## **Transition Rules**

## 1.1 Abstract Syntax

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\begin{split} R &::= D_P \ D_A \ D_V \ | R_1 \ R_2 \\ S &::= x := a \ | \ r[a_1] := a_2 \ | \ S_1; \ S_2 \ | \ \text{if} \ b \ \text{begin} \ S \ \text{end} \ | \ \text{if} \ b \ \text{begin} \ S_1 \ \text{end} \ \text{else} \ \text{begin} \ S_2 \ \text{end} \\ & \text{while} \ b \ \text{begin} \ S \ \text{end} \ | \ \text{from} \ x := a_1 \ \text{to} \ a_2 \ \text{step} \ a_3 \ \text{begin} \ S \ \text{end} \ | \ \text{call} \ p(\vec{x}) \ | \ D_V \ | \ D_A \\ & | \ \text{switch}(a) \ \text{begin} \ \text{case} \ a_1 : \ S_1 \ \text{break}; \ \dots \ \text{case} \ a_k : \ S_k \ \text{break}; \ \text{default} : \ S \ \text{break} \ \text{end} \\ & a ::= n \ | \ x \ | \ a_1 + a_2 \ | \ a_1 - a_2 \ | \ a_1 * a_2 \ | \ a_1/a_2 \ | \ (a) \ | \ r[a_i] \\ & b ::= a_1 = a_2 \ | \ a_1 > a_2 \ | \ a_1 < a_2 \ | \ \neg b \ | \ b_1 \ \land \ b_2 \ | \ b_1 \ \lor \ b_2 \ | \ (b) \\ & D_V ::= \ \text{var} \ x := a \ | \ \varepsilon \\ & D_P ::= \ \text{func} \ p(\vec{x}) \ \text{is begin} \ S \ \text{end} \ | \ \varepsilon \\ & D_A ::= \ \text{array} \ r[a_1] \ | \ \varepsilon \end{split}
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$$[ROOT] \quad \frac{env_{V}, env_{P} \vdash \langle R_{1}, sto \rangle \rightarrow sto''}{env_{V}, env_{P} \vdash \langle R_{2}, sto'' \rangle \rightarrow sto'} \\ \frac{env_{V}, env_{P} \vdash \langle R_{2}, sto'' \rangle \rightarrow sto'}{env_{V}, env_{P} \vdash \langle R_{1}; R_{2}, sto \rangle \rightarrow sto'} \\ \langle D_{V}, env_{V}, sto \rangle \rightarrow_{DV} (env_{V}', sto'') \\ \langle D_{A}, env_{V}', sto'' \rangle \rightarrow_{DA} (env_{V}'', sto') \\ \frac{env_{V}'' \vdash \langle D_{P}, env_{P} \rangle \rightarrow_{DP} env_{P}'}{env_{V}, env_{P} \vdash \langle D_{V}, D_{A}, D_{P}, sto \rangle \rightarrow sto'}$$

Table 1.1: Root statements

[VAR-ASS] Transitioner er på formen:  $env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'$   $env_V, env_P \vdash \langle x < --a, sto \rangle \rightarrow sto[l \mapsto v]$   $\text{where } env_V, sto \vdash a \rightarrow_a v$   $\text{and } env_V \ x = l$ [ARR-ASS]  $env_V, env_P \vdash \langle r[a_1] < --a_2, sto \rangle \rightarrow sto[l_2 \mapsto v_2]$  Continued on the next page

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where env_V, sto \vdash a_1 \rightarrow_a v_1
                                                      and env_V, sto \vdash a_2 \rightarrow_a v_2
                                                      and env_V r = l_1
                                                      and l_2 = l_1 + v_1
                                                      and v_3 = sto l_1
                                                      and 1 \le v_1 \le v_3
                                           env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto''
                                           env_V, env_P \vdash \langle S_2, sto'' \rangle \rightarrow sto'
[COMP]
                                         env_V, env_P \vdash \langle S_1; S_2, sto \rangle \rightarrow sto'
                                         \frac{env_V, env_P \vdash \langle S, sto \rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \to sto'}
[IF-TRUE]
                                                      if env_V, sto \vdash b \rightarrow_b true
[IF-FALSE]
                                        env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                      if env_V, sto \vdash b \rightarrow_b false
                                        \frac{env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'}
[IF-ELSE-TRUE]
                                                      if env_V, sto \vdash b \rightarrow_b true
                                        \frac{env_V, env_P \vdash \langle S_2, sto \rangle \to sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \to sto'}
[IF-ELSE-FALSE]
                                                      if env_V, sto \vdash b \rightarrow_b false
                                                           env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto''
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto'' \rangle \rightarrow sto'
[WHILE-TRUE]
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'
                                                      if env_V, sto \vdash b \rightarrow_b true
[WHILE-FALSE]
                                        env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                                      if env_V, sto \vdash b \rightarrow_b false
                                                                        env_V, env_P \vdash \langle S, sto[l \mapsto v_1] \rangle \rightarrow sto''
                                                  \langle \text{from } x < --a_1 + a_3 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto'' \to sto'
[FROM-TRUE]
                                         env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'
                                                      where env_V, sto \vdash a_1 \rightarrow_a v_1
                                                      and env_V, sto \vdash a_2 \rightarrow_a v_2
                                                      and env_V, sto \vdash a_3 \rightarrow_a v_3
                                                      and v_1 \leq v_2
                                                             Continued on the next page
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$$[FROM\text{-FALSE}] \qquad env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end, } sto \rangle \rightarrow sto$$
 
