

# Transition Rules

# 1

## 1.1 Abstract Syntax

$R ::= D_P D_A D_V \mid R_1 R_2$   
 $S ::= x := a \mid r[a_1] := a_2 \mid S_1; S_2 \mid \text{if } b \text{ begin } S \text{ end} \mid \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}$   
 $\quad \text{while } b \text{ begin } S \text{ end} \mid \text{from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end} \mid \text{call } p(\vec{x}) \mid D_V \mid D_A$   
 $\quad \mid \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break end}$   
 $a ::= n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1 / a_2 \mid (a) \mid r[a_i]$   
 $b ::= a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \mid (b)$   
 $D_V ::= \text{var } x := a \mid \varepsilon$   
 $D_P ::= \text{func } p(\vec{x}) \text{ is begin } S \text{ end} \mid \varepsilon$   
 $D_A ::= \text{array } r[a_1] \mid \varepsilon$

[ROOT]	$\frac{\begin{array}{l} env_V, env_P \vdash \langle R_1, sto \rangle \rightarrow sto'' \\ env_V, env_P \vdash \langle R_2, sto'' \rangle \rightarrow sto' \end{array}}{env_V, env_P \vdash \langle R_1; R_2, sto \rangle \rightarrow sto'}$
[BLOCK]	$\frac{\begin{array}{l} \langle D_V, env_V, sto \rangle \rightarrow_{D_V} (env'_V, sto'') \\ \langle D_A, env'_V, sto'' \rangle \rightarrow_{D_A} (env''_V, sto') \\ env''_V \vdash \langle D_P, env_P \rangle \rightarrow_{D_P} env'_P \end{array}}{env_v, env_P \vdash \langle D_V D_A D_P, sto \rangle \rightarrow sto'}$

Table 1.1: Root statements

Transitioner er på formen:  $env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'$

[VAR-ASS]  $env_V, env_P \vdash \langle x \leftarrow a, sto \rangle \rightarrow sto[l \mapsto v]$

where  $env_V, sto \vdash a \rightarrow_a v$   
and  $env_V x = l$

[ARR-ASS]  $env_V, env_P \vdash \langle r[a_1] \leftarrow a_2, sto \rangle \rightarrow sto[l_2 \mapsto v_2]$

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	where $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V r = l_1$ and $l_2 = l_1 + v_1$ and $v_3 = sto l_1$ and $1 \leq v_1 \leq v_3$
[COMP]	$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'' \quad env_V, env_P \vdash \langle S_2, sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle S_1; S_2, sto \rangle \rightarrow sto'}$
[IF-TRUE]	$\frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto'}$
	if $env_V, sto \vdash b \rightarrow_b \text{true}$
[IF-FALSE]	$env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto$
	if $env_V, sto \vdash b \rightarrow_b \text{false}$
[IF-ELSE-TRUE]	$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end, } sto \rangle \rightarrow sto'}$
	if $env_V, sto \vdash b \rightarrow_b \text{true}$
[IF-ELSE-FALSE]	$\frac{env_V, env_P \vdash \langle S_2, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end, } sto \rangle \rightarrow sto'}$
	if $env_V, sto \vdash b \rightarrow_b \text{false}$
[WHILE-TRUE]	$\frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'' \quad env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end, } sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto'}$
	if $env_V, sto \vdash b \rightarrow_b \text{true}$
[WHILE-FALSE]	$env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto$
	if $env_V, sto \vdash b \rightarrow_b \text{false}$
[FROM-TRUE]	$\frac{env_V, env_P \vdash \langle S, sto[l \mapsto v_1] \rangle \rightarrow sto'' \quad \langle \text{from } x < - - a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end, } sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{from } x < - - a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end, } sto \rangle \rightarrow sto'}$
	where $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V, sto \vdash a_3 \rightarrow_a v_3$ and $v_1 \leq v_2$

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	and $l = env_V x$
[FROM-FALSE]	$env_V, env_P \vdash \langle \text{from } x < - - a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end, } sto \rangle \rightarrow sto$ where $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V, sto \vdash a_3 \rightarrow_a v_3$ and $v_1 > v_2$
[CALL]	$\frac{env'_V[\vec{z} \mapsto \vec{l}], env'_P \vdash \langle S, sto[\vec{l} \mapsto \vec{v}] \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto \rangle \rightarrow sto'}$ where $env_P p = (S, \vec{z}, env'_V, env'_P)$ and $ \vec{a}  =  \vec{z} $ and $env_V, sto \vdash a_i \rightarrow v_i$ for each $1 \leq i \leq  \vec{a} $ and $l_1 = env_V \text{ new}$ and $l_{i+1} = \text{new } l_i$ for each $1 \leq i <  \vec{a} $

Table 1.2: Statements

[SWITCH-1]

$$\frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow (sto')}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}$$

Where  $env_V, sto \vdash a \rightarrow_a v$   
 and  $env_V, sto \vdash a_1 \rightarrow_a v_1$   
 and  $v \neq v_1$

