Position of the Saline Water Interface beneath Oceanic Islands

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Abstract. If saline and fresh groundwater are assumed to be immiscible, the three-dimensional position of the saline water interface beneath an oceanic island may be described by one partial differential equation. The Dupuit assumption is used so that the governing flow equation is two-dimensional. Analytical solutions are presented for islands with certain simple geometric boundaries. Multilayered aquifers may be treated by assuming an average conductivity for all aquifer layers. The model may be solved for islands of any shape by using a numerical solution. The model has been used successfully to generate the known position of the saline water interface beneath the South Fork of Long Island, New York, and could be used in water resource studies to determine the position of the saline water interface if it were unknown.

Many oceanic islands have phreatic groundwater reservoirs with saline groundwater underlying the freshwater. A zone of dispersion with mixed salinity separates the fresh and saline groundwater zones. The thickness of the freshwater zone may be known in only a few locations. A knowledge of the position of the top of the zone of dispersion under conditions of steady flow would be useful in the placement of wells.

A number of authors have dealt with the problem of the position of the saline water interface when it appears as a wedge in a coastal aquifer [e.g., *Hubbert*, 1940; *Bear and Dagan*, 1964; *Henry*, 1964; *Glover*, 1964; *Rumer and Shiau*, 1968; *Collins and Gelhar*, 1971; *Shamir and Dagan*, 1971; *Dagan*, 1971]. These solutions are generally for vertical cross sections of aquifers with infinite length. In this paper a general solution will be developed for determining the depth of the saline water interface beneath an oceanic island of any shape.

THEORY

If the freshwater lens beneath an oceanic island is thin compared with its lateral extent and the slope of the water table is slight, groundwater flow in the aquifer will be generally horizontal. Therefore we can assume that the equipotential surfaces are vertical and that the velocity is uniform over the depth of flow. These assumptions (Dupuit's assumptions) are valid everywhere except the immediate boundary of the island, where there are significant upward components of flow. Later we will show that the error introduced at the coastline by the Dupuit approximations is small for most natural flow conditions.

Henry [1964] presented a model of a one-dimensional (infinite strip) oceanic island in which he used the Dupuit assumptions. Comparing the Dupuit model with a solution obtained by the hodograph method, he demonstrated that under natural conditions the Dupuit model yielded results that were virtually indistinguishable from those of the other model. Equations using the Dupuit assumptions have also been employed in studies of coastal saline water intrusion [e.g., *Collins and Gelhar*, 1971; *Shamir and Dagan*, 1971].

Many investigators have made the assumption that a sharp boundary exists between the fresh groundwater and the saline groundwater and that there is no flow across the boundary [e.g., *Hubbert*, 1940; *Henry*, 1964; *Glover*, 1964; *Rumer and Shiau*, 1968; *Bear and Dagan*, 1964]. Thus the effects of dispersion can be ignored in solving the equations. In natural aquifers in which the zone of dispersion is thin compared with the thickness of the freshwater lens, this assumption might not introduce too great an error. One would expect dispersion beneath an oceanic island to be greatest close to the shoreline because of tidal fluctuations, natural changes in the discharge volume of fresh groundwater, and so on. Therefore, when a sharp interface is assumed, the solutions near the shoreline would be the most approximate.

Hubbert [1940, p. 872] examined the dynamic flow of fresh groundwater over stagnant saline groundwater. A sharp interface separated them. He found that when the sea level is given the elevation of the saltwater interface at a point on the interface is

$$z_{s(x,y)} = -\left[\frac{\gamma_f}{(\gamma_s - \gamma_f)}\right] h_{s(x,y)} \quad (1)$$

where $z_{s(x,y)}$ is the elevation of the saltwater interface at point s, x, y on the saltwater interface, L; γ_s is the density of saline groundwater, ML^{-3}; γ_f is the density of fresh groundwater, ML^{-3}; and $h_{s(x,y)}$ is the hydraulic head at point x, y on the saltwater interface, L.

