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# A Basal Area Increment Model for Individual Conifers in the Northern Rocky Mountains

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**ABSTRACT.** A basal area increment model is developed and evaluated for individual trees of 11 conifer species. This distance-independent model is applicable over much of the northern Rockies, an extensive region where stands have considerable variation in species composition, age structure, and past management. The model represents the diverse ecological requirements of the various species and responds appropriately across the observed range of predictor variables. Furthermore, it is well behaved even when used for conditions found in the region but not represented in the large calibration data set (44,086 trees). In addition, it possesses desirable statistical properties such as homogeneous residual variance, minimal multicollinearity, and linear parameters. The model is validated against independent data from two sources: a research study of young managed stands (2,881 trees) and a collection of long-term permanent research plots (6,594 trees). Predictions for small suppressed trees from the permanent plots are generally too large, but there are no significant relationships between residuals and other predictor variables in either data set. An unusual feature of the model is the intentional omission of site index and age, which is motivated by the large number of irregular stands in the region. Validation tests using the permanent research plots indicate, however, that residuals are not correlated with site index and age. Furthermore, long-term projections of stand attributes are unbiased. *FOR. SCI.* 36(4):1077-1104.

**ADDITIONAL KEY WORDS.** Diameter increment, tree-growth modeling, stand projection, simulation.

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**C**ONSIDERABLE MENSURATION RESEARCH is motivated by the need for yield projections that can support forest management decisions. In the northern Rockies, planning units are generally large and include stands with diverse composition and structure. Further, there is little history from which to empirically derive yield expectations for managed stands. Thus, economic comparison of management alternatives is dependent on simulation and requires yield projection methods that are consistent and are robust to a wide range of forest conditions and management actions. However, most simulation models are constructed for narrow ecological limits and represent limited combinations of species and treatment.

The model developed here predicts individual tree basal area increment and is calibrated for 11 conifer species (Table 1) that have diverse ecological requirements. This model is unusual in that it is designed to represent all conditions and management actions that can occur in the Inland Empire, a region that encompasses 18 million forested acres in eastern Washington, northern Idaho, and western Montana. As the heart of the Stand Prognosis Model (Stage 1973, Wykoff et al. 1982, Wykoff 1986), the basal area increment model will be applied

TABLE 1.

Species for which the basal area increment model was developed.

Species	Common name	Code
<i>Pinus monticola</i> Dougl.	Western white pine	WP
<i>Larix occidentalis</i> Nutt.	Western larch	L
<i>Pseudotsuga menziesii</i> (Mirb.) Franco	Douglas-fir	DF
<i>Abies grandis</i> (Dougl.) Lindl.	Grand fir	GF
<i>Tsuga heterophylla</i> (Raf.) Sarg.	Western hemlock	WH
<i>Thuja plicata</i> Donn ex D Don	Western redcedar	C
<i>Pinus contorta</i> Dougl. ex. Loud.	Lodgepole pine	LP
<i>Picea engelmannii</i> Parry ex Engelm.	Engelmann spruce	S
<i>Abies lasiocarpa</i> (Hook.) Nutt.	Subalpine fir	AF
<i>Pinus ponderosa</i> Dougl. ex. Laws.	Ponderosa pine	PP
<i>Tsuga mertensiana</i> (Bong.) Carr.	Mountain hemlock	MH

to an extensive region with considerable variation in site, species composition, age structure, and past management.

Although the form and coefficients of previous versions of the basal area increment model have been reported elsewhere (Wykoff et al. 1982, Wykoff 1986), neither the development nor the performance and verification of the model has been presented. Thus, the objective of this paper is fourfold: (1) to describe the theoretical formulation of the model; (2) to report the calibration of the model with inventory data and the examination of statistical and behavioral properties; (3) to evaluate model performance using data from managed stands; and (4) to evaluate model performance using permanent research plots.

## THEORETICAL FORMULATION

Without regard to mathematical form, there have been two conceptual approaches to model development:

1. Predict maximum potential increment as a function of site quality and tree maturity, then adjust potential with a modifier function that reflects intertree competition (Hahn and Leary 1979, Leary and Holdaway 1979, Shifley and Brand 1984, Holdaway 1984, Monserud 1975, Alder 1979, Shifley 1987).
2. Develop a composite model that incorporates tree, stand, and site characteristics in a single equation (Cole and Stage 1972, Lemmon and Schumacher 1962, West 1980).

Martin and Ek (1984) described these respective choices as "semi-empirical" and "empirical." In practical terms, the differences in approach are mostly semantic. If relationships within a model are based on generally accepted principles of tree growth (Assmann 1970), either approach can produce acceptable behavior (Wykoff and Monserud 1988). In the first case, competition and vigor are used to explain deviations from an age and site dependent potential; in the second case, similar effects are used to explain deviation about a mean growth rate that has been corrected for other tree, site, and stand effects. Since dominant age-site relations are difficult to apply in irregular mixed-species stands, potential growth cannot be reliably estimated. Thus the second (composite) approach was chosen.

This choice avoids a sticky estimation problem. While potential is a useful construct for purposes of organizing model structure, it is exceedingly difficult to observe.

Because the model is intended for application in a regional growth projection system, the choice of variables was restricted to site, stand, and tree attributes that could be reliably obtained from stand inventories normally used in the region (e.g., USDA Forest Service 1978). Specifically, tree attributes such as species, diameter at breast height (*dbh*), crown ratio (*CR*), and height are available along with the usual array of stand density measures and site characteristics (slope, aspect, elevation, habitat type). As with most inventory data, spatial information from mapped tree locations is not available, meaning that any resultant model will be distance-independent (Munro 1974). Site index and age are not used because stands in the Northern Rockies commonly have a mixed species composition with an irregular age structure. Both site index and stand age are often meaningless concepts in this situation (Monserud 1988). Furthermore, good site index curves are not even available for all of the important species (e.g., western redcedar and western hemlock).

#### TRANSFORMATION OF THE DEPENDENT VARIABLE

The biological processes that influence tree growth are inherently nonlinear. The important considerations in parameter estimation, however, are model behavior, accuracy, and validity of statistical assumptions on the error term. Linear regression is a suitable tool for modeling growth curves if a linearizing relationship can be found between the key variables (Curtis 1967). In a study of lodgepole pine in the Northern Rocky Mountains, Cole and Stage (1972) compared inside bark radial increment, inside bark basal area increment (*bai*), and  $\ln$ -transformations of each as possible dependent variables. They concluded that an unweighted linear model, using  $\ln(bai)$  as the dependent variable, satisfied the above considerations. These findings have been supported by extensive examination of data representing the remaining northern Rocky Mountain conifers. To simplify calculations, the dependent variable in the model is the natural logarithm of 10-year periodic change in squared diameter:  $\ln(d\Delta s)$ . Note that  $d\Delta s$  is directly proportional to basal area increment and can be readily transformed to an estimate of diameter increment if diameter at the start of the growth period is known (Stage 1973).

Basal area increment is modeled as a product of three factors: tree size, site, and competition. The conceptual framework for the model is the positively skewed, unimodal diameter increment curve that is typical of tree growth processes. Diameter increment increases to a maximum early in the life of a tree and then slowly decreases, approaching zero as the tree matures. With all other factors held constant, a tree of a given size is expected to attain a larger increment on a better site. A tree with a long, healthy crown is expected to attain a larger increment than a tree with a short or sparse crown. Furthermore, growth predictions should respond to change in stand density and to the relative status of the individual tree within the stand. Open grown trees or dominant trees in closed stands should grow faster than equivalently sized trees in dense stands or in subordinate crown positions. And finally, species-specific differences in the preceding behavior should be accommodated.

## TREE SIZE EFFECTS

The logical starting point in any growth modeling effort is to determine the proper relationship between increment and size. The following linear model was found to be particularly well behaved:

$$\ln(d\dot{s}) = b_0 + b_1 \ln(dbh) + b_2 dbh^2 \quad (1)$$

When  $d\dot{s}$  is transformed to diameter increment and plotted against  $dbh$ , some of the resulting family of curves have the skewed unimodal form that is typical of increment functions (Figures 1a–1b). For  $b_2 = 0$  and  $b_1 > 1$  the derived diameter increment curve has a maximum, but the approach to zero for large values of  $dbh$  is extremely slow. The  $dbh^2$  term hastens the approach to zero for large  $dbh$  without destroying the essential advantage of the model: the variance of observed  $\ln(d\dot{s})$  is more or less constant over the observed range of  $\ln(dbh)$ . Furthermore, since the model is asymptotic to zero for large  $dbh$ , imposition of an arbitrary maximum diameter may not be required.

