Comparing the Periods of Serial and Parallel springs

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Date: 06/07/2023

ABSTRACT

This lab report investigates the application of Hooke's Law in predicting the periods of oscillating systems composed of parallel, serial, and normal springs. The derived spring constant serves as a crucial parameter in predicting the behavior of spring systems. This experiment aimed to compare the predicted periods, based on the calculated effective spring constant, with the experimental periods obtained from the conducted tests. The experimental period was expected to fall within the range of the expected error from the predictions. It was hypothesized that the period would be affected by a factor of square root 2. Experimental results for the normal, serial, and parallel spring systems yielded periods of 0.69±0.01, 0.97±0.01, and 0.49±0.02, respectively. These periods fell within the range of the predicted data. The observed results highlight the importance of accurately calculating the effective spring constant and considering the factor of the square root of 2 for identical springs.

1.0 INTRODUCTION

An oscillating system, characterized by its periodic sinusoidal motion, is a fundamental concept in the field of physics. It can be portrayed by the following equation of motion:

(1) $y(t) = Acos(\omega t + \theta) + c$

Where A is the amplitude, ω is the angular frequency, θ is the phase angle, and c is the vertical offset.

Understanding the behavior of such systems is crucial in numerous scientific and engineering applications. One approach to studying oscillating systems is through the application of Hooke's Law, which provides a valuable framework for analyzing the forces and dynamics involved.

Hooke's Law states that the force exerted by a spring is directly proportional to the displacement it experiences. Mathematically, this relationship can be expressed as:

(2) $F =- kx$,

where F represents the force applied, k is the spring constant, and x is the displacement from the equilibrium position of the spring. By manipulating Hooke's Law, it is possible to derive the spring constant, which serves as a key parameter in predicting the behavior of spring systems.

In this particular experiment, our objective was to utilize Hooke's Law and the derived spring constant to predict the periods of parallel, serial, and normal spring systems. The period of an oscillating system is defined as the time it takes for one complete cycle of motion. By accurately predicting these periods, we aimed to gain insights into the relationship between the spring constant and the resulting oscillatory behavior of various spring configurations. The motivation of this experiment is to obtain a better understanding of the complexity of modern systems, we decided to analyze the effects of different configurations of springs on the period.

(3) $k_{parallel} = k_1 + k_2$

(4) $\frac{1}{k_{serial}} = \frac{1}{k_1} + \frac{1}{k_2}$

Equations (3) and (4) portray the equations of the effective spring constant equations for parallel and serial series. In this case, k1=k2. This simplifies the effective spring constants.

(5) $k_{parallel} = 2k_1$

(6) $k_{serial} = \frac{k_1}{2}$

Equations (5) and (6) are then plugged into the period equation:

(7) $T_{normal} = 2\pi\sqrt{\frac{m}{k}}$

This yields the expected relationship between spring constant and period for the special case where $k_1 = k_2$.

(8) $T_{parallel} = \frac{T_{normal}}{\sqrt{2}}$

(9) $T_{serial} = \sqrt{2}T_{normal}$

(10) $\omega = 2\pi/T$

After analyzing the relationships and equations provided, it is hypothesized that the experimental values for the period of motion will fall within the margin of error for the predicted values. It is also predicted that the periods will be related by a factor of $\sqrt{2}$, as shown in equations (8) and (9).

To test our hypothesis and assess the validity of our predictions, we conducted a series of experiments using different spring systems. The predicted periods based on the spring constants were compared to the measured periods obtained through experimental data. By analyzing the agreement or discrepancy between the predicted and experimental results, we sought to gain a deeper understanding of the accuracy and limitations of Hooke's Law in predicting the behavior of spring systems.

2.0 METHODS

This experiment requires the use of an arduino with a bluetooth module and an ultrasonic sensor, two springs with the same spring constants, two 50 gram masses, and a stand to hang the spring-arduino system. These tools are used to monitor and collect data on the different spring-mass system configurations. The data collected will then be analyzed to find the period of each system.

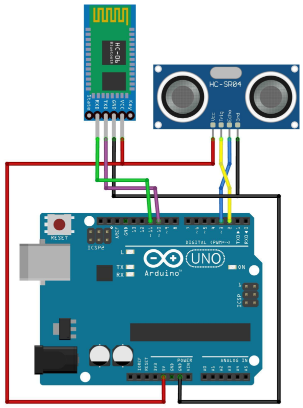


FIGURE 1: Arduino circuit diagram and wire set-up, including the ultrasonic sensor and bluetooth module.

As seen in Figure 1, the arduino is set up with both the bluetooth and the ultrasonic sensor. This eliminates the effect of a wire hanging from the arduino, which would skew the data since it adds weight and resistance. The ultrasonic sensor collects data by sending out sound pings, which reflect off of the nearest surface. It measures the amount of time it takes between sending the ping and receiving the reflected wave. This results in position versus time data.

