

Transition Rules 1

1.1 Abstract Syntax

$$\begin{aligned}
S ::= & x := a \mid r[a_1] := a_2 \mid S_1; S_2 \mid \text{if } b \text{ do } S \mid \text{if } b \text{ do } S_1 \text{ else do } S_2 \mid \text{while } b \text{ do } S \\
& \mid \text{from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ do } S \\
& \mid \text{call } p(\vec{x}) \mid \text{begin } Q \text{ end} \\
& \mid \text{switch}(a) \text{ case } a_1 : S_1 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break} \\
Q ::= & D_p Q \mid D_v Q \mid S Q \mid \varepsilon \\
a ::= & n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1 / a_2 \mid (a) \mid r[a_i] \\
b ::= & a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \mid (b) \\
D_V ::= & \text{var } x := a; D_V \mid \varepsilon \\
D_P ::= & \text{func } p \text{ is } S; D_P \mid \varepsilon \\
D_A ::= & \text{array } r[a_1]; D_A \mid \varepsilon
\end{aligned}$$

Transitioner er på formen: $\text{env}_V, \text{env}_P \vdash \langle S, \text{sto} \rangle \rightarrow \text{sto}'$

[VAR-ASS] $\text{env}_V, \text{env}_P \vdash \langle x \leftarrow a, \text{sto} \rangle \rightarrow \text{sto}[l \mapsto v]$

where $\text{env}_V, \text{sto} \vdash a \rightarrow_a v$
and $\text{env}_V x = l$

[ARR-ASS] $\text{env}_V, \text{env}_P \vdash \langle r[a_1] \leftarrow a_2, \text{sto} \rangle \rightarrow \text{sto}[l_2 \mapsto v_2]$

where $\text{env}_V, \text{sto} \vdash a_1 \rightarrow_a v_1$
and $\text{env}_V, \text{sto} \vdash a_2 \rightarrow_a v_2$
and $\text{env}_V r = l_1$
and $l_2 = l_1 + v_1$
and $v_3 = \text{sto } l_1$
and $1 \leq v_1 \leq v_3$

[COMP]
$$\frac{\begin{array}{l} \text{env}_V, \text{env}_P \vdash \langle S_1, \text{sto} \rangle \rightarrow \text{sto}'' \\ \text{env}_V, \text{env}_P \vdash \langle S_2, \text{sto}'' \rangle \rightarrow \text{sto}' \end{array}}{\text{env}_V, \text{env}_P \vdash \langle S_1; S_2, \text{sto} \rangle \rightarrow \text{sto}'}$$

Continued on the next page

[IF-TRUE]	$\frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto'}$ <p style="text-align: center;">if $env_V, sto \vdash b \rightarrow_b \text{ true}$</p>
[IF-FALSE]	$env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto$ <p style="text-align: center;">if $env_V, sto \vdash b \rightarrow_b \text{ false}$</p>
[IF-ELSE-TRUE]	$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end, } sto \rangle \rightarrow sto'}$ <p style="text-align: center;">if $env_V, sto \vdash b \rightarrow_b \text{ true}$</p>
[IF-ELSE-FALSE]	$\frac{env_V, env_P \vdash \langle S_2, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end, } sto \rangle \rightarrow sto'}$ <p style="text-align: center;">if $env_V, sto \vdash b \rightarrow_b \text{ false}$</p>
[WHILE-TRUE]	$\frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'' \quad env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end, } sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto'}$ <p style="text-align: center;">if $env_V, sto \vdash b \rightarrow_b \text{ true}$</p>
[WHILE-FALSE]	$env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end, } sto \rangle \rightarrow sto$ <p style="text-align: center;">if $env_V, sto \vdash b \rightarrow_b \text{ false}$</p>
[FROM-TRUE]	$\frac{env_V, env_P \vdash \langle S, sto[l \mapsto v_1] \rangle \rightarrow sto'' \quad \langle \text{from } x < - - a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end, } sto'' \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{from } x < - - a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end, } sto \rangle \rightarrow sto'}$ <p style="text-align: center;"> where $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V, sto \vdash a_3 \rightarrow_a v_3$ and $v_1 \leq v_2$ and $l = env_V x$ </p>
[FROM-FALSE]	$env_V, env_P \vdash \langle \text{from } x < - - a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end, } sto \rangle \rightarrow sto$ <p style="text-align: center;"> where $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V, sto \vdash a_3 \rightarrow_a v_3$ and $v_1 > v_2$ </p>
[CALL]	$\frac{env'_V[\vec{z} \mapsto \vec{l}], env'_P \vdash \langle S, sto[\vec{l} \mapsto \vec{v}] \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto \rangle \rightarrow sto'}$

Continued on the next page

	where $env_P p = (S, \vec{z}, env'_V, env'_P)$ and $ \vec{a} = \vec{z} $ and $env_V, sto \vdash a_i \rightarrow v_i$ for each $1 \leq i \leq \vec{a} $ and $l_1 = env_V \text{ new}$ and $l_{i+1} = \text{new } l_i$ for each $1 \leq i < \vec{a} $
[BLOCK]	$\frac{\begin{array}{l} \langle D_V, env_V, sto \rangle \rightarrow_{DV} (env'_V, sto'') \\ env'_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env'_P \\ env'_V, env'_P \vdash \langle S, sto'' \rangle \rightarrow sto' \end{array}}{env_V, env_P \vdash \langle \text{begin } D_V \ D_P \ S \text{ end}, sto \rangle \rightarrow sto'}$