To accommodate factors other than size, the intercept term,  $b_0$ , can be expanded to represent additional tree and stand characteristics along with site effects. As the model is multiplicative on the untransformed scale, these adjustments modify the increment prediction without affecting the basic relationship between size and increment.

## SITE EFFECTS

Site index is the variable usually selected to account for site-related variation in increment models. However, due to the large number of irregular stands in the northern Rockies, inventory data should not be expected to produce reliable site index estimates (Monserud 1988). An example of the severity of the problem is given by Hägglund (1981), who found that two-thirds of the Swedish National Forest Survey plots represented forest conditions that were unsuitable for site index measurement. But, as site index was the single best predictor of increment in his study, functions were developed to predict site index from characteristics such as location, elevation, slope, aspect, and vegetation. Predicted site index was then used as an independent variable in yield models.

As an alternative to Hägglund's approach, direct measurements of site characteristics can be used in the increment model. For Inland Douglas-fir stands with carefully determined site index (Monserud 1984), Wykoff and Monserud (1988) showed that a diameter increment model using measured site index performed no better than a model that used habitat type, location, slope, aspect, and elevation to represent site effects.

Habitat type is a land classification based on expected climax vegetation (Daubenmire and Daubenmire 1968, Pfister and Arno 1980). Habitat type can be readily determined for almost any stand within the Inland Empire and is routinely recorded in stand inventories. However, because habitat type is a discrete classification, habitat effects are most easily incorporated by estimating independent intercepts ( $HAB$ ) for individual habitat types or groups of ecologically similar types.

Due to the extensive area represented by the model, growth performance within habitat type could vary. National Forests are separated by major divides along an east-west transect and are more or less uniquely defined with regard to

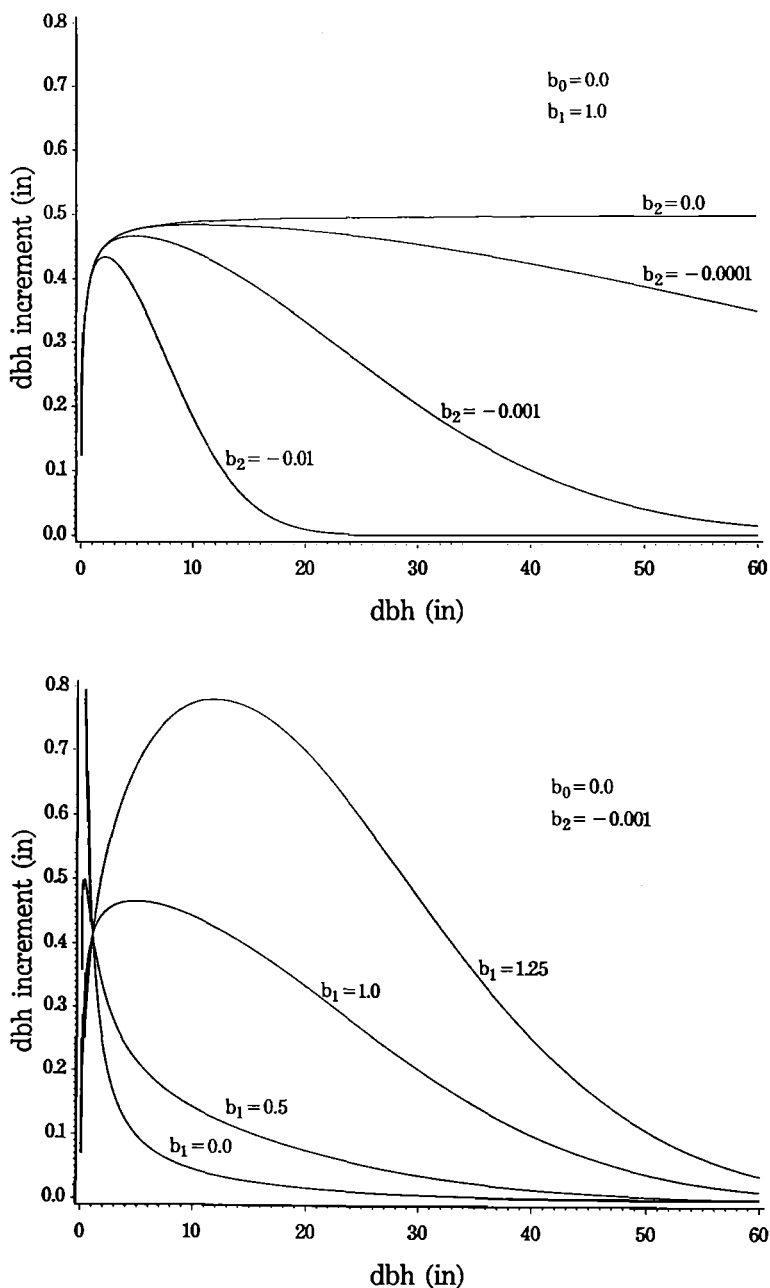


FIGURE 1. Given appropriate parameter estimates, Equation (1) can be transformed to assume the positively skewed, unimodal shape that is typical of *dbh* increment curves.

latitude. Thus, the discrete Forest boundaries provide a simple (albeit crude) representation of major climatic influences. An independent set of intercepts (*LOC*) is therefore based on groupings of National Forests. The combination of *HAB* + *LOC* results in an intercept that is specific to habitat type and National Forest.

There can be considerable geoclimatic variation remaining when the habitat type and location effects are removed. Thus, slope (*SL*), aspect (*ASP*), and elevation (*EL*) terms help refine the overall site effect.

Slope and aspect effects are based on Stage's (1976) transformation modified by inclusion of an  $SL^2$  term:

$$SA = b_3 SL[\sin(ASP)] + b_4 SL[\cos(ASP)] + b_5 SL + b_6 SL^2 \quad (2)$$

where *SA* is the combined effect attributed to slope and aspect.

This transformation has two desirable properties: it is circular, and optima exist with respect to both slope and aspect. Further, optima can vary by species, and when the aspect is level ( $SL = 0$ ), slope and aspect effects are cancelled.

Optimal growth normally occurs at the middle elevations and slows at the extremes; by including both *EL* and  $EL^2$  terms, the desired parabolic pattern can be achieved:

$$ELEV = b_7 EL + b_8 EL^2 \quad (3)$$

where *ELEV* is the effect attributed to elevation.

Substituting the linear combination of all site effects for the intercept term in Equation (1) produces an overall increment model that is conditional on site and tree size:

$$\ln(dds) = SITE + b_1 \ln(dbh) + b_2 dbh^2 \quad (4)$$

where  $SITE = HAB + LOC + ELEV + SA$ .

The resulting function is similar in structure and shape to the potential increment function described above.

#### COMPETITIVE EFFECTS

Factors were next added to represent competitive interaction between trees and the impact of management on stand development. Unfortunately, descriptions of past stand management were simply not available. Thus, treatment effects are indirectly represented by variables that reflect tree vigor, overall stand density, and relative tree size.

When a stand is thinned, stand density and the distribution of trees among size classes are changed. Over time, these changes may lead to elongation of crowns (or slow the rate of crown recession) as trees are subjected to less competition. Tree crown ratio is a measure of foliage quantity and is thus indicative of tree vigor. In relation to stand density, crown ratio also reflects the cumulative effects of past management.

Although greatly dependent on vigor, the increment attained by an individual tree is also conditioned by competition with other trees for scarce resources. The overall level of stand density is represented in the basal area increment model by crown competition factor, *CCF* (Krajicek et al. 1961). Because the crown width equations vary by species, differences in individual tree crown structure and in species composition of the stand are partially included in the density measure.

Experience with data from irregular stands with large numbers of small trees suggested that the available quadratic crown width equations overestimated small-tree crown coverage. However, by assuming an allometric relationship between

tree *dbh* and crown coverage (Curtis 1970), the contribution of small trees to *CCF* was substantially reduced (Figure 2), and the correlation between  $\ln(dds)$  and *CCF* was slightly improved. The exponents in the allometric crown coverage equations range from 1.5 to 1.8 and are in general agreement with Curtis' expectation of 1.6. For *dbh* greater than 10 in., the allometric equations are more or less coincident with the quadratic equations previously used (Figure 2).

