

Thue-Morse sequence

0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1,
1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0,
1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, ...

Replace every 0 in this sequence with 01 and 1 with 10:
you end up with the same sequence!

Solution to the "taking turns" problem: more fair
than just alternating between two sides.

The mean of the first n terms is as close to $\frac{1}{2}$ as possible.