$$\text{where } env_V, sto \vdash a_1 \rightarrow_a v_1$$
 
$$\text{and } env_V, sto \vdash a_2 \rightarrow_a v_2$$
 
$$\text{and } env_V, sto \vdash a_3 \rightarrow_a v_3$$
 
$$\text{and } v_1 > v_2$$
 
$$[CALL] \qquad \frac{env_V'[\vec{z} \mapsto \vec{l}], env_P' \vdash \langle S, sto[\vec{l} \mapsto \vec{v}] \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{call } p(\vec{a}), sto \rangle \rightarrow sto'}$$
 
$$\text{where } env_P \ p = (S, \vec{z}, env_V', env_P')$$
 
$$\text{and } |\vec{a}| = |\vec{z}|$$
 
$$\text{and } env_V, sto \vdash a_i \rightarrow v_i \text{ for each } 1 \leq i \leq |\vec{a}|$$
 
$$\text{and } l_1 = env_V \text{ new}$$
 
$$\text{and } l_{i+1} = \text{new } l_i \text{ for each } 1 \leq i < |\vec{a}|$$

Table 1.2: Statements

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env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default } : S \text{ break}; \text{ end, } sto \rangle \to sto'
                                                                                     \rightarrow sto'
                                                                          env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break; default} : S \text{ break; end, } sto \rangle
                                                                                                                                                                                                                                                                                                                                                                                                                                                                               env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'
env_V, env_P \vdash \langle S, sto \rangle \rightarrow (sto')
                                                                                                                                                                                                        Where env_V, sto \vdash a \rightarrow_a v
                                                                                                                                                                                                                                                                                  and env_V, sto \vdash a_1 \rightarrow_a v_1
                                                                                                                                                                                                                                                                                                                                                 and v \neq v_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 [SWITCH-2]
                                 [SWITCH-1]
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 $env_V, env_P \vdash (\text{switch}(a) \text{ begin case } a_1 : S_1 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default } : S \text{ break}; \text{ end, } sto \rightarrow sto'$  $env_V, env_P \vdash \langle \text{switch}(a) \text{ begin case } a_2 : S_2 \text{ break}; \dots \text{ case } a_k : S_k \text{ break}; \text{ default } : S \text{ break}; \text{ end, } sto \rangle \to sto'$ and  $env_V, sto \vdash a_1 \rightarrow_a v_1$  FiXme Fatal: kunne det ikke være bedre med "k" i stedet for 1? and  $v = v_1$ [SWITCH-3]

and  $env_V$ ,  $sto \vdash a \rightarrow_a v$ 

Where k > 0

Where k > 1and  $env_V$ ,  $sto \vdash a \rightarrow_a v$ and  $env_V$ ,  $sto \vdash a_1 \rightarrow_a v_1$ and  $v \neq v_1$ 

Table 1.3: Statements

Transitioner er på formen:  $env_V$ ,  $sto \vdash a \rightarrow_a v$ 

$$[NUM] \quad env_V, sto \vdash n \to_a v$$

if 
$$\mathcal{N}[[n]] = v$$

[VAR] 
$$env_V, sto \vdash x \rightarrow_a v$$

if 
$$env_V x = l$$
  
and  $sto l = v$ 

[ADD] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$$

where 
$$v = v_1 + v_2$$

[SUB] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$$

where 
$$v = v_1 - v_2$$

[MULT] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$$

where 
$$v = v_1 \cdot v_2$$

[DIV] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1/a_2 \rightarrow_a v}$$

where 
$$v = v_1/v_2$$

[PAR] 
$$\frac{env_V, sto \vdash a_1 \to_a v_1}{env_V, sto \vdash (a_1) \to_a v_1}$$

[ARR] 
$$env_V, sto \vdash r[a_1] \rightarrow_a a_2$$

where 
$$env_V$$
,  $sto \vdash a_1 \rightarrow_a v_1$   
and  $env_V$ ,  $sto \vdash a_2 \rightarrow_a v_2$   
and  $env_V r = l$   
and  $sto l = v_3$   
and  $0 < v_1 \le v_3$   
and  $sto(l + v_1) = v_2$ 