The force potential Φ^* (dimensions L^2T^{-2}) is related to the hydraulic head by the equality

$$\Phi^* = gh \quad (2)$$

where g is the acceleration of gravity, LT^{-2}. Equation 1 is a general formula where the equipotential surfaces are curved, so that a point $f(x, y)$ on the water table vertically above a point $s(x, y)$ may lie in a different equipotential surface and thus have a different hydraulic head. However, since by the Dupuit assumptions the equipotential surfaces are vertical, any point on the saltwater interface $z_{s(x,y)}$ must lie in the same potential surface as the point on the water table vertically above it $z_{f(x,y)}$, and

$$h_{s(x,y)} = h_{f(x,y)} \quad (3)$$

Consider the control volume shown in Figure 1. This figure represents a vertical section of the porous media that is saturated with fresh groundwater. The upper surface is the water table, and the lower surface is the saltwater interface. The x–y plane represents mean sea level. At a given point x_i, y_j, the total thickness of the zone of freshwater flow is equal to

$$h_{(i,j)} + [\gamma_f/(\gamma_s - \gamma_f)]h_{(i,j)}$$

The first part of the above expression represents the vertical distance from the water table to mean sea level, and the second part represents the vertical distance from mean sea level to the saltwater interface.

The horizontal velocity components are

$$u = -K\ \delta h/\delta x \qquad v = -K\ \delta h/\delta y \quad (4)$$

where K is the hydraulic conductivity, LT^{-1}. The flow rate through the left face of the control volume in the x direction is equal to the area

$$\{\langle h\rangle + [\gamma_f/(\gamma_s - \gamma_f)]\langle h\rangle\}\ dy$$

where $\langle h\rangle$ is the average value of h along the x axis in the length dy times the velocity u. The flow rate along the x axis per unit width in the x direction is designated as q_x.

The flow rate through the left face is

$$q_{x+dx}\ dy = -K\left[\left(\langle h\rangle + \frac{\gamma_f}{\gamma_s - \gamma_f}\langle h\rangle\right)\frac{\delta\langle h\rangle}{\delta x}\right]_{x+dx} dy \quad (5)$$

Similarly, it can be shown that the flow rate through the right side of the control volume is equal to the expression

$$q_x\ dy = -K\left[\left(\langle h\rangle + \frac{\gamma_f}{\gamma_s - \gamma_f}\langle h\rangle\right)\frac{\delta\langle h\rangle}{\delta x}\right]_x dy \quad (6)$$

The difference in flow per unit time between the amount of fluid flowing in the x direction

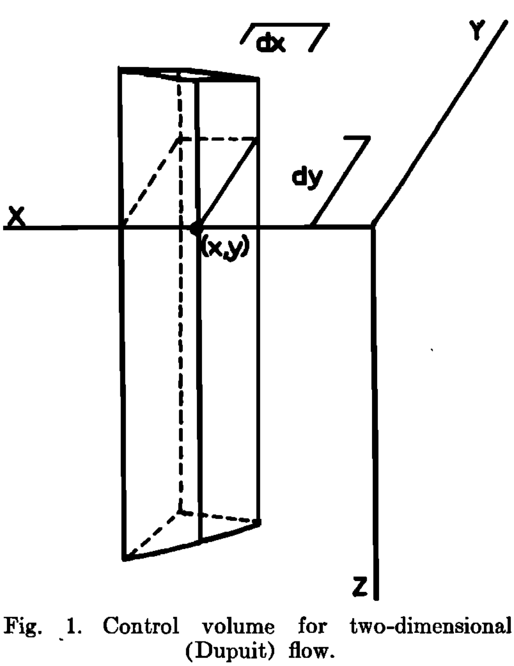

Fig. 1. Control volume for two-dimensional (Dupuit) flow.

