Transition Rules 1

1.1 Abstract Syntax

$$S ::= x := a \mid r[a_1] := a_2 \mid S_1;\ S_2 \mid \text{if } b \text{ begin } S \text{ end} \mid \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}$$

$$\text{while } b \text{ begin } S \text{ end} \mid \text{from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end} \mid \text{call } p(\vec{x}) \mid D_V \mid D_A \mid D_P$$

$$\mid \text{switch}(a) \text{ begin case } a_1 :\ S_1 \text{ break}; \ \ldots \ \text{case} a_k :\ S_k \text{ break}; \text{ default} :\ S \text{ break end}$$

$$a ::= n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1/a_2 \mid (a) \mid r[a_i]$$

$$b ::= a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1 \ \wedge \ b_2 \mid b_1 \ \vee \ b_2 \mid (b)$$

$$D_V ::= \text{var } x := a$$

$$D_P ::= \text{func } p \text{ is begin } S \text{ end} \mid \text{func } p(\vec{x}) \text{ is begin } S \text{ end}$$

$$D_A ::= \text{array } r[a_1]$$

[VAR-ASS] $env_V, env_P \vdash \langle x < -- a, sto \rangle \to sto[l \mapsto v]$

where $env_V, sto \vdash a \to_a v$
and $env_V\ x = l$

[ARR-ASS] $env_V, env_P \vdash \langle r[a_1] < -- a_2, sto \rangle \to sto[l_2 \mapsto v_2]$

where $env_V, sto \vdash a_1 \to_a v_1$
and $env_V, sto \vdash a_2 \to_a v_2$
and $env_V\ r = l_1$
and $l_2 = l_1 + v_1$
and $v_3 = sto\ l_1$
and $1 \le v_1 \le v_3$

[COMP]
$$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \to sto''\qquad env_V, env_P \vdash \langle S_2, sto'' \rangle \to sto'}{env_V, env_P \vdash \langle S_1; S_2, sto \rangle \to sto'}$$

[IF-TRUE]
$$\frac{env_V, env_P \vdash \langle S, sto \rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \to sto'}$$

if $env_V, sto \vdash b \to_b$ true
Continued on the next page

[IF-FALSE] $env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto$

           if $env_V, sto \vdash b \rightarrow_b$ false

[IF-ELSE-TRUE]

$$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'}$$

           if $env_V, sto \vdash b \rightarrow_b$ true

[IF-ELSE-FALSE]

$$\frac{env_V, env_P \vdash \langle S_2, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'}$$

           if $env_V, sto \vdash b \rightarrow_b$ false

[WHILE-TRUE]

$$\frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'' \qquad env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'}$$

           if $env_V, sto \vdash b \rightarrow_b$ true

[WHILE-FALSE] $env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto$

           if $env_V, sto \vdash b \rightarrow_b$ false

[FROM-TRUE]

$$\frac{env_V, env_P \vdash \langle S, sto[l \mapsto v_1] \rangle \rightarrow sto'' \qquad \langle \text{from } x < -- a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto'' \rightarrow sto'}{env_V, env_P \vdash \langle \text{from } x < -- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'}$$

           where $env_V, sto \vdash a_1 \rightarrow_a v_1$
           and $env_V, sto \vdash a_2 \rightarrow_a v_2$
           and $env_V, sto \vdash a_3 \rightarrow_a v_3$
           and $v_1 \le v_2$
           and $l = env_V x$

[FROM-FALSE] $env_V, env_P \vdash \langle \text{from } x < -- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto \rangle \rightarrow sto$

           where $env_V, sto \vdash a_1 \rightarrow_a v_1$
           and $env_V, sto \vdash a_2 \rightarrow_a v_2$
           and $env_V, sto \vdash a_3 \rightarrow_a v_3$
           and $v_1 > v_2$

[CALL]

$$\frac{env_V'[\vec{z} \mapsto \vec{l}], env_P' \vdash \langle S, sto[\vec{l} \mapsto \vec{v}] \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto \rangle \rightarrow sto'}$$

           where $env_P \, p = (S, \vec{z}, env_V', env_P')$
           and $|\vec{a}| = |\vec{z}|$
           and $env_V, sto \vdash a_i \rightarrow v_i$ for each $1 \le i \le |\vec{a}|$
           and $l_1 = env_V$ new
           Continued on the next page

and $l_{i+1} = \text{new } l_i$ for each $1 \leq i < |\vec{a}|$

Table 1.1: Statements

[SWITCH-1]
$$\frac{envv, envp \vdash \langle S, sto \rangle \rightarrow (sto')}{envv, envp \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; default} : S \text{ break; end}, sto \rangle \rightarrow sto'}$$

Where $envv, sto \vdash a \rightarrow_a v$
and $envv, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

