Transition Rules 1

1.1 Abstract Syntax

$$\begin{aligned}
R &::= D_P\ D_A\ D_V\ | R_1\ R_2 \\
S &::= x := a \mid r[a_1] := a_2 \mid S_1;\ S_2 \mid \text{if } b \text{ begin } S \text{ end} \mid \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end} \\
&\quad \text{while } b \text{ begin } S \text{ end} \mid \text{from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end} \mid \text{call } p(\vec{x}) \mid D_V \mid D_A \\
&\quad \mid \text{switch}(a) \text{ begin case } a_1 :\ S_1 \text{ break};\ \ldots \text{ case } a_k :\ S_k \text{ break};\ \text{default} :\ S \text{ break end} \\
a &::= n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1/a_2 \mid (a) \mid r[a_i] \\
b &::= a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1\ \wedge\ b_2 \mid b_1\ \vee\ b_2 \mid (b) \\
D_V &::= \text{var } x := a \mid \varepsilon \\
D_P &::= \text{ func } p(\vec{x}) \text{ is begin } S \text{ end} \mid \varepsilon \\
D_A &::= \text{array } r[a_1] \mid \varepsilon
\end{aligned}$$

$$[\text{BLOCK}] \quad \frac{\begin{array}{c}\langle D_V, env_V, sto\rangle \rightarrow_{DV} (env'_V, sto'') \\ \langle D_A, env'_V, sto''\rangle \rightarrow_{DA} (env''_V, sto') \\ env''_V \vdash \langle D_P, env_P\rangle \rightarrow_{DP} env'_P\end{array}}{env_V, env_P \vdash \langle\ D_V\ D_A\ D_P, sto\rangle \rightarrow sto'}$$

$$[\text{ROOT}] \quad \frac{\begin{array}{c}env_V, env_P \vdash \langle R_1, sto\rangle \rightarrow sto'' \\ env_V, env_P \vdash \langle R_2, sto''\rangle \rightarrow sto'\end{array}}{env_V, env_P \vdash \langle R_1; R_2, sto\rangle \rightarrow sto'}$$

Table 1.1: Root statements

Transitions are on the form: $env_V, env_P \vdash \langle S, sto\rangle \rightarrow sto'$

[VAR-ASS] $env_V, env_P \vdash \langle x <-- a, sto\rangle \rightarrow sto[l \mapsto v]$

where $env_V, sto \vdash a \rightarrow_a v$
and $env_V\ x = l$

[ARR-ASS] $env_V, env_P \vdash \langle r[a_1] <-- a_2, sto\rangle \rightarrow sto[l_2 \mapsto v_2]$

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where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V \; r = l_1$
and $l_2 = l_1 + v_1 + 1$
and $v_3 = sto \; l_1$
and $0 \leq v_1 \leq v_3$

[COMP]
$$\frac{env_V, env_P \vdash \langle S_1, sto\rangle \rightarrow sto'' \quad env_V, env_P \vdash \langle S_2, sto''\rangle \rightarrow sto'}{env_V, env_P \vdash \langle S_1; S_2, sto\rangle \rightarrow sto'}$$

[IF-TRUE]
$$\frac{env_V, env_P \vdash \langle S, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto\rangle \rightarrow sto'}$$

if $env_V, sto \vdash b \rightarrow_b \text{true}$

[IF-FALSE]
$$env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto\rangle \rightarrow sto$$

if $env_V, sto \vdash b \rightarrow_b \text{false}$

[IF-ELSE-TRUE]
$$\frac{env_V, env_P \vdash \langle S_1, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto\rangle \rightarrow sto'}$$

if $env_V, sto \vdash b \rightarrow_b \text{true}$

[IF-ELSE-FALSE]
$$\frac{env_V, env_P \vdash \langle S_2, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto\rangle \rightarrow sto'}$$

if $env_V, sto \vdash b \rightarrow_b \text{false}$

[WHILE-TRUE]
$$\frac{env_V, env_P \vdash \langle S, sto\rangle \rightarrow sto'' \quad env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto''\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto\rangle \rightarrow sto'}$$

if $env_V, sto \vdash b \rightarrow_b \text{true}$

[WHILE-FALSE]
$$env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto\rangle \rightarrow sto$$

if $env_V, sto \vdash b \rightarrow_b \text{false}$

[FROM-TRUE]
$$\frac{env_V, env_P \vdash \langle S, sto[l \mapsto v_1]\rangle \rightarrow sto'' \quad \langle \text{from } x <-- a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto'' \rightarrow sto'}{env_V, env_P \vdash \langle \text{from } x <-- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto\rangle \rightarrow sto'}$$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V, sto \vdash a_3 \rightarrow_a v_3$
and $v_1 \leq v_2$

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and $l = env_V x$

[FROM-FALSE] $env_V, env_P \vdash \langle \text{from } x <-- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto\rangle \rightarrow sto$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V, sto \vdash a_3 \rightarrow_a v_3$
and $v_1 > v_2$