through the left face and the amount flowing through the right face is

$$dq_x = (q_{x+dx} - q_x)\,dy \tag{7}$$

Lagrange's formula is

$$f(x + dx) - f(x) = f'(\gamma)\,dx \tag{8}$$

where γ is a number with a value between x and $x + dx$. The change in flow rate in the direction of the x axis can be given by a truncated Taylor's expansion as

$$(q_{x+dx} - q_x)\,dy = \left(\frac{\delta q_x}{\delta x}\right) dx\,dy$$

$$= -K\frac{\delta}{\delta x}\left[\left(\langle h\rangle + \frac{\gamma_f}{\gamma_s - \gamma_f}\langle h\rangle\right)\frac{\delta\langle h\rangle}{\delta x}\right] dx\,dy \tag{9}$$

In the y direction the flow is given by the similar expression

$$\left(\frac{\delta q_y}{\delta y}\right) dy\,dx = -K\frac{\delta}{\delta y}$$

$$\cdot\left[\left(\langle\langle h\rangle\rangle + \frac{\gamma_f}{\gamma_s - \gamma_f}\langle\langle h\rangle\rangle\right)\frac{\delta\langle\langle h\rangle\rangle}{\delta y}\right] dy\,dx \tag{10}$$

where $\ll h \gg$ is the average value of h along the x axis in the length dx.

For the steady flow of an incompressible fluid through an incompressible porous media, the law of continuity requires the change in flow rate through the sides of the control volume to be equal to the inflow or outflow through the top and bottom plus any change in the thickness of the saturated prism of the porous media. Since we have made the assumption that there is no flow across the saline water interface, the accretion due to the recharge of precipitation across the phreatic surface represents an addition to the control volume. The average daily accretion W (dimensions LT^{-1}) is a steady state addition to the control volume, represented as a source within the control volume equal to $W\,dx\,dy$.

By continuity, the sum of the change in flow rate through the sides is equal to $W\,dx\,dy$, so that

$$(\delta q_x/\delta x)\,dx\,dy + (\delta q_x/\delta y)\,dx\,dy = W\,dx\,dy$$

This equality can also be expressed in terms of the hydraulic head as

$$K\left(1 + \frac{\gamma_f}{\gamma_s - \gamma_f}\right)\frac{\delta}{\delta x}\left(h\frac{\delta h}{\delta x}\right)$$

$$+ K\left(1 + \frac{\delta_f}{\delta_s - \delta_f}\right)\frac{\delta}{\delta y}\left(h\frac{\delta h}{\delta y}\right) = -W \tag{11}$$

Equation 11 may be rewritten in the form

$$K\left(1 + \frac{\gamma_f}{\gamma_s - \gamma_f}\right)\left(\frac{\delta^2 h^2}{\delta x^2} + \frac{\delta^2 h^2}{\delta y^2}\right) = -2W \tag{12}$$

Because we have made the assumption that equipotential surfaces are vertical, there are no vertical components of flow. Multilayered aquifers can be considered by using an average value of the horizontal conductivities of the various aquifer layers K_{avg}. For the two-layered aquifer the average conductivity may be found from

$$K_{avg} = \frac{K_1(b_1 + h) + K_2\{[\gamma_f/(\gamma_s - \gamma_f)]h - b_1\}}{h + [\gamma_f/(\gamma_s - \gamma_f)]h} \tag{13}$$

where K_1 and K_2 are the conductivities of the first and second layers and b_1 is the thickness below mean sea level of the upper aquifer layer.

