1 Transition Rules

1.1 Abstract Syntax

$$
\begin{aligned}
S ::= {} & x := a \mid r[a_1] := a_2 \mid S_1;\ S_2 \mid \text{if } b \text{ do } S \mid \text{if } b \text{ do } S_1 \text{ else do } S_2 \mid \text{while } b \text{ do } S \\
& \mid \text{from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ do } S \\
& \mid \text{call } p(\vec{x}) \mid \text{begin } Q \text{ end} \\
& \mid \text{switch}(a) \text{ case } a_1 :\ S_1 \text{ break; } \ldots \text{ case} a_k :\ S_k \text{ break; default} :\ S \text{ break} \\
Q ::= {} & D_p\ Q | D_v\ Q | S\ Q | \varepsilon \\
a ::= {} & n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1 / a_2 \mid (a) \mid r[a_i] \\
b ::= {} & a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1\ \wedge\ b_2 \mid b_1\ \vee\ b_2 \mid (b) \\
D_V ::= {} & \text{var } x := a;\ D_V \mid \varepsilon \\
D_P ::= {} & \text{func } p \text{ is } S;\ D_P \mid \varepsilon \\
D_A ::= {} & \text{array } r[a_1];\ D_A \mid \varepsilon
\end{aligned}
$$

Transitioner er på formen: $env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'$

[VAR-ASS] $env_V, env_P \vdash \langle x <-- a, sto \rangle \rightarrow sto[l \mapsto v]$

where $env_V, sto \vdash a \rightarrow_a v$
and $env_V\ x = l$

[ARR-ASS] $env_V, env_P \vdash \langle r[a_1] <-- a_2, sto \rangle \rightarrow sto[l_2 \mapsto v_2]$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V\ r = l_1$
and $l_2 = l_1 + v_1$
and $v_3 = sto\ l_1$
and $1 \leq v_1 \leq v_3$

[COMP]
$$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'' \quad env_V, env_P \vdash \langle S_2, sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle S_1; S_2, sto \rangle \rightarrow sto'}$$

Continued on the next page

[IF-TRUE]
$$\frac{env_V, env_P \vdash \langle S, sto\rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto\rangle \to sto'}$$
if $env_V, sto \vdash b \to_b \text{true}$

[IF-FALSE]
$$env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto\rangle \to sto$$
if $env_V, sto \vdash b \to_b \text{false}$

[IF-ELSE-TRUE]
$$\frac{env_V, env_P \vdash \langle S_1, sto\rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto\rangle \to sto'}$$
if $env_V, sto \vdash b \to_b \text{true}$

[IF-ELSE-FALSE]
$$\frac{env_V, env_P \vdash \langle S_2, sto\rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto\rangle \to sto'}$$
if $env_V, sto \vdash b \to_b \text{false}$

[WHILE-TRUE]
$$\frac{\begin{array}{c} env_V, env_P \vdash \langle S, sto\rangle \to sto'' \\ env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto''\rangle \to sto' \end{array}}{env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto\rangle \to sto'}$$
if $env_V, sto \vdash b \to_b \text{true}$

[WHILE-FALSE]
$$env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto\rangle \to sto$$
if $env_V, sto \vdash b \to_b \text{false}$

[FROM-TRUE]
$$\frac{\begin{array}{c} env_V, env_P \vdash \langle S, sto[l \mapsto v_1]\rangle \to sto'' \\ \langle \text{from } x < -- a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto''\rangle \to sto' \end{array}}{env_V, env_P \vdash \langle \text{from } x < -- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto\rangle \to sto'}$$
where $env_V, sto \vdash a_1 \to_a v_1$
and $env_V, sto \vdash a_2 \to_a v_2$
and $env_V, sto \vdash a_3 \to_a v_3$
and $v_1 \leq v_2$
and $l = env_V x$

[FROM-FALSE]
$$env_V, env_P \vdash \langle \text{from } x < -- a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto\rangle \to sto$$
where $env_V, sto \vdash a_1 \to_a v_1$
and $env_V, sto \vdash a_2 \to_a v_2$
and $env_V, sto \vdash a_3 \to_a v_3$
and $v_1 > v_2$

[CALL]
$$\frac{env'_V[\vec{z} \mapsto \vec{l}], env'_P \vdash \langle S, sto[\vec{l} \mapsto \vec{v}]\rangle \to sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto\rangle \to sto'}$$

Continued on the next page

$$\begin{array}{l}\text{where } env_P\ p = (S, \vec{z}, env'_V, env'_P)\\ \text{and } |\vec{a}| = |\vec{z}|\\ \text{and } env_V, sto \vdash a_i \to v_i \text{ for each } 1 \le i \le |\vec{a}|\\ \text{and } l_1 = env_V \text{ new}\\ \text{and } l_{i+1} = \text{new } l_i \text{ for each } 1 \le i < |\vec{a}|\end{array}$$

[BLOCK]
$$\frac{\begin{array}{c}\langle D_V, env_V, sto\rangle \to_{DV} (env'_V, sto'')\\ env'_V \vdash \langle D_P, env_P\rangle \to_{DP} env'_P\\ env'_V, env'_P \vdash \langle S, sto''\rangle \to sto'\end{array}}{env_V, env_P \vdash \langle \text{begin } D_V\ D_P\ S \text{ end}, sto\rangle \to sto'}$$

