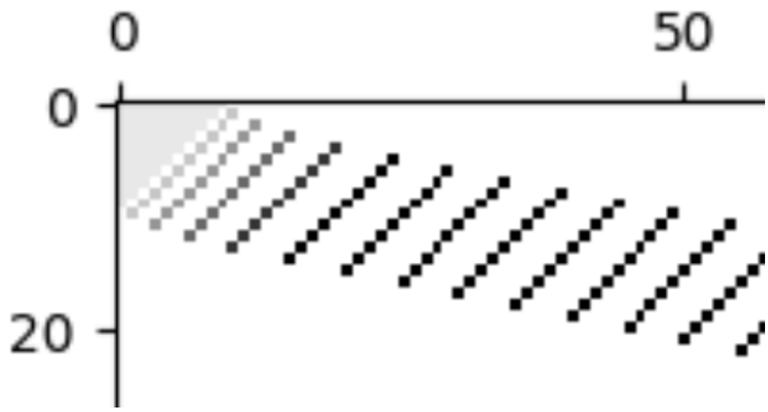
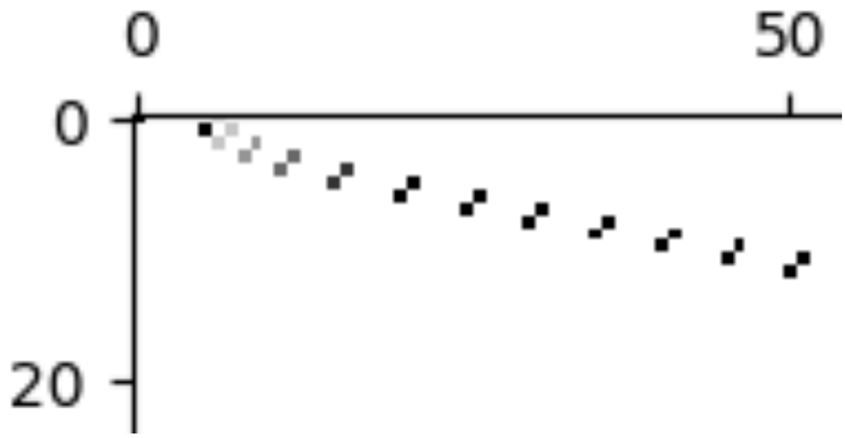
Traffic Flow

Task 1: A First CA Model

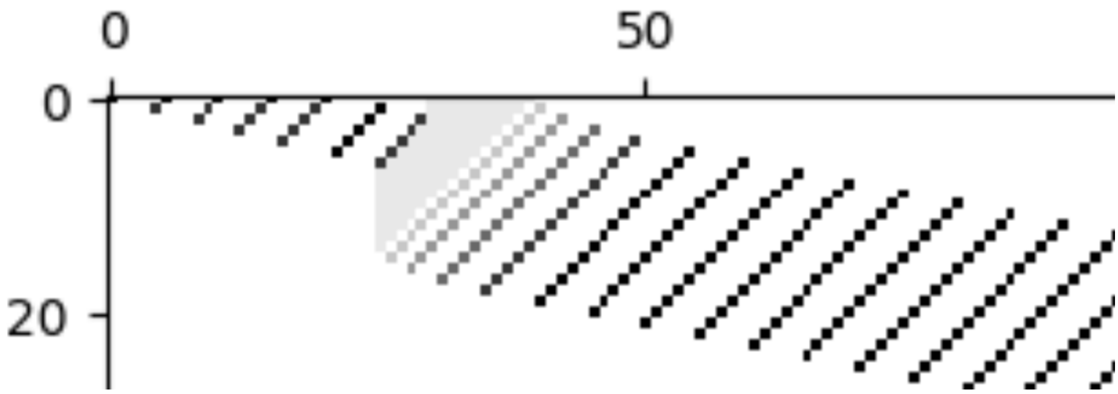
1. We shall verify the correctness of our implementation by comparing the result of the simulation on a set of test cases with the predicted behavior. Recall that lighter grey cells correspond to slower moving cars, and darker cells correspond to faster moving cars.
   Let's consider the case in which there are 10 cars bumper to bumper at the beginning of our road. The predicted behavior is that the cars in front will begin to accelerate one by one, while the cars in the back remain in standstill until there is distance between them and the car in front of them. In the end, since there are only 10 cars on the road, we should observe all 10 cars in free traffic flow, moving uniformly at the maximum velocity. This predicted behavior matches what is demonstrated in the simulation:

   Let's consider the case in which a fast moving car is approaching a slow moving car from behind. According to the rules of the simulation, we would expect the slow moving car to eventually accelerate, but if the fast moving car approaches too quickly then it will decelerate accordingly. This predicted behavior matches what is demonstrated in the simulation:

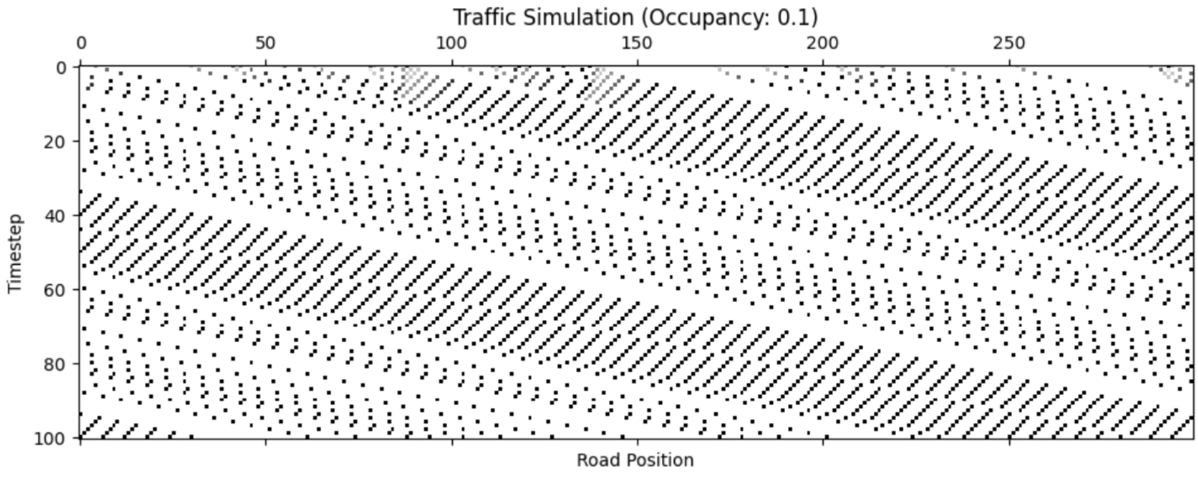


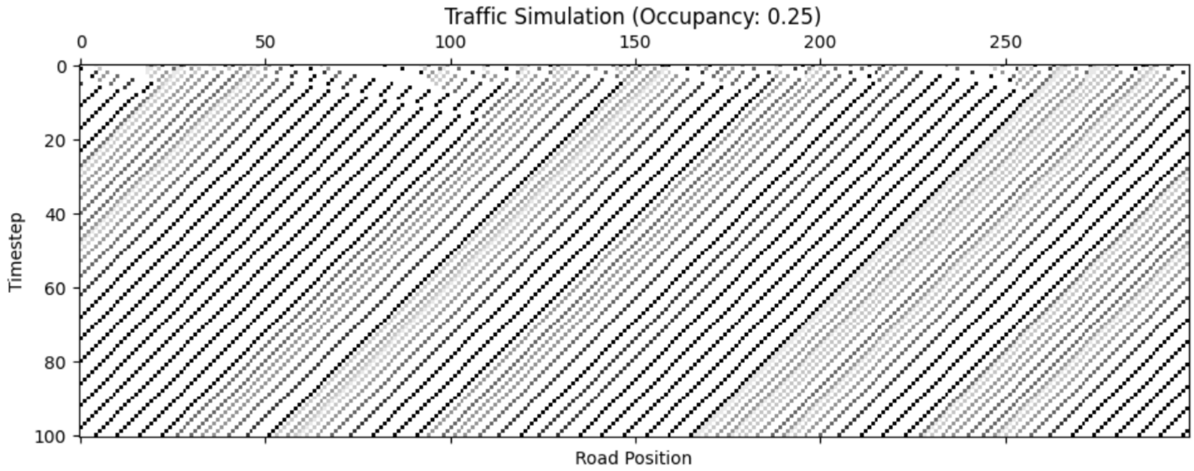
Let’s combine these two cases and consider the case in which there are 10 cars bumper to bumper in a traffic in the middle of the road. Meanwhile, 5 cars approach from behind. The predicted behavior of the jam itself should match the behavior exhibited in the first case. If the traffic jam does not resolve by the time the 5 approaching cars reach the last car in the jam, they too will slow down and become part of the traffic jam. In the end, since there are only 15 cars on the road, we should observe all 15 cars in free traffic flow, moving uniformly at the maximum velocity. This predicted behavior matches what is demonstrated in the simulation:

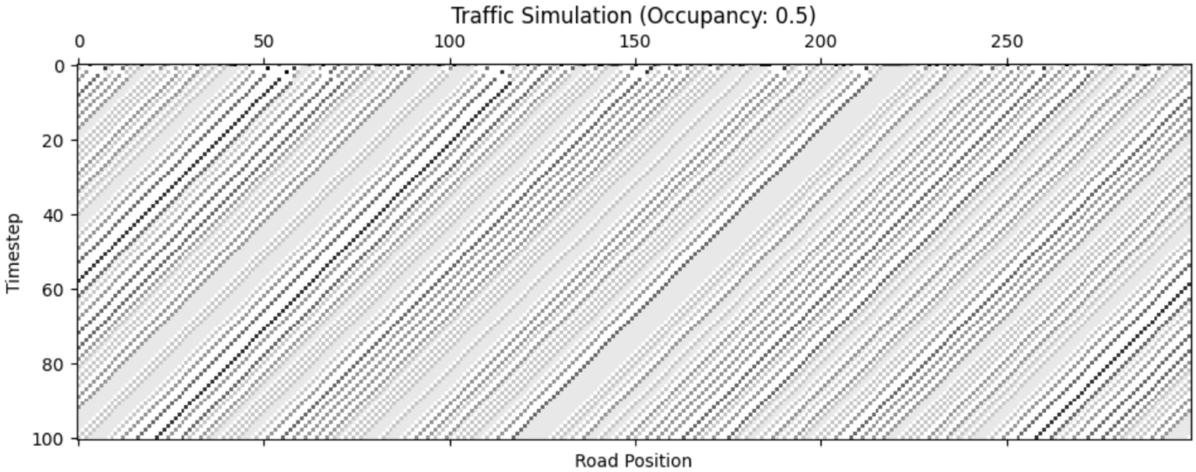


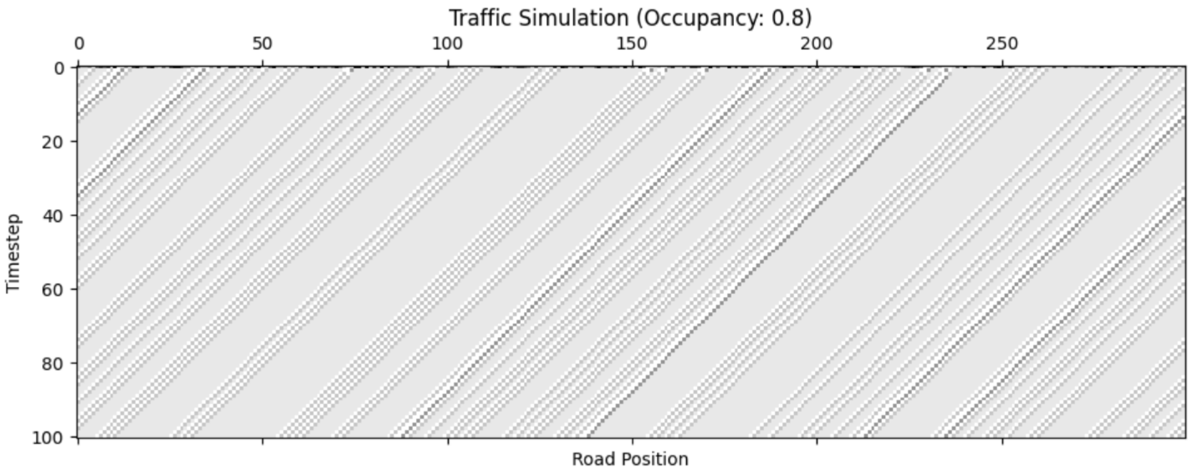
There are “elementary test cases” for one-lane traffic that can be combined to form all other possible scenarios of one-lane traffic. These include a slow moving car behind a fast moving car, a fast moving car behind a slow moving car, a sequence of slow moving cars, and a sequence of fast moving cars. The test cases above demonstrate that our model matches the predicted behavior for each of these elementary test cases. Since any initial condition is a sequence of these elementary test cases, our model will maintain its correctness for any scenario of one-lane traffic.

2. In the case of 10% occupancy, each vehicle has on average 10 cells to itself. As described by the textbook, 6 cells per vehicle are needed to maintain maximum velocity. 10 cells is plenty above this threshold, thus the end behavior of the 10% occupancy simulation is steady state free traffic flow for all the cars, after a few initial timesteps for clusters of cars to spread out.
   In the case of 25% occupancy, each vehicle has on average 4 cells to itself. Since this is under the 6 cell threshold, we begin to see uniformly moving traffic jams forming. Each diagonal strip of lighter grey cells represents a traffic jam shockwave, traveling backwards in space over time as cars in front accelerate and leave the jam while new cars join the jam from behind.
   In the case of 50% occupancy, we see thicker bands of standstill traffic begin to form. However, there is still enough space between the bands for vehicles to begin to speed up before they come to a stop again.
   In the case of 80% occupancy, thick bands of standstill traffic become the predominant feature of the model. The bands are much thicker than when occupancy was at 50%, and there is much less space between them for cars to begin to speed up. This makes sense, as each vehicle has less than 2 cells to itself on average. Thus, the only statistically sustainable velocity is 1 cell per timestep.





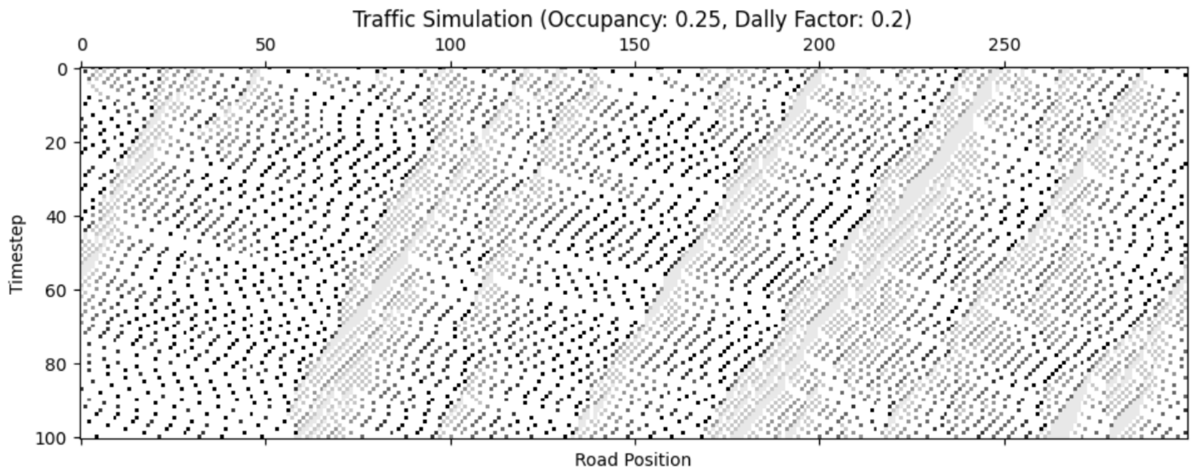




Task 2: Stochastic Behavior

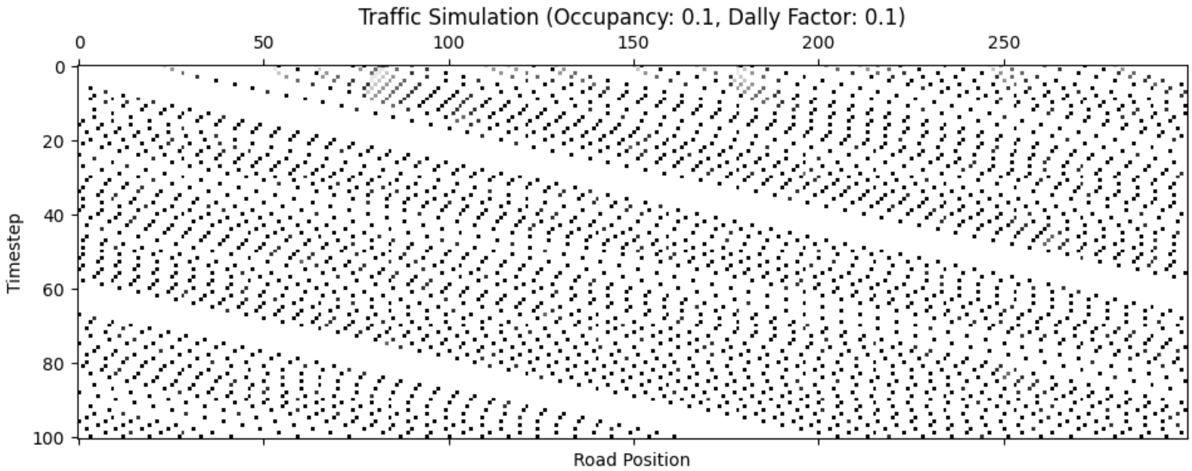
1. Comparing the behavior of 25% occupancy with and without a dally factor of 0.2, we can clearly see that the dally factor adds some random disruption to the very linear traffic shockwaves exhibited in the model

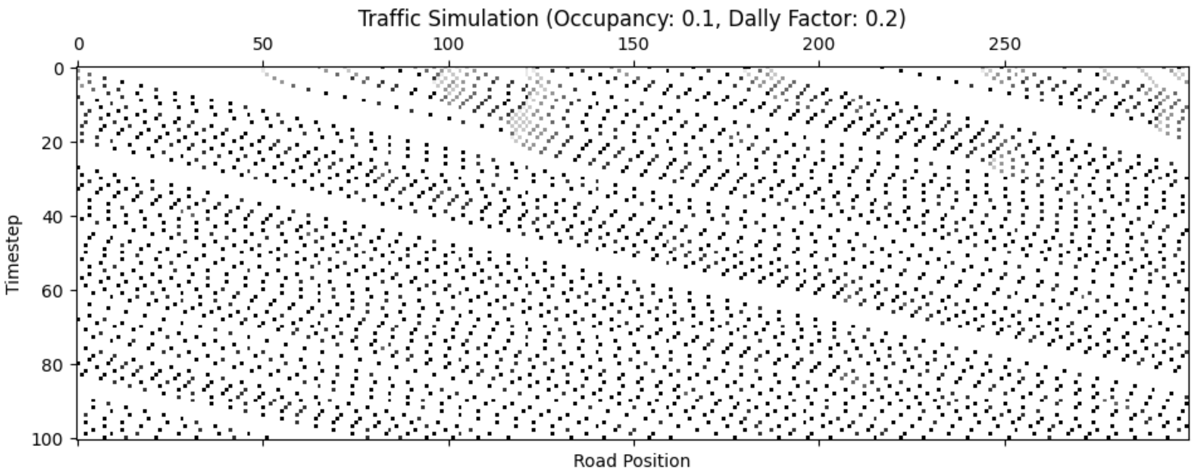
without dallying. This model seems to paint a much more realistic picture of how cars respond to other cars in traffic, with ununiform speeding up and slowing down. The results of the simulation agree with and depict the textbook's reasons for adding this dally factor: to account for random delays when accelerating, overbraking when decelerating, and dallying on an open road.