The flow equation can be expressed as

$$\frac{\delta^2 h^2}{\delta x^2} + \frac{\delta^2 h^2}{\delta y^2} = \frac{-2W}{K_{avg}\{1 + [\gamma_f/(\gamma_s - \gamma_f)]\}} \tag{14}$$

ANALYTICAL SOLUTIONS

Infinite strip island. For an infinite strip island all flow is normal to the y axis. Flow is thus only a function of x and can be expressed as

$$K\left(1 + \frac{\gamma_f}{\gamma_s - \gamma_f}\right)\frac{\delta}{\delta x}\left(h\frac{\delta h}{\delta x}\right) = -W \tag{15}$$

If the axes of the infinite strip island are selected so that the y axis bisects the island, the boundary conditions are $dh/dx = 0$ at $x = 0$ and $h = 0$ at $x = \pm a$. The resultant solution to (15) is

$$h^2 = \frac{W(a^2 - x^2)}{K\{1 + [\gamma_f/(\gamma_s - \gamma_f)]\}} \tag{16}$$

This equation is similar to one derived by *Jacob* [1943] and is identical to an equation presented by *Henry* [1964].

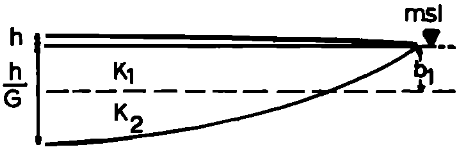

Fig. 2. Cross section of the half width of a two-layered infinite strip aquifer.

For an infinite strip island, multilayered aquifers may be evaluated by considering each individual layer. Equation 16 may be used when the interface is in the upper layer, but when it is in the lower aquifer layer a modified equation must be used (Figure 2).

A control volume for a two-layered aquifer system is given in Figure 3. Flow along the x axis through the left face of the control volume toward the origin is expressed as

$$q_{x+dx}\,dy = -K_1\left[(h+b_1)\frac{\delta h}{\delta x}\right]_{x+dx} dy - K_2\left[\left(\frac{\gamma_f}{\gamma_s-\gamma_f}h - b_1\right)\frac{\delta h}{\delta x}\right]_{x+dx} dy \quad (17)$$

Flow through the right face toward the origin in the x direction is given by

$$q_x\,dy = -K_1\left[(h+b_1)\frac{\delta h}{\delta x}\right]_x dy - K_2\left[\left(\frac{\gamma_f}{\gamma_s-\gamma_f}h - b_1\right)\frac{\delta h}{\delta x}\right]_x dy \quad (18)$$

The difference in flow between the two faces is given by

$$\frac{\delta q_x}{\delta x}\,dx\,dy = -K_1\frac{\delta}{\delta x}\left[(h+b_1)\frac{\delta h}{\delta x}\right] dx\,dy - K_2\frac{\delta}{\delta x}\left[\left(\frac{\gamma_f}{\gamma_s-\gamma_f}h - b_1\right)\frac{\delta h}{\delta x}\right] dx\,dy \quad (19)$$

By continuity, (19) is equal to the steady state accretion to the control volume $W\,dx\,dy$, and

$$K_1\left[\frac{d}{dx}\left(h\frac{dh}{dx}\right) + b_1\frac{d}{dx}\left(\frac{dh}{dx}\right)\right] + K_2\left[\left(\frac{\gamma_f}{\gamma_s-\gamma_f}\right)\frac{d}{dx}\left(h\frac{dh}{dx}\right) - b_1\frac{d}{dx}\left(\frac{dh}{dx}\right)\right] = -W \quad (20)$$

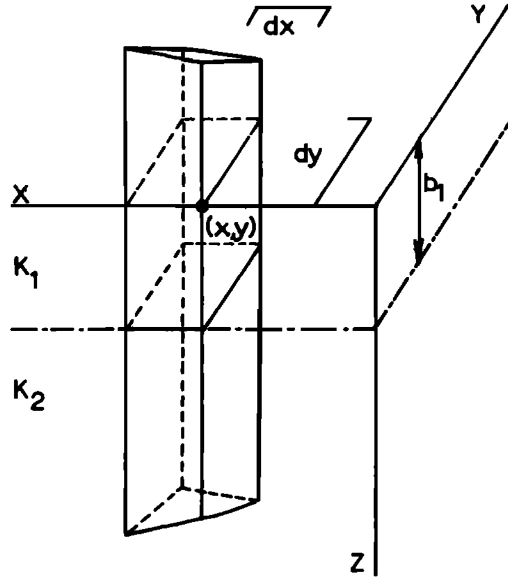

Fig. 3. Control volume for two-dimensional (Dupuit) flow in a two-layered aquifer.