[SWITCH-2]
$$\frac{envv, envp \vdash \langle S_1, sto \rangle \rightarrow sto'}{envv, envp \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; } \ldots \text{ case } a_k : S_k \text{ break; default} : S \text{ break; end}, sto \rangle \rightarrow sto'}$$

Where $k > 0$
and $envv, sto \vdash a \rightarrow_a v$
and $envv, sto \vdash a_1 \rightarrow_a v_1$
and $v = v_1$

[SWITCH-3]
$$\frac{envv, envp \vdash \langle \text{switch}(a) \text{ begin case } a_2 : S_2 \text{ break; } \ldots \text{ case } a_k : S_k \text{ break; default} : S \text{ break; end}, sto \rangle \rightarrow sto'}{envv, envp \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; } \ldots \text{ case } a_k : S_k \text{ break; default} : S \text{ break; end}, sto \rangle \rightarrow sto'}$$

Where $k > 1$
and $envv, sto \vdash a \rightarrow_a v$
and $envv, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

Table 1.2: Statements

Transitioner er på formen: $env_V, sto \vdash a \rightarrow_a v$

[NUM] $env_V, sto \vdash n \rightarrow_a v$

               if $\mathcal{N}[[n]] = v$

[VAR] $env_V, sto \vdash x \rightarrow_a v$

               if $env_V\ x = l$
               and $sto\ l = v$

[ADD] $\dfrac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$

               where $v = v_1 + v_2$

[SUB] $\dfrac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$

               where $v = v_1 - v_2$

[MULT] $\dfrac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$

               where $v = v_1 \cdot v_2$

[DIV] $\dfrac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1/a_2 \rightarrow_a v}$

               where $v = v_1/v_2$

[PAR] $\dfrac{env_V, sto \vdash a_1 \rightarrow_a v_1}{env_V, sto \vdash (a_1) \rightarrow_a v_1}$

[ARR] $env_V, sto \vdash r[a_1] \rightarrow_a a_2$

               where $env_V, sto \vdash a_1 \rightarrow_a v_1$
               and $env_V, sto \vdash a_2 \rightarrow_a v_2$
               and $env_V\ r = l$
               and $sto\ l = v_3$
               and $0 < v_1 \leq v_3$
               and $sto(l + v_1) = v_2$

Table 1.3: Arithmetic expressions

Transitioner på formen: $env_V, sto \vdash b \rightarrow_b t$

[EQUAL-TRUE] $\dfrac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{true}}$
Continued on the next page

$$\text{if } v_1 = v_2$$

[EQUAL-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{false}}$$

$$\text{if } v_1 \neq v_2$$

[GRT-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{true}}$$

$$\text{if } v_1 > v_2$$

[GRT-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{false}}$$

$$\text{if } v_1 \not> v_2$$

[LESS-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$$

$$\text{if } v_1 < v_2$$

[LESS-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{false}}$$

$$\text{if } v_1 \not< v_2$$

[NOT-1] $$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$$

[NOT-2] $$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$$

[AND-TRUE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{true}}$$

[AND-FALSE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{false}}$$

$$\text{where } i \in 1, 2$$

[OR-TRUE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{true}}$$

$$\text{where } i \in 1, 2$$

[OR-FALSE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{false}}$$

Continued on the next page

[PAR-BOOL]
$$\frac{env_V, sto \vdash b \rightarrow_b v}{env_V, sto \vdash (b) \rightarrow_b v}$$

Table 1.4: Boolean expressions

Transitioner på formen: $\langle D_V, env_V, sto \rangle \rightarrow$

[VAR-DEC]
$$\frac{\langle D_V, env_V'', sto[l \mapsto v] \rangle \rightarrow_{DV} (env_V', sto')}{\text{var } x < - - a, env_V, sto \rangle \rightarrow_{DV} (env_V', sto'}$$

where $env_V, sto \vdash a \rightarrow_a v$
and $l = env_V$ next
and $env_V'' = env_V[x \mapsto l][\text{next} \mapsto \text{ne}$

Transitioner på formen: $env_V \vdash \langle D_P, env_P$

[FUNC-DEC]
$$\frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, env_V, env_P)] \rangle -}{env_V \vdash \langle \text{proc } p \text{ is begin } S \text{ end}, env_P \rangle \rightarrow_L}$$

[FUNC-PARA-DEC]
Transitioner på formen: $\langle D_A, env_V, sto \rangle \rightarrow_{DA} (env_V', sto')$

[ARRAY-DEC]
$$\frac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto}{\langle \text{array } r[a_1], env_V, sto \rangle \rightarrow_{DA} (}$$

where $env_V, sto \vdash a_1 \rightarrow_a v$
and $l = env_V \text{next}$
and $v > 0$

Table 1.5: Declarations