alpha\rangle e^{2i\chi}\right)$$

$$= E_0\left(\alpha^2 e^{-2i\chi} + \alpha^{*2} e^{2i\chi} + \langle\alpha|\underline{aa^\dagger}|\alpha\rangle + \langle\alpha|a^\dagger a|\alpha\rangle\right.$$

$$[a^\dagger, a] = 1 \Rightarrow aa^\dagger - a^\dagger a = 1 \Rightarrow aa^\dagger = 1 + a^\dagger a$$

$$= E_0^2\left(\alpha^2 e^{-2i\chi} + \alpha^{*2} e^{2i\chi} + \langle\alpha|\alpha\rangle + 2\langle\alpha|a^\dagger a|\alpha\rangle\right)$$

$$= E_0^2\left(\quad + 1 + 2|\alpha|^2\right)$$

$$= E_0^2\left(1 + 4|\alpha|^2 \sin^2(\chi+\phi)\right)$$

this term should be $\cos^2(\chi+\phi)$

$$(\Delta E)^2 = \langle E^2\rangle - \langle E\rangle^2$$

$$= E_0^2\left(1 + 4|\alpha|^2\sin^2(\chi+\phi)\right) - E_0^2 4|\alpha|^2\cos^2(\chi+\phi).$$

$$= E_0^2 + 4E_0^2\cos(2(\chi+\phi)). \Rightarrow E_0^2$$

(b) $E_0 = \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2}$ $\quad E_0 \propto \sqrt{\omega}$, $E_0 \propto \frac{1}{\sqrt{V}}$.

(c) $H = \hbar\omega(a^\dagger a + \frac{1}{2})$.

$H^2 = \hbar^2\omega^2 (a^\dagger a + \frac{1}{2})^2 = \hbar^2\omega^2 (a^\dagger a\, a^\dagger a + a^\dagger a + \frac{1}{4})$.

Fock st.

$\langle n|H|n\rangle = \hbar\omega(n + \frac{1}{2}) \leftarrow$

$\langle n|H^2|n\rangle = (\hbar\omega)^2 (\langle n|\hat{n}^2|n\rangle + \langle n|\hat{n}|n\rangle + \frac{1}{4})$

$= (\hbar\omega)^2 (n^2 + n + \frac{1}{4}) \leftarrow$

$(\Delta H)^2 = \langle H^2\rangle - \langle H\rangle^2 = 0$

· Coherent state:

$\langle\alpha|a^\dagger a|\alpha\rangle = \alpha^2$

$\langle\alpha|H|\alpha\rangle = \hbar\omega(|\alpha|^2 + \frac{1}{2})$

$\langle\alpha|H^2|\alpha\rangle = (\hbar\omega)^2 (\langle\alpha|a^\dagger a\, a^\dagger a|\alpha\rangle + \langle\alpha|a^\dagger a|\alpha\rangle + \frac{1}{4})$

$= (\hbar\omega)^2 (|\alpha|^4 + 2|\alpha|^2 + \frac{1}{4})$

$\langle H\rangle^2 = (|\alpha|^4 + |\alpha|^2 + \frac{1}{4})(\hbar\omega)^2$

$(\Delta H)^2 = \langle H^2\rangle - \langle H\rangle^2 = (\hbar\omega)^2 (|\alpha|^4 + 2|\alpha|^2 + \frac{1}{4}) - (\hbar\omega)^2 (|\alpha|^4 + |\alpha|^2 + \frac{1}{4}) = \hbar^2\omega^2 |\alpha|^2$

$$A\cos(\omega t+\phi) = A\left(\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)\right)$$

$$= a\cos(\omega t) + b\sin(\omega t)$$

$$a = A\cos(\phi)$$

$$b = -A\sin(\phi)$$

$$\hat{x} = \sqrt{\frac{\hbar}{2\omega}}\,(a + a^{+})$$

$$\hat{p} = -i\sqrt{\frac{\hbar\omega}{2}}\,(a - a^{+})$$

$$\langle\hat{x}\rangle_\alpha = \langle\alpha|\,\hat{x}\,|\alpha\rangle = \sqrt{\frac{\hbar}{2\omega}}\left(\langle a\rangle_\alpha + \langle a^{+}\rangle_\alpha\right)$$

$$\alpha^2 + \alpha^{*2} + 2|\alpha|^2$$

$$= \sqrt{\frac{\hbar}{2\omega}}\,(\alpha + \alpha^*) \rightarrow$$

$$\langle\hat{p}\rangle \qquad = -i\sqrt{\frac{\hbar\omega}{2}}\,(\alpha - \alpha^*)$$

$$\alpha^* \quad \alpha$$

$$\langle\alpha|aa^{+}|\alpha\rangle \qquad \langle\alpha|a^{+}a|\alpha\rangle$$

$$\langle\hat{x}^2\rangle = \frac{\hbar}{2\omega}\left(\langle aa^{+}\rangle + \langle a^{+}a\rangle + \langle a^2\rangle + \langle a^{+2}\rangle\right)$$

$$\alpha^2$$

$$= \frac{\hbar}{2\omega}\left(\underbrace{1 + 2|\alpha|^2}_{-} + \alpha^2 + \alpha^{*2}\right) \leftarrow$$

$$[a, a^{+}] = 1$$

$$aa^{+} - a^{+}a = 1$$

$$aa^{+} = a^{+}a + 1$$

$$\langle\alpha|\,a^{+}\,\hat{a}^{+}\,|\alpha\rangle$$

$$= \alpha^*\langle\alpha|$$

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{\hbar}{2\omega}\,(1)}$$

$$\langle p^2\rangle = \left\langle -\frac{\hbar\omega}{2}(a-a^\dagger)^2\right\rangle = \frac{-\hbar\omega}{2}\left(-1-2|\alpha|^2 + \alpha^2+\alpha^{*2}\right)$$

$$\langle p\rangle = -i\sqrt{\frac{\hbar\omega}{2}}\left(\alpha-\alpha^*\right)$$