The axes are selected so that the y axis lies on one of the shorelines, and the half width of the island is a. The boundary conditions are $dh/dx = 0$ at $x = a$ and $h = 0$ at $x = 0$. The solution to (20) is

$$h^2\left[\tfrac{1}{2}K_1 + \tfrac{1}{2}K_2\left(\frac{\gamma_f}{\gamma_s-\gamma_f}\right)\right] + h(b_1K_1 - b_1K_2) = W(ax - \tfrac{1}{2}x^2) \quad (21)$$

Circular island. Many oceanic islands tend to be somewhat circular in shape. Equation 14 may be expressed in polar coordinates as

$$\frac{\delta^2 h^2}{\delta r^2} + \frac{\delta h^2}{r\,\delta r} + \frac{\delta^2 h^2}{r^2\,\delta\theta^2} = \frac{-2W}{K_{\text{avg}}\{1 + [\gamma_f/(\gamma_s-\gamma_f)]\}} \quad (22)$$

where r is the radius and θ is the angle. If the aquifer possesses an axis of radial symmetry, (22) becomes

$$\frac{\delta^2 h^2}{\delta r^2} + \frac{\delta h^2}{r\,\delta r} = \frac{-2W}{K_{\text{avg}}\{1 + [\gamma_f/(\gamma_s-\gamma_f)]\}} \quad (23)$$

With the origin of radial symmetry at the center of the circular island and the radius of the island R (dimensions L), the boundary conditions are $dh/dr = 0$ at $r = 0$ and $h = 0$ at

$r = R$. The solution to (23) is

$$h^2 = \frac{W(r^2 - R^2)}{2K_{avg}\{1 + [\gamma_f/(\gamma_s - \gamma_f)]\}} \quad (24)$$

NUMERICAL SOLUTION

Since most oceanic islands are irregularly shaped, the governing flow equation cannot be solved directly. Numerical analysis may be used to approximate Poisson's equation for problems with irregular boundaries. The differential equation may be replaced by the difference equations [*Hildebrand,* 1952]

$$\frac{\delta^2\phi}{\delta x^2} = \frac{\phi(x + n, y) + \phi(x - n, y) - 2\phi(x, y)}{n^2} \quad (25)$$

$$\frac{\delta^2\phi^2}{\delta y^2} = \frac{\phi(x, y + n) + \phi(x, y - n) - 2\phi(x,y)}{n^2} \quad (26)$$

where n (dimensions L) is the difference between nodal points on a grid. Equation 14 may be approximated by the difference equation

$$\tfrac{1}{4}[h^2(x + n, y) + h^2(x - n, y) + h^2(x, y + n) + h^2(x, y - n)] - \tfrac{1}{4}n^2\left\{\frac{-2W}{K_{avg}[\gamma_f/(\gamma_s - \gamma_f)]}\right\} = h^2(x, y) \quad (27)$$

As the value of n decreases, the grid becomes finer, and the value of h^2 computed by the difference method approaches the value of h^2 that would be obtained by a corresponding analytical solution. The model is a form of Dirichlet's problem; i.e., given a set of values $\phi(B)$ on the boundary, find the value of $\phi(x, y)$ at all points within the boundary.

For convenience, consider the boundary of an oceanic island to be located at the coastline. The ocean is a hydrologic sink in which the freshwater head h is always 0. If tidal effects are not considered, mean sea level forms a constant head boundary. A convenient computer program for this problem can be written by assigning nonzero values of h to nodal points in the interior of the island. The boundary nodes and all points outside the boundary can then be assigned a value of h equal to 0. By successive iterations of nonzero nodal points, the program can converge to some preassigned point. Programs could be designed for the steady state analysis of phreatic aquifers of oceanic islands of any shape with any number and thickness of aquifers. The hydraulic conductivity of each aquifer could be entered separately at each nodal point to correspond to field conditions. The amount of recharge could also be varied spatially to reflect the distribution of precipitation, evapotranspiration, and runoff.