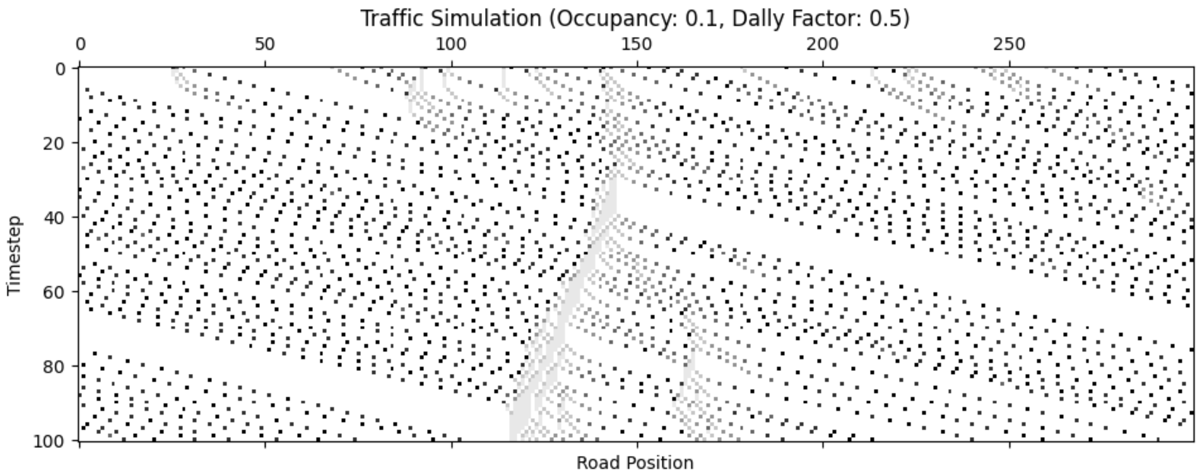


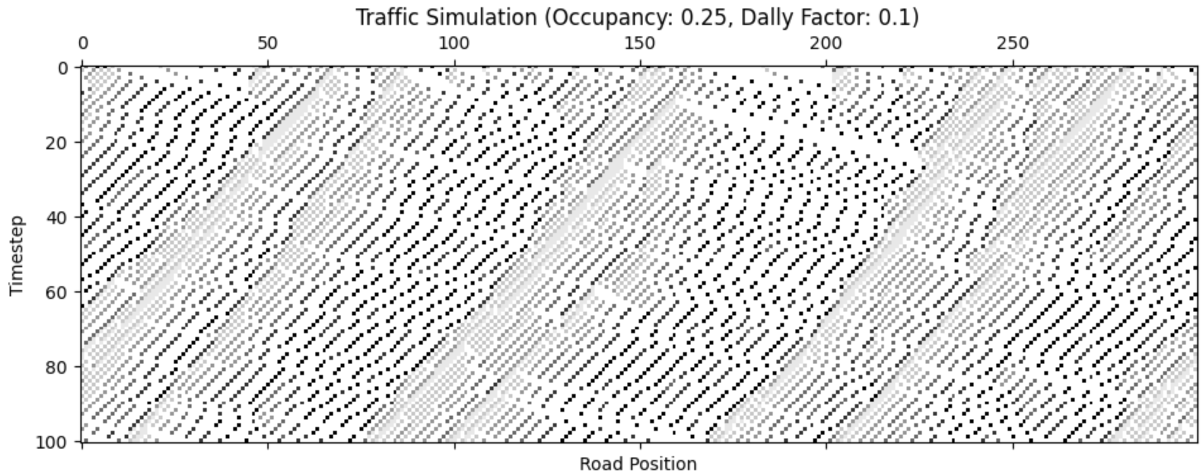
2. The simulation with 10% occupancy and a dally factor of 0.1 sees the least deviation from the model without any dallying at all. At this occupancy, each vehicle has on average 10 cells to itself. Even if a vehicle dallies, there is ample space for it to accelerate back to maximum velocity before it interferes with the vehicles behind it. Thus, there is little deviation to the simple free-flowing traffic we saw without any dallying at all. As we increase the dally factor however, it takes longer for vehicles to recover maximum velocity after dallying, and the likelihood of dallying increases. Thus as the dally factor increases, both the impact and frequency of dallying increases. At p=0.2, we see a much slower pickup rate at the beginning of the simulation before the model reaches free-flowing traffic. At p=0.5, the free-flowing traffic is punctuated by randomly appearing traffic jams, where dallying has accumulated and delayed a segment of traffic. However, given the low occupancy, these traffic jams are sparse and the dominant feature of the model is still free flowing traffic.
   Even while keeping the dally factor constant at 0.1, we see greater deviations from the dallyless model as we increase the occupancy. As each vehicle has less empty cells around it, the impact of even occasional dallying is propagated much further, since cars are required to respond to dallying in front them much quicker.

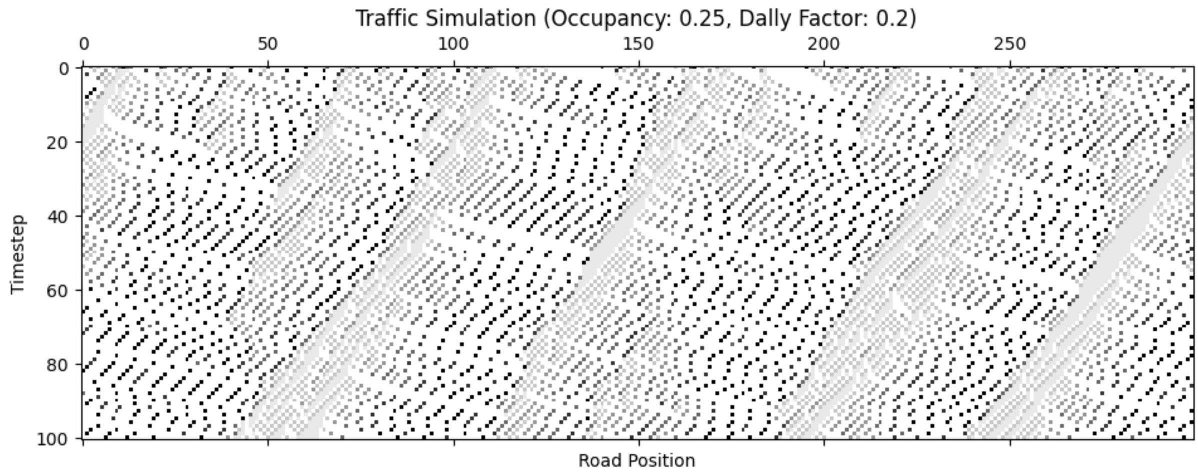
Without dallying, the models of occupancies of 25% and above become quite boring, with very stratified and predictable layers of heavier and lighter traffic. As we introduce dallying, the linear shockwaves of traffic are disrupted more and more, producing more realistic models of how cars respond to higher densities of traffic. It is also intuitive to say that as the occupancy increases, a higher dally factor is needed for a more realistic simulation. On the road, the more cars we interact with around us the higher the likelihood is of random braking, overbraking, and dallying.
At occupancy levels that have dense traffic standstills without dallying, there are instances where introducing dallying actually keeps more cars in motion at a time. This is because without dallying, a car will continue to accelerate towards the maximum velocity until there is another car within 5 cells in front of it. However, when dallying is introduced, the time it takes for a car to reach maximum velocity is increased. Since the car is traveling slower for a longer period of time, it will take more timesteps for it to reach the back of a traffic jam. Those extra timesteps slow the backwards propagation of a traffic jam.

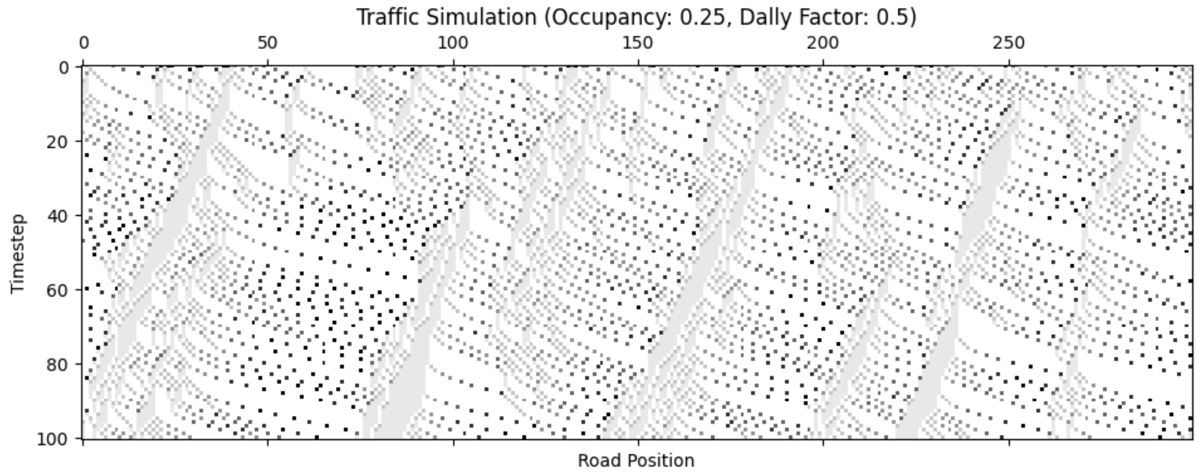


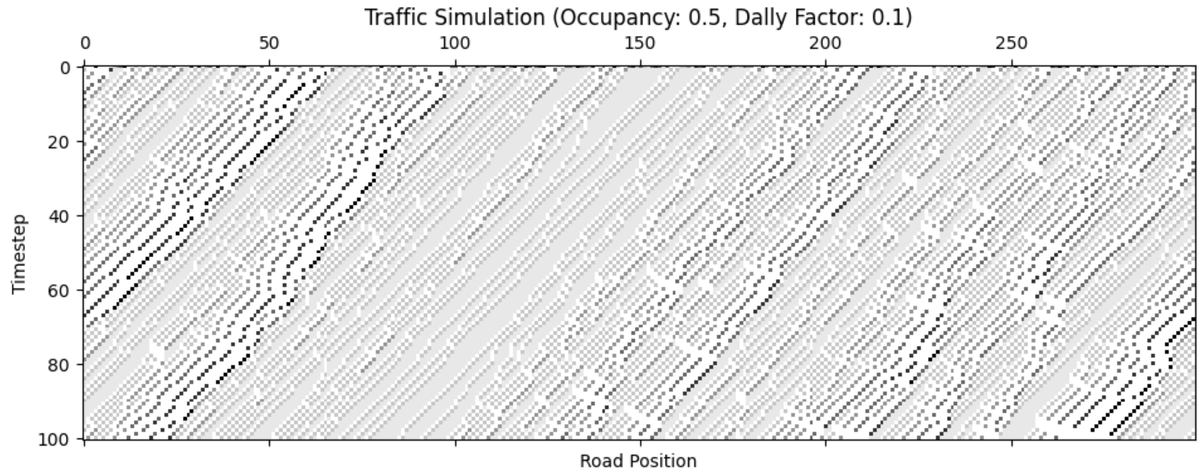



Traffic Simulation (Occupancy: 0.1, Dally Factor: 0.5)
Timestep
Road Position
0
50
100
150
200
250
0
20
40
60
80
100


Traffic Simulation (Occupancy: 0.25, Dally Factor: 0.1)
Timestep
Road Position
0
50
100
150
200
250
0
20
40
60
80
100


Traffic Simulation (Occupancy: 0.25, Dally Factor: 0.2)
Timestep
Road Position
0
50
100
150
200
250
0
20
40
60
80
100


Traffic Simulation (Occupancy: 0.25, Dally Factor: 0.5)
0 50 100 150 200 250
0 20 40 60 80 100
Timestep
Road Position


