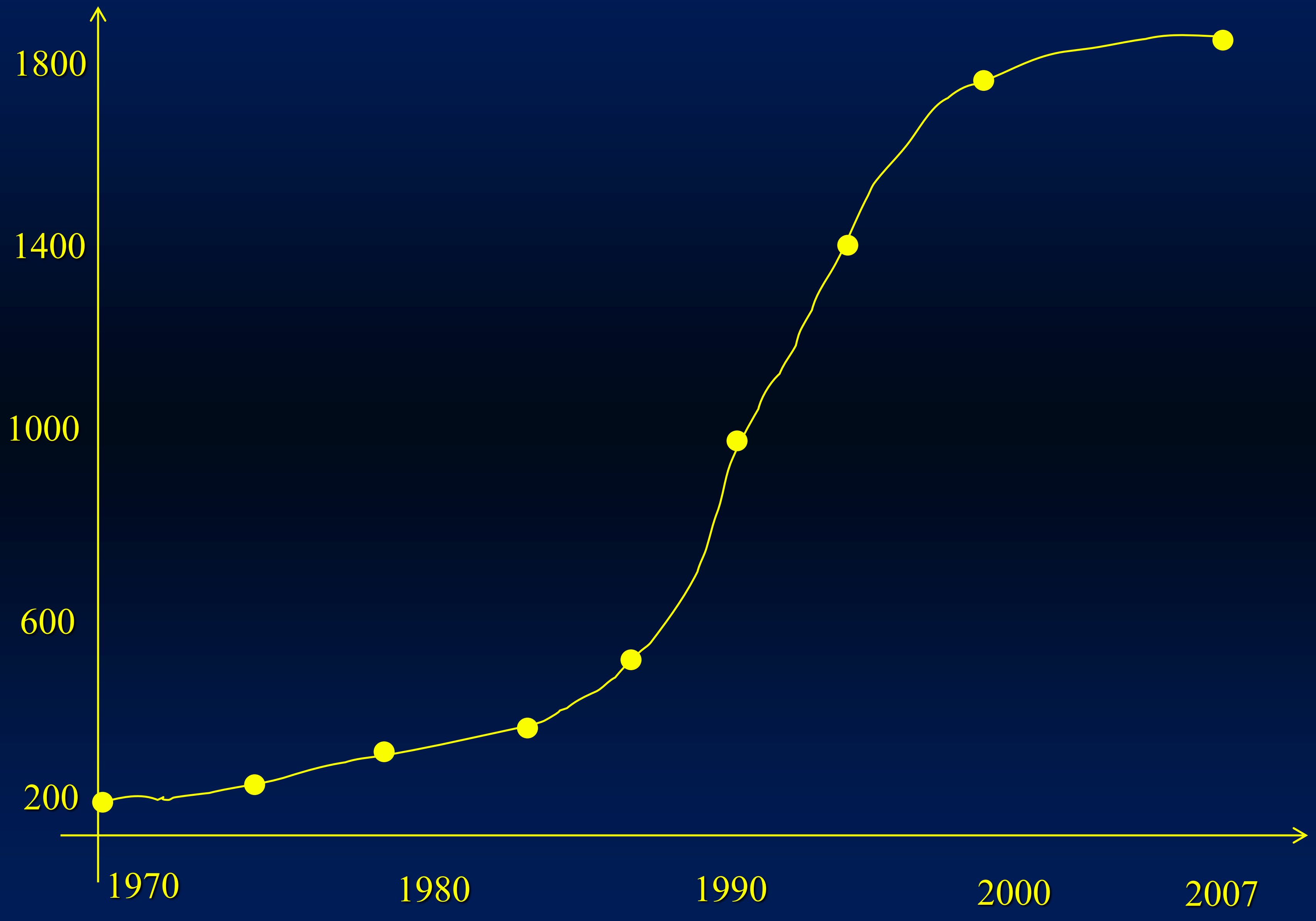
DESCRIPTION AND VALIDATION OF MC SIMULATORS IN PET

PUBLISHED PAPERS ON MC methods in Medical Radiation Physics



I. MONTE CARLO METHODS

- THEORY OF MONTE CARLO METHODS
- RANDOM NUMBER GENERATOR
- SAMPLING
- ESTIMATION OF PRECISION
- PHOTON TRANSPORT AND INTERACTIONS

