Digits

Multiplicative persistence

Multiply *n*'s digits together to get a new number. Repeat until you get a single digit. The number of steps taken is called the *multiplicative persistence* of *n*.

e.g. $987 \rightarrow 504 \rightarrow 0$. Multiplicative persistence = 2.

What's the smallest number with multiplicative persistence 3?
What's the multiplicative persistence of your birthday, in DDMMYYYY form?

Harshad numbers

18 is a Harshad number because the sum of its digits is $1 + 8 = 9$, and 18 is divisible by 9.
19 is *not* a Harshad number because the sum of its digits is $1 + 9 = 10$, and 19 is not divisible by 10.

What are the first 10 Harshad numbers?
Can you find three consecutive Harshad numbers?

Palindromes

A *palindrome* is a word that looks the same when read backwards or forwards.
12321 is a palindromic number.
How many 5-digit palindromic numbers are there?
How many *n*-digit palindromic numbers are there?

Folding paper

Fold in half

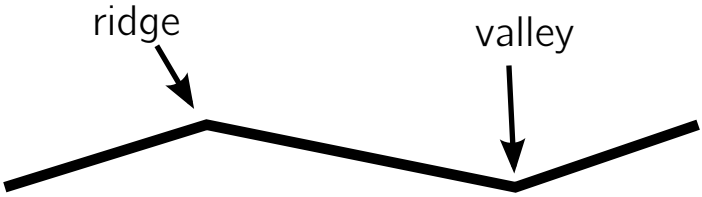
Take a piece of paper and fold it in half twice.
How many different ways are there of doing it?
How many different ways are there of folding a piece of paper in half n times?

Fold in half and cut the corners off

Take a piece of paper and fold it in half twice, always folding the longest side.
Then, cut off each of the 4 corners.
When you unfold it, there's a hole in the middle.
How many holes when you fold 3 times? 4? n?

Ridges and valleys

Take a long piece of paper, and fold the right edge onto the left. Repeat this, never turning the paper.
When you unfold it, you get a sequence of ridges (1s) and valleys (0s).
What sequence do you get after 2 folds? 3? n?



Odds and ends

Think like a Roman

What's this integer sequence? What's the pattern?

I	II	III	IV	V	VI	VII	VIII	IX	X
I	II	III	III	II	III	IV	V	III	II

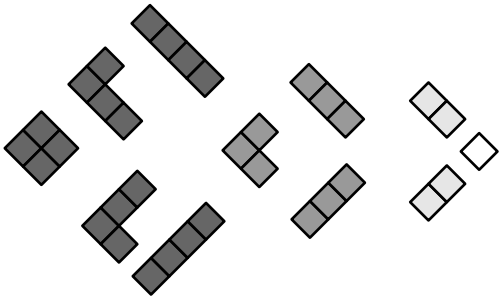
The Collatz conjecture

Pick a number n.
If n is even, divide it by 2.
If n is odd, multiply it by 3 and add 1.
Repeat until you get to 1.
The number of steps taken is n's Collatz number.

What are the Collatz numbers of the numbers 1 to 10?
Can you find a three-digit number which takes 7 steps?

Partitions

How many ways can you write n as the sum of smaller numbers?



Playing card sequences

Ways of ordering

Take the Ace, 2 and 3 of Hearts. How many different ways can you order them?
How about the Ace, 2, 3 and 4?
Keep increasing the number of cards.
How many ways can you order all 52 cards?

Digit cards

You can "write" the number 2,103 by putting the 2, 10 and 3 cards next to each other.
Which numbers can you make using just the Hearts?
What's the biggest number you can make using all 36 number cards?

Deal into two piles

Again, take the Ace, 2, 3 and 4 of Hearts.
Deal them alternately into two piles, then pick the piles up and put them together.
How many times do you need to repeat this before the cards are reversed from their original order?
How many times with 6 cards? With 8? With n?

Polygon numbers

Triangle numbers

Draw 1 dot, then a row of 2 dots next to it, and so on...

You draw 1 dot on the first go, 3 on the second, 6 on the third.
How many dots do you draw on the n^{th} go?

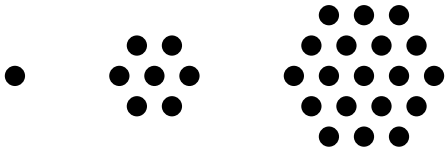
Square numbers

How many dots are there in an n×n square of dots?

Is there a link with the triangle numbers? Why?

Hexagon numbers

Draw a dot then surround it with 6 evenly spaced dots.
Keep adding more layers of dots.



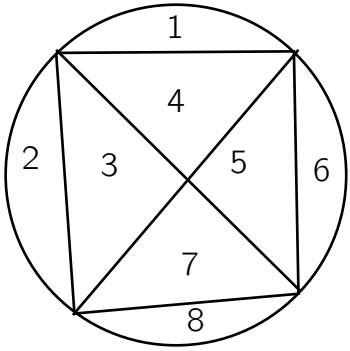
How many dots in the third picture? In the n^{th}?
Is there a link with the triangle or square numbers?

The Law of Small Numbers

Circle crossings

Mark 6 points on a circle, and join them all with straight lines. How many different regions are there?

4 points: **8** regions



Partial sums

Write down the numbers, delete every other one, then write the partial sums underneath:

1	~~2~~	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~	11
1		4		9		16		25		36

Start again, but delete every third one, then every second partial sum, then take partial sums again:

1	2	~~3~~	4	5	~~6~~	7	8	~~9~~	10	11
1	~~3~~		7	~~12~~		19	~~27~~		37	~~48~~
1			8			27			64	

Is there a pattern?