Traffic Simulation (Occupancy: 0.5, Dally Factor: 0.1)
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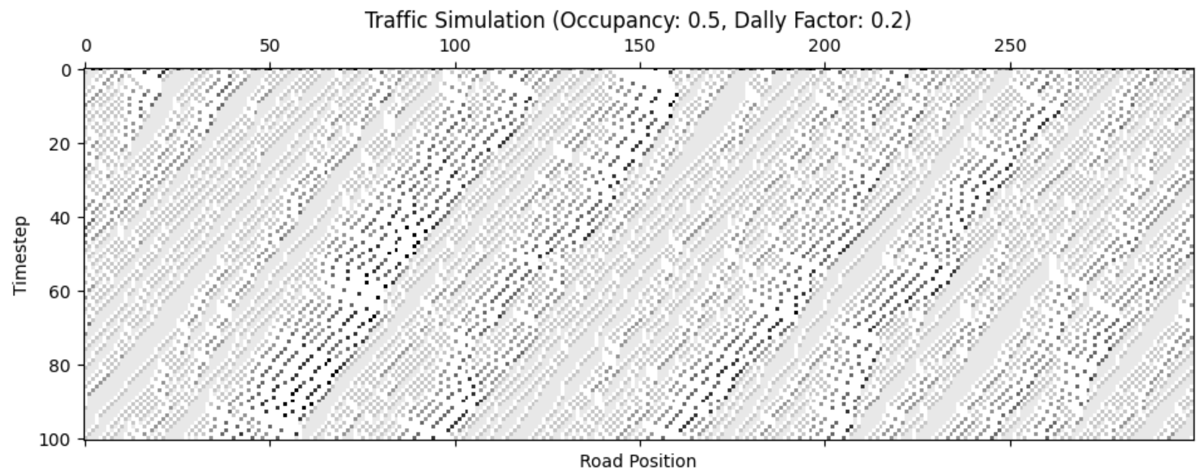
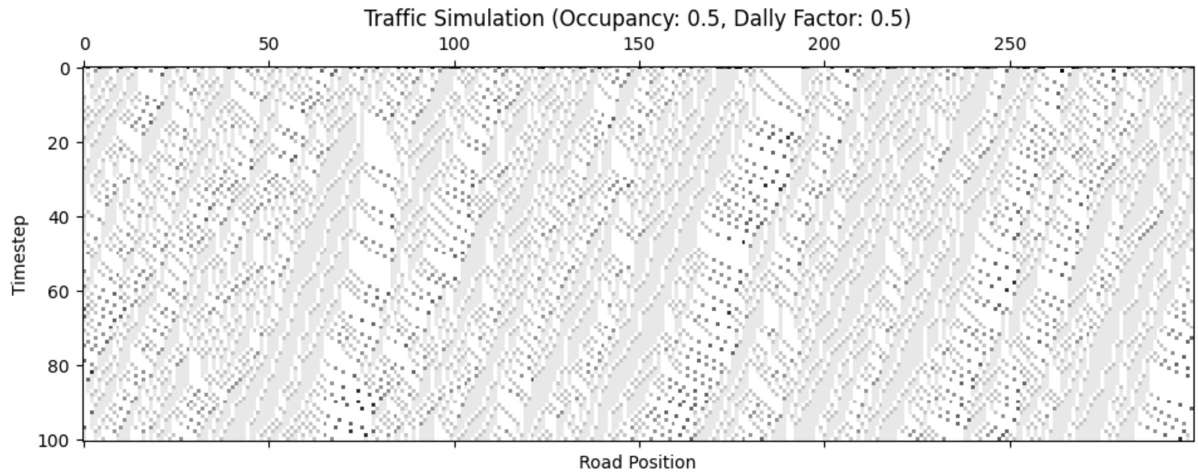
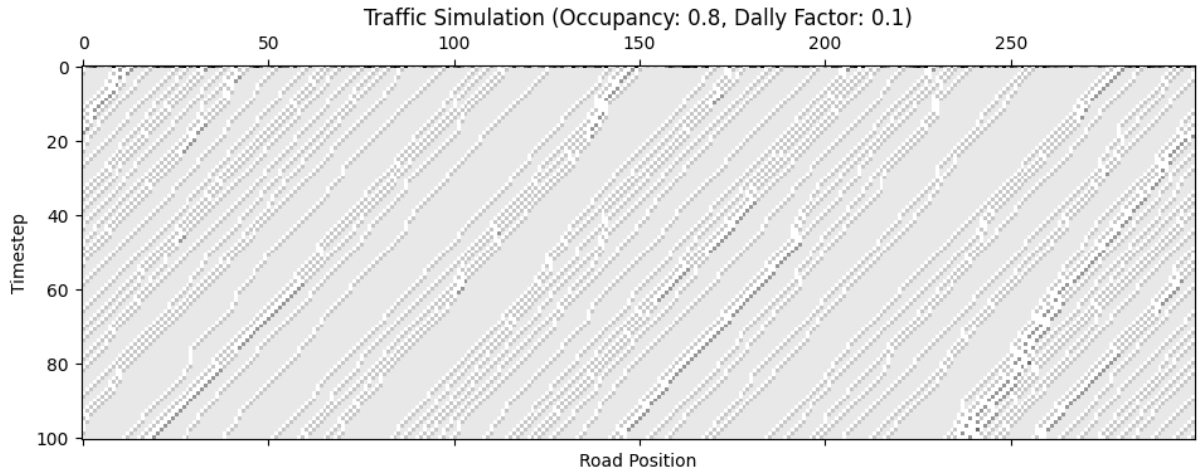
0 20 40 60 80 100
Timestep
Road Position

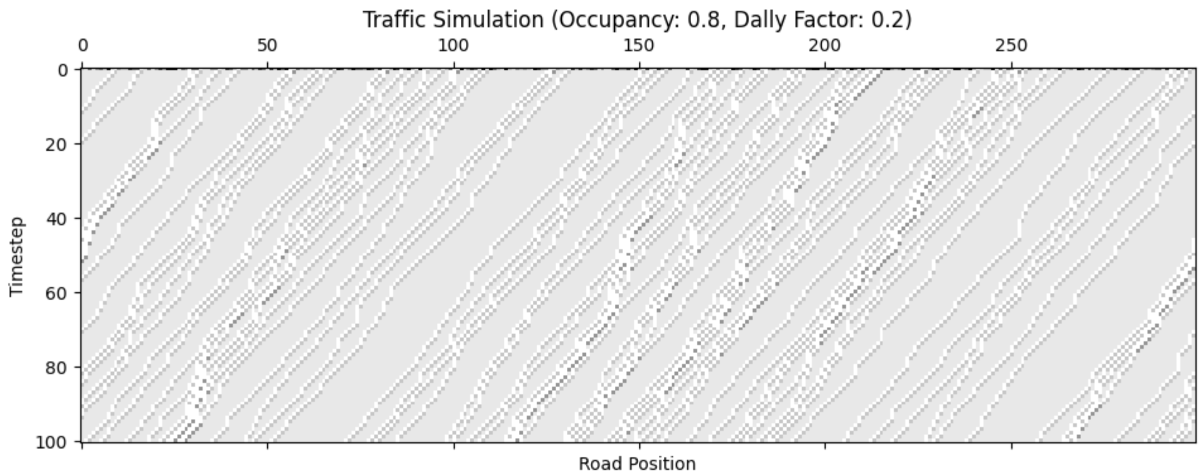

Traffic Simulation (Occupancy: 0.5, Dally Factor: 0.2)
0 50 100 150 200 250
0 20 40 60 80 100
Timestep
Road Position



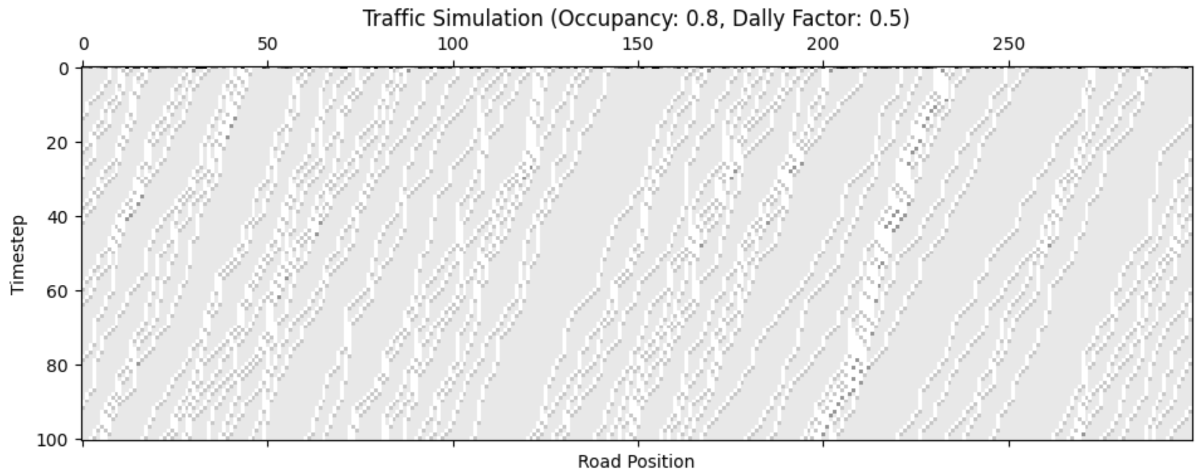
Traffic Simulation (Occupancy: 0.5, Dally Factor: 0.5)
Timestep
Road Position



Traffic Simulation (Occupancy: 0.8, Dally Factor: 0.1)
Timestep
Road Position



Traffic Simulation (Occupancy: 0.8, Dally Factor: 0.2)
Timestep
Road Position



Task 3: Two Lane Traffic

Algorithm:

Assume that $v_{L,\,max} \geq v_{R,\,max}$.

Assume that a car will not switch from lane L to lane R if it's current velocity is greater than $v_{R,\,max}$.

Velocity update for vehicle i located at cell k in lane R (R_i denotes the vehicle, $R[k]$ denotes the cell):

1. **Accelerate:** $v_{R,\,i} := \min\{v_{R,\,i} + 1, v_{R,\,max}\}$
2. **Lane Change:**
    a. **If:**
        i. **Need to Decelerate:**
            1. $v_{R,\,i} > d(R_i,\, R_{i+1})$
        ii. **Check Lane L Conditions:**
            1. $L[k]$ must not be occupied
            2. $L_i =$ hypothetical new vehicle at $L[k]$
            3. $L_h,\, L_j =$ cars immediately before and after L_i
            4. $v_{R,\,i} \leq d(L_i,\, L_j)$
            5. $v_{L,\,h} \leq d(L_h, L_i)$
        iii. **Caution for Other Lane Changes:**

1. $v_{R,\,i-1} \leq d(R_{i-1},\, R_i)$

b. **Then:**

i. Move car from $R[k]$ to $L[k]$

3. **Decelerate:**

a. **If:**

i. No lane change occurred

ii. $v_{R,\,i} > d(R_i,\, R_{i+1})$

b. **Then:** $v_{R,\,i} := d(R_i,\, R_{i+1})$

4. **Dally:**

a. If no lane change occurred:

i. $v_{R,\,i} := \max\{v_{R,\,i} - 1,\, 0\}$ with probability $p < 1$

b. If lane change occurred:

i. $v_{L,\,i} := \max\{v_{L,\,i} - 1,\, 0\}$ with probability $p < 1$

Velocity update for vehicle i located at cell k in lane L (L_i denotes the vehicle, $L[k]$ denotes the cell):

1. Symmetric algorithm as with lane R, but with additional lane change condition that

$$v_{L,\,i} \leq v_{R,\,max}$$

Move: All vehicles i move forward v_i cells.