Crown ratio and *CCF* reflect tree vigor, treatment history, and overall stand density. The increment attained by an individual tree is also dependent on its competitive status relative to neighboring trees. Stage (1973) proposed percentile in the distribution of tree basal area (*PCT*) as an indicator of relative dominance. In undisturbed stands, *PCT* reflects the competitive status of the individual tree. However, when trees are removed from below, the value of *PCT* is reduced for most residual trees. The partial effect of change in *PCT* is to reduce the increment predicted after thinning. This counterintuitive thinning response is avoided by expressing relative size in terms of basal area in larger trees (*BL*), which describes a tree's competitive status more accurately. As *BL* decreases, the predicted increment increases. Following a thinning, *BL* is thus either unchanged (thinning from below) or reduced (thinning from above) and predicted increment will either remain the same or increase. Further, because the measure of relative size is tied to stand density, dominance is less of a factor in increment predictions in sparsely stocked stands; the more open-grown the tree, the less it is influenced by competitors.

### Proportion of acre covered by open – grown crown

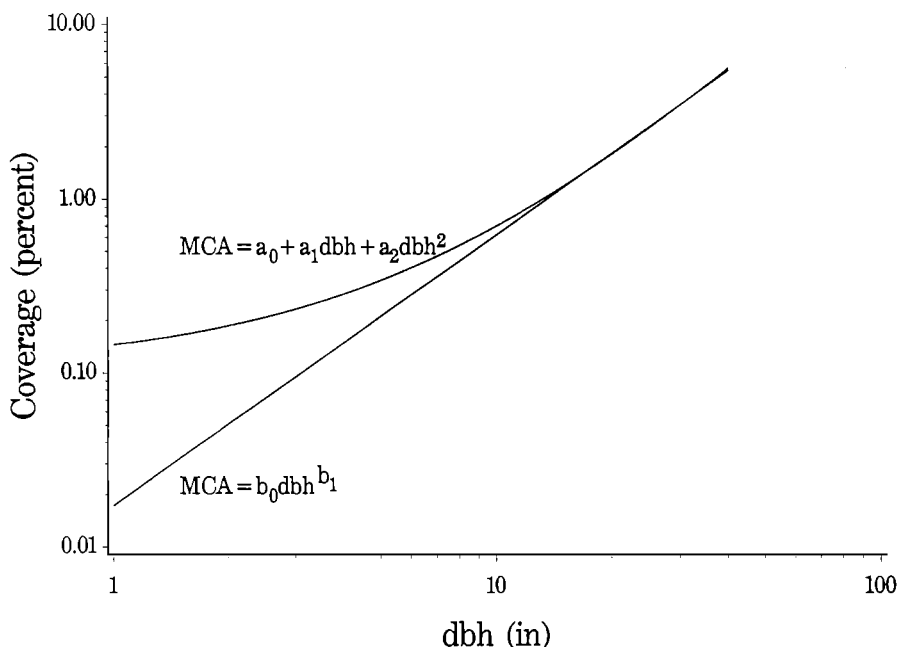


FIGURE 2. Comparison of quadratic and allometric equations for computing individual tree contribution to stand *CCF*.

*CR*, *CCF*, and *BL* are combined to produce a competition effect (*COMP*):

$$COMP = b_9 CR + b_{10} CR^2 + b_{11} CCF + b_{12} BL \quad (5)$$

The addition of the competition effect to the potential increment equation [Equation (4)] produces a biologically rational model that is simple in concept yet can accommodate extensive variation in stand structure and site conditions:

$$\ln(dds) = COMP + SITE + b_1 \ln(dbh) + b_2 dbh^2 \quad (6)$$

The flexibility of the formulation is further enhanced by estimating habitat specific coefficients for the *CCF* term ( $b_{11}$ ) and location specific coefficients for the  $dbh^2$  term ( $b_{12}$ ). Thus, both potential growth and competition modifier are influenced by the carrying capacity of the site, and the resulting growth curves can be polymorphic in form.

As specified above, the model has many variables. Yet each is intended to reflect a factor that exerts an independent and important influence on tree growth. Given the mathematical complexity, parameters must be estimated from a large database that samples the range of variation in stand, site, and tree conditions. Further, results must be carefully examined to assure that parameters are biologically consistent and that estimated effects are actually independent.

## THE DATA

Stage (1977) discussed the various types of forestry data and their relevance to growth model development. Inventory plots usually sample the range of variation in environment, pest populations, site quality, competition, genetic composition, stand structure, and treatment history that have shaped the overall condition of the forest. And when inventory data are used to develop a model, predictor variables are automatically measured to the same standards for model development and application. The primary disadvantage of inventory data is that they usually sample a short time period and thus may not contain information on long-term stand development.

In contrast, when growth data from remeasured plots span a relatively long period of observation, they are generally concentrated on a few sites and represent limited treatments. Furthermore, if the period of measurement is sufficiently long for model development purposes, the treatments represented are probably no more current than those represented in inventory data. Permanent research plots, however, are generally well suited for tracking long-term model reliability and for evaluating model performance relative to specific treatments.

### TIMBER MANAGEMENT PLANNING INVENTORY DATA (INV)

The INV data represent the 10 National Forests in the Inland Empire as measured during the period 1971 to 1974. These inventories are intended to provide an unbiased sampling of the stands that make up a forest (Stage and Alley 1972). A total of 3,900 stands were sampled using point sampling, resulting in measurements of 200,000 individual trees of 11 species. Nearly one-quarter of these were growth sample trees (*GSTs*) and were bored to determine 10-year inside-bark *dbh* increment (measured to the nearest 0.1 in.). Total height (to nearest foot), crown



ratio (to nearest 10% class), species, *dbh* (to nearest 0.1 in.), and tree condition were also recorded. Complete field procedures are described in the Northern Region stand examination handbook (USDA Forest Service 1978).

#### MANAGED REGENERATION STAND DEVELOPMENT DATA (REGEN)

Sites for the REGEN study were selected to represent young, managed stands with relatively uniform site conditions and homogeneous size-class and species distributions. The study design included three age classes, three location classes, and separate strata for clearcut, seed tree, shelterwood, and selection regeneration methods. Each stratum included both thinned and unthinned stands. The study included 160 stands distributed throughout northern Idaho and eastern Washington. Ground sampling procedures were very similar to those used to collect the INV data.

#### PERMANENT RESEARCH PLOTS (PERM)

In contrast to the above data sets, the PERM data were extracted from a collection of 102 permanent fixed-area plots that vary in size from 0.05 to 2.2 ac. Even though there are nominally 102 plots, many were established to test alternative treatments on the same site, and the passage of time has substantially blurred the distinction between treatments. A majority of these plots are concentrated in the Deception Creek and Priest River Experimental Forests in northern Idaho. Most were established to monitor silvicultural methods or to provide data for construction of normal yield tables for the western white pine type. The oldest plots were established in 1914 and have been remeasured every 5 to 10 years for 70 years; on the average, plots have been followed for about 39 years. At each remeasurement, species, *dbh* (to nearest 0.1 in.), and condition were recorded for all trees, and height (to nearest foot) was recorded for a subsample of trees (about 25 trees of each species present). Crown ratio (to nearest 10% class) was recorded for each tree at each remeasurement since 1970. Inside bark *dbh* increments were estimated by subtraction and proportional adjustment for bark growth (see Wykoff et al. 1982).

For most of the analyses using PERM data, only the data for growth periods ending in the 1970s are used. This period most closely corresponds to the inventory dates for the REGEN and INV samples. Thus, comparisons will have minimal confounding due to differences in climate. In addition, all trees from this period have measured crown ratios.

#### DATA COMPARISONS

The most important distinction between the three data sets is the extent of sampling (Table 2). The INV data are probably most representative of overall forest conditions on National Forest lands within the Inland Empire. The sample includes nearly 44,000 *GSTs* and represents 32 habitat types and 11 conifer species while extensively sampling all 10 National Forests in the area.

The PERM and REGEN samples are much smaller in size and, on the average, there are more sample trees per stand. Further, REGEN and PERM stands represent only the more mesic habitat types, predominantly the grand fir, western hemlock, and western redcedar habitat series. There are no REGEN stands

TABLE 2.

Number of growth sample trees (GSTs) by species for each of the three data collections. Also shown are total number of stands sampled and averaged number of GSTs per stand.