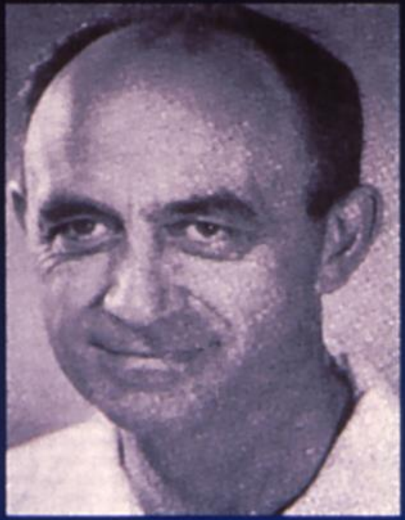
MONTE CARLO METHOD IN PET

INDEPENDENT MODELLING OF DIFFERENT RADIATION ACCORDING TO FIXED PROBABILITY (CROSS SECTION DATA)

BIRTH OF MONTE CARLO METHODS

1770- Compte de Buffon (*Historie Naturelle*)

[Random sampling for mathematical problems]



1856- E. Fermi

[Neutron moderation]

1940- Mauchly, Von Neumann
[first electronic computer:ENIAC]

1945- Monte Carlo methods

1950- first software programs in machine code
1960- in Fortran language

thermonuclear and fission devices,
cosmic ray showers,
partial differential equations

1963- Nuclear Medicine- H.O.Anger
performance parameters of new scintillation camera

RANDOM NUMBER GENERATORS

REAL PROCESSES

-radioactive decay

ALGORITHMS

-linear congruential generators

$R_{j+1} = (a\, R_j + c) \bmod m \qquad 0 < R_j \leq m - 1$

R_0 seed

$I_j = R_j / m \qquad 0 < I_j \leq 1$

period $< m = 2^k$

-subctractive method

-Fibonacci

-bit manipulation

SAMPLING x

$$\int_a^b f(x)dx = 1 \qquad a < x < b$$

f: probability density function

$$\int_a^x f(t)dt = F(x) \qquad 0 < F(x) < 1$$

F: cumulative probability function $\rightarrow$ x ?

ESTIMATION OF PRECISION σ

Central limit theorem:

x : random variable

f(x): probability density function

The distribution of Y (average of n measurements of x) approaches a normal distribution, centred in $< x >$ with $\sigma = \sigma_f / \sqrt{n}$

- $< x >$ finite
- statistically independent x measurements
- n large

T: CPU time/event

$$e = \frac{1}{\sigma^2 T}$$

PHOTON TRANSPORT

- photon generated from a radioactive source

  initial data: energy, position, emission direction

- sampled distance to:

  next interacion $$x = \frac{\ln(R)}{\mu}$$

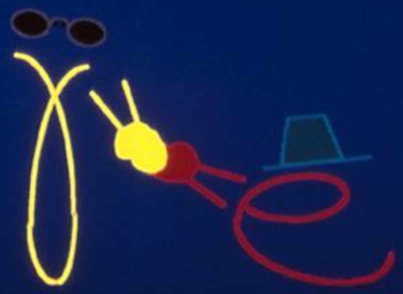
  (no interaction probability at x: $e^{-\mu x}$)

  border of region x_{border}

if $x < x_{border}$ interaction is simulated

else transport is simulated and

PHOTON INTERACTIONS



- photoelectric effect
- Rayleigh scatter (Thompson)
- Compton scatter (Klein-Nishina)
- Pair production

DATA NEEDED:

photon interaction table(Z, energy)
cross section data (Z, energy)
fluoroscence data
Bremsstrahlung probability

