## Abstract syntax

```
\begin{split} S ::= x := a \mid r[a_1] := a_2 \mid S_1; \ S_2 \mid \text{if } b \text{ do } S \mid \text{if } b \text{ do } S_1 \text{ else do } S_2 \mid \text{ while } b \text{ do } S \\ \mid \text{ from } x := a_1 \text{ to } a_2 \text{ step } a_3 \text{ do } S \mid \text{call } p(\vec{x}) \mid \text{ begin } D_V \ D_P \ S \text{ end} \\ a ::= n \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \mid a_1/a_2 \mid (a) \\ b ::= a_1 = a_2 \mid a_1 > a_2 \mid a_1 < a_2 \mid \neg b \mid b_1 \ \land \ b_2 \mid b_1 \ \lor \ b_2 \mid (b) \\ D_V ::= \text{var } x := a; \ D_V \mid \varepsilon \\ D_P ::= \text{proc } p \text{ is } S; \ D_P \mid \varepsilon \\ D_A ::= \text{array } r[a_1] := a_2; \ D_A \mid \varepsilon \end{split}
```

```
Transitioner er på formen: env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'
                                         env_V, env_P \vdash \langle x < --a, sto \rangle \rightarrow sto[l \mapsto v]
[VAR-ASS]
                                         where env_V, sto \vdash a \rightarrow_a v
                                         and env_V x = l
                                         env_V, env_P \vdash \langle r[a_1] < --a_2, sto \rangle \rightarrow sto[l \mapsto v_2]
[ARR-ASS]
                                         where env_V, sto \vdash a_1 \rightarrow_a v_1
                                         and env_V, sto \vdash a_2 \rightarrow_a v_2
                                         and ???[r[v_1] = l]
                                            env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto''
                                         \frac{env_V, env_P \vdash \langle S_2, sto'' \rangle \to sto'}{env_V, env_P \vdash \langle S_1; S_2, sto \rangle \to sto'}
[COMP]
                                         \frac{env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'}
[IF-TRUE]
                                         if env_V, sto \vdash b \rightarrow_b \text{TRUE}
[IF-FALSE]
                                         env_V, env_P \vdash \langle \text{if } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                         if env_V, sto \vdash b \rightarrow_b \text{FALSE}
                                                                         env_V, env_P \vdash \langle S_1, sto \rangle \rightarrow sto'
[IF-ELSE-TRUE]
                                         env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'
                                         if env_V, sto \vdash b \rightarrow_b \text{TRUE}
                                         \frac{env_V, env_P \vdash \langle S_2, sto \rangle \rightarrow sto'}{env_V, env_P \vdash \langle \text{if } b \text{ begin } S_1 \text{ end else begin } S_2 \text{ end}, sto \rangle \rightarrow sto'}
[IF-ELSE-FALSE]
                                         if env_V, sto \vdash b \rightarrow_b \text{FALSE}
                                                            env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto''
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto'' \rangle \rightarrow sto'
[WHILE-TRUE]
                                          env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto'
                                         if env_V, sto \vdash b \rightarrow_b \text{TRUE}
[WHILE-FALSE]
                                         env_V, env_P \vdash \langle \text{while } b \text{ begin } S \text{ end}, sto \rangle \rightarrow sto
                                         if env_V, sto \vdash b \rightarrow_b \text{FALSE}
```

Tabel 1: Statements

$$[FROM-TRUE] \qquad \overline{env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end} \rangle \rightarrow sto'}$$

$$\text{where } env_V, sto \vdash a_1 \rightarrow_a v_1$$

$$\text{and } env_V, sto \vdash a_2 \rightarrow_a v_2$$

$$\text{and } env_V, sto \vdash a_3 \rightarrow_a v_3$$

$$\text{and } v_1 \leq v_2$$

$$[FROM-FALSE] \qquad \overline{env_V, env_P \vdash \langle \text{from } x < --a_1 \text{ to } a_2 \text{ step } a_3 \text{ begin } S \text{ end} \rangle \rightarrow sto'}$$