Species	INV	REGEN	PERM
WP	1805	239	1895
L	4645	461	558
DF	11522	791	477
GF	4683	708	1574
WH	1774	124	594
C	2441	240	1319
LP	6952	168	71
S	3092	—	92
AF	4248	—	5
PP	1989	150	9
MH	935	—	—
Total	44086	2881	6594
Total stands	3900	160	102
GSTs/stand	11	18	65

on the Lolo, Bitterroot, or Flathead National Forests, while the PERM stands occur almost exclusively on the Kaniksu, Clearwater, and Coeur d'Alene National Forests. Thus, although useful for evaluating many aspects of model behavior, they would not, of themselves, be adequate for model calibration.

Other differences are apparent from the distributions of tree and stand attributes. The PERM stands can be characterized as dense, and as evidenced by relatively small average *dbh* increments and crown ratios, they exhibit poor vigor. Mature to overmature white pine and western larch currently dominate these stands but, due to poor growth and accelerated mortality, are rapidly losing ground to a tolerant and more vigorous understory. In contrast, the REGEN sample represents young stands with moderate densities. Stand basal areas are low, and all species exhibit good vigor and rapid growth. The INV stands are intermediate with regard to stand characteristics and measures of tree vigor.

The INV sample has two shortcomings. First, the data are predominantly from unmanaged stands and there is a strong possibility of dependence between stand density and site effects. Second, because trees were sampled proportional to their basal area, there are relatively few small trees in subordinate crown positions in the data. The REGEN data afford a good test of site-density dependencies while the PERM data, with high densities and relatively large numbers of small trees, allow for testing of suppression effects.

## ANALYSIS

### INITIAL CALIBRATION

With the exception of crown ratio, the start of growth period values were used for all variables for all analyses. For the INV data, tree size and stand density vari-

ables were backdated by subtracting measured increments for growth sample trees and by applying a stand average basal area growth ratio to backdate the diameters of non-GSTs. Trees that died in the previous 5 years were included in the initial stand density estimates with a weight of 2 and assuming no increment for the period. Calculations for backdating diameter, stand density and mortality are described by Wykoff et al. (1982).

Crown ratio could not be backdated with available data, although a model (Hatch 1980) could have been applied to predict change. Under most conditions, however, crown ratio changes slowly over time and bias introduced by not backdating is probably negligible. In most cases, periodic predictions from the crown change model are substantially less than the error associated with measurement.

The REX regression program (Grosenbaugh 1967) was used to independently estimate model parameters for each of the 11 species using the INV data. Based on analysis of residuals, the model performed well for all species and behavior relative to the independent variables agreed with the previously outlined behavioral expectations.

#### TESTING FOR APPLICATION TO YOUNG MANAGED STANDS

Prediction errors were computed for individual trees from the REGEN data. For most species, fit was not practically different than the fit to the INV data. White pine and larch models, however, consistently overpredicted increments, but there were no specific trends relative to predictor variables (Wykoff 1983).

The model was further evaluated by fitting REGEN residuals to all possible combinations of the four variables (*dbh*, *CR*, *BL*, and *CCF*) that are most sensitive to stand management. An interaction between *BL* and *dbh* explained a significant proportion of the variation in residuals for all 11 species. Thus, an interaction term,  $BL/\ln(dbh + 1)$ , was added to the model (Wykoff 1986). This interaction allows the effect of relative size to vary with changes in the distribution of diameters.

Parameters for the revised model were again estimated from the INV data. For each species, terms were selectively removed from the model and change in mean squared error was computed. When deletion of a variable resulted in a reduction in mean squared error, the variable was removed from the model. Model residuals were then evaluated by constructing scatterplots with midmean lines (Cleveland and Kleiner 1975) for both the REGEN and the INV data. REGEN scatterplots indicated a slight improvement in overall model performance.

Finally, new parameters were estimated from the combined INV and REGEN data (Wykoff 1986). As evidenced by plots of residual  $\ln(dds)$  vs.  $\ln(dbh)$  and habitat type for Douglas-fir (Figures 3 and 4), the resulting models were unbiased relative to predictor variables, and prediction errors were homogeneously distributed. Similar patterns of residuals were observed for all species and all predictor variables.

#### TESTS FOR MULTICOLLINEARITY

Regardless of apparent robustness, with a model that includes as many as 27 parameters for an individual species, there remains considerable potential for dependence among predictor variables. Parsimony of the model was evaluated for

## Douglas – fir

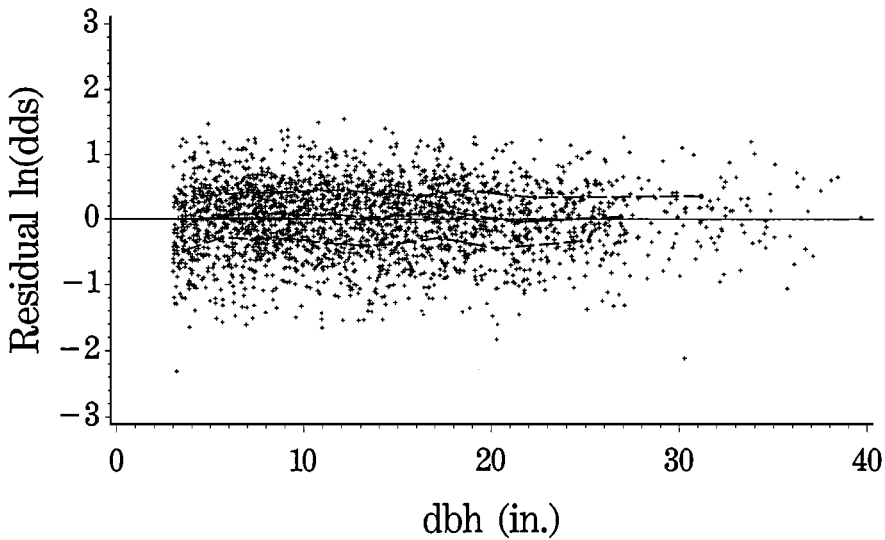


FIGURE 3. Distribution of  $\ln(dds)$  vs.  $\ln(dbh)$  for Douglas-fir from the combined INV and REGEN data collection (20% sample).

each species by individually omitting each variable and computing Mallows'  $C_p$  (Mallows 1973, Draper and Smith 1981) for each truncated model. For this analysis, collections of dummy variables such as the habitat type effects were removed from the model as a group.

## Douglas – fir

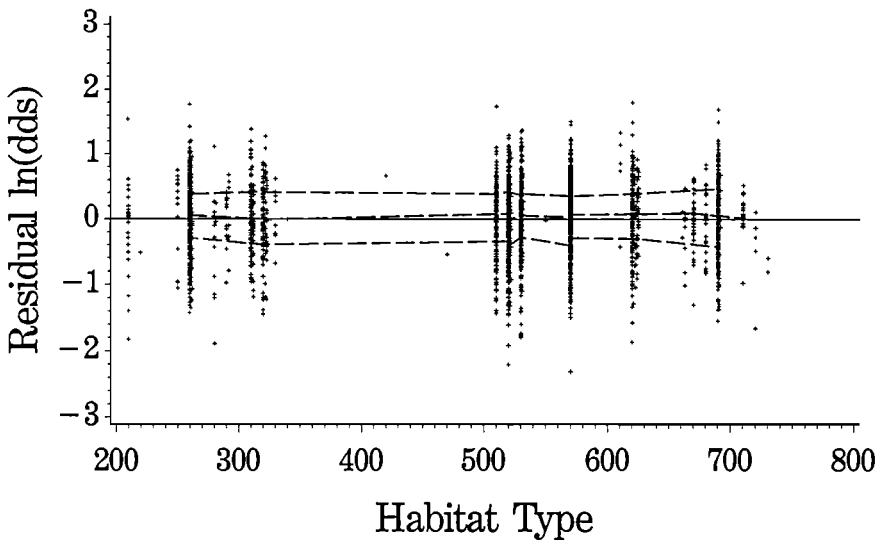


FIGURE 4. Distribution of predicted  $\ln(dds)$  vs. habitat type for Douglas-fir from the combined INV and REGEN collections (20% sample).

A variable was deemed to be expendable when  $C_p$  computed with the variable removed was less than the remaining number of parameters. Based on this criterion,  $CR^2$  was excluded for five species,  $SL^2$  and  $BL$  for three species each, and  $EL$  and  $CCF$  for one species each. In each case, the associated regression coefficient was statistically significant but not practically different from zero. The variables were removed from the model with virtually no impact on the associated fit statistics and no apparent loss of predictive capability.

