P4

Transition Rules

1.1 Abstract Syntax

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S ::= x := a \mid r[a_1] := a_2 \mid S_1; \ S_2 \mid \text{if } b \text{ begin } S \text{ end } | \text{ if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end} while b \text{ begin } S \text{ end } | \text{ from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end } | \text{ call } p(\vec{x}) \mid D_V \mid D_A \mid D_P | \text{ switch}(a) \text{ begin case } a_1 : \ S_1 \text{ break}; \ \dots \text{ case} a_k : \ S_k \text{ break}; \text{ default } : \ S \text{ break end} a ::= n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1/a_2 \mid (a) \mid r[a_i] b ::= a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1 \land b_2 \mid b_1 \lor b_2 \mid (b) D_V ::= \text{var } x := a D_P ::= \text{func } p \text{ is begin } S \text{ end } \mid \text{func } p(\vec{x}) \text{ is begin } S \text{ end} D_A ::= \operatorname{array} r[a_1]
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Transitioner er på formen: $env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'$ [VAR-ASS] $env_V, env_P \vdash \langle x < --a, sto \rangle \rightarrow sto[l \mapsto v]$ where env_V , $sto \vdash a \rightarrow_a v$ and $env_V x = l$ $env_V, env_P \vdash \langle r[a_1] < --a_2, sto \rangle \rightarrow sto[l_2 \mapsto v_2]$ [ARR-ASS] where env_V , $sto \vdash a_1 \rightarrow_a v_1$ and env_V , $sto \vdash a_2 \rightarrow_a v_2$ and $env_V r = l_1$ and $l_2 = l_1 + v_1$ and $v_3 = sto l_1$ and $1 \le v_1 \le v_3$ $env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto''$ $env_V, env_P \vdash \langle S_2, sto'' \rangle \rightarrow sto'$ [COMP] $\overline{env_V, env_P \vdash \langle S_1; S_2, sto \rangle \rightarrow sto'}$ $env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'$ [IF-TRUE] $env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'$ Continued on the next page

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if env_V, sto \vdash b \rightarrow_b true
[IF-FALSE]
                                         env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                       if env_V, sto \vdash b \rightarrow_b false
                                         \frac{env_V, env_P \vdash \langle S_1, sto \rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \to sto'}
[IF-ELSE-TRUE]
                                                       if env_V, sto \vdash b \rightarrow_b true
                                         \frac{env_V, env_P \vdash \langle S_2, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'}
[IF-ELSE-FALSE]
                                                       if env_V, sto \vdash b \rightarrow_b false
                                                            env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto''
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto'' \rangle \rightarrow sto'
[WHILE-TRUE]
                                          env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'
                                                       if env_V, sto \vdash b \rightarrow_b true
[WHILE-FALSE]
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                       if env_V, sto \vdash b \rightarrow_b false
                                                                         env_V, env_P \vdash \langle S, sto[l \mapsto v_1] \rangle \rightarrow sto''
                                                   \langle \text{from } x < --a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto'' \to sto'
[FROM-TRUE]
                                         env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'
                                                       where env_V, sto \vdash a_1 \rightarrow_a v_1
                                                       and env_V, sto \vdash a_2 \rightarrow_a v_2
                                                       and env_V, sto \vdash a_3 \rightarrow_a v_3
                                                       and v_1 \leq v_2
                                                       and l = env_V x
[FROM-FALSE]
                                         env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                       where env_V, sto \vdash a_1 \rightarrow_a v_1
                                                       and env_V, sto \vdash a_2 \rightarrow_a v_2
                                                       and env_V, sto \vdash a_3 \rightarrow_a v_3
                                                       and v_1 > v_2
                                         \frac{env'_V[\vec{z} \mapsto \vec{l}], env'_P \vdash \langle S, sto[\vec{l} \mapsto \vec{v}] \rangle \to sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto \rangle \to sto'}
[CALL]
                                                       where env_P p = (S, \vec{z}, env'_V, env'_P)
                                                       and |\vec{a}| = |\vec{z}|
                                                              Continued on the next page
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and env_V , $sto \vdash a_i \rightarrow v_i$ for each $1 \leq i \leq |\vec{a}|$ and $l_1 = env_V$ new and $l_{i+1} = \text{new } l_i$ for each $1 \leq i < |\vec{a}|$

Table 1.1: Statements

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env_V, env_P \vdash (\text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default } : S \text{ break}; \text{ end, } sto \rightarrow sto'
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default } : S \text{ break}; \text{ end, } sto \rangle \to sto'
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 : S_2 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default } : S \text{ break}; \text{ end, } sto \rangle \to sto'
                                                                                  \rightarrow sto'
                                                                       env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; default} : S \text{ break; end, } sto \rangle
                                                                                                                                                                                                                                                                                                                                                                                                                                                               env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'
env_V, env_P \vdash \langle S, sto \rangle \rightarrow (sto')
                                                                                                                                                                                                 Where env_V, sto \vdash a \rightarrow_a v
                                                                                                                                                                                                                                                                         and env_V, sto \vdash a_1 \rightarrow_a v_1
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         and env_V, sto \vdash a \rightarrow_a v
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     and env_V, sto \vdash a \rightarrow_a v
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 Where k > 0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Where k > 1
                                                                                                                                                                                                                                                                                                                                      and v \neq v_1
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       and v \neq v_1
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                [SWITCH-2]
                                [SWITCH-1]
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Table 1.2: Statements