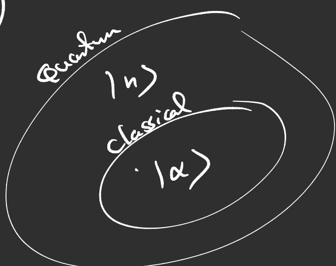
$$-\frac{\hbar\omega}{2}\left(\alpha^2+\alpha^{*2}-2|\alpha|^2\right)$$

$$\Delta p = \sqrt{\frac{\hbar\omega}{2}}$$

$$\Delta x\,\Delta p = \sqrt{\frac{\hbar}{2\cancel{\omega}}}\sqrt{\frac{\hbar\cancel{\omega}}{2}} = \underline{\frac{\hbar}{2}}$$ {minimum uncertainty}

0 {classically}

c)



III. (a) $$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle$$

$$\langle n|n'\rangle = \langle 0|\frac{\hat{a}^n}{\sqrt{n!}}\cdot\frac{\hat{a}^{\dagger n'}}{\sqrt{n'!}}|0\rangle$$

$$= \frac{1}{\sqrt{n!n'!}}\langle 0|\underbrace{\hat{a}\cdots\hat{a}}_{n}\cdot\underbrace{\hat{a}^\dagger\cdots\hat{a}^\dagger}_{n'}|0\rangle$$

$$[\hat{a},\hat{a}^\dagger]=1 \qquad aa^\dagger = a^\dagger a+1$$

$$= \frac{1}{\sqrt{n!n'!}} \langle 0| \underbrace{\hat{a} \cdots \hat{a}}_{n-1} (\hat{a}^\dagger \hat{a} + 1) \underbrace{\hat{a}^\dagger \cdots \hat{a}^\dagger}_{n'-1} |0\rangle$$

$$= \frac{1}{\sqrt{n!n'!}} \langle 0| \underbrace{\hat{a} \cdots \hat{a}}_{n-1} \hat{a}^\dagger \hat{a} \underbrace{\hat{a}^\dagger \cdots \hat{a}^\dagger}_{n'-1} |0\rangle$$

$$+ \frac{1}{\sqrt{n!n'!}} \langle 0| \underbrace{\hat{a} \cdots \hat{a}}_{n-1} \underbrace{\hat{a}^\dagger \cdots \hat{a}^\dagger}_{n'-1} |0\rangle$$

$$= \sum \langle 0| \hat{a}^\dagger \hat{a}^\dagger \cdots \hat{a} \hat{a} |0\rangle$$

$$\langle 0| \hat{a} \hat{a}^\dagger |0\rangle = \langle 0| \hat{a}^\dagger \hat{a} |0\rangle + 1 = 1$$

$$\langle 0| \hat{a} \hat{a}^\dagger \hat{a}^\dagger |0\rangle = \langle 0| \hat{a}^\dagger \hat{a} \hat{a}^\dagger |0\rangle + \langle 0| \hat{a}^\dagger |0\rangle$$

$$= \langle 0| \hat{a}^\dagger \hat{a}^\dagger a |0\rangle + \langle 0| \hat{a}^\dagger |0\rangle + \langle 0| \hat{a}^\dagger |0\rangle$$

$$= 0$$

(b) $$|\{n_i\}\rangle = \prod_i \frac{(\hat{a}^\dagger)^{n_i}}{\sqrt{n_i!}} |0\rangle_i$$

$$\langle \{n_i\} | \{n_i'\} \rangle = \prod_i \delta_{n_i, n_i'}$$

(c) $\hat{a}^{\dagger}$

$$|\psi\rangle = \sum_n c_n |n\rangle$$

$$\hat{a}^{\dagger} |\psi\rangle = \sum_n c_n \hat{a}^{\dagger} |n\rangle$$

$$= \sum_n c_n \sqrt{n+1}\, |n+1\rangle$$

$$\hat{a}^{\dagger} |\psi\rangle = \alpha \sum_n c_n |n\rangle$$

$\underline{c_n \alpha = c_{n-1} \sqrt{n-1}}$ for $n \neq 0$

$|0\rangle$ disappears

(d) $\langle\alpha| \hat{a}^{\dagger} \hat{a} |\alpha\rangle = |\alpha|^2$

(e) $$e^{\alpha \hat{a}^{\dagger}} = \sum_{n=0} \frac{(\alpha \hat{a}^{\dagger})^n}{n!}$$

$$e^{\alpha \hat{a}^{\dagger} - \frac{1}{2}|\alpha|^2} |0\rangle = e^{-\frac{1}{2}|\alpha|^2} e^{\alpha \hat{a}^{\dagger}} |0\rangle$$

$$= e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0} \frac{\alpha^n \hat{a}^{\dagger n}}{n!} |0\rangle$$

$$\sqrt{n!}\ |n\rangle$$

$$= e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$= |\alpha\rangle$$