Table 1.1: Statements

[SWITCH-1]	$\frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow (sto')}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}$ <p>Where $env_V, sto \vdash a \rightarrow_a v$ and $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $v \neq v_1$</p>
[SWITCH-2]	$\frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}$ <p>Where $k > 0$ and $env_V, sto \vdash a \rightarrow_a v$ and $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $v = v_1$</p>
[SWITCH-3]	$\frac{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 : S_2 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; } \dots \text{ case } a_k : S_k \text{ break; default : } S \text{ break; end, } sto \rangle \rightarrow sto'}$ <p>Where $k > 1$ and $env_V, sto \vdash a \rightarrow_a v$ and $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $v \neq v_1$</p>

Table 1.2: Statements

Transitioner er på formen: $env_V, sto \vdash a \rightarrow_a v$

[NUM]	$env_V, sto \vdash n \rightarrow_a v$ if $\mathcal{N}[[n]] = v$
[VAR]	$env_V, sto \vdash x \rightarrow_a v$ if $env_V x = l$ and $sto l = v$
[ADD]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$ where $v = v_1 + v_2$
[SUB]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$ where $v = v_1 - v_2$
[MULT]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$ where $v = v_1 \cdot v_2$
[DIV]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 / a_2 \rightarrow_a v}$ where $v = v_1 / v_2$
[PAR]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1}{env_V, sto \vdash (a_1) \rightarrow_a v_1}$
[ARR]	$env_V, sto \vdash r[a_1] \rightarrow_a a_2$ where $env_V, sto \vdash a_1 \rightarrow_a v_1$ and $env_V, sto \vdash a_2 \rightarrow_a v_2$ and $env_V r = l$ and $sto l = v_3$ and $0 < v_1 \leq v_3$ and $sto(l + v_1) = v_2$

Table 1.3: Arithmetic expressions

Transitioner på formen: $env_V, sto \vdash b \rightarrow_b t$

[EQUAL-TRUE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{true}}$
--------------	---

Continued on the next page

	if $v_1 = v_2$
[EQUAL-FALSE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{false}}$
	if $v_1 \neq v_2$
[GRT-TRUE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{true}}$
	if $v_1 > v_2$
[GRT-FALSE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{false}}$
	if $v_1 \not> v_2$
[LESS-TRUE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$
	if $v_1 < v_2$
[LESS-FALSE]	$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{false}}$
	if $v_1 \not< v_2$
[NOT-1]	$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$
[NOT-2]	$\frac{env_V, sto \vdash b \rightarrow_b \text{true}}{env_V, sto \vdash !b \rightarrow_b \text{false}}$
[AND-TRUE]	$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{true}}$
[AND-FALSE]	$\frac{env_V, sto \vdash b_i \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \wedge b_2 \rightarrow_b \text{false}}$
	where $i \in 1, 2$
[OR-TRUE]	$\frac{env_V, sto \vdash b_i \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{true}}$
	where $i \in 1, 2$
[OR-FALSE]	$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{false}}{env_V, sto \vdash b_1 \vee b_2 \rightarrow_b \text{false}}$

Continued on the next page

$$[\text{PAR-BOOL}] \quad \frac{env_V, sto \vdash b \rightarrow_b v}{env_V, sto \vdash (b) \rightarrow_b v}$$

Table 1.4: Boolean expressions

Transitioner på formen: $\langle D_V, env_V, sto \rangle \rightarrow_{DV} (env'_V, sto')$

$$[\text{VAR-DEC}] \quad \frac{\langle D_V, env''_V, sto[l \mapsto v] \rangle \rightarrow_{DV} (env'_V, sto')}{\text{var } x < - - a; D_V, env_V, sto \rangle \rightarrow_{DV} (env'_V, sto')}$$

where $env_V, sto \vdash a \rightarrow_a v$
and $l = env_V \text{ next}$
and $env''_V = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$

$$[\text{EMPTY-VAR}] \quad \langle \varepsilon, env_V, sto \rangle \rightarrow_{DV} (env_V, sto)$$

Transitioner på formen: $env_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env'_P$

$$[\text{FUNC-DEC}] \quad \frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, env_V, env_P)] \rangle \rightarrow_{DP} env'_P}{env_V \vdash \langle \text{proc } p \text{ is } S; D_P, env_P \rangle \rightarrow_{DP} env'_P}$$

$$[\text{EMPTY-FUNC}] \quad env_V \vdash \langle \varepsilon, env_P \rangle \rightarrow_{DP} env'_P$$

Transitioner på formen: $\langle D_A, env_V, sto \rangle \rightarrow_{DA} (env'_V, sto')$

$$[\text{ARRAY-DEC}] \quad \frac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l + v + 1], sto[l \mapsto v] \rangle \rightarrow_{DA} (env'_V, sto')}{\langle r[a_1], D_A, env_V, sto \rangle \rightarrow_{DA} (env'_V, sto')}$$

where $env_V, sto \vdash a_1 \rightarrow_a v$
and $l = env_V \text{ next}$
and $l > 0$

$$[\text{EMPTY-ARRAY}] \quad \langle \varepsilon, env_V, sto \rangle \rightarrow_{DA} (env_V, sto)$$

Table 1.5: Declarations