Next, variance inflation factors (VIFs) were computed (Marquardt 1970). All VIFs for  $\ln(dbh)$  were greater than 5, and the VIFs for  $BL$  and  $BL/\ln(dbh + 1)$  consistently exceeded 20. The large VIFs associated with the two  $BL$  terms (see Snee 1973 and Marquardt 1970 for discussion of appropriate limits) suggest potential for poor extrapolative behavior.

Marquardt (1970), Snee (1973), and Hocking (1976), among others, proposed ridge regression as a mechanism for evaluating dependencies among predictor variables. A ridge trace was produced for the white pine, Douglas-fir, and western redcedar from the combined INV and REGEN datasets (Figure 5). In all cases, the ridge trace stabilized with  $k$  in the range of 0.05 to 0.15, and when the coefficient for the  $BL$  term was essentially zero. Thus, the  $BL$  term was eliminated from the model. Variance inflation factors for remaining variables were less than 6.0 and, with the exception of the  $\ln(dbh)$  term in the grand fir, lodgepole pine, mountain hemlock and ponderosa pine models, all VIFs were within the range suggested by Snee (1973). For western redcedar, removal of the  $BL$  term resulted in a 4.4% increase in mean squared error. For the remaining species, the change in mean squared error was less than 1% and probably of no practical significance.

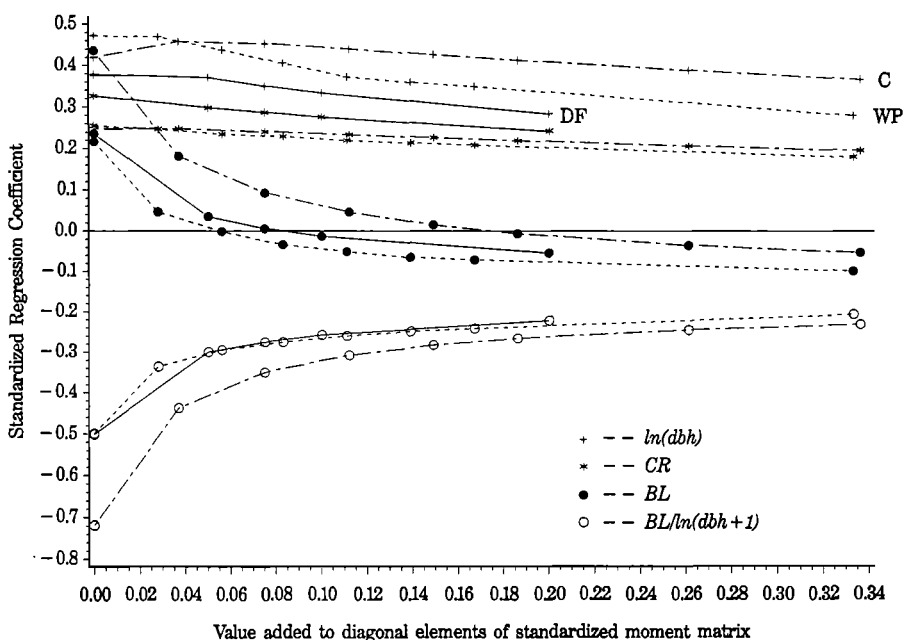


FIGURE 5. Ridge traces for selected variables for western redcedar (C), western white pine (WP), and Douglas-fir (DF).

## THE REVISED MODEL

Based on the behavioral considerations, regression analyses, and diagnostics discussed above, the following model was judged an adequate representation of the extensive INV and REGEN data:

$$\begin{aligned}\ln(dds) = & b_0 + b_1 \ln(dbh) + b_2 dbh^2 + b_3 SL[\cos(ASP)] \\ & + b_4 SL[\sin(ASP)] + b_5 SL + b_6 SL^2 + b_7 EL + b_8 EL^2 \\ & + b_9 CR + b_{10} CR^2 + b_{11} BL/\ln(dbh + 1) + b_{12} CCF \quad (7)\end{aligned}$$

where

*dbh* = Tree diameter outside bark at breast height (in.).

*BL* = Total basal area in trees with larger *dbh* than the subject tree (ft<sup>2</sup>/ac).

*CR* = Ratio of live crown length to total tree height.

*CCF* = Crown Competition Factor (modified as discussed above).

*SL* = Average slope percent for the stand.

*ASP* = Average aspect for the stand (radians).

*EL* = Average elevation for the stand (100 ft).

*b<sub>i</sub>* = Estimated regression coefficients (see Appendix); *b<sub>0</sub>* is dependent on habitat type and location, *b<sub>2</sub>* is dependent on location, and *b<sub>12</sub>* is dependent on habitat type.

## VALIDATION USING INDIVIDUAL TREE DATA FROM THE PERMANENT RESEARCH PLOTS

Regardless of behavioral attributes or apparent model robustness, the ultimate test of model reliability is performance against independent data. Residuals were examined for individual trees in the PERM data for the measurement period most closely corresponding to the timing of the INV inventory. The data were not screened for damage due to insects, pathogens, or natural accidents; all trees that survived to the end of the growth period were used for model evaluation.

Numbers of observations for lodgepole pine, Engelmann spruce, subalpine fir, ponderosa pine, and mountain hemlock were inadequate for meaningful comparisons. For the six other species, regressions against residual  $\ln(dds)$  were computed for each of the independent variables in the model and for stand basal area and site index (Haig 1932). Regressions were also computed for residual *dds* and residual *dbh* increment versus *dbh*. Scatterplots with midmean lines were produced for each regression and, in addition, residual  $\ln(dds)$  was plotted against  $\ln(dbh)$  for three classes of basal area percentile.

Based on a comparison of fit statistics, the model appears to behave reasonably for all species except larch. In general, the error variances are larger for predictions of the PERM data, but the proportion of total variation explained by the model is about the same for the PERM data and the combined INV and REGEN data. The increased error variances are in part attributable to the methods used to estimate inside bark increment. Successive tapings with adjustment for bark growth are less precise than increment borings and will result in larger measurement errors.

For western larch, there were no appreciable trends between predictor variables and residuals but there was a substantial overprediction bias. When the bias is removed by adding the mean residual to each prediction, the variation in residuals explained for the PERM data (39%) is comparable to the original fit to the INV and REGEN data (44%). With an average 10-year diameter increment of 0.3 in. and little variation, the PERM larch data probably provide a poor basis for judging model validity.

In general, simple linear regressions explained little of the variation between PERM residuals and predictor variables (Table 3). Notably, at most 2.8% of residual variation is explained by site index, and the distribution of residuals relative to site index is unbiased (Figure 6). Even assuming that all 102 plots are independent, the correlation between mean residual and site index is not significant at the 95% level for any species. However, for each of the species except for larch there are significant relationships between residuals and tree size and relative size variables. As evidenced by comparison of residual plots for different percentile classes (Figure 7), the model substantially overpredicts increments for suppressed small diameter trees.

Several factors may be involved in this apparent model shortcoming. The range in *BL* is considerably smaller for the INV data than for the PERM data, and the *BL* relationship could be poorly defined. Or, the model form may simply lack the flexibility needed to allow for extreme variation in both tree size and level of suppression. This latter possibility was addressed by estimating model coefficients from the PERM data for grand fir and western white pine. These regressions explained 77 and 68% of the variation in  $\ln(dds)$ , respectively. Further, the bias relative to tree size was eliminated (Figure 8), and there were no apparent residual patterns relative to any of the other predictor variables. Thus, the model form appears to have adequate flexibility, suggesting that the calibration data were critically lacking in small suppressed trees. Based on the inventory design, this is not a startling conclusion. The INV growth sample trees were selected with probability proportional to tree basal area, and thus small, suppressed trees could be underrepresented in the calibration data. Further, increments were measured

TABLE 3.

Proportion of variation in residual  $\ln(dds)$  for PERM data explained by simple regression against selected variables.

