

# Bayesian Data Analysis - Assignment 4

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## General information

- The recommended tool in this course is R (with the IDE R-Studio). You can download R [here](#) and R-Studio [here](#). There are tons of tutorials, videos and introductions to R and R-Studio online. You can find some initial hints [here](#).
- You can write the report with your preferred software, but the outline of the report should follow the instruction in the R markdown template that can be found [here](#).
- Report all results in a single, **anonymous** \*.pdf -file and return it to [peer-grade.io](#).
- The course has its own R package with data and functionality to simplify coding. To install the package just run the following:
  1. `install.packages("remotes")`
  2. `remotes::install_github("avehtari/BDA_course_Aalto",  
subdir = "rpackage")`
- Many of the exercises can be checked automatically using the R package `markmyassignment`. Information on how to install and use the package can be found [here](#).
- Additional self study exercises and solutions for each chapter in BDA3 can be found [here](#).
- We collect common questions regarding installation and technical problems in a course Frequently Asked Questions (FAQ). This can be found [here](#).
- If you have any suggestions or improvements to the course material, please feel free to create an issue or submit a pull request to the public repository!!

## Information on this assignment

This exercise is related to Chapters 3 and 10. The maximum amount of points from this assignment is 6.

**Reading instructions:** Chapters 3 and 10 in BDA3, see reading instructions [here](#) and [here](#)

**Grading instructions:** The grading will be done in peergrade. All grading questions and evaluations for exercise 4 can be found [here](#)

**Reporting accuracy:** As many significant digits as justified by the Monte Carlo error and posterior accuracy.

To use markmyassignment for this assignment, run the following code in R:

```
> library(markmyassignment)
> exercise_path <-
+   "https://github.com/avehtari/BDA_course_Aalto/blob/master/exercises/tests/ex4.yml"
> set_assignment(exercise_path)
> # To check your code/functions, just run
> mark_my_assignment()
```

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1. (**Bioassay model and importance sampling**). In this exercise, you will use a dose-response relation model that is used in Section 3.7 of the course book and in the chapter reading instructions [here](#). The used likelihood is the same, but we will use a different prior distribution on the parameters  $\alpha$  and  $\beta$ .

- a) Construct a bivariate normal distribution as prior distribution for  $(\alpha, \beta)$ . The marginal distributions are  $\alpha \sim N(0, 2^2)$ ,  $\beta \sim N(10, 10^2)$  with correlation  $\text{corr}(\alpha, \beta) = 0.5$ . Report the mean and covariance of the bivariate normal distribution.

**Hint!** The mean and covariance of the bivariate normal distribution are a length-2 vector and a  $2 \times 2$  matrix. The elements of the covariance matrix can be computed using the relation of correlation and covariance.

- b) Implement a function in R for computing the **logarithm** of the density of the prior distribution in a) for arbitrary values of  $\alpha$  and  $\beta$ . Below is an example of how the function should be named and work if you want to check them with `markmyassignment`.

```
> p_log_prior(alpha = 3, beta = 9)
[1] -6.296435
```

**Hint!** Use function `dmvnorm` from the `aaltobda` package. We use logarithms for better numerical accuracy in later questions.

- c) Implement a function in R for computing the **logarithm** of the density of the posterior for arbitrary values of  $\alpha$  and  $\beta$  and data  $x$ ,  $y$  and  $n$ . Below is an example of how the function should be named and work if you want to check them with `markmyassignment`.

```
> library(aaltobda)
> data("bioassay")
> p_log_posterior(alpha = 3,
+                 beta = 9,
+                 x = bioassay$x,
+                 y = bioassay$y,
+                 n = bioassay$n)
[1] -15.78798
```

**Hint!** Equation (3.16) in the course book. The **logarithm** of the prior density was already implemented in b). For computing the **logarithm** of the likelihood, use the `bioassaylp` function from the `aaltobda` package. The data can be loaded with the R command `data("bioassay")`.

**Hint!** Logarithm of the product of two densities is the sum of the log-densities, i.e.  $\log ab = \log a + \log b$ .

- d) Plot the posterior density in a grid of points ( $\alpha \in [-4, 4]$ ,  $\beta \in [-10, 30]$ ) using the `bioassay_posterior_density_plot` function from the `aaltobda` package. Pass the `p_log_posterior` function you implemented in c) as argument to `bioassay_posterior_density_plot`.
- e) Implement two functions in: 1) A function for computing the log importance ratios (log importance weights) for draws from the prior distribution in 1a) when the target distribution is the posterior distribution. 2) A function for

exponentiating the log importance ratios and normalizing them to sum to one. Below is a test example, the functions can also be tested with `markmyassignment`. **Note!** The values below are *only* a test case, you need to use correct draws for `alpha` and `beta` in the final report.

```
> a <- c(1.896, -3.6, 0.374, 0.964, -3.123, -1.581)
> b <- c(24.76, 20.04, 6.15, 18.65, 8.16, 17.4)
> log_importance_weights(alpha = a, beta = b)
[1] -8.95 -23.47 -6.02 -8.13 -16.61 -14.57
> normalized_importance_weights(alpha = a, beta = b)
[1] 0.045 0.000 0.852 0.103 0.000 0.000
```

**Hint!** Equation (10.3) in the course book.

- f) Sample draws of  $\alpha$  and  $\beta$  from the prior distribution from 1a). Implement a function for computing the posterior mean using importance sampling, and report the obtained mean.

**Hint!** Use the function `rmvnorm` from the `aaltobda` package. Below is an example how the function would work with the example values for `alpha` and `beta` above. **Note!** The values below are *only* a test case, you need to use correct draws for `alpha` and `beta` in the final report.

```
> posterior_mean(alpha = a, beta = b)
[1] 0.503 8.275
```

- g) Using the importance ratios, compute the effective sample size  $S_{\text{eff}}$  and report it. If  $S_{\text{eff}}$  is less than 1000, repeat f) with more draws. **Note!** The values below are *only* a test case, you need to use correct draws for `alpha` and `beta` in the final report.

```
> S_eff(alpha = a, beta = b)
[1] 1.354
```

**Hint!** Equation (10.4) in the course book.

- **Note!** *BDA3 1st (2013) and 2nd (2014) printing have an error for  $\tilde{w}(\theta^s)$  used in the effective sample size equation (10.4). The normalized weights equation should not have the multiplier  $S$  (the normalized weights should sum to one). Errata for the book [http://www.stat.columbia.edu/~gelman/book/errata\\_bda3.txt](http://www.stat.columbia.edu/~gelman/book/errata_bda3.txt). The later printings and slides have the correct equation.*

- h) Use importance resampling without replacement to obtain a posterior sample of size 1000 of  $\alpha$  and  $\beta$  and plot a scatterplot of the obtained posterior sample.
- i) Using the posterior sample obtained via importance resampling, report an estimate for  $p(\beta > 0 | x, n, y)$ , that is, the probability that the drug is harmful.
- j) Using the posterior sample obtained via importance resampling, draw a histogram of the draws from the posterior distribution of the LD50 conditional on  $\beta > 0$ .

**Hint!** See Figure 3.4 and corresponding section in the course book. You can plot a basic histogram with R using the library `ggplot2` and the command `ggplot() + geom_histogram(aes(ld50))`