- simulation ends if

  photon absorption occurs

  energy of secondary particles < cut-off value

  photon or secondary particles leave area of interest

- position
- emission direction
- kind of interaction

are extracted RANDOMLY

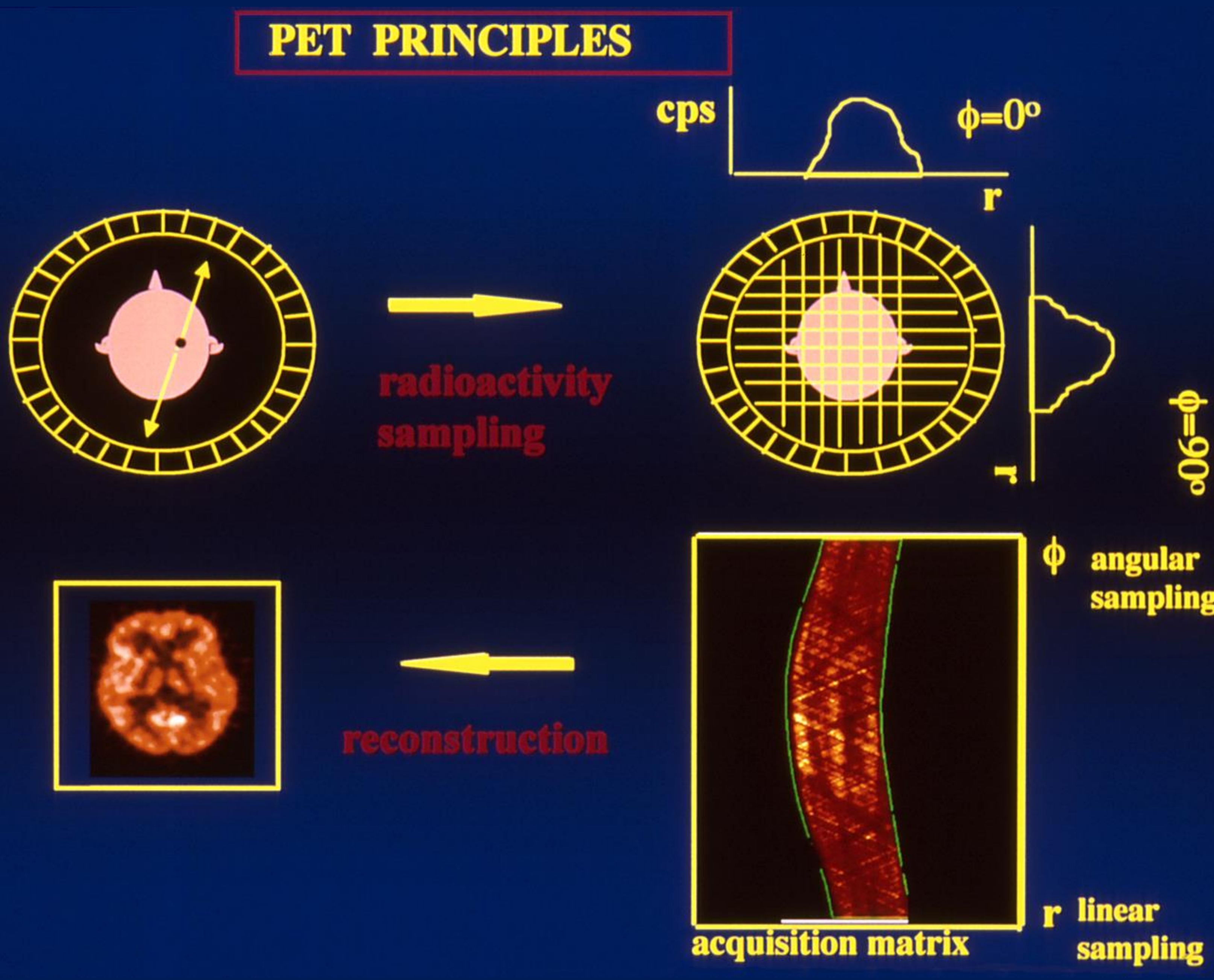
ADVANTAGES OF MONTE CARLO METHODS

- independent control of selected type of interactions

DISADVANTAGES OF MONTE CARLO METHODS

- data amount and computation time

MC METHODS IN PET


PET PRINCIPLES
cps
φ=0º
r
radioactivity
sampling
r
φ=90º
φ
angular
sampling
reconstruction
r
linear
sampling
acquisition matrix