Variable	Species					
	WP	L	DF	GF	WH	C
$\ln(dbh)$	0.134	0.022	0.133	0.268	0.196	0.188
<i>Site Index</i>	0.011	0.000	0.025	0.001	0.000	0.028
<i>PCT</i>	0.064	0.009	0.118	0.253	0.019	0.106
<i>EL</i>	0.002	0.016	0.059	0.001	0.053	0.014
<i>CR</i>	0.027	0.000	0.000	0.000	0.015	0.028
<i>BL</i>	0.036	0.008	0.070	0.163	0.042	0.002
<i>CCF</i>	0.000	0.000	0.064	0.022	0.080	0.032
<i>BA</i>	0.010	0.000	0.026	0.011	0.137	0.033
<i>SL</i>	0.007	0.053	0.000	0.000	0.045	0.000
<i>ASP</i>	0.017	0.050	0.000	0.014	0.120	0.000

## Western White Pine

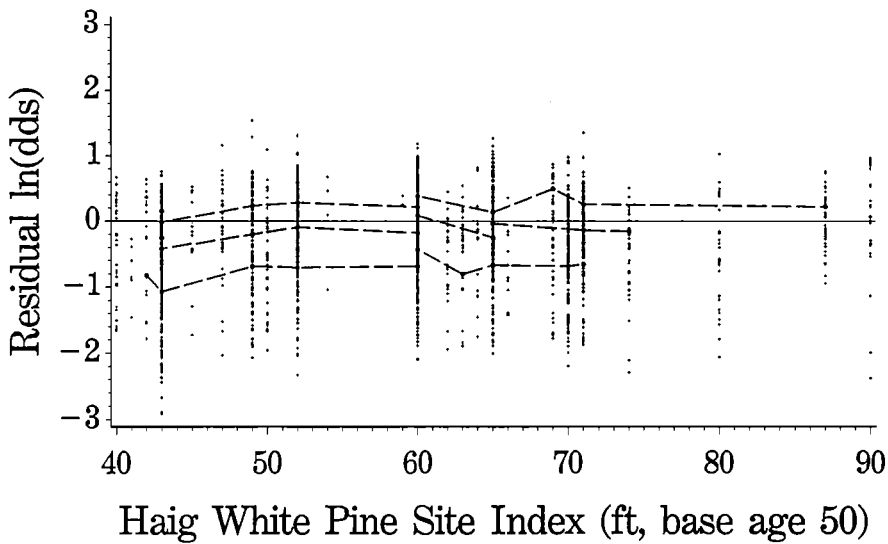


FIGURE 6. Scatterplots showing distribution of residual  $\ln(dds)$  vs. site index for white pine from the PERM collection. Similar trends were seen for other species.

from cores and the small suppressed trees that were available could have missing rings.

### LONG-TERM PROJECTIONS FOR THE PERMANENT RESEARCH PLOTS

The final basal area increment model was next inserted into the Prognosis Model, and projections were made for each permanent research plot. The average length of projection was approximately 39 years. Variation arising from mortality was eliminated by using a thinning algorithm to reproduce the observed mortality. For each species on each plot, diameter increment data for the first measurement period were used to adjust the model intercept to eliminate bias in predicted  $\ln(dds)$ . This adjustment is a standard feature of Prognosis; measured increment data provide a mechanism for representing site-to-site variation in expected growth (Stage 1973, 1981). As reported by Stage and Renner (1988), bias in stand basal area, quadratic mean diameter, and in the 10, 50, and 90 percentile points of the diameter distribution was relatively small, although the predicted diameter distribution appeared to be slightly more peaked than the observed distribution. Further, none of the biases were statistically related to site index, initial height, or relative density.

To expand on the Stage and Renner analysis, mean bias was computed for each of the PERM plots, for each measurement period, for each species with five or more increment observations. In no case, however, were there sufficient observations for subalpine fir or mountain hemlock. When species were pooled across all measurement periods, there was minimal bias; the weighted average  $\ln(dds)$  prediction was 98% of the observed value.

Individually, however, predicted  $\ln(dds)$  was significantly biased ( $P = 0.05$ ) for



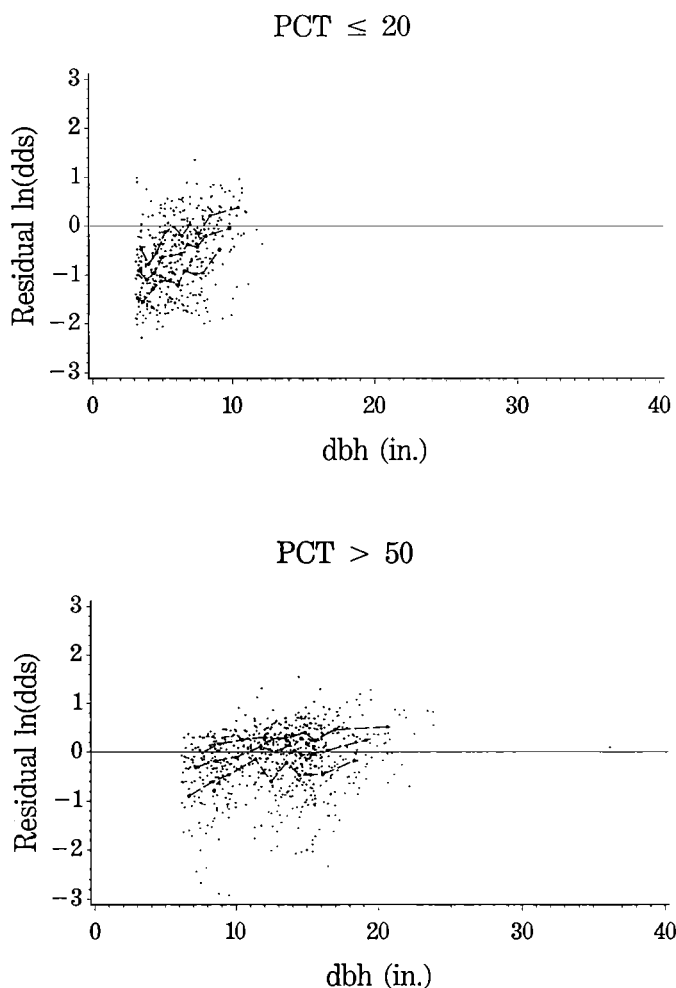


FIGURE 7. Scatterplots showing residual  $\ln(dds)$  vs.  $\ln(dbh)$  for PERM western white pine for two classes of basal area percentile ( $PCT \leq 20$  and  $PCT > 50$ ).

all species except for western hemlock, ponderosa pine, and spruce. Furthermore, a regression on some combination of white pine site index (Haig 1932), age, year of measurement, and stand basal area explained a significant proportion of the bias. Thus, it appears as though the increment model might be improved by inclusion of site index and age as predictor variables. In the irregular stands that are typical of the Inland Empire, however, observations of past increment are considerably more reliable than are estimates of site index and age. And, because long-term projections of stand attributes were unbiased when past increment was used to adjust model intercepts, site index and age do not appear to be necessary variables.

## DISCUSSION

In general, diameter increment predictions relative to tree size have the desired unimodal, positively skewed shape; expected increments are greatest for domi-

## Western White Pine

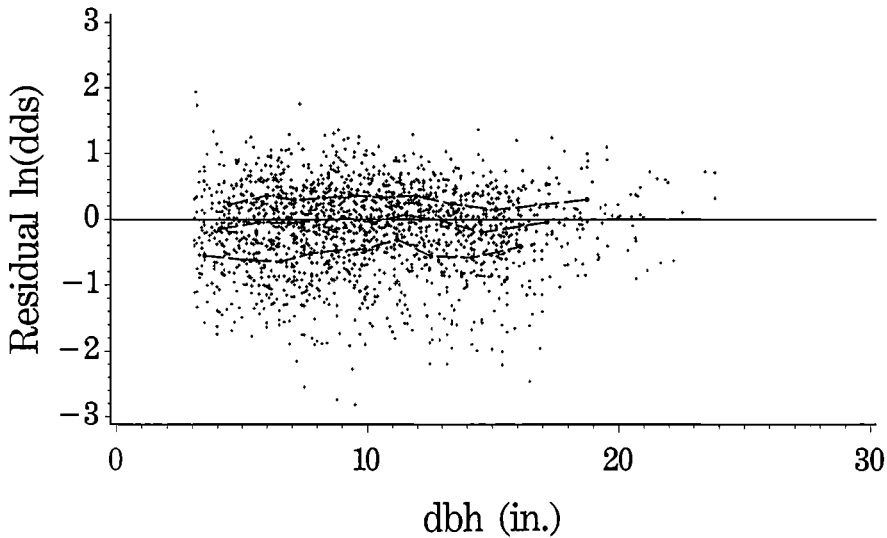


FIGURE 8. Scatterplot showing residual  $\ln(dds)$  vs.  $\ln(dbh)$  for the PERM data when model parameters were estimated using only data from the PERM collection.

nant, full-crowned trees in low density stands and least for suppressed, short crowned trees in high density stands (Figure 9). Furthermore, species differences within the model are consistent with our understanding of forest ecology within the Inland Empire.