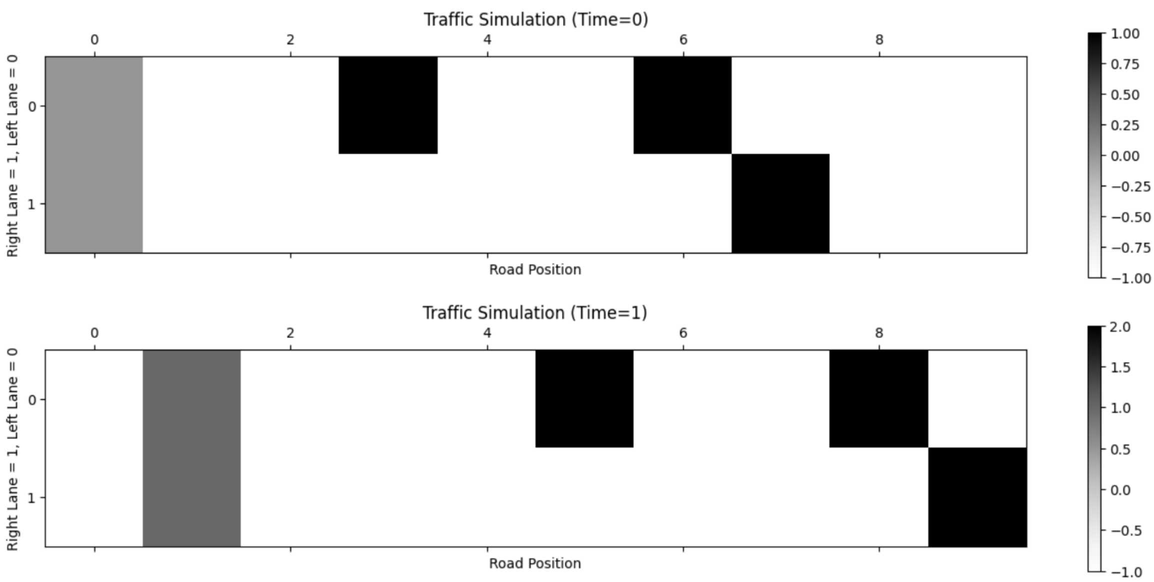
English Explanation:

After the accelerate step but before the decelerate step, we first want to check if a lane change is possible. First, we need to make sure a lane change is necessary, given the assumption of lane inertia, so we check that the car will otherwise have to decelerate if it does not change lanes. Second, if the car is currently traveling in the left lane at a velocity greater than the maximum velocity allowed in the right lane, it will not change lanes either, since changing lanes will not prevent the car from having to decelerate. Once these conditions are established, we need to ensure the safety of the lane change. First, the cell in the other lane that the car will change into must be unoccupied. Second, the car must not produce any collisions in the scenario of a lane change. Thus, the distance between the spot that the current car may change into and the location of the car in the other lane directly behind that spot must be greater than the speed of that car in the other lane. Likewise, the switching car must currently be traveling at a speed less than or equal to the distance between the spot it would change into and the car in the other lane directly in front of it. Finally, the car must consider the possibility of another car in its lane also switching lanes. If the car in the same lane behind the switching car is traveling at a speed greater than the distance between them, the car in front must not switch, since if they both switched lanes a collision would result. Due to the way this is handled, we need not consider the behavior of the car in front in the same lane, since it will consider the behavior of our car and prevent colliding with it. After the lane switching logic, if no lane switch was performed, the car will decelerate as per usual. All cars will then have a random chance to dally, before moving forward according to their velocity.

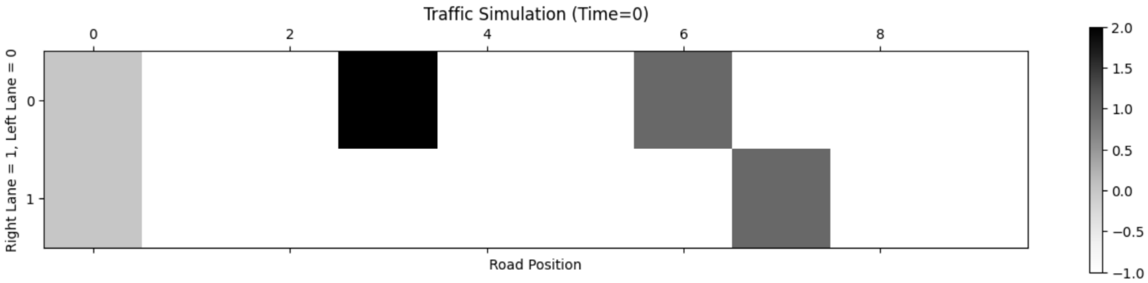
Testing and Verification

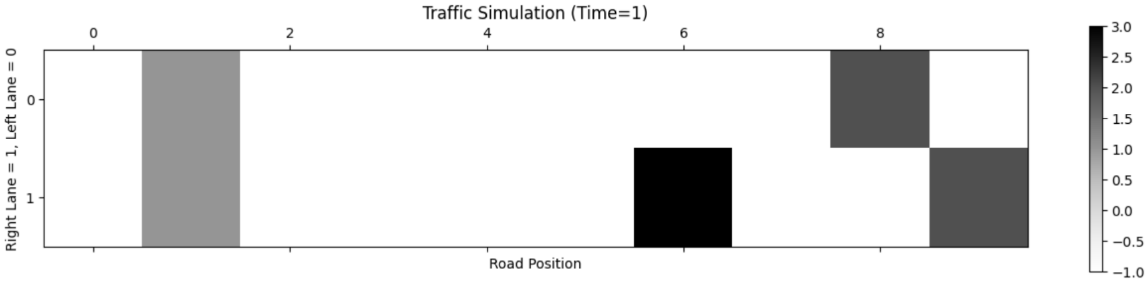
We will use a set of unit tests to verify that the lane switching logic is correct. These unit tests should cover all possible types of scenarios in which a car does or does not switch lanes.

Case 1: No need to lane change

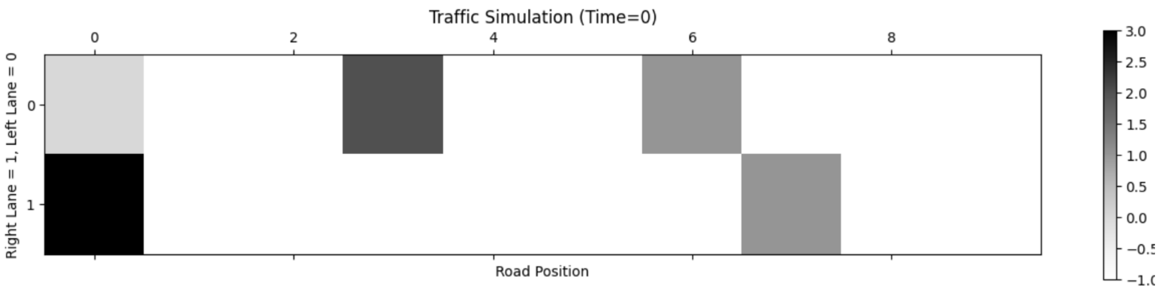


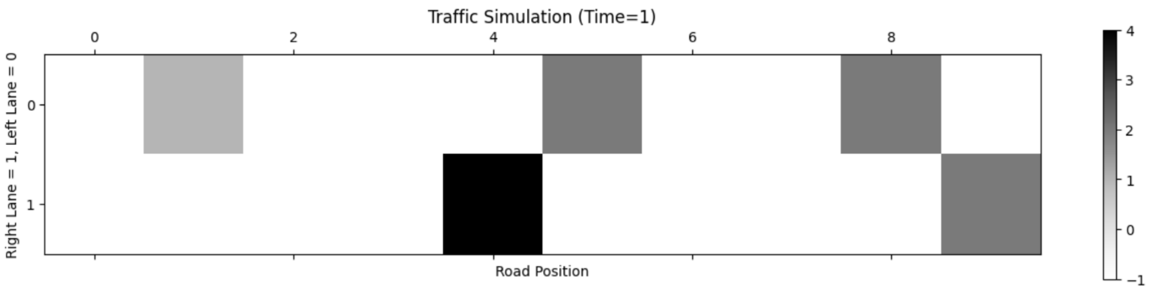
Case 2: No cars interfering with lane change



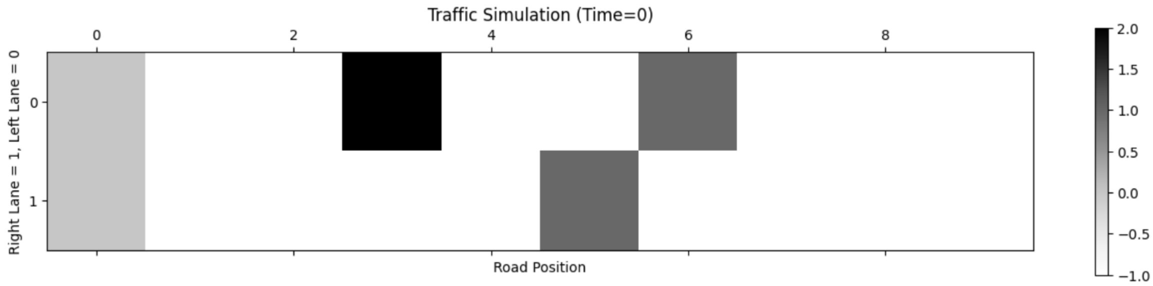


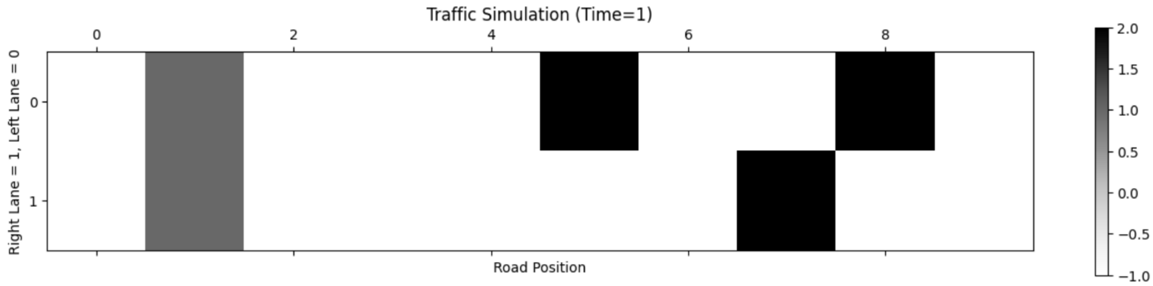
Case 3: Car behind in the other lane moving too fast



