(copied from Andrew's handwritten notes)

1 Computing with infinite rewriting systems

Let Γ be a (finite or infinite) alphabet.

Let $S \subseteq \Gamma^* \times \Gamma^*$.

In order to work with S we require that:

- S is a recursive subset of $\Gamma^* \times \Gamma^*$,

- $L = \{l \in \Gamma^* : (l, r) \in S\}$ is a recursive subset of Γ^*

Then S is a rewriting system on Γ

In Cain's papers S may be infinite but is always length-reducing; so if L is recursive so is S.

When working with a pregroup (if no finite generating set is chosen) the set Γ is infinite, usually.

2 Knuth-Bendix procedures

In Diekert-Duncan-Myasnikov a Knuth-Bendix procedure for geodesically perfect systems is described; where S is finite.

Can such a process be devised in cases where say S and L are regular sets ?

Starting with a rewriting system with S and L regular, but s not confluent, does a Knuth-Bendix process preserve regularity?

What about other properties (instead of regular)?

3 Rewriting systems and pregroups

As in DDM triangular geodesically perfect rewriting systems correspond to pregroups.

Does there exist a KB-type procedure that constructs a triangular geodesic system from an arbitrary triangular system?

In any case what happens to these pregroup rewriting systems when specific generating sets are used for the pregroups?

For instance what happens to the system for a free product with amalgamation if a finite generating set is chosen for the factors and for the amalgamated subgroups?

Ditto HHN extensions.

Ditto general fundamental group of graph of groups.

There are known finite convergent rewriting systems for several classes of groups. These are not usuall triangular.

What happens if the corresponding presentations are triangulated – can this be done so that a triangular geodesic system is produced?

e.g. Coxeter groups,

partially commutative groups,

polycyclic groups.

If so describe the corresponding pregroups (it may be that only the length-reducing part of the system need be triangular; but this would require some additional condition on the length-preserving part.)

4 Geodesic, preperfect and geodesically perfect systems

Find geodesically perfect systems for:

- hyperbolic groups

- 1 relator groups

- fully residually free groups

- polycyclic groups

- surface groups (hyperbolic)

(This is really part of the question about KB procedures for group systems)

What can be said about geodesically perfect (geodesic, or preperfect) systems where L is regular or context-free etc?

(In the spirit of Cain's result that if S is convergent and L regular monadic then the group or monoid is hyperbolic)

5 Cain's results

The 'Gilman conjecture' is that if a group can be presented by a finite convergent weight-reducing system then it is "plain". A *plain* group is a free product of finite and free groups.

Are the hyperbolic groups appearing in Cain's papers all plain? If so this is probably what is really happening. If not - fame beckons (at least for Cain)

Do the results of Cain shed any light at all on this conjecture?

Cain told Andrew that he thinks that his results show that there exists a hyperbolic monoid which has no finite convergent rewriting system.

Confirm this.

Any possibility of extending to groups?