Trees tend to grow faster and attain larger overall size in northern Idaho than in Montana. In northern Idaho, annual precipitation is higher and growing seasons are relatively long. Sites to the north are usually cooler, and sites to the east are cooler and drier. Model location coefficients are in agreement with these climatic trends. For all species, the location intercept is largest for a north central Idaho forest, usually the St. Joe National Forest, and smallest for one of the forests in western Montana. The intercepts for the Kaniksu (northern Idaho) and the Colville (eastern Washington) National Forests are usually between these extremes.

Effects of habitat type are not as apparent. The grand fir, western redcedar, and western hemlock habitat types are the most productive and in general have the largest habitat intercepts. Habitat effects, however, may be confounded with other site effects such as elevation, slope, aspect, and location, and rankings change by species.

For most species, maximum predictions were associated with moderate slopes (25 to 40%). However, with all other effects held constant, there are large between-species differences in the aspect and elevation at which maximum increment occurs (Figure 10). The magnitude of behavioral differences between species would appear to support independent estimation of aspect, slope, and elevation coefficients.

Although cause and effect cannot be assessed from the above analyses, many

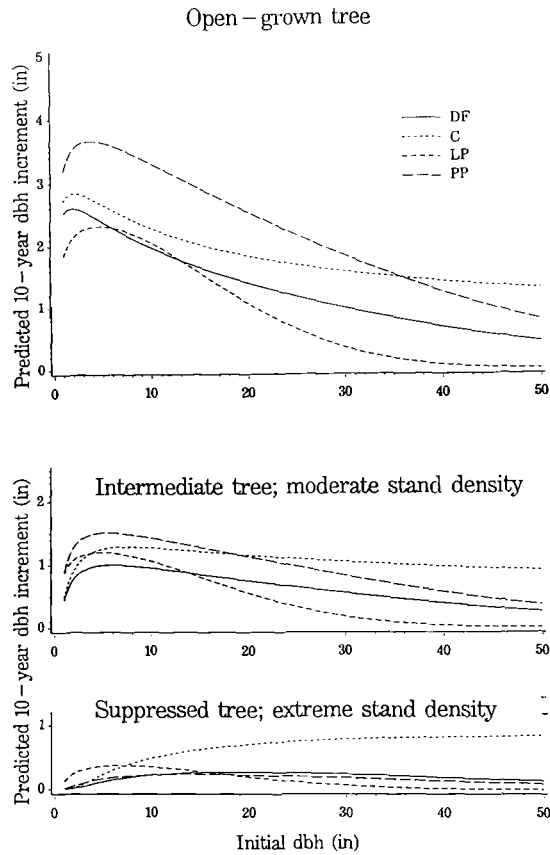


FIGURE 9. Predicted *dbh* increment vs. *dbh* for selected species for open-grown trees ( $CCF = 50$ ); trees at the 50th percentile in moderately dense stands ( $CCF = 150$ ); and trees at the 20th percentile in extremely dense stands ( $CCF = 400$ ).

of the ecological characteristics of the various species are consistent with model coefficients. Ponderosa pine is less tolerant of shading than other species, and the ponderosa pine model is the most sensitive with regard to relative size ( $BL$ ) and stand density ( $CCF$ ). Western redcedar is a long-lived species, capable of attaining large diameters, and is not as responsive to increasing size or stand density as are the other species. This behavior is associated with relatively weak  $dbh^2$  and  $BL$  effects. In contrast, lodgepole pine is a short-lived species that does not attain large diameters in the Inland Empire, and the lodgepole  $dbh^2$  coefficient is an order of magnitude larger than the western redcedar coefficient. The lodgepole  $dbh$  increment function has a decided peak relative to  $dbh$  with a maximum at  $dbh$  less than 8 in. The western redcedar function is extremely flat, with a maximum near 14 in. (Figure 9).

The robustness of the model formulation has been further demonstrated by extension of the model to different geographical regions. Similar models have been developed for most of the inland forest types in the Pacific Northwest and the Northern Rocky Mountains (Stiff et al. 1982, Johnson 1987). Although most model coefficients differ among regions, essentially the same variables are used as

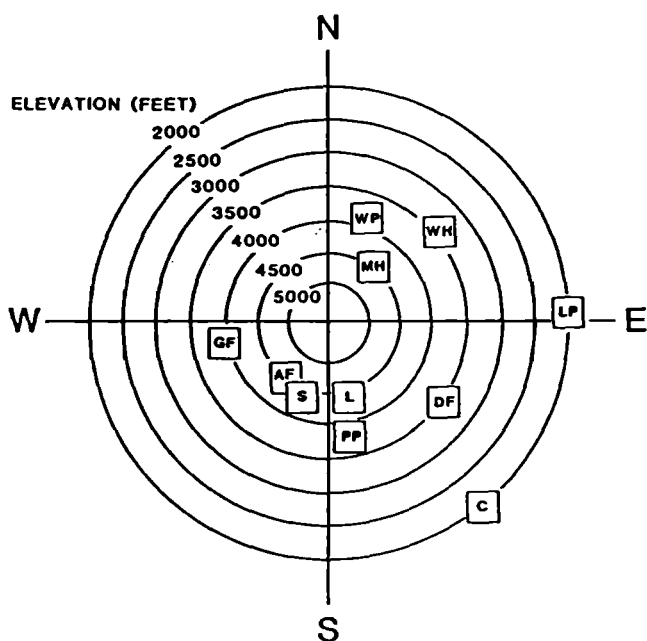


FIGURE 10. Elevation and aspect associated with maximum predicted increment with all other effects held constant. Species codes are listed in Table 1.

predictors, and there are no changes in coefficients that alter the basic form of the curve relative to *dbh* or model behavior relative to changes in stand density and structure.

## CONCLUSIONS

The model developed in these analyses appears to be well behaved and robust. Essentially the same equation was suitable for all 11 species and has been found to apply in other regions as well. Further, based on comparison of predictions with observed increments from managed stands, management effects seem to be adequately represented by the model.

Analyses of residuals from permanent research plots suggest that predictions are too large for small suppressed trees. However, the model form fit well to the PERM data, so the difficulty probably lies with an inadequacy in the INV and REGEN data used for calibration. In any case, this apparent shortcoming should not limit the usefulness of the model for projecting inventory data or for projecting data from managed stands. Further, there are no indications from the PERM data that model performance would be enhanced by inclusion of a site index variable given that increment data are available for model calibration.

Although there are many model variables, parameters have been estimated from an extensive data base and have been shown to be parsimonious and relatively free of multicollinearity. With the exception of suppressed trees in dense stands, the model will probably accommodate most conditions found within the Inland Empire.

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## APPENDIX

Table A1 presents coefficients estimated for the final model from the combined INV and REGEN data. Habitat and location intercepts are grouped into classes, and the classes are mapped onto habitat type and National Forest in Tables A2 and A3, respectively. The  $dbh^2$  coefficients are dependent on location, and classes are defined by National Forest and species in Table A4. The  $CCF$  coefficients are dependent on habitat type and classes are defined by habitat type and species in Table A5.

As an example, assume that basal area increment predictions are desired for western larch on a *Pseudotsuga menziesii*/*Physocarpus malvaceus* (PSME/PHMA) habitat type within the Coeur d'Alene National Forest. This prediction would require habitat intercept class 5 (0.054), location intercept class 2 (0.088),  $dbh^2$  coefficient class 1 ( $-0.424 \times 10^{-3}$ ) and  $CCF$  coefficient class 3 ( $-0.044$ ).

TABLE A1.  
Coefficients for the Inland Empire basal area increment models [see Equation (7)].

Variable	Class	Species <sup>a</sup>										
		WP	L	DF	GF	WH	C	LP	S	AF	PP	MH
Habitat type <sup>b</sup> (HAB)	1	0.869	0.171	0.189	0.297	0.583	0.996	0.797	-0.707	-0.960	1.173	-1.680
	2	0.766	0.300	-0.140			0.698	0.702	-0.310	-0.722	0.741	-1.521
	3		0.246	0.002				0.668	-0.830	-0.568	0.520	
	4		0.495					0.403	-0.656	-0.819		
	5		0.054					0.567		-1.238		
	6									-1.102		
Location <sup>c</sup> (LOC)	1	0.188	0.218	0.525	0.465	0.461	0.505	0.437	0.266	0.426	0.239	0.125
	2	0.0	0.088	0.360	0.303	0.131	0.155	0.211	-0.163	0.142	0.562	0.481
	3		0.322	0.224	-0.136	0.0	0.333	0.149	0.0	-0.132	0.429	0.0
	4		0.0	0.627	0.202		0.0	0.0		0.0		
	5			0.0	0.610							
	6				0.0							
ln(dbh) CR		0.664	0.629	0.693	0.810	0.693	0.756	0.882	0.806	0.853	0.658	0.898
		1.107	1.092	2.044	1.932	1.380	1.372	1.855	1.271	0.531	1.315	1.284





TABLE A2.

