Reinforcement Learning, Lecture 4: Monte Carlo Methods [1]

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[1] Many slides adapted from R. Sutton's course, D. Silver's course, as well as previous courses given at University of Stuttgart by D. Hennes, M. Toussaint, H. Ngo, and V. Ngo.

Outline

Monte Carlo Integration

- **Estimate** integral

$$\mathbb{E}\left[f(x)\right] = \int f(x)p(x)dx$$

- **Draw samples** $x_i \overset{\text{i.i.d.}}{\sim}$ [2] $p(x)$ and approximate the integral as

$$\hat{f} \approx \frac{1}{L}\sum_{l=1}^{L} f(x_l)$$

- The **empirical mean estimator** $\hat{f}$ converges to the true mean $\mathbb{E}\left[f(x)\right]$ as the number of samples L increases (law of large numbers)

[2] i.i.d. means Independent and Identically Distributed: each random variable has the same probability distribution as the others and all are mutually independent

Monte Carlo Integration

Estimation of the expected output of a black box function f w.r.t. some distribution over inputs

Assume we have access to

- Samples (i.i.d.) from prior distribution over states
- Black box function that encodes environment and returns sequence of experience

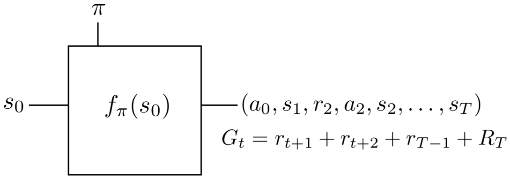


Figure: Black box view on RL

Use Monte Carlo methods to estimated the expected output or a function of it, e.g. the expected cumulative reward.

Monte Carlo methods in RL

- **Learn** value function from experience
- **Discover** optimal policies
- **Blackbox view** does not require knowledge of the environment: $p(s'|s,a)$ & $r(s,a,s')$
- *Experience:* sample sequences of states, actions, rewards: $S_1, A_1, R_2, S_2, A_2, \ldots$
  - real experience: interaction with the environment
  - simulated experience: interaction with a simulator
- Achieve optimal behavior

Monte Carlo principle

- Divide experience into episodes
- All episodes must terminate!
- Keep estimates of value function / policy
- Update estimate at end of each episode
- MC vs. DP:
  - update every episode vs. every step
  - average total return vs. bootstrapping (average 1-step lookahead)

Returns

- Return at time t: $G_t = R_{t+1} + R_{t+2} + \ldots R_{T-1} + R_T$
- Total sum of immediate rewards up to and including terminal transition at $t = T$
- *Idea:* average returns over many episodes starting from same state s
  - gives the value function $v_\pi(s)$ for that state and policy π
  - $v_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s]$ is the expected cumulative future discounted reward
  - $\gamma = 1$ as every episode terminates and we update at end of epsiode

Monte Carlo learning of v_π

- Learn v_π from experience
- Generate many *plays*/episodes starting from state s
- Observe total reward of each *play*
- Average over many *plays*

1. **Initialize:**

   $\pi \leftarrow$ policy to be evaluated
   $V \leftarrow$ arbitrary state-value function
   $Returns(s) \leftarrow$ empty list for all $s \in S$

2. **Repeat forever:**

   Generate episode using π
   for all state s in episode **do**
       $G \leftarrow$ return following the first occurrence of s
       Append G to $Returns(s)$
       $V(s) \leftarrow average(Returns(s))$
   end for

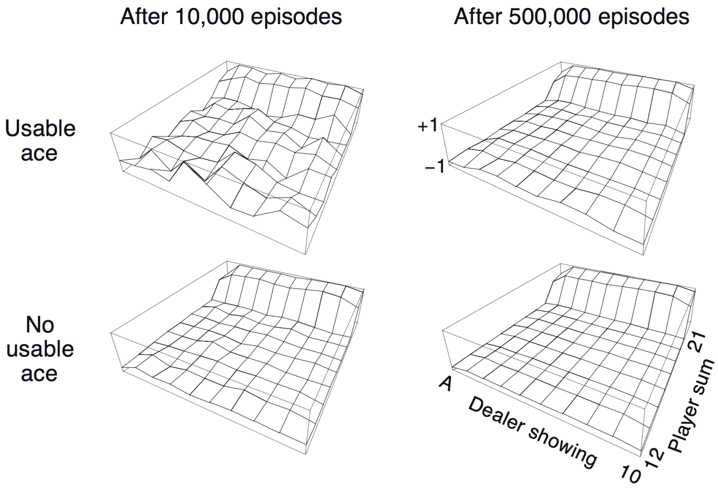
Backup diagram

- Entire episode included
- Only single choice considered at each state
  - Non-averaging as with DP
- Thus, there will be an explore/exploit dilemma
- No bootstrap from successor states' values
- Value is estimated by mean return