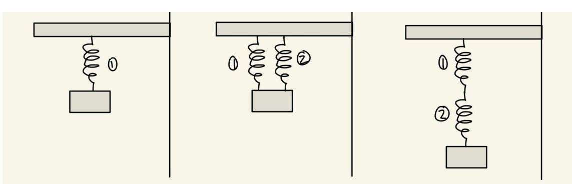
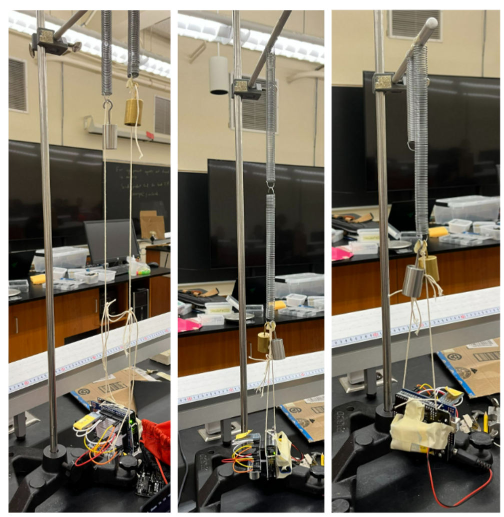


FIGURE 2: Conceptual set up for the experiment. From left to right, there are the normal, parallel, and serial spring configurations, with a mass attached at the bottom.

The conceptual set up, depicted in Figure 2, shows the three mass-spring configurations tested in this experiment. The mass is hung from the springs, which are hung from the stand. The single spring system is used to set a control, or baseline set up, which the other systems will be compared to.

Before the experiment is conducted, it is important to measure the spring constant of each spring. For this process, different masses are hung from the spring and the associated spring displacement is measured. The mass versus displacement plot will appear linear. By Hooke's Law, the slope of this plot is the spring constant.

In order to simplify this experimental process, a few assumptions must be made about the setup. First, it is assumed that the springs are approximately massless, and there is no distortion in the spring. Additionally, it is assumed that there is no air resistance as the mass oscillates. These assumptions allow for the equations, in Section 1.0, to be valid.



A B C

FIGURE 3: This figure depicts all three experimental set-ups. Part (A) shows the parallel configuration, part (B) shows the serial configuration, and part (C) shows the normal configuration. The arduino is hung from the springs with two additional 50 gram masses.

The experimental set-up, shown in Figure 3, slightly differs from the experimental set-up in Figure 2. Two additional 50 gram masses were added to each spring system. This is done to allow for the series system to oscillate properly. The arduino's mass alone was too small to account for the increased effective spring constant for the serial system. In order to keep each trial system consistent, the additional masses were included in the normal and parallel systems as well. Additionally, the parallel series set-up is tied in a way so that the springs are not at an angle. This is important for the precision of the data, since the springs need to be parallel and the ultrasonic sensor needs to face the ground.

To reduce noise in the data, each trial for each configuration had the same initial displacement of two centimeters. Once the mass is released, the arduino code instructs the ultrasonic sensor to collect position and time data for the oscillation. This data is then uploaded and analyzed using python code.

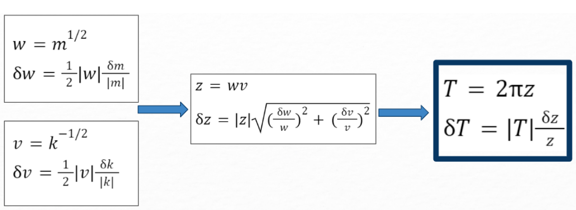


FIGURE 4: This diagram displays the error propagation technique used for the period predictions. If the experimental values fall within this range, the hypothesis is confirmed.

Figure 4 demonstrates the process used to find the errors associated with the predicted periods. This will be utilized to determine whether the hypothesis is supported by the data.

3.0 RESULTS

As an overview, the goal is to compare the periods of oscillation between the different configurations using the angular frequency as collected from the best-fit sine function. The following results verify the special case for Equations (3) and (4) by numerically and visually exemplifying the difference by factors of $\sqrt{2}$

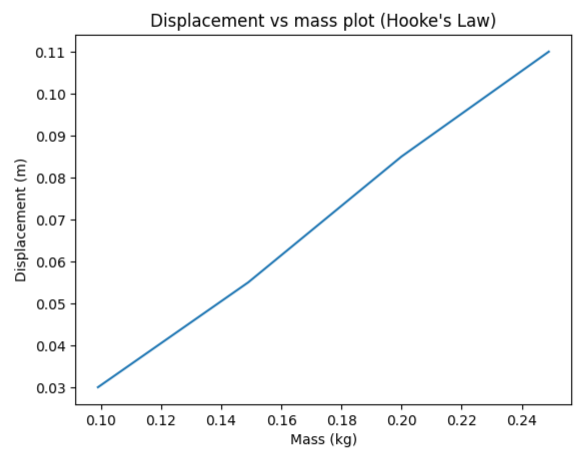


FIGURE 5: Hooke's Law for Normal Configuration. The slope of the line represents the spring constant for the single springs.