II. DEVELOPMENT OF A MONTE CARLO SOFTWARE PACKAGE FOR POSITRON EMISSION TOMOGRAPHY

- MONTE CARLO SIMULATION OF A 3D PET SCANNER

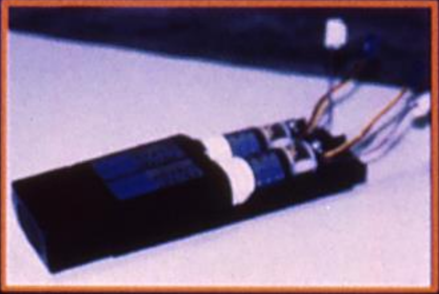
- MONTE CARLO SIMULATION OF PET RADIOACTIVE SOURCES

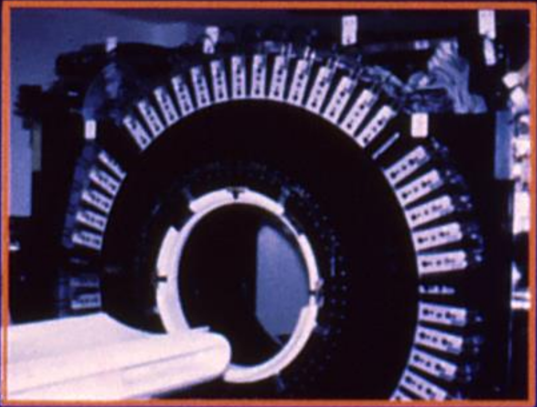
- SIMULATION OF PET RADIOACTIVE EVENT

THE PET SYSTEM
GE ADVANCE

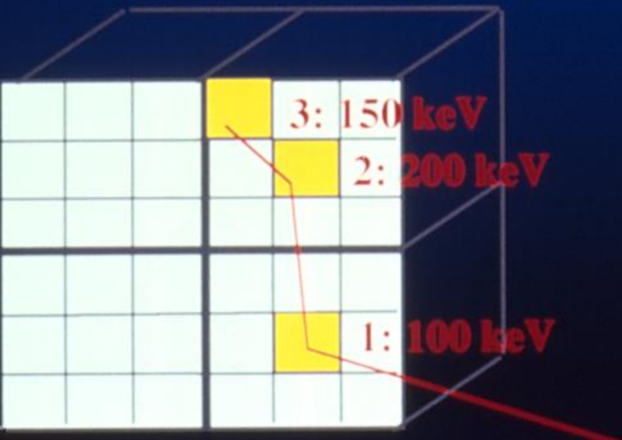
Detection rings : 18
Crystal medium: BGO
Crystals/ring : 672
Crystal dim. : 4 x 8 x 30 mm^3
Detection unit: 6 x 6 crystals
Coincidence crystals : 283
Transaxial FOV : 55 cm
Axial FOV : 15.4 cm
External shields: W, z = 3.5cm
r_i = 36.5, r_e = 59.5

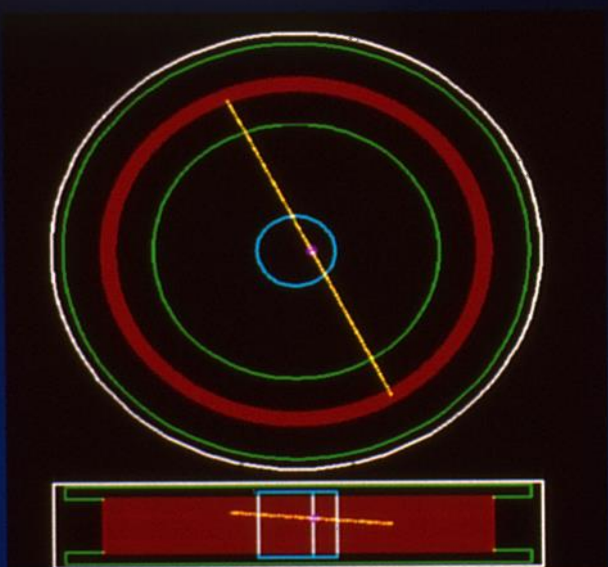
Time window: 12.5 ns
Energy window: 300 - 650 keV
Energy resolution: 33%





SIMULATION OF A 3D PET SCANNER





DETECTION UNIT: block of crystals coupled to PMTs

- the position of the event is given by the crystal in which the foton lost the major fraction of energy (e.g. 2)
- the energy of the event is the sum of the energies lost in the block (e.g. 450 keV)