Blackjack example

- **Objective:** your card sum greater than the dealer's without exceeding 21
- **Actions**
  - *stick* (no more cards)
  - *hit* (receive another card)
- **States**
  - current sum (12-21), we do not consider cases bellow 12 $\rightarrow$ always hit
  - dealer's showing card (A-10)
  - usable ace?
- **Reward:** $+1$ for winning, 0 for a draw, -1 for losing
- **Policy:** stick if sum ≥ 20, else hit
- No discounting $(\gamma = 1)$

Wikipedia


After 10,000 episodes
After 500,000 episodes
Usable ace
No usable ace
+1
−1
A
Dealer showing
10
12
Player sum
21

Recap: policy iteration

- Alternating policy **evaluation** and policy **improvement**
- **Policy evaluation:** estimate v_π for fixed π
- **Policy improvement:** determine greedy policy π' w.r.t. to v_π
- Iterate until optimal value function & policy is reached
- We can use *Monte Carlo* instead of policy *evaluation* in policy *iteration*
- MC estimates the value function given a policy

First–visit vs. every–visit MC

- Fist–visit MC:
  - Estimate $v_\pi(s)$ as the average of returns following **first** visits to s
- Every–visit MC:
  - Estimate $v_\pi(s)$ as the average of returns following **every** visit to s
- Both strategies converge to $v_\pi(s)$ as the number of visits to s goes to infinity

Properties of MC

- Estimates of v for each state are independent
  - no bootstrapping!
  - thus no need to learn about all states
- Compute time is independent of $|\mathcal{S}|$
- If only a few states are relevant, we can generate episodes from those states and ignore the value of others
- No need to know the full model $p(s'|s,a)$ and $r(s,a,s')$
- Learning from real/simulated experience
- Often (i.e. in games) it is possible to generate transitions from p without actually having to express p explicitly
- Less harmed by violating Markov property

Estimating q–values

- Same principle as for v_π

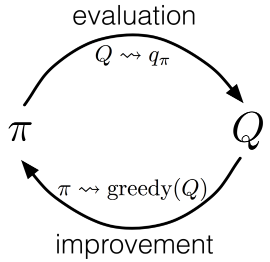
$$q_\pi(s,a) = \mathbb{E}_\pi\Big[G_t \mid S_t = s, A_t = a\Big]$$

- Update estimate $q_\pi(s,a)$ by averaging returns following first visit to that state–action pair (s,a)
- *Warning:* if the policy is deterministic, some (s,a) pairs may never be visited

Monte Carlo control

- **MC policy iteration step:** policy evaluation using MC methods
- **Policy improvement step:** greed w.r.t. to action–value

$$\pi_0 \xrightarrow{\text{E}} q_{\pi_0} \xrightarrow{\text{I}} \pi_1 \xrightarrow{\text{E}} q_{\pi_1} \xrightarrow{\text{I}} \pi_2 \xrightarrow{\text{E}} \dots \xrightarrow{\text{I}} \pi_* \xrightarrow{\text{E}} q_*$$



Greedy policy

- For any action–value function q_π, the corresponding greedy policy is:

$$\pi'(s) = \arg\max_a q_\pi(s, a)$$

- *Policy improvement* is simply constructing each π_{k+1} as the greedy policy w.r.t. to q_{π_k}

Convergence of MC control

$$\begin{aligned} q_{\pi_k}(s, \pi_{k+1}(s)) &= q_{\pi_k}(s, \arg\max_a q_{\pi_k}(s,a)) \\ &= \max_a q_{\pi_k}(s,a) \\ &\geq q_{\pi_k}(s, \pi_k(s)) \\ &= v_{\pi_k}(s) \end{aligned}$$

- Thus π_{k+1} must be equal or better than π_k
- Assumes *exploring starts* and *infinite number of episodes* for MC policy evaluation

Monte Carlo ES (exploring starts)

1. **Initialize** for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$

   $Q(s, a) \leftarrow$ arbitrary
   $\pi(s) \leftarrow$ arbitrary
   $Returns(s, a) \leftarrow$ empty list

