Transition Rules

1.1 Abstract Syntax

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\begin{aligned} R &::= D_P \mid D_A \mid D_V \mid R_1 \mid R_2 \mid \varepsilon \\ S &::= x := a \mid r[a_1] := a_2 \mid S_1; \mid S_2 \mid \text{if } b \text{ begin } S \text{ end } | \text{ if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end} \\ & \text{while } b \text{ begin } S \text{ end } | \text{ from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end } | \text{ call } p(\vec{x}) \mid D_V \mid D_A \\ & | \text{ switch}(a) \text{ begin case } a_1 : \mid S_1 \text{ break}; \dots \text{ case } a_k : \mid S_k \text{ break}; \text{ default } : \mid S \text{ break end} \end{aligned}
a ::= n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1/a_2 \mid (a) \mid r[a_i]
b ::= a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1 \land b_2 \mid b_1 \lor b_2 \mid (b)
D_V ::= \text{var } x := a
D_P ::= \text{func } p(\vec{x}) \text{ is begin } S \text{ end}
D_A ::= \text{array } r[a_1]
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[COMP]
$$\frac{env_V \vdash \langle R_1, sto \rangle \to sto''}{\frac{env_V, env_P \vdash \langle R_2, sto'' \rangle \to sto'}{env_V, env_P \vdash \langle R_1, R_2, sto \rangle \to sto'}}$$

Table 1.1: Root statements

[VAR-ASS] Transitioner er på formen: $env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'$ $env_V, env_P \vdash \langle x < --a, sto \rangle \rightarrow sto[l \mapsto v]$ $where <math>env_V, sto \vdash a \rightarrow_a v$ and $env_V x = l$ $env_V, env_P \vdash \langle r[a_1] < --a_2, sto \rangle \rightarrow sto[l_2 \mapsto v_2]$ $where <math>env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V r = l_1$ and $l_2 = l_1 + v_1$ Continued on the next page

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and v_3 = sto l_1
                                                      and 1 \le v_1 \le v_3
                                           env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto''
                                          env_V, env_P \vdash \langle S_2, sto'' \rangle \rightarrow sto'
[COMP]
                                         \overline{env_V, env_P \vdash \langle S_1; S_2, sto \rangle \rightarrow sto'}
                                                      env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'
[IF-TRUE]
                                         env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'
                                                      if env_V, sto \vdash b \rightarrow_b true
[IF-FALSE]
                                        env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                      if env_V, sto \vdash b \rightarrow_b false
                                        \frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'}
[IF-ELSE-TRUE]
                                                      if env_V, sto \vdash b \rightarrow_b true
                                        \frac{env_V, env_P \vdash \langle S_2, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'}
[IF-ELSE-FALSE]
                                                      if env_V, sto \vdash b \rightarrow_b false
                                                           env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto''
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto'' \rangle \rightarrow sto'
[WHILE-TRUE]
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'
                                                      if env_V, sto \vdash b \rightarrow_b true
[WHILE-FALSE]
                                        env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                      if env_V, sto \vdash b \rightarrow_b false
                                                                        env_V, env_P \vdash \langle S, sto[l \mapsto v_1] \rangle \rightarrow sto''
                                                  \langle \text{from } x < --a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto'' \to sto'
[FROM-TRUE]
                                        \overline{env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'}
                                                      where env_V, sto \vdash a_1 \rightarrow_a v_1
                                                      and env_V, sto \vdash a_2 \rightarrow_a v_2
                                                      and env_V, sto \vdash a_3 \rightarrow_a v_3
                                                      and v_1 \leq v_2
                                                      and l = env_V x
[FROM-FALSE]
                                        env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                      where env_V, sto \vdash a_1 \rightarrow_a v_1
                                                             Continued on the next page
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and
$$env_V$$
, $sto \vdash a_2 \rightarrow_a v_2$
and env_V , $sto \vdash a_3 \rightarrow_a v_3$
and $v_1 > v_2$

$$\frac{env_V'[\vec{z} \mapsto \vec{l}], env_P' \vdash \langle S, sto[\vec{l} \mapsto \vec{v}] \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto \rangle \rightarrow sto'}$$
where $env_P \ p = (S, \vec{z}, env_V', env_P')$
and $|\vec{a}| = |\vec{z}|$
and env_V , $sto \vdash a_i \rightarrow v_i$ for each $1 \le i \le |\vec{a}|$
and $l_1 = env_V$ new
and $l_{i+1} = \text{new } l_i$ for each $1 \le i < |\vec{a}|$

Table 1.2: Statements

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env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default } : S \text{ break}; \text{ end, } sto \rangle \to sto'
                                                                                      \rightarrow sto'
                                                                           env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; default} : S \text{ break; end, } sto \rangle
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'
env_V, env_P \vdash \langle S, sto \rangle \rightarrow (sto')
                                                                                                                                                                                                          Where env_V, sto \vdash a \rightarrow_a v
                                                                                                                                                                                                                                                                                      and env_V, sto \vdash a_1 \rightarrow_a v_1
                                                                                                                                                                                                                                                                                                                                                     and v \neq v_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           [SWITCH-2]
                                     [SWITCH-1]
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 $env_V, env_P \vdash (\text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default} : S \text{ break}; \text{ end}, sto \rightarrow sto'$ $env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 : S_2 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default} : S \text{ break}; \text{ end, } sto \rangle \to sto'$ and $env_V, sto \vdash a_1 \rightarrow_a v_1$ FiXme Fatal: kunne det ikke være bedre med "k" i stedet for 1? and $v = v_1$ [SWITCH-3]

and env_V , $sto \vdash a \rightarrow_a v$

Where k > 0

Where k > 1and env_V , $sto \vdash a \rightarrow_a v$ and env_V , $sto \vdash a_1 \rightarrow_a v_1$ and $v \neq v_1$

