1 Transition Rules

1.1 Abstract Syntax

$$
\begin{aligned}
S ::=\ & x := a \mid r[a_1] := a_2 \mid S_1;\ S_2 \mid \text{if } b \text{ do } S \mid \text{if } b \text{ do } S_1 \text{ else do } S_2 \mid \text{while } b \text{ do } S \\
& \mid \text{from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ do } S \\
& \mid \text{switch}(a) \text{ case } a_1 :\ S_1 \text{ break};\ \ldots \text{ case} a_k :\ S_k \text{ break};\ \text{default} :\ S \text{ break} \\
& \mid \text{call } p(\vec{x}) \mid \text{begin } D_V\ D_A\ D_P\ S \text{ end} \\
a ::=\ & n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1 / a_2 \mid (a) \mid r[a_i] \\
b ::=\ & a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1 \ \wedge\ b_2 \mid b_1 \ \vee\ b_2 \mid (b) \\
D_V ::=\ & \text{var } x := a;\ D_V \mid \varepsilon \\
D_P ::=\ & \text{func } p \text{ is } S;\ D_P \mid \varepsilon \\
D_A ::=\ & \text{array } r[a_1];\ D_A \mid \varepsilon
\end{aligned}
$$

Transitioner er på formen: $env_V, env_P \vdash \langle S, sto\rangle \to sto'$

[VAR-ASS] $env_V, env_P \vdash \langle x <-- a, sto\rangle \to sto[l \mapsto v]$

where $env_V, sto \vdash a \to_a v$
and $env_V\ x = l$

[ARR-ASS] $env_V, env_P \vdash \langle r[a_1] <-- a_2, sto\rangle \to sto[l_2 \mapsto v_2]$

where $env_V, sto \vdash a_1 \to_a v_1$
and $env_V, sto \vdash a_2 \to_a v_2$
and $env_V\ r = l_1$
and $l_2 = l_1 + v_1$
and $v_3 = sto\ l_1$
and $1 \leq v_1 \leq v_3$

[COMP] $$\frac{\begin{array}{c} env_V, env_P \vdash \langle S_1, sto\rangle \to sto'' \\ env_V, env_P \vdash \langle S_2, sto''\rangle \to sto' \end{array}}{env_V, env_P \vdash \langle S_1; S_2, sto\rangle \to sto'}$$

[IF-TRUE] $$\frac{env_V, env_P \vdash \langle S, sto\rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto\rangle \to sto'}$$

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if $env_V, sto \vdash b \rightarrow_b$ true

[IF-FALSE] $\quad env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto\rangle \rightarrow sto$

if $env_V, sto \vdash b \rightarrow_b$ false

[IF-ELSE-TRUE] $$\frac{env_V, env_P \vdash \langle S_1, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto\rangle \rightarrow sto'}$$

if $env_V, sto \vdash b \rightarrow_b$ true

[IF-ELSE-FALSE] $$\frac{env_V, env_P \vdash \langle S_2, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto\rangle \rightarrow sto'}$$

if $env_V, sto \vdash b \rightarrow_b$ false

[WHILE-TRUE] $$\frac{\begin{array}{c} env_V, env_P \vdash \langle S, sto\rangle \rightarrow sto'' \\ env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto''\rangle \rightarrow sto' \end{array}}{env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto\rangle \rightarrow sto'}$$

if $env_V, sto \vdash b \rightarrow_b$ true

[WHILE-FALSE] $\quad env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto\rangle \rightarrow sto$

if $env_V, sto \vdash b \rightarrow_b$ false

[FROM-TRUE] $$\frac{\begin{array}{c} env_V, env_P \vdash \langle S, sto[l \mapsto v_1]\rangle \rightarrow sto'' \\ \langle \text{from } x <-- a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto'' \rightarrow sto' \end{array}}{env_V, env_P \vdash \langle \text{from } x <-- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto\rangle \rightarrow sto'}$$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V, sto \vdash a_3 \rightarrow_a v_3$
and $v_1 \leq v_2$
and $l = env_V x$

[FROM-FALSE] $\quad env_V, env_P \vdash \langle \text{from } x <-- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto\rangle \rightarrow sto$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V, sto \vdash a_3 \rightarrow_a v_3$
and $v_1 > v_2$

Table 1.1: Statements

[SWITCH-1]
$$\frac{env_V, env_P \vdash \langle S, sto\rangle \rightarrow (sto')}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 :\ S_1 \text{ break};\ \text{default} :\ S \text{ break};\ \text{end}, sto\rangle \rightarrow sto'}$$

Where $k > 0$
and $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

[SWITCH-2]
$$\frac{env_V, env_P \vdash \langle S_1, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 :\ S_1 \text{ break};\ \dots\ \text{case } a_k :\ S_k \text{ break};\ \text{default} :\ S \text{ break};\ \text{end}, sto\rangle \rightarrow sto'}$$