Map of habitat type onto habitat intercept class by species.

Habitat Type	Species <sup>a</sup>										
	WP	L	DF	GF	WH	C	LP	S	AF	PP	MH
PIPO/AGSP <sup>b</sup>	— <sup>c</sup>	—	—	—	—	—	—	—	—	1	—
PIPO/SYAL	—	—	—	—	—	—	—	—	—	1	—
PSME/AGSP	—	—	3	—	—	—	—	—	—	3	—
PSME/VACA	—	5	3	—	—	—	5	—	—	2	—
PSME/PHMA	2	5	3	1	—	2	5	4	6	3	—
PSME/VAGL	—	5	3	—	—	—	1	—	—	—	—
PSME/LIBO	—	5	3	—	—	—	2	—	—	2	—
PSME/SYAL	—	5	3	—	—	—	1	—	6	2	—
PSME/CARU	—	5	3	—	—	—	5	4	6	3	—
PSME/CAGE	—	—	1	—	—	—	—	—	—	3	—
PICEA/CLUN	—	1	—	—	—	—	5	—	—	—	—
PICEA/LIBO	—	1	3	—	—	—	5	4	—	—	—
ABGR/XETE	—	2	1	1	—	2	2	1	6	2	—
ABGR/CLUN	1	1	1	1	1	2	2	1	1	2	—
THPL/CLUN	1	2	1	1	1	2	3	4	2	2	2
THPL/OPHO	—	—	—	—	1	1	—	2	3	—	—
THSE/CLUN	1	3	1	1	1	2	3	4	4	3	2
ABLA/OPHO	—	—	3	—	—	—	—	2	3	—	—
ABLA/CLUN	1	2	1	1	—	2	3	1	1	2	2
ABLA/VACA	—	—	—	—	—	—	4	—	—	—	—
ABLA/LIBO	—	2	2	—	—	—	4	4	6	—	—
ABLA/MEFE	1	1	1	1	—	1	3	4	6	—	1
TSME/MEFE	1	1	1	1	—	—	4	4	6	—	2
ABLA/XETE	2	1	3	1	—	—	5	4	6	3	—
TSME/XETE	2	5	3	1	—	—	5	4	6	—	2
ABLA/VASC	—	4	3	—	—	—	4	4	1	—	—
ABLA/LUHI	—	—	2	—	—	—	4	3	5	—	2
PIAL-ABLA	—	—	—	—	—	—	—	—	5	—	2
OTHER	2	5	3	1	1	2	5	4	6	3	2

<sup>a</sup> Species codes are defined in Table 1.<sup>b</sup> Habitat type (h.t.) codes as defined in Pfister et al. (1977; Table 1, p. 6–7). The codes are abbreviations for the scientific names of species representing the climax overstory/understory unions; PIPO/AGSP is the code for the *Pinus ponderosa*/*Agropyron spicatum* h.t.<sup>c</sup> Where no class is given, the combination of species and habitat type did not occur in the calibration data.

TABLE A3.

Map of National Forest onto location intercept class by species.

National Forest	Species <sup>a</sup>										
	WP	L	DF	GF	WH	C	LP	S	AF	PP	MH
Bitterroot	— <sup>b</sup>	1	5	6	—	—	4	3	4	1	—
Clearwater	2	1	1	1	—	1	1	1	1	2	1
Coeur d'Alene	2	2	2	2	1	1	1	1	2	2	1
Colville	2	2	3	2	3	2	2	3	2	1	—
Flathead	2	2	3	3	—	2	4	2	3	4	—
Kaniksu	2	2	2	2	3	3	3	3	3	3	—
Kootenai	2	4	3	4	3	4	3	3	4	1	3
Lolo	2	4	5	6	—	3	4	3	4	4	1
Nez Perce	—	3	1	2	—	1	2	1	2	3	—
St. Joe	1	1	4	5	2	1	2	1	1	2	2

<sup>a</sup> Species codes are given in Table 1.<sup>b</sup> Where no class is given, the combination of species and forest did not occur in the calibration data.

TABLE A4.

Map of National Forest onto  $dbh^2$  coefficient class by species.

National Forest	Species <sup>a</sup>										
	WP	L	DF	GF	WH	C	LP	S	AF	PP	MH
Bitterroot	— <sup>b</sup>	1	1	1	—	—	1	1	1	1	—
Clearwater	2	1	2	1	—	2	2	2	2	1	1
Coeur d'Alene	2	1	2	1	1	1	2	1	1	1	1
Colville	2	1	2	1	2	1	1	1	2	1	—
Flathead	1	1	3	2	—	1	1	1	1	2	—
Kaniksu	2	1	1	2	1	1	2	3	1	2	—
Kootenai	1	1	3	3	1	2	3	2	2	1	1
Lolo	1	1	1	1	—	1	1	1	1	1	2
Nez Perce	—	1	1	2	—	2	4	1	1	1	—
St. Joe	2	2	3	1	2	1	1	1	2	1	2

<sup>a</sup> Species codes are given in Table 1.<sup>b</sup> Where no class is given, the combination of species and forest did not occur in the calibration data.

TABLE A5.  
Map of habitat type onto CCF coefficient class by species.

Habitat Type	Species <sup>a</sup>										
	WP	L	DF	GF	WH	C	LP	S	AF	PP	MH
PIPO/AGSP <sup>b</sup>	— <sup>c</sup>	—	—	—	—	—	—	—	—	1	—
PIPO/SYAL	—	—	—	—	—	—	—	—	—	1	—
PSME/AGSP	—	—	4	—	—	—	—	—	—	2	—
PSME/VACA	—	3	4	—	—	—	1	—	—	3	—
PSME/PHMA	2	3	4	2	—	2	4	4	2	2	—
PSME/VAGL	—	3	4	—	—	—	2	—	—	—	—
PSME/LIBO	—	3	4	—	—	—	4	—	—	3	—
PSME/SYAL	—	3	1	—	—	—	3	—	2	3	—
PSME/CARU	—	3	2	—	—	—	1	4	2	2	—
PSME/CAGE	—	—	4	—	—	—	—	—	—	1	—
PICEA/CLUN	—	3	—	—	—	—	1	—	—	—	—
PICEA/LIBO	—	3	4	—	—	—	4	4	—	—	—
ABGR/XETE	—	1	2	2	—	2	4	1	2	2	—
ABGR/CLUN	2	3	1	2	1	2	4	1	1	4	—
THPL/CLUN	2	3	4	2	1	1	3	2	1	4	1
THPL/OPHO	—	—	—	—	1	2	—	4	1	—	—
TSHE/CLUN	2	3	3	2	1	2	3	1	1	2	1
ABLA/OPHO	—	—	4	—	—	—	—	4	1	—	—
ABLA/CLUN	2	2	4	2	—	2	3	2	1	4	1
ABLA/VACA	—	—	—	—	—	—	4	—	—	—	—
ABLA/LIBO	—	1	1	—	—	—	1	3	1	—	—
ABLA/MEFE	2	3	3	2	—	1	4	4	1	—	1
TSME/MEFE	1	3	2	1	—	—	1	2	2	—	1
ABLA/XETE	1	1	4	1	—	—	4	4	2	4	—
TSME/XETE	2	3	4	2	—	—	4	4	1	—	1
ABLA/VASC	—	3	4	—	—	—	1	2	1	—	—
ABLA/LUHI	—	—	2	—	—	—	4	4	1	—	1
PIAL-ABLA	—	—	—	—	—	—	—	—	2	—	1
OTHER	2	3	4	2	1	2	4	4	2	4	1

<sup>a</sup> Species codes are defined in Table 1.

<sup>b</sup> Habitat type (h.t.) codes as defined in Pfister et al. (1977; Table 1, p. 6-7). The codes are abbreviations for the scientific names of species representing the climax overstory/understory unions; PIPO/AGSP is the code for the *Pinus ponderosa*/*Agropyron spicatum* h.t.

<sup>c</sup> Where no class is given, the combination of species and habitat type did not occur in the calibration data.