Table 1.4: Arithmetic expressions

Transitioner på formen:  $env_V$ ,  $sto \vdash b \rightarrow_b$  bool

$$[\text{EQUAL-TRUE}] \quad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{ true}} \\ Continued \ on \ the \ next \ page$$

if 
$$v_1 = v_2$$

[EQUAL-FALSE] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 = a_2 \rightarrow_b \text{ false}}$$

if 
$$v_1 \neq v_2$$

[GRT-TRUE] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{ true}}$$

if 
$$v_1 > v_2$$

$$[\text{GRT-FALSE}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 > a_2 \rightarrow_b \text{ false}}$$

if 
$$v_1 \not> v_2$$

[LESS-TRUE] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{true}}$$

if 
$$v_1 < v_2$$

$$[\text{LESS-FALSE}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 < a_2 \rightarrow_b \text{ false}}$$

if 
$$v_1 \not< v_2$$

[NOT-1] 
$$\frac{env_V, sto \vdash b \to_b \text{ true}}{env_V, sto \vdash !b \to_b \text{ false}}$$

[NOT-2] 
$$\frac{env_V, sto \vdash b \to_b \text{ false}}{env_V, sto \vdash !b \to_b \text{ true}}$$

$$[\text{AND-TRUE}] \qquad \frac{env_V, sto \vdash b_1 \rightarrow_b \text{true} \quad env_V, sto \vdash b_2 \rightarrow_b \text{true}}{env_V, sto \vdash b_1 \land b_2 \rightarrow_b \text{true}}$$

[AND-FALSE] 
$$\frac{env_V, sto \vdash b_i \to_b \text{ false}}{env_V, sto \vdash b_1 \land b_2 \to_b \text{ false}}$$

where 
$$i \in 1, 2$$

[OR-TRUE] 
$$\frac{env_V, sto \vdash b_i \to_b \text{true}}{env_V, sto \vdash b_1 \lor b_2 \to_b \text{true}}$$

where 
$$i \in 1, 2$$

[OR-FALSE] 
$$\frac{env_V, sto \vdash b_1 \rightarrow_b \text{ false} \quad env_V, sto \vdash b_2 \rightarrow_b \text{ false}}{env_V, sto \vdash b_1 \lor b_2 \rightarrow_b \text{ false}}$$

Continued on the next page

[PAR-BOOL] 
$$\frac{env_V, sto \vdash b \to_b v}{env_V, sto \vdash (b) \to_b v}$$

Table 1.5: Boolean expressions

Transitioner på formen:  $\langle D_V, env_V, sto \rangle \rightarrow_{DV} (env_V', sto')$ 

[VAR-DEC] 
$$\frac{\langle D_V, env_V'', sto[l \mapsto v] \rangle \to_{DV} (env_V', sto')}{\text{var } x < --a, env_V, sto \rangle \to_{DV} (env_V', sto')}$$

where  $env_V$ ,  $sto \vdash a \rightarrow_a v$ and  $l = env_V$  next and  $env_V'' = env_V[x \mapsto l][\text{next} \mapsto \text{new } l]$ 

[EMPTY-VAR]  $\langle \varepsilon, env_V, sto \rangle \rightarrow_{DV} (env_V, sto)$ 

Transitioner på formen:  $env_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env_P'$ 

 $[PROC-PARA-DEC] \quad \frac{env_V \vdash \langle D_P, env_P[p \mapsto (S, \vec{x}, env_V, env_P)] \rangle \rightarrow_{DP} env_P'}{env_V \vdash \langle \text{function } p \text{ using}(\text{var } \vec{x}) \text{ begin } S \text{ end, } env_P \rangle \rightarrow_{DP} env_P'}$ 

[EMPTY-PROC]  $env_V \vdash \langle \varepsilon, env_P \rangle \rightarrow_{DP} env_P'$ 

Transitioner på formen:  $\langle D_A, env_V, sto \rangle \rightarrow_{DA} (env_V', sto')$ 

 $[\text{ARRAY-DEC}] \qquad \qquad \frac{\langle D_A, env_V[r \mapsto l, \text{next} \mapsto l+v+1], sto[l \mapsto v] \rangle \rightarrow_{DA} (env_V', sto')}{\langle \text{array } r[a_1], env_V, sto \rangle \rightarrow_{DA} (env_V', sto')}$ 

where  $env_V$ ,  $sto \vdash a_1 \rightarrow_a v$ and  $l = env_V \text{next}$ and v > 0

[EMPTY-ARRAY]  $\langle \varepsilon, env_V, sto \rangle \rightarrow_{DA} (env_V, sto)$ 

Table 1.6: Declarations