DETECTION SYSTEM

- rings of crystals
- two external shield rings

SIMULATION OF PET RADIOACTIVE SOURCES

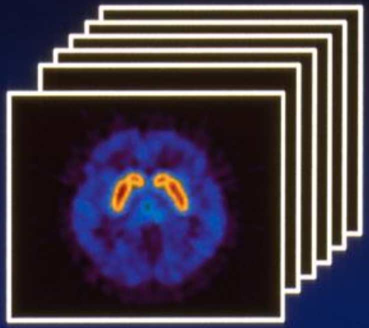
RADIOACTIVITY DISTRIBUTION

ANALYTICAL SOURCES

described by analytical function in the 3D space (i.e. Points, lines, spheres, cylinders...) filled with uniform radioactivty concentration

VOXELIZED SOURCES

described by matrices representing radioactivity concentration i.e. PET EMISSION IMAGES



SIMULATION OF PET RADIOACTIVE SOURCES

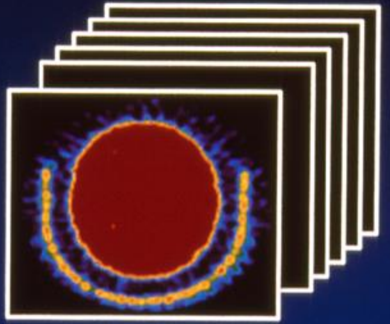
MEDIA DISTRIBUTION

ANALYTICAL SOURCES

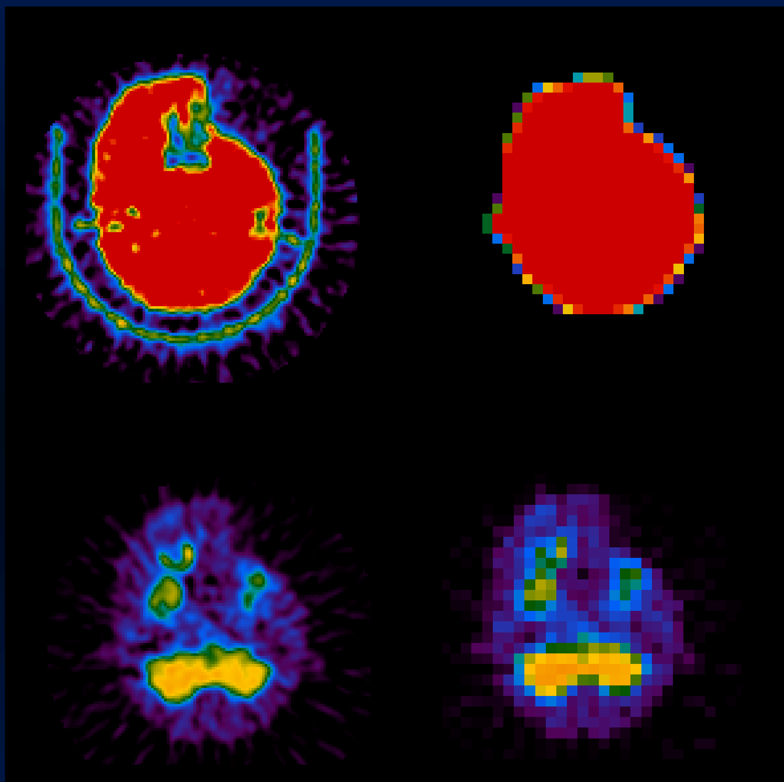
described by analytical function in the 3D space (i.e. Points, lines, spheres, cylinders...) associated to a defined medium (e.g. water...)

VOXELIZED SOURCES

described by matrices representing media attenuation coefficient i.e. PET TRASMISSION IMAGES