[BLOCK]
$$\frac{\langle Q, env_P, env_V, sto\rangle \to_Q (env'_V, env'_P, sto'')\rangle \to sto'}{env_V, env_P \vdash \langle \text{begin } Q \text{ end}, sto\rangle \to sto'}$$

Table 1.1: Statements

FiXme Fatal:
OPDATER BLO
skriv om Q

[SWITCH-1] $$\frac{env_V, env_P \vdash \langle S, sto\rangle \rightarrow (sto')}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : \ S_1 \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}$$

Where $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

[SWITCH-2] $$\frac{env_V, env_P \vdash \langle S_1, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : \ S_1 \text{ break; } \ldots \text{ case } a_k : \ S_k \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}$$

Where $k > 0$
and $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v = v_1$

[SWITCH-3] $$\frac{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 : \ S_2 \text{ break; } \ldots \text{ case } a_k : \ S_k \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : \ S_1 \text{ break; } \ldots \text{ case } a_k : \ S_k \text{ break; default : } S \text{ break; end}, sto\rangle \rightarrow sto'}$$

Where $k > 1$
and $env_V, sto \vdash a \rightarrow_a v$
and $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $v \neq v_1$

Table 1.2: Statements

Transitioner er på formen: $env_V, sto \vdash a \rightarrow_a v$

[NUM] $env_V, sto \vdash n \rightarrow_a v$

if $\mathcal{N}[[n]] = v$

[VAR] $env_V, sto \vdash x \rightarrow_a v$

if $env_V\ x = l$
and $sto\ l = v$

[ADD] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$$

where $v = v_1 + v_2$

[SUB] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$$

where $v = v_1 - v_2$

[MULT] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$$

where $v = v_1 \cdot v_2$

[DIV] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 / a_2 \rightarrow_a v}$$

where $v = v_1 / v_2$

[PAR] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1}{env_V, sto \vdash (a_1) \rightarrow_a v_1}$$

[ARR] $env_V, sto \vdash r[a_1] \rightarrow_a a_2$

where $env_V, sto \vdash a_1 \rightarrow_a v_1$
and $env_V, sto \vdash a_2 \rightarrow_a v_2$
and $env_V\ r = l$
and $sto\ l = v_3$
and $0 < v_1 \leq v_3$
and $sto(l + v_1) = v_2$

Table 1.3: Arithmetic expressions

Transitioner på formen: $env_V, sto \vdash b \rightarrow_b t$

[EQUAL-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{true}}$$

Continued on the next page

if $v_1 = v_2$

[EQUAL-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{false}}$$

if $v_1 \neq v_2$

[GRT-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{true}}$$

if $v_1 > v_2$

[GRT-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{false}}$$

if $v_1 \ngtr v_2$

[LESS-TRUE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$$

if $v_1 < v_2$

[LESS-FALSE] $$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{false}}$$

if $v_1 \nless v_2$

[NOT-1] $$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$$

[NOT-2] $$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$$

[AND-TRUE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{true}}$$

[AND-FALSE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{false}}$$

where $i \in 1, 2$

[OR-TRUE] $$\frac{env_V, sto \vdash b_i \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{true}}$$

where $i \in 1, 2$

[OR-FALSE] $$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{false}}$$

Continued on the next page

[PAR-BOOL] $$\frac{env_V, sto \vdash b \rightarrow_b v}{env_V, sto \vdash (b) \rightarrow_b v}$$

Table 1.4: Boolean expressions

Transitioner på formen: $\langle D_V, env_V, sto\rangle \rightarrow_{DV} (env'_V, sto')$

[VAR-DEC] $$\frac{\langle D_V, env''_V, sto[l \mapsto v]\rangle \rightarrow_{DV} (env'_V, sto')}{\text{var } x <\!- - a; D_V, env_V, sto\rangle \rightarrow_{DV} (env'_V, sto')}$$

where $env_V, sto \vdash a \rightarrow_a v$
and $l = env_V$ next
and $env''_V = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$

[EMPTY-VAR] $\langle \varepsilon, env_V, sto\rangle \rightarrow_{DV} (env_V, sto)$

Transitioner på formen: $env_V \vdash \langle D_P, env_P\rangle \rightarrow_{DP} env'_P$

[FUNC-DEC] $$\frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, env_V, env_P)]\rangle \rightarrow_{DP} env'_P}{env_V \vdash \langle \text{proc } p \text{ is } S; D_P, env_P\rangle \rightarrow_{DP} env'_P}$$

[EMPTY-FUNC] $env_V \vdash \langle \varepsilon, env_P\rangle \rightarrow_{DP} env'_P$

Transitioner på formen: $\langle D_A, env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')$

[ARRAY-DEC] $$\frac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto v]\rangle \rightarrow_{DA} (env'_V, sto')}{\langle \text{array } r[a_1], D_A, env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')}$$

where $env_V, sto \vdash a_1 \rightarrow_a v$
and $l = env_V$next
and $v > 0$

[EMPTY-ARRAY] $\langle \varepsilon, env_V, sto\rangle \rightarrow_{DA} (env_V, sto)$

Table 1.5: Declarations