$$\text{where } env_V, sto \vdash a_1 \rightarrow_a v_1$$

$$\text{and } env_V, sto \vdash a_2 \rightarrow_a v_2$$

$$\text{and } env_V, sto \vdash a_3 \rightarrow_a v_3$$

$$\text{and } v_1 > v_2$$

$$[CALL] \qquad \overline{env_V, env_P \vdash \langle \text{call } p(\vec{x}), sto \rangle \rightarrow sto'}$$

$$\frac{\langle D_V, env_V, sto \rangle \rightarrow_{DV} (env_V', sto'')}{env_V' \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env_P'}$$

$$env_V' \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env_P'$$

$$env_V' env_P' \vdash \langle S, sto'' \rangle \rightarrow sto'}$$

$$[BLOK] \qquad \overline{env_V, env_P \vdash \langle \text{begin } D_V D_P S \text{ end}, sto \rangle \rightarrow sto'}$$

Tabel 2: Statements continued

Transitioner er på formen: 
$$env_V, sto \vdash a \rightarrow_a v$$

[NUM] 
$$env_V, sto \vdash n \rightarrow_a v$$
 if  $\mathcal{N}[[n]] = v$ 

$$[\text{VAR}] \qquad env_V, sto \vdash x \to_a v$$
 if  $env_V \ x = l$  and  $sto \ l = v$ 

$$[ADD] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 + a_2 \rightarrow_a v}$$
 where  $v = v_1 + v_2$ 

$$[SUB] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 - a_2 \rightarrow_a v}$$

where 
$$v = v_1 - v_2$$

[MULT] 
$$\frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1 \cdot a_2 \rightarrow_a v}$$

where 
$$v = v_1 \cdot v_2$$

$$[\text{DIV}] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1 \quad env_V, sto \vdash a_2 \rightarrow_a v_2}{env_V, sto \vdash a_1/a_2 \rightarrow_a v}$$

where 
$$v = v_1/v_2$$

$$[PAR] \qquad \frac{env_V, sto \vdash a_1 \rightarrow_a v_1}{env_V, sto \vdash (a_1) \rightarrow_a v_1}$$

Tabel 3: Arithmic expressions

$$[EQUAL-TRUE] \qquad \frac{env_{V}, sto \vdash a_{1} \rightarrow_{a} v_{1} \quad env_{V}, sto \vdash a_{2} \rightarrow_{a} v_{2}}{env_{V}, sto \vdash a_{1} = a_{2} \rightarrow_{b} TRUE}$$

$$if \ v_{1} = v_{2}$$

$$[EQUAL-FALSE] \qquad \frac{env_{V}, sto \vdash a_{1} \rightarrow_{a} v_{1} \quad env_{V}, sto \vdash a_{2} \rightarrow_{a} v_{2}}{env_{V}, sto \vdash a_{1} = a_{2} \rightarrow_{b} FALSE}$$

$$if \ v_{1} \neq v_{2}$$

$$[GRT-TRUE] \qquad \frac{env_{V}, sto \vdash a_{1} \rightarrow_{a} v_{1} \quad env_{V}, sto \vdash a_{2} \rightarrow_{a} v_{2}}{env_{V}, sto \vdash a_{1} > a_{2} \rightarrow_{b} TRUE}$$

$$if \ v_{1} > v_{2}$$

$$[GRT-FALSE] \qquad \frac{env_{V}, sto \vdash a_{1} \rightarrow_{a} v_{1} \quad env_{V}, sto \vdash a_{2} \rightarrow_{a} v_{2}}{env_{V}, sto \vdash a_{1} > a_{2} \rightarrow_{b} FALSE}$$

$$if \ v_{1} \neq v_{2}$$

$$[LES-TRUE] \qquad \frac{env_{V}, sto \vdash a_{1} \rightarrow_{a} v_{1} \quad env_{V}, sto \vdash a_{2} \rightarrow_{a} v_{2}}{env_{V}, sto \vdash a_{1} < a_{2} \rightarrow_{b} TRUE}$$