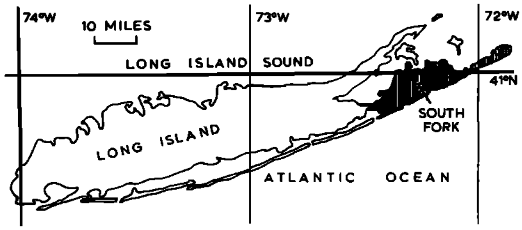

Fig. 4. The location of the South Fork of Long Island, New York.

SALINE WATER INTERFACE BENEATH THE SOUTH FORK OF LONG ISLAND, NEW YORK

The South Fork of Long Island, New York, is an oceanic island, being separated from the rest of Long Island by a sea level canal (Figure 4). Saline groundwater not only surrounds the island but underlies it as well. A program of test well drilling from 1967 to 1969 provided data on the depth to the saline water interface beneath the center of the island. The interface is not a sharp boundary but a zone of diffusion about 50 feet thick. Additional data were ob-

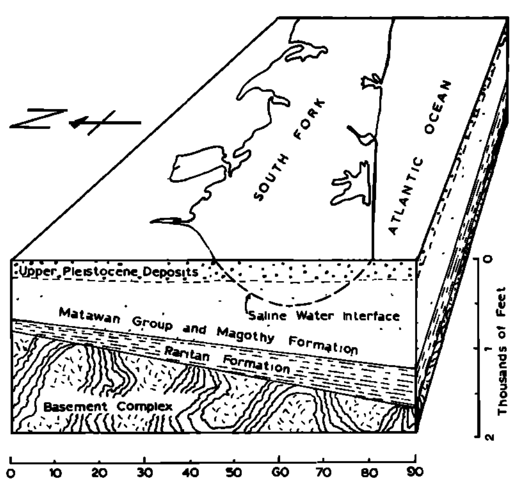

Fig. 5. A hydrogeologic cross section of the South Fork.

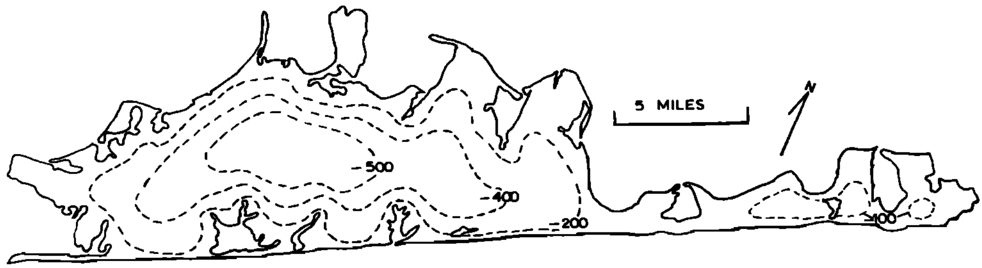

Fig. 6. The depth below mean sea level (in feet) to the saline water interface beneath the South Fork based on computer simulation.

tained from well logs and records of wells drilled closer to the shoreline.

A series of unconsolidated Pleistocene and Cretaceous sediments underlies the area. The freshwater reservoir is confined to the two uppermost units (Figure 5). The Glacial aquifer consists of a series of stratified upper Pleistocene sand and gravel deposits. The underlying beds have been designated as the Magothy Formation and Matawan Group, undifferentiated. The water-bearing layers of fine sand and silt are known as the Magothy aquifer. The average horizontal hydraulic conductivities of the Glacial and the Magothy aquifers are 150 and 70 ft/day, respectively [*Fetter*, 1971]. The Glacial aquifer extends to an average depth of 150 feet below sea level.