2. **Repeat forever:**

   Choose $S_0 \in \mathcal{S}, A_0 \in \mathcal{A}(S_0)$ s.t. all pairs have probability > 0
   Generate episode starting from (S_0, A_0) following π
   for all pair (s, a) in episode **do**
   $\quad G \leftarrow$ return following the first occurrence of (s, a)
   $\quad$ Append G to $Returns(s, a)$
   $\quad Q(s, a) \leftarrow average(Returns(s, a))$
   end for
   for all s in episode **do**
   $\quad \pi(s) \leftarrow \arg\max_a Q(s, a)$
   end for

On–policy Monte Carlo control

- **On-policy:** learn about policy currently used to generate experience
- We must explore
- How do we avoid the assumption of exploring starts?
- E.g., using ϵ–greedy or softmax policies, i.e., $\pi(s, a) > 0$ for all (s, a)

On-policy Monte Carlo control

1. **Initialize** for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
   $Q(s,a) \leftarrow$ arbitrary
   $Returns(s,a) \leftarrow$ empty list
   $\pi(a \mid s) \leftarrow$ arbitrary ϵ-soft policy
2. **Repeat forever:**
   Generate episode using π
   for all pair (s,a) in episode **do**
   $G \leftarrow$ return following the first occurrence of (s,a)
   Append G to $Returns(s,a)$
   $Q(s,a) \leftarrow average(Returns(s,a))$
   end for
   for all s in episode **do**
   $a^* \leftarrow \arg\max_a Q(s,a)$
   for all a in $\mathcal{A}(s)$ **do**
   $$\pi(a \mid s) \leftarrow \begin{cases} 1 - \epsilon + \frac{\epsilon}{|\mathcal{A}(s)|} & \text{if } a = a^* \\ \frac{\epsilon}{|\mathcal{A}(s)|} & \text{if } a \neq a^* \end{cases}$$
   end for
   end for

Off–policy Monte Carlo control

Learn the value of the **target policy** π from experience generated using a **behavior policy** μ

- For example, π is the **greedy policy** (thus ultimately the optimal policy), while μ is an exploring (e.g. softmax) policy
- In general, we only require that μ generates behavior that *covers*/includes π

$$\pi(a \mid s) > 0 \Rightarrow \mu(a \mid s) > 0 \; \forall s, a$$

- *Idea:* **importance sampling**
  - weight each return by the ratio of the probabilities of the trajectory under the two policies

Importance sampling

- **Target distribution** $p(x)$ from which it's complicated to fraw samples
- **Proposal distribution** $q(x)$ from which it's easy to draw samples
- We need to be able to evaluate $p(x)$ numerically

$$\mathbb{E}_{p(x)}[f(x)] = \int f(x)p(x)dx = \int f(x)\frac{p(x)}{q(x)}q(x)dx = \mathbb{E}_{q(x)}\left[f(x)\frac{p(x)}{q(x)}\right]$$
$$\approx \frac{1}{L}\sum_{l=1}^{L} f(x_l)\underbrace{\frac{p(x_l)}{q(x_l)}}_{w_l} \quad \text{,with samples } x_l \overset{i.i.d.}{\sim} q(x)$$

- The ratio w_l is called importance weight
- Choice of **proposal distribution** $q(x)$ is crucial for efficiency

Importance sampling for off policy prediction

Consider the trajectory $\psi = (a_t, s_{t+1}, a_{t+1}, \dots, s_T)$

$$p_t^T = \frac{Pr\{\psi \mid \pi\}}{Pr\{\psi \mid \mu\}} = \frac{\prod_{k=t}^{T-1} \pi(a_k \mid s_k) p(s_{k+1} \mid s_k, a_k)}{\prod_{k=t}^{T-1} \mu(a_k \mid s_k) p(s_{k+1} \mid s_k, a_k)} = \prod_{k=t}^{T-1} \frac{\pi(a_k \mid s_k)}{\mu(a_k \mid s_k)}$$

Ordinary importance sampling

$$V(s) = \frac{\sum_{t \in \mathcal{T}(s)} p_t^{T(t)} G_t}{|\mathcal{T}(s)|}$$

Weighted importance sampling

$$V(s) = \frac{\sum_{t \in \mathcal{T}(s)} p_t^{T(t)} G_t}{\sum_{t \in \mathcal{T}(s)} p_t^{T(t)}}$$

Notation: Time step numbering increases across episodes boundaries

- $\tau(s)$ denotes the set of all time steps in which state s is visited
- $T(t)$ the first time of termination following time t

Summary

- Monte Carlo has several advantages over dynamic programming:
  - can learn directly from experience
  - no need for full models
  - less harmed by violating Markov property
- MC methods provide an alternative to policy evaluation
- MC requires sufficient exploration
- On–policy vs. off-policy methods
- Importance sampling for off–policy