Where $k > 1$
and $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v = v_1$

[SWITCH-3]
$$\frac{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 :\ S_2 \text{ break};\ \dots\ \text{case } a_k :\ S_k \text{ break};\ \text{default} :\ S \text{ break};\ \text{end}, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 :\ S_1 \text{ break};\ \dots\ \text{case } a_k :\ S_k \text{ break};\ \text{default} :\ S \text{ break};\ \text{end}, sto\rangle \rightarrow sto'}$$

Where $k > 1$
and $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

[CALL]
$$\frac{env'_V[\vec{z} \mapsto \vec{l}], env'_P \vdash \langle S, sto[\vec{l} \mapsto \vec{v}]\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto\rangle \rightarrow sto'}$$

where $env_P\ p = (S, \vec{z}, env'_V, env'_P)$
and $|\vec{a}| = |\vec{z}|$

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$$\text{and } env_V, sto \vdash a_i \rightarrow v_i \text{for each } 1 \leq i \leq |\vec{a}|$$
$$\text{and } l_1 = env_V \text{ new}$$
$$\text{and } l_{i+1} = l_i \text{ for each } 1 < i < |\vec{a}|$$

$$[\text{BLOCK}] \quad \frac{\langle D_V, env_V, sto \rangle \rightarrow_{DV} (env'_V, sto'') \quad env'_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env'_P \quad env'_V, env'_P \vdash \langle S, sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{begin } D_V \ D_P \ S \text{ end}, sto \rangle \rightarrow sto'}$$

Table 1.2: Statements

Transitioner er på formen: $env_V, sto \vdash a \rightarrow_a v$

[NUM] $env_V, sto \vdash n \rightarrow_a v$

if $\mathcal{N}[[n]] = v$

[VAR] $env_V, sto \vdash x \rightarrow_a v$

if $env_V\ x = l$
and $sto\ l = v$

[ADD] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$$

where $v = v_1 + v_2$

[SUB] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$$

where $v = v_1 - v_2$

[MULT] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$$

where $v = v_1 \cdot v_2$

[DIV] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 / a_2 \rightarrow_a v}$$

where $v = v_1 / v_2$

[PAR] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1}{env_V, sto \vdash (a_1) \rightarrow_a v_1}$$

[ARR] $env_V, sto \vdash r[a_1] \rightarrow_a a_2$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V\ r = l$
and $sto\ l = v_3$
and $0 < v_1 \leq v_3$
and $sto(l + v_1) = v_2$

Table 1.3: Arithmetic expressions

Transitioner på formen: $env_V, sto \vdash b \rightarrow_b t$

[EQUAL-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{true}}$$

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if $v_1 = v_2$

[EQUAL-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{false}}$$

if $v_1 \neq v_2$

[GRT-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{true}}$$

if $v_1 > v_2$

[GRT-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{false}}$$

if $v_1 \not> v_2$

[LESS-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$$

if $v_1 < v_2$

[LESS-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{false}}$$

if $v_1 \not< v_2$

[NOT-1] $$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$$

[NOT-2] $$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$$

[AND-TRUE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{true}}$$

[AND-FALSE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{false}}$$

where $i \in 1, 2$

[OR-TRUE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{true}}$$

where $i \in 1, 2$

[OR-FALSE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{false}}$$

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[PAR-BOOL]	$\dfrac{env_V, sto \vdash b \rightarrow_b v}{env_V, sto \vdash (b) \rightarrow_b v}$

Table 1.4: Boolean expressions

	Transitioner på formen: $\langle D_V, env_V, sto\rangle \rightarrow_{DV} (env'_V, sto')$
[VAR-DEC]	$\dfrac{\langle D_V, env''_V, sto[l \mapsto v]\rangle \rightarrow_{DV} (env'_V, sto')}{\text{var } x <--a; D_V, env_V, sto\rangle \rightarrow_{DV} (env'_V, sto')}$ where $env_V, sto \vdash a \rightarrow_a v$ and $l = env_V$ next and $env''_V = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$
[EMPTY-VAR]	$\langle \varepsilon, env_V, sto\rangle \rightarrow_{DV} (env_V, sto)$
	Transitioner på formen: $env_V \vdash \langle D_P, env_P\rangle \rightarrow_{DP} env'_P$
[FUNC-DEC]	$\dfrac{env_V \vdash \langle D_P, env_P[p \mapsto (S, env_V, env_P)]\rangle \rightarrow_{DP} env'_P}{env_V \vdash \langle \text{proc } p \text{ is } S; D_P, env_P\rangle \rightarrow_{DP} env'_P}$
[EMPTY-FUNC]	$env_V \vdash \langle \varepsilon, env_P\rangle \rightarrow_{DP} env'_P$
	Transitioner på formen: $\langle D_A, env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')$
[ARRAY-DEC]	$\dfrac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto v]\rangle \rightarrow_{DA} (env'_V, sto')}{\langle r[a_1], D_A, env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')}$ where $env_V, sto \vdash a_1 \rightarrow_a v$ and $l = env_V \text{next}$ and $l > 0$
[EMPTY-ARRAY]	$\langle \varepsilon, env_V, sto\rangle \rightarrow_{DA} (env_V, sto)$

Table 1.5: Declarations