[CALL] $$\frac{env'_V[\vec{z} \mapsto \vec{l}], env'_P \vdash \langle S, sto[\vec{l} \mapsto \vec{v}]\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto\rangle \rightarrow sto'}$$

where $env_P\ p = (S, \vec{z}, env'_V, env'_P)$
and $|\vec{a}| = |\vec{z}|$
and $env_V, sto \vdash a_i \rightarrow v_i$ for each $1 \leq i \leq |\vec{a}|$
and $l_1 = env_V$ new
and $l_{i+1} =$ new l_i for each $1 \leq i < |\vec{a}|$

Table 1.2: Statements

[SWITCH-1]
$$\frac{env_V, env_P \vdash \langle S, sto\rangle \rightarrow (sto')}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : \ S_1 \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}$$

Where $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

[SWITCH-2]
$$\frac{env_V, env_P \vdash \langle S_1, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : \ S_1 \text{ break; } \ldots \text{ case } a_k : \ S_k \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}$$

Where $k > 0$
and $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v = v_1$

[SWITCH-3]
$$\frac{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 : \ S_2 \text{ break; } \ldots \text{ case } a_k : \ S_k \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : \ S_1 \text{ break; } \ldots \text{ case } a_k : \ S_k \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}$$

Where $k > 1$
and $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

Table 1.3: Statements

Transitions are on the form: $env_V, sto \vdash a \rightarrow_a v$

[NUM] $env_V, sto \vdash n \rightarrow_a v$

if $\mathcal{N}[[n]] = v$

[VAR] $env_V, sto \vdash x \rightarrow_a v$

if $env_V\ x = l$
and $sto\ l = v$

[ADD] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$$

where $v = v_1 + v_2$

[SUB] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$$

where $v = v_1 - v_2$

[MULT] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$$

where $v = v_1 \cdot v_2$

[DIV] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 / a_2 \rightarrow_a v}$$

where $v = v_1 / v_2$

[PAR] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1}{env_V, sto \vdash (a_1) \rightarrow_a v_1}$$

[ARR] $env_V, sto \vdash r[a_1] \rightarrow_a a_2$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V\ r = l$
and $sto\ l = v_3$
and $0 < v_1 \leq v_3$
and $sto(l + v_1) = v_2$

Table 1.4: Arithmetic expressions

Transitions are on the form: $env_V, sto \vdash b \rightarrow_b$ bool

[EQUAL-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{true}}$$

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if $v_1 = v_2$

[EQUAL-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{false}}$$

if $v_1 \neq v_2$

[GRT-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{true}}$$

if $v_1 > v_2$

[GRT-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{false}}$$

if $v_1 \not> v_2$

[LESS-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$$

if $v_1 < v_2$

[LESS-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{false}}$$

if $v_1 \not< v_2$

[NOT-1] $$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$$

[NOT-2] $$\frac{env_V, sto \vdash b \rightarrow_b \text{false}}{env_V, sto \vdash !b \rightarrow_b \text{true}}$$

[AND-TRUE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{true}}$$

[AND-FALSE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{false}}$$

where $i \in 1, 2$

[OR-TRUE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{true}}$$

where $i \in 1, 2$

[OR-FALSE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{false}}$$

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[PAR-BOOL]	$\dfrac{env_V, sto \vdash b \rightarrow_b v}{env_V, sto \vdash (b) \rightarrow_b v}$

Table 1.5: Boolean expressions

	Transitions are on the form: $\langle D_V, env_V, sto\rangle \rightarrow_{DV} (env'_V, sto')$
[VAR-DEC]	$\dfrac{\langle D_V, env''_V, sto[l \mapsto v]\rangle \rightarrow_{DV} (env'_V, sto')}{\text{var } x <--a, env_V, sto\rangle \rightarrow_{DV} (env'_V, sto')}$
	where $env_V, sto \vdash a \rightarrow_a v$ and $l = env_V$ next and $env''_V = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$
[EMPTY-VAR]	$\langle \varepsilon, env_V, sto\rangle \rightarrow_{DV} (env_V, sto)$
	Transitions are on the form: $env_V \vdash \langle D_P, env_P\rangle \rightarrow_{DP} env'_P$
[PROC-PARA-DEC]	$\dfrac{env_V \vdash \langle D_P, env_P[p \mapsto (S, \vec{x}, env_V, env_P)]\rangle \rightarrow_{DP} env'_P}{env_V \vdash \langle \text{function } p \text{ using(var } \vec{x}) \text{ begin } S \text{ end, } env_P\rangle \rightarrow_{DP} env'_P}$
[EMPTY-PROC]	$env_V \vdash \langle \varepsilon, env_P\rangle \rightarrow_{DP} env'_P$
	Transitions are on the form: $\langle D_A, env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')$
[ARRAY-DEC]	$\dfrac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto v]\rangle \rightarrow_{DA} (env'_V, sto')}{\langle \text{array } r[a_1], env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')}$
	where $env_V, sto \vdash a_1 \rightarrow_a v$ and $l = env_V \text{next}$ and $v > 0$
[EMPTY-ARRAY]	$\langle \varepsilon, env_V, sto\rangle \rightarrow_{DA} (env_V, sto)$

Table 1.6: Declarations