Table 1.3: Statements

Transitioner er på formen: env_V , $sto \vdash a \rightarrow_a v$

[NUM]
$$env_V, sto \vdash n \rightarrow_a v$$

if
$$\mathcal{N}[[n]] = v$$

[VAR]
$$env_V, sto \vdash x \rightarrow_a v$$

if
$$env_V x = l$$

and $sto l = v$

[ADD]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$$

where
$$v = v_1 + v_2$$

[SUB]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$$

where
$$v = v_1 - v_2$$

[MULT]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$$

where
$$v = v_1 \cdot v_2$$

[DIV]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1/a_2 \rightarrow_a v}$$

where
$$v = v_1/v_2$$

[PAR]
$$\frac{env_V, sto \vdash a_1 \to_a v_1}{env_V, sto \vdash (a_1) \to_a v_1}$$

[ARR]
$$env_V, sto \vdash r[a_1] \rightarrow_a a_2$$

where
$$env_V$$
, $sto \vdash a_1 \rightarrow_a v_1$
and env_V , $sto \vdash a_2 \rightarrow_a v_2$
and $env_V r = l$
and $sto l = v_3$
and $0 < v_1 \le v_3$
and $sto(l + v_1) = v_2$

Table 1.4: Arithmetic expressions

Transitioner på formen: env_V , $sto \vdash b \rightarrow_b$ bool

$$[\text{EQUAL-TRUE}] \quad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{ true}} \\ Continued \ on \ the \ next \ page$$

if
$$v_1 = v_2$$

[EQUAL-FALSE]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{ false}}$$

if
$$v_1 \neq v_2$$

[GRT-TRUE]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{ true}}$$

if
$$v_1 > v_2$$

$$[\text{GRT-FALSE}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{false}}$$

if
$$v_1 \not> v_2$$

$$[\text{LESS-TRUE}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$$

if
$$v_1 < v_2$$

$$[\text{LESS-FALSE}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{ false}}$$

if
$$v_1 \not< v_2$$

[NOT-1]
$$\frac{env_V, sto \vdash b \to_b \text{ true}}{env_V, sto \vdash !b \to_b \text{ false}}$$

[NOT-2]
$$\frac{env_V, sto \vdash b \to_b \text{ false}}{env_V, sto \vdash !b \to_b \text{ true}}$$

[AND-TRUE]
$$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \land b_2 \rightarrow_b \text{true}}$$

[AND-FALSE]
$$\frac{env_V, sto \vdash b_i \to_b \text{ false}}{env_V, sto \vdash b_1 \land b_2 \to_b \text{ false}}$$

where
$$i \in 1, 2$$

[OR-TRUE]
$$\frac{env_V, sto \vdash b_i \to_b \text{ true}}{env_V, sto \vdash b_1 \lor b_2 \to_b \text{ true}}$$

where
$$i \in 1, 2$$

[OR-FALSE]
$$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{ false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{ false}}{env_V, sto \vdash b_1 \lor b_2 \rightarrow_b \text{ false}}$$

Continued on the next page

[PAR-BOOL]
$$\frac{env_V, sto \vdash b \to_b v}{env_V, sto \vdash (b) \to_b v}$$

Table 1.5: Boolean expressions

Transitioner på formen: $\langle D_V, env_V, sto \rangle \rightarrow_{DV} (env_V', sto')$

[VAR-DEC]
$$\frac{\langle D_V, env_V'', sto[l \mapsto v] \rangle \to_{DV} (env_V', sto')}{\operatorname{var} x < --a, env_V, sto \rangle \to_{DV} (env_V', sto')}$$

where env_V , $sto \vdash a \rightarrow_a v$ and $l = env_V$ next and $env_V'' = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$

Transitioner på formen: $env_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env_P'$

$$[PROC\text{-PARA-DEC}] \quad \frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, \vec{x}, env_V, env_P)] \rangle \rightarrow_{DP} env_P'}{env_V \vdash \langle \text{function } p \text{ using}(\text{var } \vec{x}) \text{ begin } S \text{ end, } env_P \rangle \rightarrow_{DP} env_P'}$$

Transitioner på formen: $\langle D_A, env_V, sto \rangle \rightarrow_{DA} (env_V', sto')$

[ARRAY-DEC]
$$\frac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto v] \rangle \rightarrow_{DA} (env_V', sto')}{\langle \text{array } r[a_1], env_V, sto \rangle \rightarrow_{DA} (env_V', sto')}$$

where $env_V, sto \vdash a_1 \rightarrow_a v$ and $l = env_V$ next and v > 0

Table 1.6: Declarations