	Spring Constant (N/m)
Normal	18.18 ± 0.40
Serial	9.09 ± 0.20
Parallel	36.36 ± 0.81

TABLE 1: The effective spring constants for each corresponding configuration is determined.

Figure 5 displays the slope from which the spring constant is extracted. Using Equations (5) and (6), the following values errors are calculated in Table 1. Each constant corresponds to the slope of their respective mass-displacement graph.

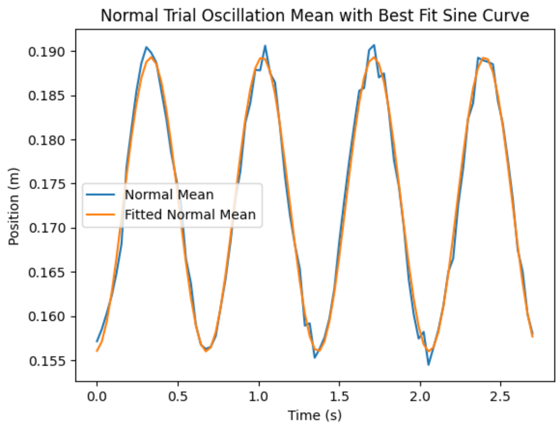


FIGURE 6: The mean for the normal configuration data is plotted with the calculated best fit function. This process is repeated for the other two configurations

Each dataset is converted to SI units of position in meters and time in seconds. It is then further refined by filtering out 4 periods of oscillation, starting at 0 seconds.

The means for each configuration are calculated by standardizing the values into fixed-sized window frames and averaging the results. A sine function, as seen in Figure 6, is then fitted to obtain the optimal angular frequency and period. Figure 7 displays the fitted function for the means of the three configurations.

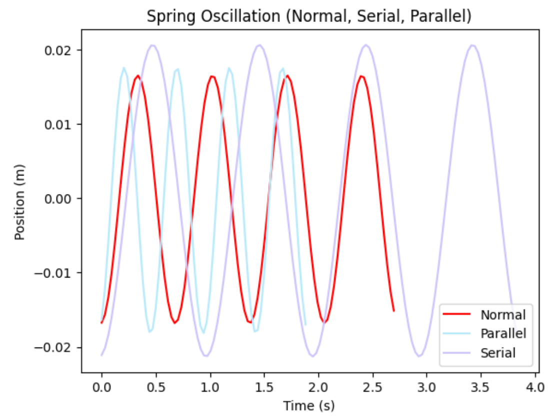


FIGURE 7: The relationship between the period of motion for different spring configurations is visually evident. They are related by factors of square-root two.

The fitted functions for each configuration are as follows:

$$y_n(t) = 0.02cos(9.11t + 4.81) + 0.17$$
$$y_s(t) = 0.02cos(12.9t + 5.16) + 0.14$$
$$y_p(t) = 0.02cos(6.38t + 4.85) + 0.14$$

The terms in their respective orders correspond to the amplitude, angular frequency, phase, and offset as shown in Figure 7.

Period (s)	**Predicted**	**Experimental**
Normal	0.68 ± 0.01	0.69± 0.01
Serial	0.96 ± 0.01	0.97± 0.01
Parallel	0.48 ± 0.01	0.49± 0.02

TABLE 2: This table compares the predicted and experimental values for the period of motion for different spring configurations.

Using the angular frequency obtained from the best-fit function and Equation (10), Table 2 is generated and displays the experimental data collected.

The following data agrees with our predicted hypothesis for the period of oscillation as the experimental data falls under the range of expected error.

4.0 CONCLUSIONS

In order to change the effective spring constant of the system, springs with the same spring constants were added in series and parallel configurations. As the results show, the spring constant significantly affects the period of motion for the system in a predictable manner.

The central goal of this experiment is to determine whether the predicted periods of different mass-spring configurations match the experimental values obtained. The periods are predicted to be related by factors of $\sqrt{2}$ for the special case where $k_1 = k_2$. The normal period is plugged into equations (8) and (9), yielding:

$$T_{parallel} = \frac{0.68}{\sqrt{2}} = 0.481s$$

$$T_{serial} = \sqrt{2} * (0.68) = 0.962s$$

These values match those in Table 2, verifying the predicted relationship.

The experimental values fall within the predicted range, and the period values verify the predicted relationship; therefore, the hypothesis is verified.

Although air resistance was assumed to be negligible, the experiment was not conducted in a vacuum, so it could have affected the experiment. This could be a source of error, which would have slowed the oscillation and affected the period.

Additionally, for future experiments it would be important to include springs with different spring constants. This experiment utilizes the special case where the spring constants are equal, which means that this experiment can not verify the effective spring constant equations. Future experiments with different spring constants won't have the same period relationship, but it will be able to verify equations (3) and (4).

REFERENCES

UCLA, Department of Physics and Astronomy. (2023). *Unit 3 - oscillations (Winter 2023)*. Physics 4. Retrieved May 17, 2023, from https://www.uclaphysics4labs.org/unit-3---oscillations-spring-2023.html