The average annual precipitation of 45 inches represents the only natural source of freshwater. By applying the Thornthwaite method of computing evapotranspiration [*Thornthwaite and Mather*, 1955, 1957], the actual evapotranspiration was determined to be about one-half of the precipitation. A comparison with studies made elsewhere on Long Island suggests that direct runoff is probably about 3–5% of precipitation [*Pluhowski and Kantrowitz*, 1964; *Warren et al.*, 1968]. The rest of the precipitation, about 21 inches annually, recharges the water table. Groundwater withdrawn by pumping is for the most part artificially recharged through cesspools. Gaged streamflow, consumptive withdrawals, and the direct evaporation of groundwater account for about 10% of the groundwater accretion. The rest, an average of 135,000 ac ft/yr, is discharged as undersea outflow at the perimeter of the island.

A computer program was designed that used a grid of nodal points at a spacing of 1000 feet. The actual boundary of the South Fork was closely approximated. The density of saline groundwater from a well on the South Fork was 1.024 g/ml [*Fetter*, 1971]. The chloride con-

TABLE 1. Comparison of the Measured Elevation to the Top of the Zone of Diffusion beneath the South Fork with the Computed Elevation of the Saline Water Interface

Location on Figure 7	New York State Well Number	Measured Elevation of the Top of the Zone of Diffusion	Computed Elevation of the Saline Water Interface
A	S-18819	100	...*
B	S-31037-T	350	452
C	S-33922-T	600	584
D	unknown	425	420
E	unknown	150	...*
F	S-31053-T	450	431
G	S-30227	160	150
H	S-31735-T	120	127

The measured elevation of the top of the zone of diffusion and the computed elevation of the saline water interface are in feet below mean sea level.

* Wells A and E are located at the coastline; therefore the model does not provide a solution, since one of the boundary conditions is that the head at the coastline is always equal to 0.

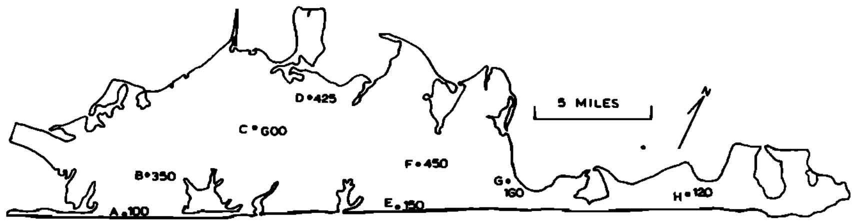

Fig. 7. The locations of wells in which the depth to the saline water interface was measured. The letters represent the wells, and the numbers are the depths in feet below mean sea level to the zone of diffusion.

tent of the well was 17,500 mg/l, which is slightly less than the 17,800 mg/l [*Perlmutter and DeLuca,* 1963] in the Atlantic Ocean nearby. The density of saline groundwater was therefore assumed to be the generally accepted value of 1.025 g/ml. With the values of the physical parameters as given above, the depth to the saline water interface was generated. A contour map of the interface was made from the data (Figure 6).

A comparison of the measured depths to the top of the zone of diffusion with the computed depths to the saline water interface at the same location is shown in Table 1. (The locations of the wells are shown in Figure 7.) Other than well S-31037-T the computed depths are within 6% of the measured depths. Well S-31037-T is located at an existing well field in which pumpage during the summer months averages about 1 million gpd. The measured depth to the top of the zone of diffusion is almost 100 feet less than the computed depth to the saline water interface. Withdrawal from the well field has evidently resulted in an upward movement of the zone of diffusion.

SALINE WATER INTERFACE AT THE SHORELINE

One of the consequences of using the Dupuit assumptions is that the model does not provide a subsurface outflow face for the discharge of fresh groundwater to the sea. Solutions have been given by a number of authors for a defined subsea outflow face.