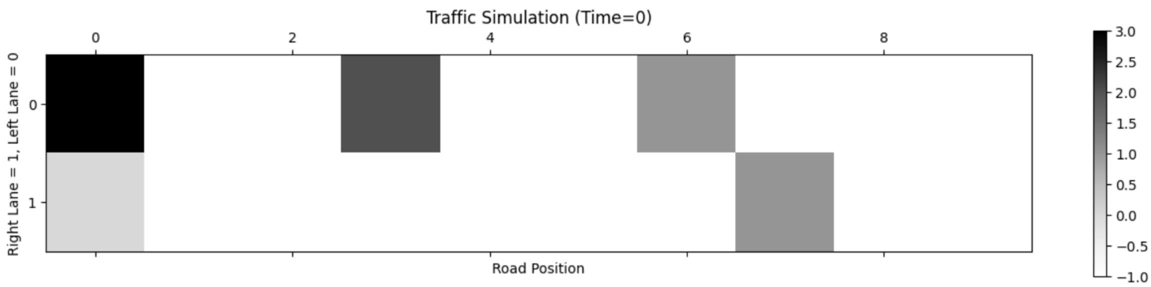


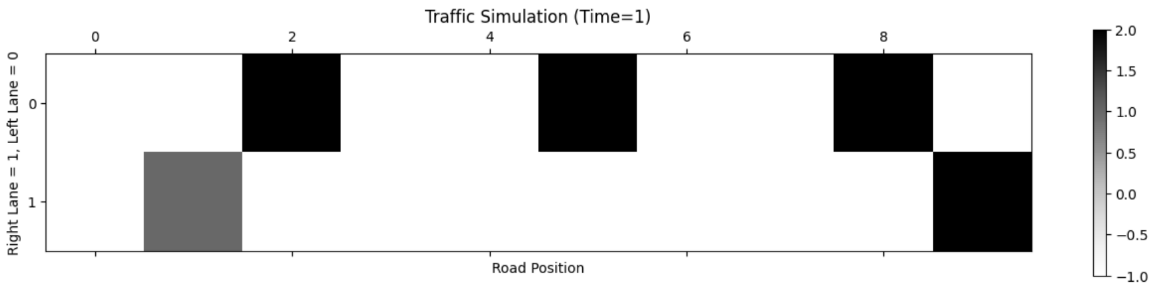
Case 4: Car in front in the other lane too close



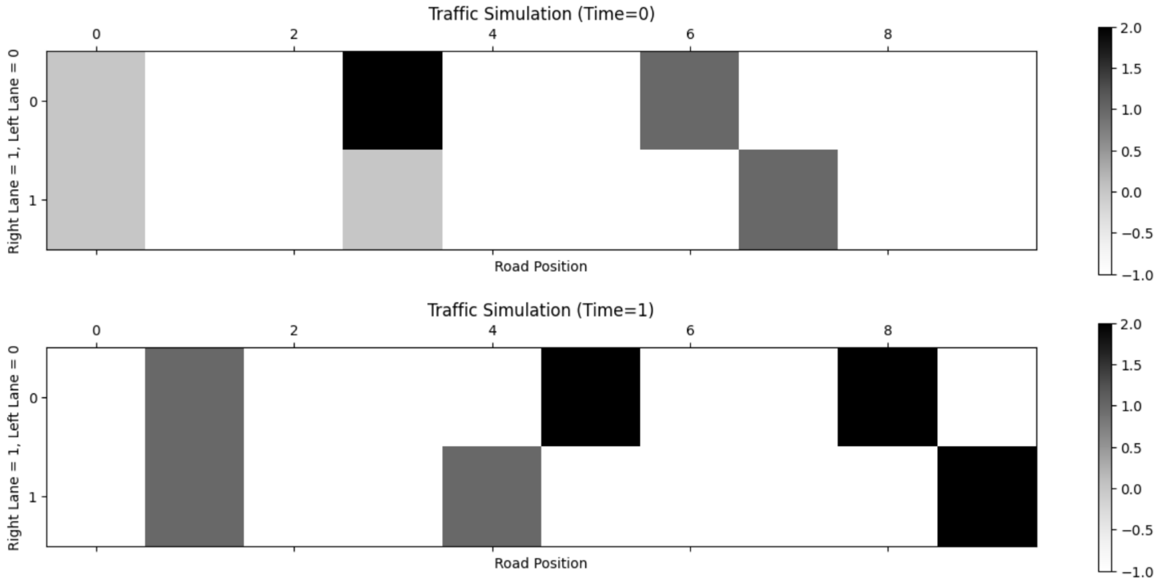


Case 5: Car behind in the same lane moving too fast

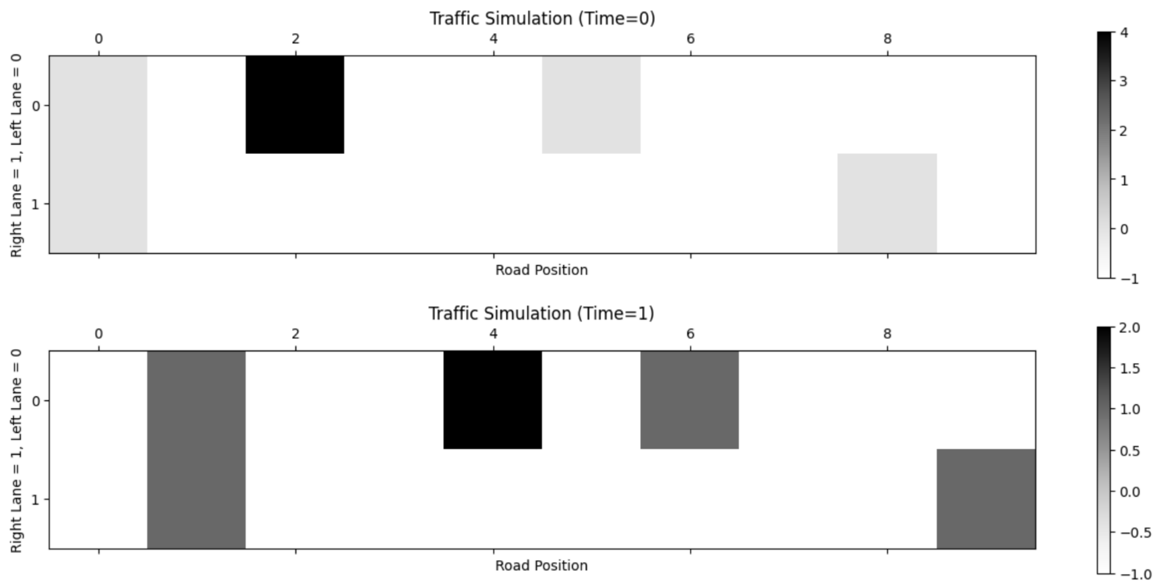




Case 6: No space in other lane to switch



Case 7: Car in left lane traveling at velocity greater than the right lane's maximum velocity
(v_{R, max} = 4)

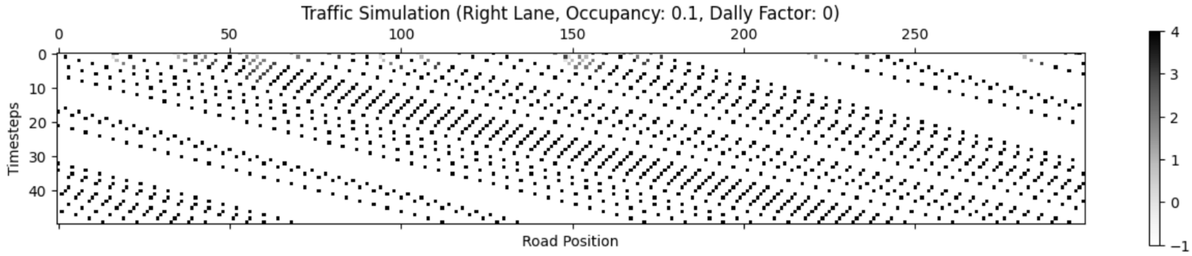


Cases 1-6 were tested symmetrically on cars wanting to switch from the right lane to the left lane. Case 7 only applies in the left-to-right case.

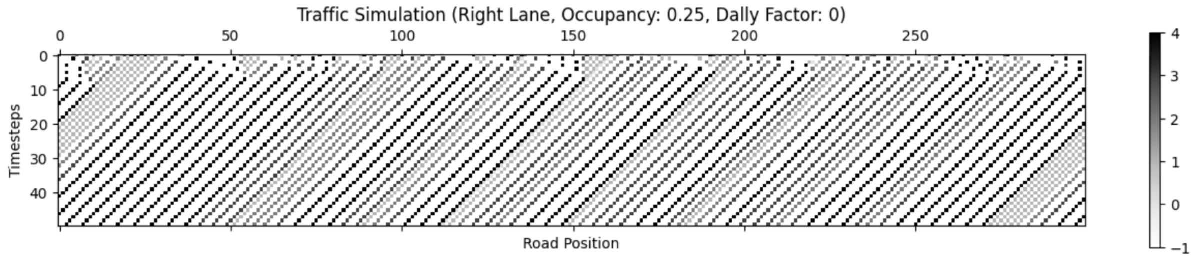
Since each of these cases perform correctly independently, we can verify that the implementation is correct, since any model consists of a combination of these cases.

