The all 7s sequence

```
7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, ...
```

Decimal expansion of 7/9.

Final digit of $16^{2^n}+1$, for all n.

It's just the number 7, forever.

n factorial

```
1, 1, 2, 6, 24, 120, 720, 5040, 40320,
362880, 3628800, 39916800, 479001600, …
```

Written $n!$, this is the product of the numbers $1 \ldots n$.

The number of different ways of arranging n objects.

There are exactly 10! seconds in a week. *(Prove it!)*

First appeared in a book written in 300AD.

The happy numbers

```
1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49,
68, 70, 79, 82, 86, 91, 94, 97, 100, 103,
109, 129, 130, 133, 139, 167, 176, 188, …
```

Add up the squares of each of the digits of n.
If repeatedly doing this eventually reaches 1, n is happy.

Example:
$19 \to 82 \to 68 \to 100 \to 1$, so 19 is happy.

Numbers which never reach 1 are called *unhappy.*

Mersenne primes

```
2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107,
127, 521, 607, 1279, 2203, 2281, 3217,
4253, 4423, 9689, 9941, 11213, 19937, …
```

Prime numbers p such that 2^p - 1 is also prime.

First 8 terms computed by Marin Mersenne, a monk, in the 17^{th} century.

The Great Internet Mersenne Prime Search, *GIMPS*, recently found a Mersenne prime with 22,338,618 digits!

Not McNugget numbers

```
1, 2, 3, 4, 5, 7, 8, 10, 11, 13, 14, 16,
17, 19, 22, 23, 25, 28, 31, 34, 37, 43
```

Chicken McNuggets used to come in boxes of 6, 9 or 20.
These are the numbers of nuggets you couldn't order as any combination of boxes.
You could make every number larger than 43.

Since the introduction of the Happy Meal with 4 nuggets, the biggest non-McNugget number is now 11.

The perfect numbers

```
6, 28, 496, 8128, 33550336, 8589869056,
137438691328, 2305843008139952128,
2658455991569831744654692615953842176, …
```

n such that *n* is equal to the sum of its divisors.

Example:

1, 2 and 3 divide into 6.

1+2+3 = 6.

Nobody knows if there are any odd perfect numbers!

Recamán's sequence

```
0, 1, 3, 6, 2, 7, 13, 20, 12, 21, 11, 22,
10, 23, 9, 24, 8, 25, 43, 62, 42, 63, 41,
18, 42, 17, 43, 16, 44, 15, 45, 14, 46, …
```

a(0) = 0;
a(n+1) = a(n-1)-n if positive and not already in,
otherwise a(n) = a(n-1)+n.

Nobody knows if every number eventually appears!

The square numbers

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
121, 144, 169, 196, 225, 256, 289, 324,
361, 400, 441, 484, 529, 576, 625, 676, …

$a(n) = n^2$.

The area of an $n \times n$ square.

The first sequence ever computed by an electronic computer, *EDSAC*, on May 6th 1949.

Thue-Morse sequence

```
0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1,
1, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0,
1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, 0, …
```

Replace every `0` in this sequence with `01` and `1` with `10`: you end up with the same sequence!

Solution to the "taking turns" problem: more fair than just alternating between two sides.

The mean of the first n terms is as close to ½ as possible.

Wieferich primes

```
1093, 3511, ?
```

Prime numbers p such that p^2 divides $2^{p-1} - 1$.

Nobody knows what the next one is, but there are believed to be infinitely many.

A supercomputer has proved that there are no other Wieferich primes less than 497,000,000,000,000,000.

Powers of 2

```
1, 2, 4, 8, 16, 32, 64, 128, 256, 1024,
2048, 4096, 8192, 16384, 32768, 65536,
131072, 262144, 524288, 1048576, …
```

$a(n) = 2^n$.

If you placed one grain of rice on the first square of a chess board and doubled the number on each subsequent square, the last one would have **18,446,744,073,709,551,616** grains of rice on it!

Easy phone numbers

```
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 14,
15, 21, 22, 23, 24, 25, 26, 32, 35, 36,
41, 42, 44, 45, 47, 48, 51, 52, 53, 54, …
```

A number is "easy" to dial if each pair of adjacent digits are adjacent on the standard keypad:

```
1 2 3
4 5 6
7 8 9
  0
```