Rumer and Shiau [1968] examined the position of the saline water interface at the coastline of an infinitely long aquifer system in which there was no vertical recharge to the aquifer system. The solution that they gave for an unconfined homogeneous coastal aquifer can be expressed in dimensionless form as

$$Z' = (G + 1)\Phi'\Psi' + G\Phi'$$

$$X' = \tfrac{1}{2}(G + 1)(\Phi'^2 - \Psi'^2) - G\Psi' \qquad (28)$$

The dimensionless constants are $Z' = (KGz)/Q$, $\Phi' = \phi/Q$, $X' = (KGx)/Q$, and $\Psi' = \psi/Q$, where

- K, hydraulic conductivity, LT^{-1};
- Q, discharge of freshwater, L^3T^{-1};
- ψ, stream function;
- ϕ, velocity potential, L^2T^{-1};
- G, $(\gamma_s - \gamma_f)/\gamma_f$.

Along the interface the boundary conditions can be expressed as $X' = -1$ and $Z' = -\phi$.

If we select an infinite strip island with a width of 36,000 feet and an average daily groundwater recharge rate of 0.00525 ft/day, the freshwater discharge past the coastline will be 94 ft^3/day/foot of coastline. Given an aquifer conductivity of 150 ft/day and values for γ_f and γ_s of 1.000 and 1.025 g/ml, the width of the outflow face computed by Rumer and Shiau's method is 12.6 feet. This interface is shown as a solid line on Figure 8.

The saline water interface for the given infinite strip island was computed by using (16), which employs the Dupuit assumption. This interface is the dashed line on Figure 8. Other than at the immediate coastline the two interfaces are only about 5 feet apart. The values of aquifer conductivity and recharge are those used for the South Fork analysis, and the width of the South Fork has a maximum of about 36,000 feet. Thus under natural conditions of fresh groundwater discharge the use of the Dupuit assumptions does not introduce a significant error in the computation of the position of the saline water interface.

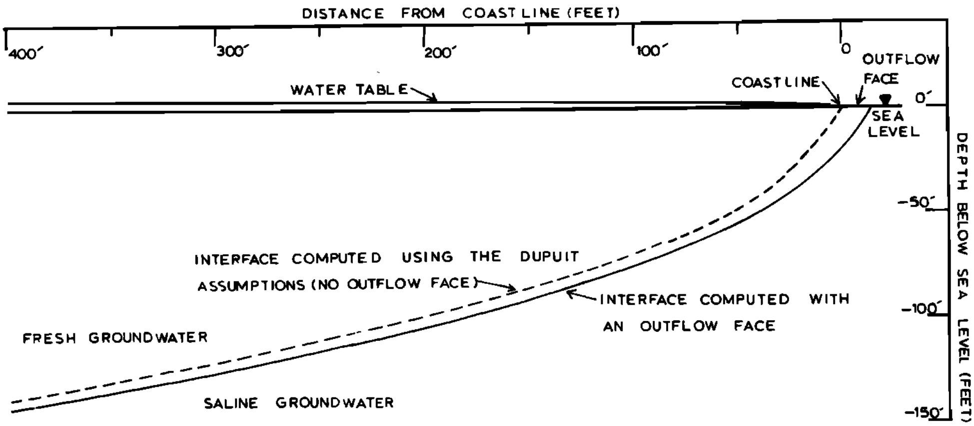

Fig. 8. A comparison of the position of the saline water interface beneath a coastline computed with and without an undersea outflow face.

SUMMARY AND CONCLUSIONS

An equation describing the two-dimensional steady state position of the saline water interface in a phreatic aquifer beneath an oceanic island has been developed. This equation can be solved analytically for certain simple boundary value problems. Numerical solutions can be obtained on a digital computer for oceanic islands of any shape. The model was successfully used to generate the known position of the saline water interface beneath the South Fork of Long Island, New York, and could be used in the water resource evaluation of oceanic islands. When the location of the saline water interface is unknown or known in only a few places, its position can be determined.

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