Transitioner er på formen: env_V , $sto \vdash a \rightarrow_a v$

$$[NUM] \quad env_V, sto \vdash n \to_a v$$

if
$$\mathcal{N}[[n]] = v$$

[VAR]
$$env_V, sto \vdash x \rightarrow_a v$$

if
$$env_V x = l$$

and $sto l = v$

[ADD]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$$

where
$$v = v_1 + v_2$$

[SUB]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$$

where
$$v = v_1 - v_2$$

[MULT]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$$

where
$$v = v_1 \cdot v_2$$

[DIV]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1/a_2 \rightarrow_a v}$$

where
$$v = v_1/v_2$$

[PAR]
$$\frac{env_V, sto \vdash a_1 \to_a v_1}{env_V, sto \vdash (a_1) \to_a v_1}$$

[ARR]
$$env_V, sto \vdash r[a_1] \rightarrow_a a_2$$

where
$$env_V, sto \vdash a_1 \rightarrow_a v_1$$

and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V r = l$
and $sto l = v_3$
and $0 < v_1 \le v_3$
and $sto(l + v_1) = v_2$

Table 1.3: Arithmetic expressions

Transitioner på formen: $env_V, sto \vdash b \rightarrow_b t$

$$[\text{EQUAL-TRUE}] \quad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{ true}} \\ Continued \ on \ the \ next \ page$$

if
$$v_1 = v_2$$

[EQUAL-FALSE]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{ false}}$$

if
$$v_1 \neq v_2$$

[GRT-TRUE]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{ true}}$$

if
$$v_1 > v_2$$

$$[\text{GRT-FALSE}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{ false}}$$

if
$$v_1 \not> v_2$$

[LESS-TRUE]
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$$

if
$$v_1 < v_2$$

$$[\text{LESS-FALSE}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{ false}}$$

if
$$v_1 \not< v_2$$

[NOT-1]
$$\frac{env_V, sto \vdash b \to_b \text{ true}}{env_V, sto \vdash !b \to_b \text{ false}}$$

[NOT-2]
$$\frac{env_V, sto \vdash b \to_b \text{ true}}{env_V, sto \vdash !b \to_b \text{ false}}$$

$$[\text{AND-TRUE}] \qquad \frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \land b_2 \rightarrow_b \text{true}}$$

[AND-FALSE]
$$\frac{env_V, sto \vdash b_i \to_b \text{ false}}{env_V, sto \vdash b_1 \land b_2 \to_b \text{ false}}$$

where
$$i \in 1, 2$$

[OR-TRUE]
$$\frac{env_V, sto \vdash b_i \to_b \text{true}}{env_V, sto \vdash b_1 \lor b_2 \to_b \text{true}}$$

where
$$i \in 1, 2$$

[OR-FALSE]
$$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{ false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{ false}}{env_V, sto \vdash b_1 \lor b_2 \rightarrow_b \text{ false}}$$

Continued on the next page

[PAR-BOOL]
$$\frac{env_V, sto \vdash b \to_b v}{env_V, sto \vdash (b) \to_b v}$$

Table 1.4: Boolean expressions

Transitioner på formen: $\langle D_V, env_V, sto \rangle \rightarrow_{DV} (env_V', sto')$

[VAR-DEC]
$$\frac{\langle D_V, env_V'', sto[l \mapsto v] \rangle \to_{DV} (env_V', sto')}{\operatorname{var} x < --a, env_V, sto \rangle \to_{DV} (env_V', sto')}$$

where env_V , $sto \vdash a \rightarrow_a v$ and $l = env_V$ next and $env_V'' = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$

Transitioner på formen: $env_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env_P'$

[FUNC-DEC]
$$\frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, env_V, env_P)] \rangle \rightarrow_{DP} env_P'}{env_V \vdash \langle \operatorname{proc} p \text{ is begin } S \text{ end}, env_P \rangle \rightarrow_{DP} env_P'}$$

$$[\text{FUNC-PARA-DEC}] \quad \frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, \vec{x}, env_V, env_P)] \rangle \rightarrow_{DP} env_P'}{env_V \vdash \langle \text{func } p(\text{var } \vec{x}) \text{ is begin } S \text{ end, } env_P \rangle \rightarrow_{DP} env_P'}$$

Transitioner på formen: $\langle D_A, env_V, sto \rangle \rightarrow_{DA} (env_V', sto')$

[ARRAY-DEC]
$$\frac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto v] \rangle \rightarrow_{DA} (env_V', sto')}{\langle \text{array } r[a_1], env_V, sto \rangle \rightarrow_{DA} (env_V', sto')}$$

where env_V , $sto \vdash a_1 \rightarrow_a v$ and $l = env_V$ next and v > 0

Table 1.5: Declarations