

2. Optimization to Problems

Thursday, April 16, 2020 7:18 AM

Review

Greedy Algorithm

Pros:

- Easy to implement
- Very Fast
- If a problem fits the **greedy property**, you can take a shortcut: the globally optimal solution can be arrived at by making locally optimal choice (i.e. greedy choice property)

Cons:

- Doesn't actually solve the problem (may or may not be optimal)
- We don't know how close to optimal it is

Search Trees

Using a Search Tree to Implement the Greedy Algorithm

Search tree enumerates possibilities. Using a root to draw out consideration for each of the two element options (children). You run the **recursive** function until you reach the leaf (end)

Left First Depth First

Traversing a tree by all the left-hand side (taking all your options)

- The levels reflect the number of choices
- Number of node = 2^{n+1}

Use

- It's helpful to list all potential outcomes at every point that you need to make a decision-- all outcomes are unique because it captures everything you take and don't take (inside and outside the bag)
- You can then weigh the value and constraint at the end (implementing the greedy algorithm)
- It doesn't actually build a tree, but just stores each legal solution at every iteration and updates the best solution when it finds one

Implementation

Header for Decision Tree Implementation

```
def maxVal(toConsider, avail):  
    """Assumes toConsider a list of items,  
        avail a weight  
    Returns a tuple of the total value of a  
        solution to 0/1 knapsack problem and  
        the items of that solution"""
```

toConsider. Those items that nodes higher up in the tree (corresponding to earlier calls in the recursive call stack) have not yet considered

avail. The amount of space still available

Body of maxVal (without comments)

```
if toConsider == [] or avail == 0:
    result = (0, ())
elif toConsider[0].getUnits() > avail:
    result = maxVal(toConsider[1:], avail)
else:
    nextItem = toConsider[0]
    withVal, withToTake = maxVal(toConsider[1:],
                                avail - nextItem.getUnits())
    withVal += nextItem.getValue()
    withoutVal, withoutToTake = maxVal(toConsider[1:], avail)
    if withVal > withoutVal:
        result = (withVal, withToTake + (nextItem,))
    else:
        result = (withoutVal, withoutToTake)
return result
```

Does not actually build search tree

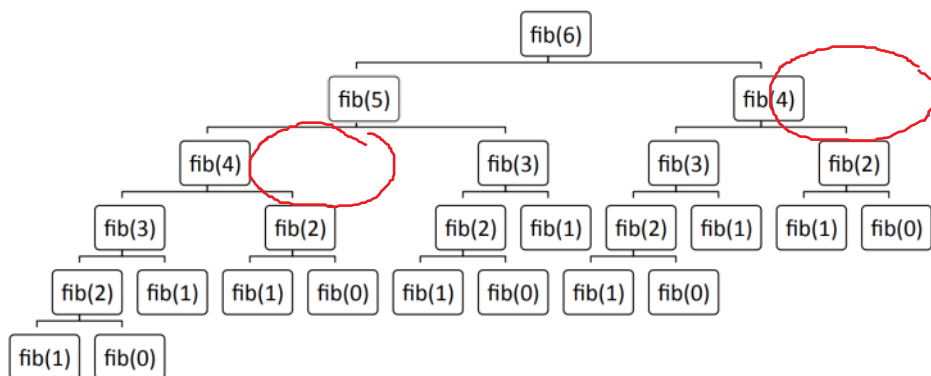
Local variable `result` records best solution found so far

Limitation

Search trees can get really slow past a certain number of options.

The call tree shows that it can be a wasteful algorithm because it does the same calculations over and over again (i.e. lots of duplicate calculations)

Call Tree for Recursive Fibonacci(6) = 13



Solution: you store the answer and look it up when you need it (this is the foundation of **dynamic programming**)

- Aka Memo-ization (creating a memo)
- You save time by trading space (negligible because it saves you a lot of time without a lot of space)

Dynamic Programming

The problems it can help is when problems have optimal substructure and overlapping subproblems

■ **Optimal substructure:** a globally optimal solution can be found by combining optimal solutions to local subproblems

- For $x > 1$, $\text{fib}(x) = \text{fib}(x - 1) + \text{fib}(x - 2)$

■ **Overlapping subproblems:** finding an optimal solution involves solving the same problem multiple times

- Compute $\text{fib}(x)$ or many times

Laymen's Term:

- You can solve two independent problems and then putting them together
- Creating a memo is less effective if you don't solve the same problems multiple times

Example

- Dynamic cannot help us with Merge sort: they're independent, but they're different sort problems. In other words, when you separate the problems, there's no overlapping subproblems
- Search tree help with dynamic programming?
 - Does have optimal substructure: choosing between the left branch or the right branch
 - Overlapping subproblems: Non of the branch looks identical, so though you can have dynamic programmed algorithm but you won't see a speed up

Link

<https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-0002-introduction-to-computational-thinking-and-data-science-fall-2016/lecture-videos/lecture-2-optimization-problems/>