$$if \ v_{1} < v_{2}$$

$$[LES-FALSE] \qquad \frac{env_{V}, sto \vdash a_{1} \rightarrow_{a} v_{1} \quad env_{V}, sto \vdash a_{2} \rightarrow_{a} v_{2}}{env_{V}, sto \vdash a_{1} < a_{2} \rightarrow_{b} FALSE}$$

$$if \ v_{1} \neq v_{2}$$

$$if \ v_{1} \neq v_{2}$$

Transitioner på formen:  $env_V$ ,  $sto \vdash b \rightarrow_b t$ 

Tabel 4: Boolean expressions

$$[\text{NOT-1}] \qquad \frac{env_V, sto \vdash b \to_b \text{ TRUE}}{env_V, sto \vdash !b \to_b \text{ FALSE}}$$

$$[\text{NOT-2}] \qquad \frac{env_V, sto \vdash b \to_b \text{ TRUE}}{env_V, sto \vdash !b \to_b \text{ FALSE}}$$

$$[\text{AND-TRUE}] \qquad \frac{env_V, sto \vdash b_1 \to_b \text{ TRUE}}{env_V, sto \vdash b_1 \land b_2 \to_b \text{ TRUE}}$$

$$[\text{AND-FALSE}] \qquad \frac{env_V, sto \vdash b_1 \to_b \text{ FALSE}}{env_V, sto \vdash b_1 \land b_2 \to_b \text{ FALSE}}$$

$$\text{where } i \in 1, 2$$

$$[\text{OR-TRUE}] \qquad \frac{env_V, sto \vdash b_1 \to_b \text{ TRUE}}{env_V, sto \vdash b_1 \lor b_2 \to_b \text{ TRUE}}$$

$$\text{where } i \in 1, 2$$

$$[\text{OR-FALSE}] \qquad \frac{env_V, sto \vdash b_1 \to_b \text{ FALSE}}{env_V, sto \vdash b_2 \to_b \text{ FALSE}}$$

$$env_V, sto \vdash b_1 \lor b_2 \to_b \text{ FALSE}}$$

$$[\text{PAR-BOOL}] \qquad \frac{env_V, sto \vdash b \to_b v}{env_V, sto \vdash b \to_b v}$$

Tabel 5: Boolean expressions continued

$$[VAR-DECL] \begin{tabular}{l} & Transitioner på formen: $\langle D_V, env_V, sto \rangle \to_{DV} (env_V', sto')$ \\ & \frac{\langle D_V, env_V'', sto[l \mapsto v] \rangle \to_{DV} (env_V', sto')}{var \ x < --a; D_V, env_V, sto \rangle \to_{DV} (env_V', sto')}$ \\ & where \ env_V, sto \vdash a \to_a v$ \\ & and \ l = env_V \ next$ \\ & and \ env_V'' = env_V[x \mapsto l][next \mapsto new \ l]$ \\ [EMPTY-VAR] \begin{tabular}{l} & \langle \varepsilon, env_V, sto \rangle \to_{DV} (env_V, sto) \\ & \frac{env_V \vdash \langle D_P, env_P \models \langle S, env_V, env_P \rangle) \to_{DP} env_P'}{env_V \vdash \langle D_P, env_P \rangle \to_{DP} env_P'} \\ & \frac{env_V \vdash \langle D_P, env_P \models \langle S, env_V, env_P \rangle) \to_{DP} env_P'}{env_V \vdash \langle Proc \ p \ is \ S; D_P, env_P \rangle \to_{DP} env_P'} \\ & env_V \vdash \langle \varepsilon, env_P \rangle \to_{DP} env_P' \\ [ARRAY-DECL] \end{tabular}$$

Tabel 6: Declarations

[EMPTY-ARRAY]