Experiments

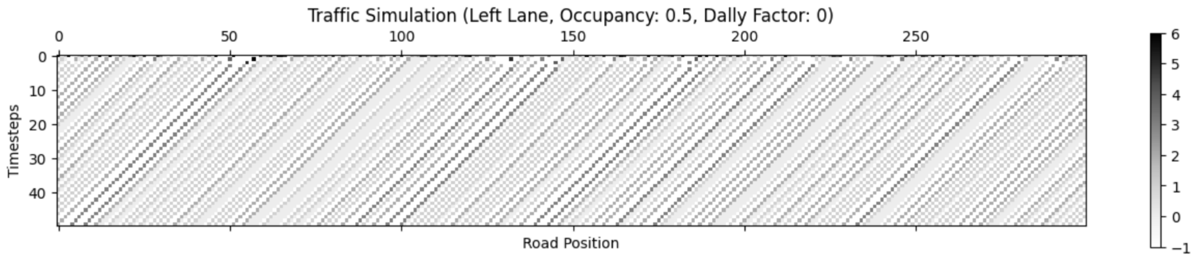
Number of lane switches: 11

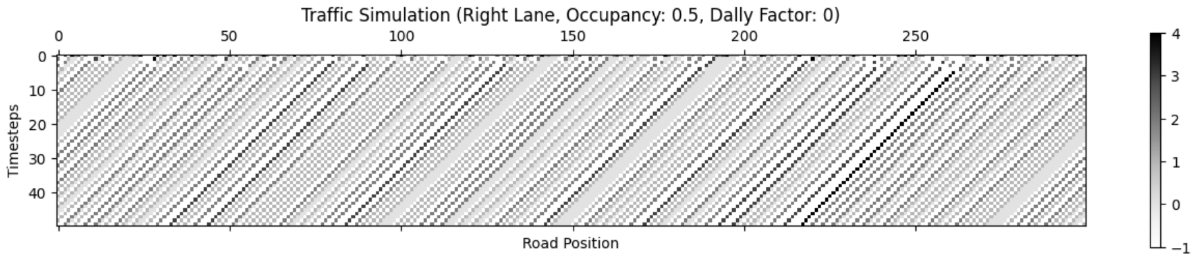


Number of lane switches: 9



Number of lane switches: 2





```
Number of lane switches: 0
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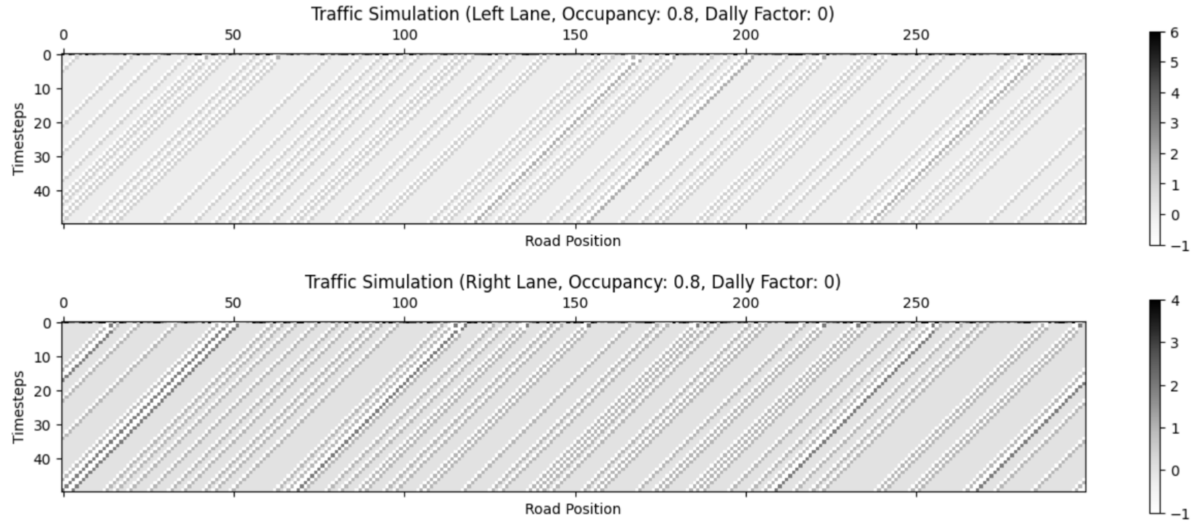


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Number of lane switches: 24
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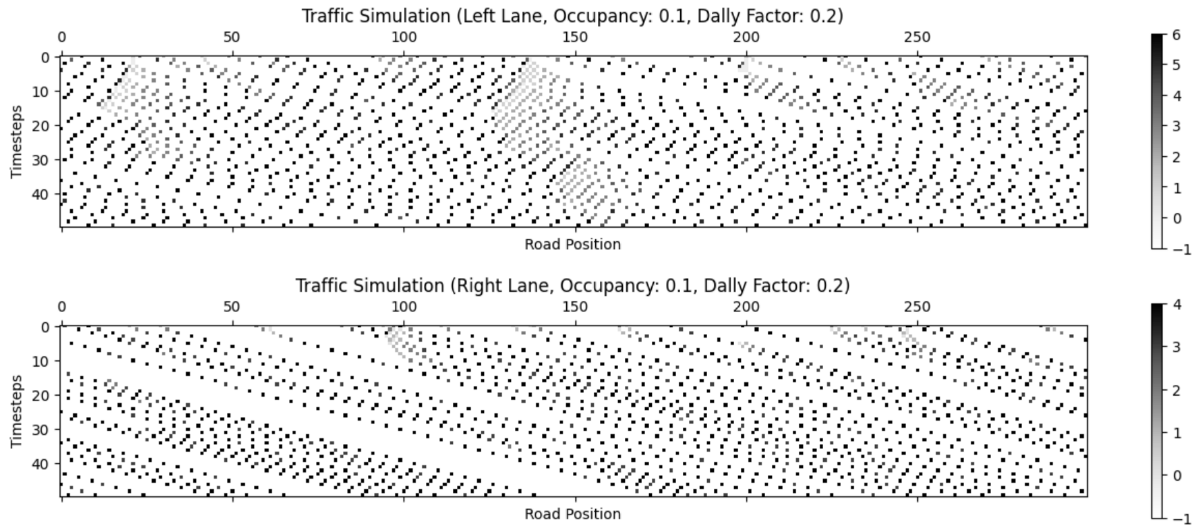


```
Number of lane switches: 100
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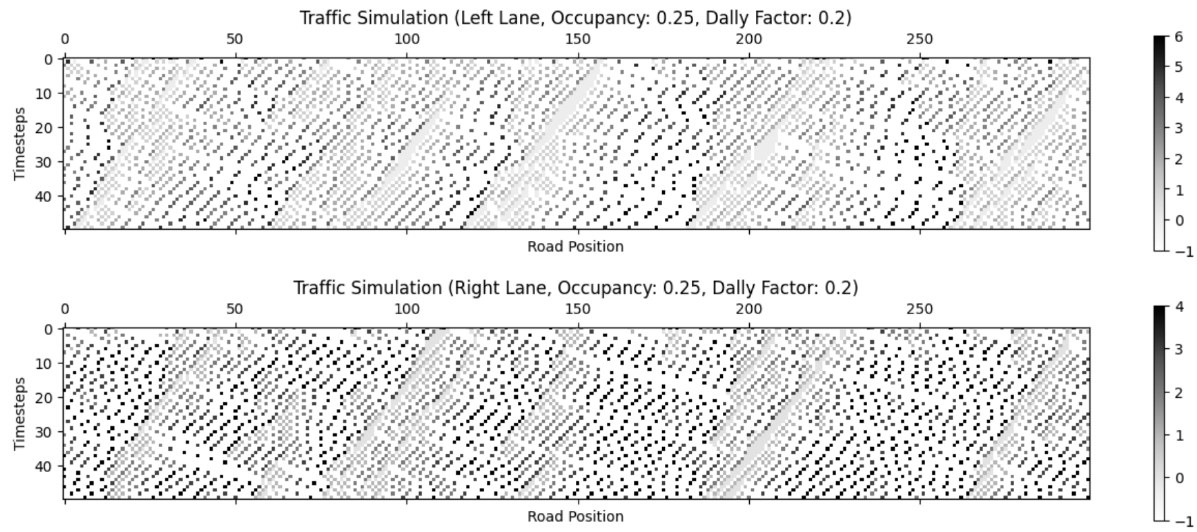

```
Number of lane switches: 75
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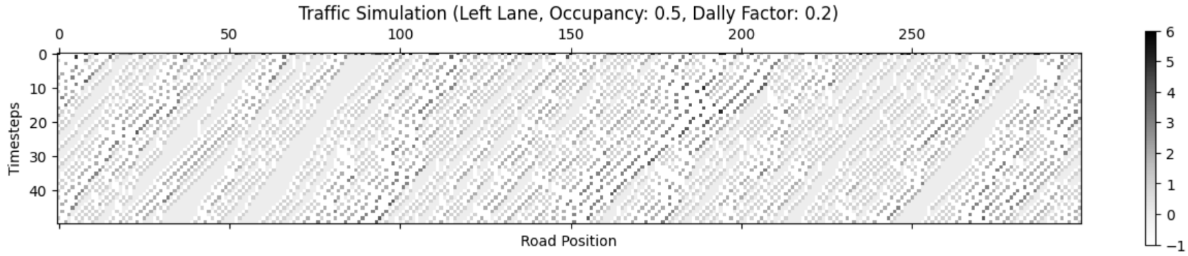

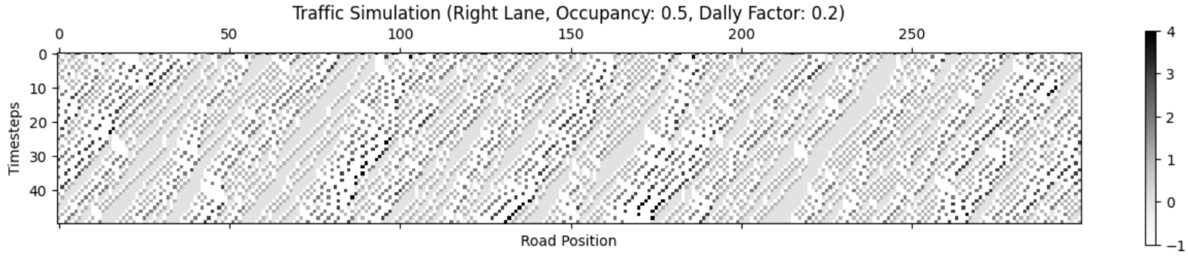


```
Number of lane switches: 6
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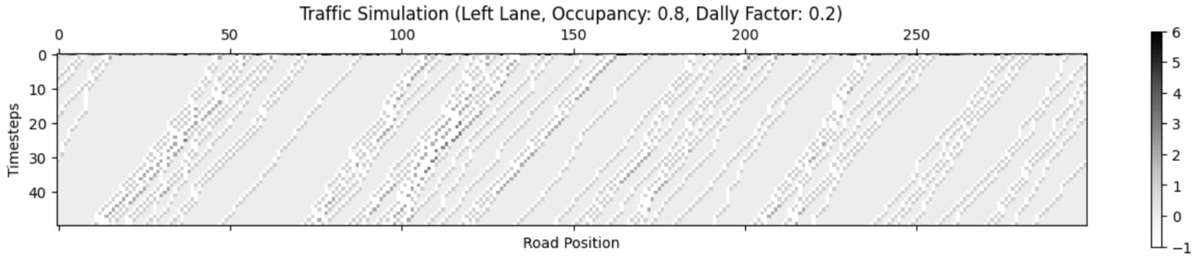