[SWITCH-2]

$$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}$$

Where  $k > 0$   
 and  $env_V, sto \vdash a \rightarrow_a v$   
 and  $env_V, sto \vdash a_1 \rightarrow_a v_1$  **FiXme Fatal: kunne det ikke være bedre med "k" i stedet for 1?**  
 and  $v = v_1$

[SWITCH-3]

$$\frac{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 : S_2 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}$$

Where  $k > 1$   
 and  $env_V, sto \vdash a \rightarrow_a v$   
 and  $env_V, sto \vdash a_1 \rightarrow_a v_1$   
 and  $v \neq v_1$

Table 1.3: Statements

Transitioner er på formen:  $env_V, sto \vdash a \rightarrow_a v$

[NUM]	$env_V, sto \vdash n \rightarrow_a v$ if $\mathcal{N}[[n]] = v$
[VAR]	$env_V, sto \vdash x \rightarrow_a v$ if $env_V x = l$ and $sto l = v$
[ADD]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$ where $v = v_1 + v_2$
[SUB]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$ where $v = v_1 - v_2$
[MULT]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$ where $v = v_1 \cdot v_2$
[DIV]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 / a_2 \rightarrow_a v}$ where $v = v_1 / v_2$
[PAR]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1}{env_V, sto \vdash (a_1) \rightarrow_a v_1}$
[ARR]	$env_V, sto \vdash r[a_1] \rightarrow_a a_2$ where $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V r = l$ and $sto l = v_3$ and $0 < v_1 \leq v_3$ and $sto(l + v_1) = v_2$

Table 1.4: Arithmetic expressions

Transitioner på formen:  $env_V, sto \vdash b \rightarrow_b \text{bool}$

[EQUAL-TRUE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{true}}$
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	if $v_1 = v_2$
[EQUAL-FALSE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{false}}$
	if $v_1 \neq v_2$
[GRT-TRUE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{true}}$
	if $v_1 > v_2$
[GRT-FALSE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{false}}$
	if $v_1 \not> v_2$
[LESS-TRUE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$
	if $v_1 < v_2$
[LESS-FALSE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{false}}$
	if $v_1 \not< v_2$
[NOT-1]	$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$
[NOT-2]	$\frac{env_V, sto \vdash b \rightarrow_b \text{false}}{env_V, sto \vdash !b \rightarrow_b \text{true}}$
[AND-TRUE]	$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{true}}$
[AND-FALSE]	$\frac{env_V, sto \vdash b_i \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{false}}$
	where $i \in 1, 2$
[OR-TRUE]	$\frac{env_V, sto \vdash b_i \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{true}}$
	where $i \in 1, 2$
[OR-FALSE]	$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{false}}$

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$$[\text{PAR-BOOL}] \quad \frac{env_V, sto \vdash b \rightarrow_b v}{env_V, sto \vdash (b) \rightarrow_b v}$$

Table 1.5: Boolean expressions

Transitioner på formen:  $\langle D_V, env_V, sto \rangle \rightarrow_{DV} (env'_V, sto')$

$$[\text{VAR-DEC}] \quad \frac{\langle D_V, env''_V, sto[l \mapsto v] \rangle \rightarrow_{DV} (env'_V, sto')}{\text{var } x < - - a, env_V, sto \rangle \rightarrow_{DV} (env'_V, sto')}$$

where  $env_V, sto \vdash a \rightarrow_a v$   
and  $l = env_V \text{ next}$   
and  $env''_V = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$

$$[\text{EMPTY-VAR}] \quad \langle \varepsilon, env_V, sto \rangle \rightarrow_{DV} (env_V, sto)$$

Transitioner på formen:  $env_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env'_P$

$$[\text{PROC-PARA-DEC}] \quad \frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, \vec{x}, env_V, env_P)] \rangle \rightarrow_{DP} env'_P}{env_V \vdash \langle \text{function } p \text{ using } (\text{var } \vec{x}) \text{ begin } S \text{ end, } env_P \rangle \rightarrow_{DP} env'_P}$$

$$[\text{EMPTY-PROC}] \quad env_V \vdash \langle \varepsilon, env_P \rangle \rightarrow_{DP} env'_P$$

Transitioner på formen:  $\langle D_A, env_V, sto \rangle \rightarrow_{DA} (env'_V, sto')$

$$[\text{ARRAY-DEC}] \quad \frac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto v] \rangle \rightarrow_{DA} (env'_V, sto')}{\langle \text{array } r[a_1], env_V, sto \rangle \rightarrow_{DA} (env'_V, sto')}$$

where  $env_V, sto \vdash a_1 \rightarrow_a v$   
and  $l = env_V \text{ next}$   
and  $v > 0$

$$[\text{EMPTY-ARRAY}] \quad \langle \varepsilon, env_V, sto \rangle \rightarrow_{DA} (env_V, sto)$$

Table 1.6: Declarations