Arrays 1

We will define **Boa** which is **Bip** with arrays. We assume that the set of locations to be the natural numbers, i.e. $\mathbf{Loc} = \mathbb{N}$, and we can then find the location k positions from the location l as the location $l + k$

1.1 Abstract syntax

$$
\begin{aligned}
S &::= \dots \mid x[a] := a \mid \text{begin } D_V\ D_A\ D_P\ S \text{ end} \\
a &::= \dots \mid x[a] \\
D_A &::= \text{array } x \text{ of } [1..n];\ D_A \mid \varepsilon
\end{aligned}
$$

1.2 Statements

The transition system for the statements are as follows: $(\Gamma_{\mathbf{Stm}}, \rightarrow, \mathrm{T}_{\mathbf{Stm}})$, where the configurations are: $\Gamma_{\mathbf{Stm}} = \mathbf{Stm} \times \mathbf{Sto} \cup \mathbf{Sto}$ and the end configurations are: $\mathrm{T}_{\mathbf{Stm}} = \mathbf{Sto}$. The transition rules are on the form: $env_V, env_P \vdash \langle S, sto \rangle \rightarrow sto'$

$$
\begin{aligned}
&[\text{ARR-ASS}] \quad env_V, env_P \vdash \langle x[a_1] := a_2, sto \rangle \rightarrow sto[l_2 \mapsto v_2] \\
&\qquad \text{where } env_V, sto \vdash a_1 \rightarrow_a v_1 \\
&\qquad \text{and } env_V, sto \vdash a_2 \rightarrow_a v_2 \\
&\qquad \text{and } env_V\ x = l_1 \\
&\qquad \text{and } l_2 = l_1 + v_1 \\
&\qquad \text{and } v_3 = sto\ l_1 \\
&\qquad \text{and } 1 \leq v_1 \leq v_3
\end{aligned}
$$

$$
[\text{BLOCK}] \quad \frac{\begin{array}{c} \langle D_V, env_V, sto \rangle \rightarrow_{DV} (env'_V, sto'') \\ \langle D_A, env'_V, sto'' \rangle \rightarrow_{DA} (env''_V, sto^3) \\ env''_V \vdash \langle D_P, env_P \rangle \rightarrow_{DP} env'_P \\ env''_V, env'_P \vdash \langle S, sto^3 \rangle \rightarrow sto' \end{array}}{env_v, env_P \vdash \langle \text{begin } D_V\ D_A\ D_P\ S \text{ end}, sto \rangle \rightarrow sto'}
$$

1.3 Arithmetic Expressions

The transition system for the arithmetic expressions are as follows: $(\Gamma_{\mathbf{Aexp}}, \rightarrow_a, \mathrm{T}_{\mathbf{Aexp}})$, where the configurations are: $\Gamma_{\mathbf{Aexp}} = \mathbf{Aexp} \cup \mathbb{Z}$ and the end configurations are: $\mathrm{T}_{\mathbf{Aexp}} = \mathbb{Z}$. The transition rules are on the form: $env_V, sto \vdash a \rightarrow_a v$.

$$[\text{ARR}] \quad env_V, sto \vdash x[a_1] \rightarrow_a a_2$$

$$\begin{aligned}
&\text{where } env_V, sto \vdash a_1 \rightarrow_a v_1\\
&\text{and } env_V, sto \vdash a_2 \rightarrow_a v_2\\
&\text{and } env_V\ x = l\\
&\text{and } sto\ l = v_3\\
&\text{and } 0 < v_1 \leq v_3\\
&\text{and } sto(l + v_1) = v_2
\end{aligned}$$

1.4 Array Declaration

The transition system for the array declaration are as follows: $(\Gamma_{\mathbf{ErkA}}, \rightarrow_{DA}, \mathrm{T}_{\mathbf{ErkA}})$, where the configurations are: $\Gamma_{DV} = (\mathbf{ErkA} \times \mathbf{EnvV} \times \mathbf{Sto}) \cup (\mathbf{EnvV} \times \mathbf{Sto}), \mathrm{T}_{DV} = \mathbf{EnvV} \times \mathbf{Sto}$ and the end configurations are: $\mathrm{T}_{\mathbf{ErkV}} = \mathbf{EnvV} \times \mathbf{Sto}$. The transition rules are on the form: $\langle D_A, env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')$.

$$[\text{ARRAY-DEC}] \quad \frac{\langle D_A, env_V[x \mapsto l,\ \text{next} \mapsto l + v + 1], sto[l \mapsto v]\rangle \rightarrow_{DA} (env'_V, sto')}{\langle \text{array } x \text{ of } [1..n], D_A, env_V, sto\rangle \rightarrow_{DA} (env'_V, sto')}$$

$$\begin{aligned}
&\text{where } env_V, sto \vdash n \rightarrow_a v\\
&\text{and } l = env_V \text{ next}\\
&\text{and } n > 0
\end{aligned}$$

$$[\text{EMPTY-ARRAY}] \quad \langle \varepsilon, env_V, sto\rangle \rightarrow_{DA} (env_V, sto)$$