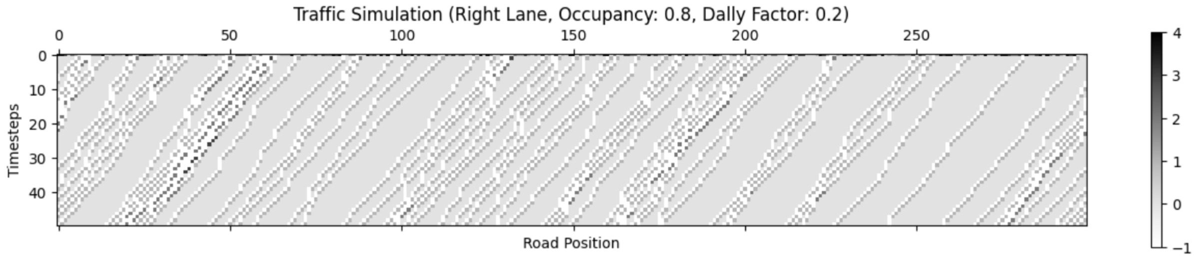


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Number of lane switches: 52
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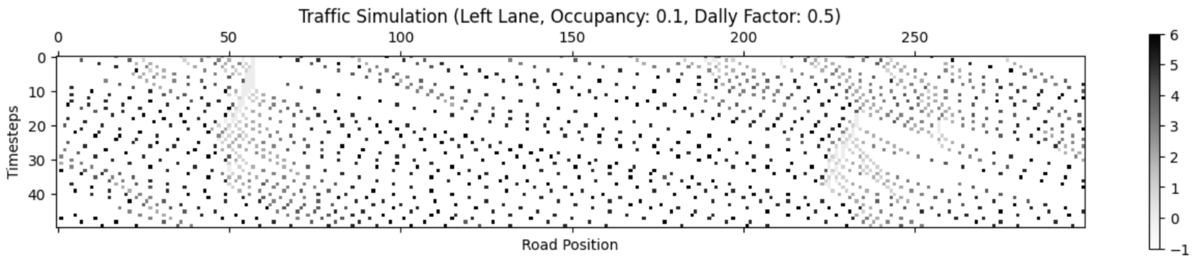
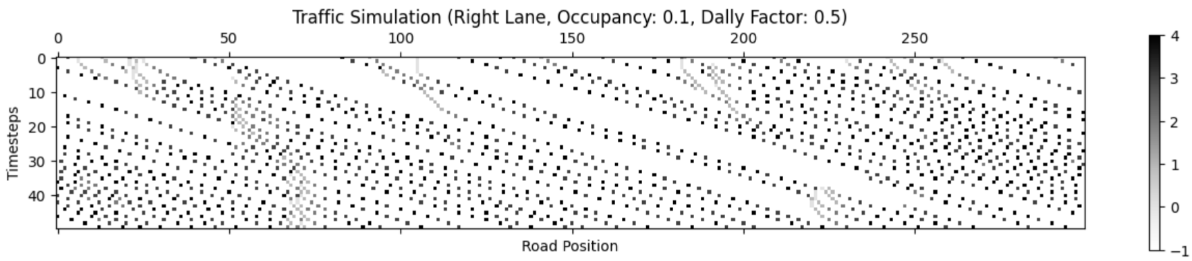
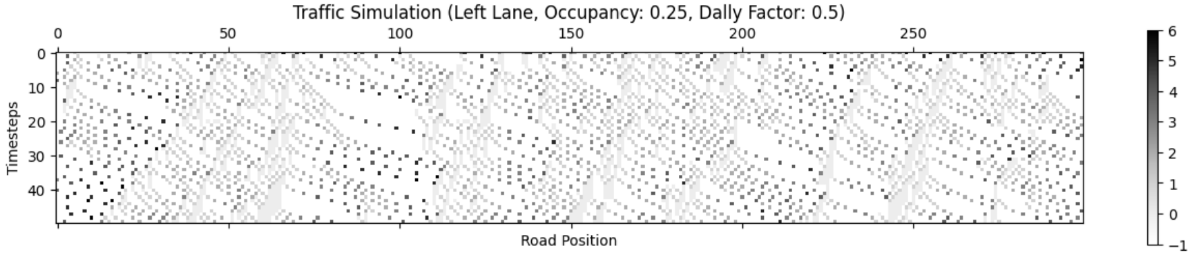
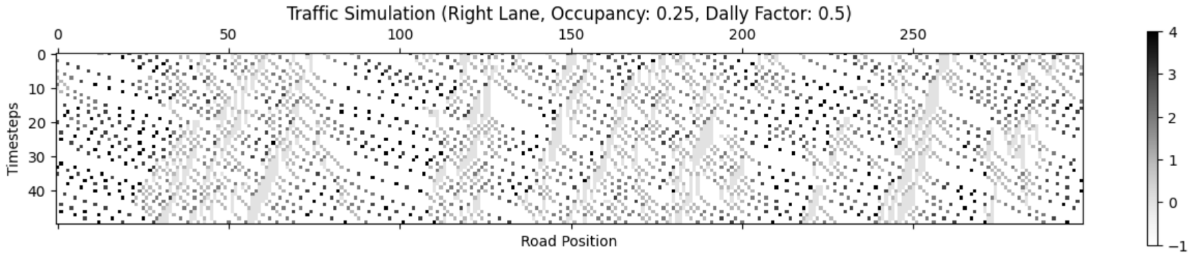


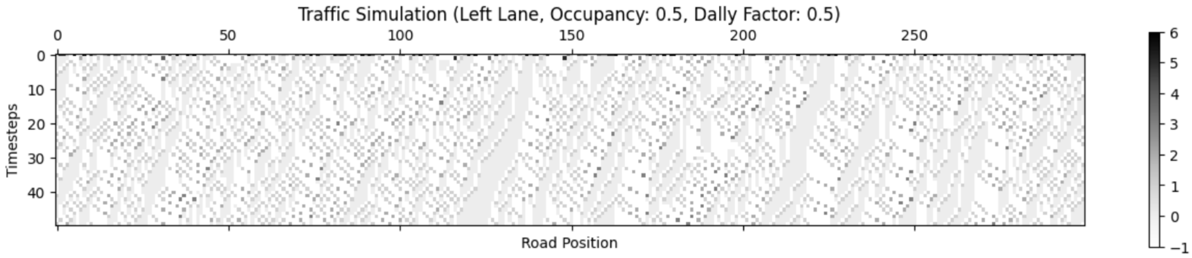


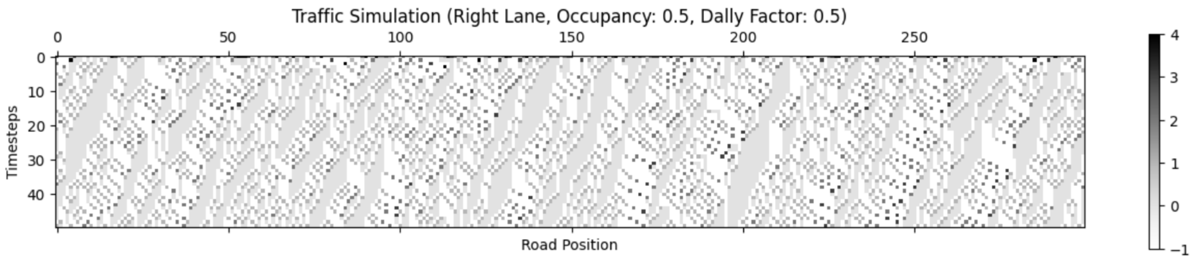
```
Number of lane switches: 283
```



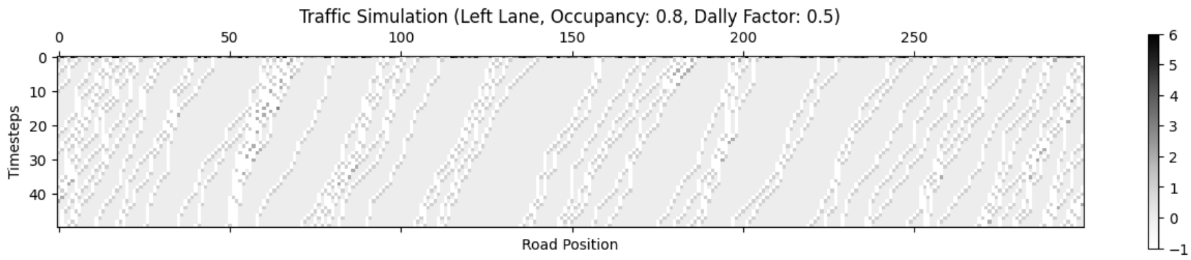


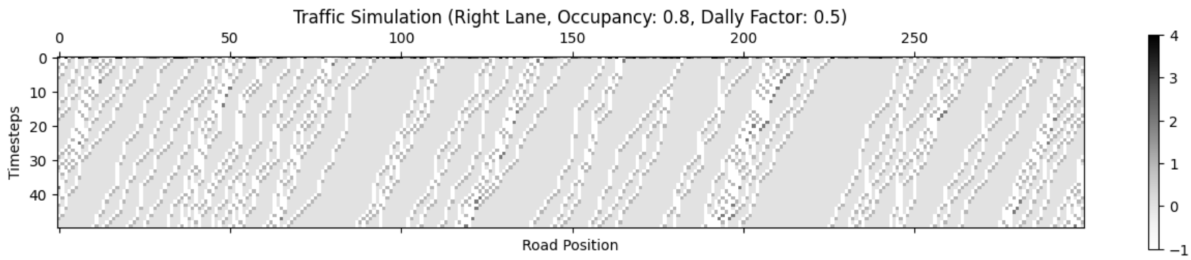

```
Number of lane switches: 237
```






```
Number of lane switches: 14
```





Summarize observations

With no dallying, lane switching only occurs in the first few timesteps, if at all, for all occupancy levels. Once free flowing traffic is achieved or a linearly stratified traffic pattern emerges, no more lane switching occurs. We also observe that the number of lane switches at no dallying generally decreases as occupancy increases. This is because as the streets become more crowded, the chances of performing a safe lane change decrease significantly. This corroborates our real life expectations.

As we introduce dallying, new trends emerge. First, at all occupancy levels, the number of lane changes increases as the dally factor increases (between p=0 and p=0.5). We also see lane switches occurring throughout the duration of the simulation rather than just at the first few timesteps. This is because a dallying car disrupts the linear traffic stratification, and may introduce a need for cars behind it to switch lanes. Whether or not a lane switch is possible depends on the occupancy. We see the most lane switches occurring at occupancy levels of 25% or 50%, compared to 10% or 80%. It is likely that occupancy levels between 25% and 50% are a sweet spot where the traffic is dense enough such that many lane switches are considered, but sparse enough such that those lane switches are actually safe to perform. 10% occupancy may be too sparse to necessitate as many lane changes, while 80% occupancy may be too dense to safely perform many lane changes. Thus, these patterns also corroborate our expectations of reality.