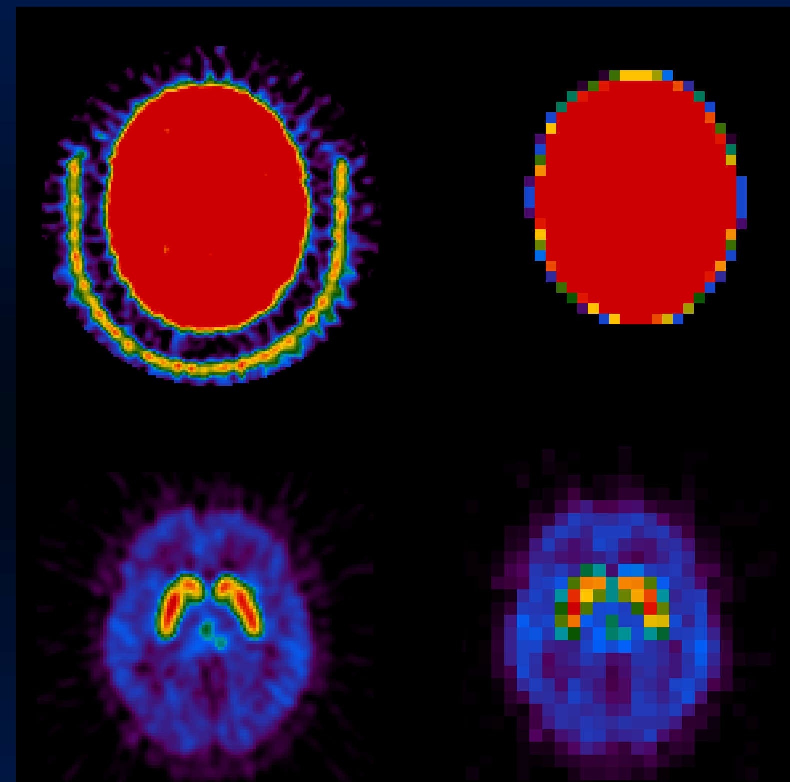


PRE-PROCESSING FOR CLINICAL SIMULATIONS



TR

EM



18F-FDG
RABDOSARCOMA

18F-FESP
DEPRESSO

THE MONTE CARLO SIMULATOR PET-EGS
HUMAN TISSUES

TISSUE
- composition, weight %
- density

→ EGS4 → photon interaction cross section

Element composition, weigth % (from : MIRD phantom)

ELEMENT	BONE $d = 1.5\ g/cm^3$	LUNG $d = 0.3\ g/cm^3$	TISSUE $d = 1.0\ g/cm^3$
H	7.04	10.21	10.47
C	22.79	10.01	23.02
O	48.56	75.96	63.21
N	3.87	2.80	2.34
P	6.94	—	—
Ca	9.91	—	—
others (neglected)	0.89	1.02	0.96

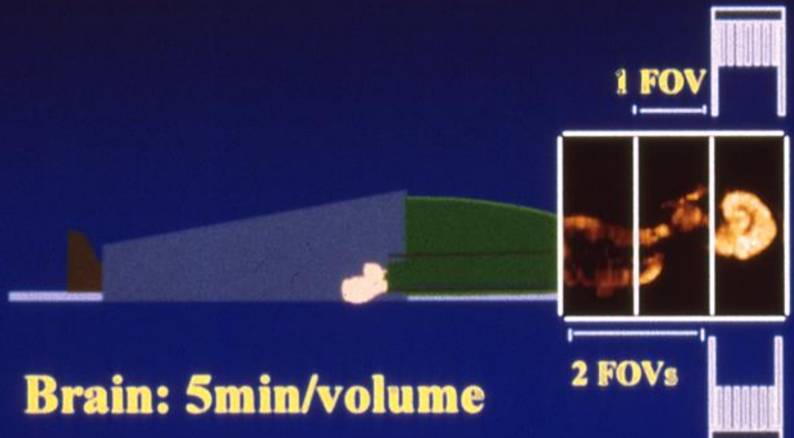
PHANTOM AND CLINICAL STUDIES

- **PHANTOMS**

Uniform Cylinder (UC)

Line Source (LS)

- **CLINICAL STUDIES** Multiple -Volume 2D ^{18}FDG PET



Brain: 5min/volume

Thorax: 5min/volume

SIMULATION OF RADIOACTIVE EVENT

PHYSICAL MODEL

True coincidence → Event detection of two photons from the same annihilation no scatter in the source medium.

Scatter coincidence → Event detection of two photons from the same annihilation scatter in the source medium :

Klein-Nishina formula

$$\frac{d\sigma}{d\Omega} = Zr_0^{\,2}\left(\frac{1}{1+\alpha(1-\cos\theta)}\right)\left(\frac{1+\cos^2\theta}{2}\right)\left(1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos 2\theta)[1+\alpha(1-\cos\theta)]}\right)$$

θ = scatter angle ; $\alpha = \frac{h\nu}{m_0c^2}$; r_0 = classical electron radius ; Z = target atomic number

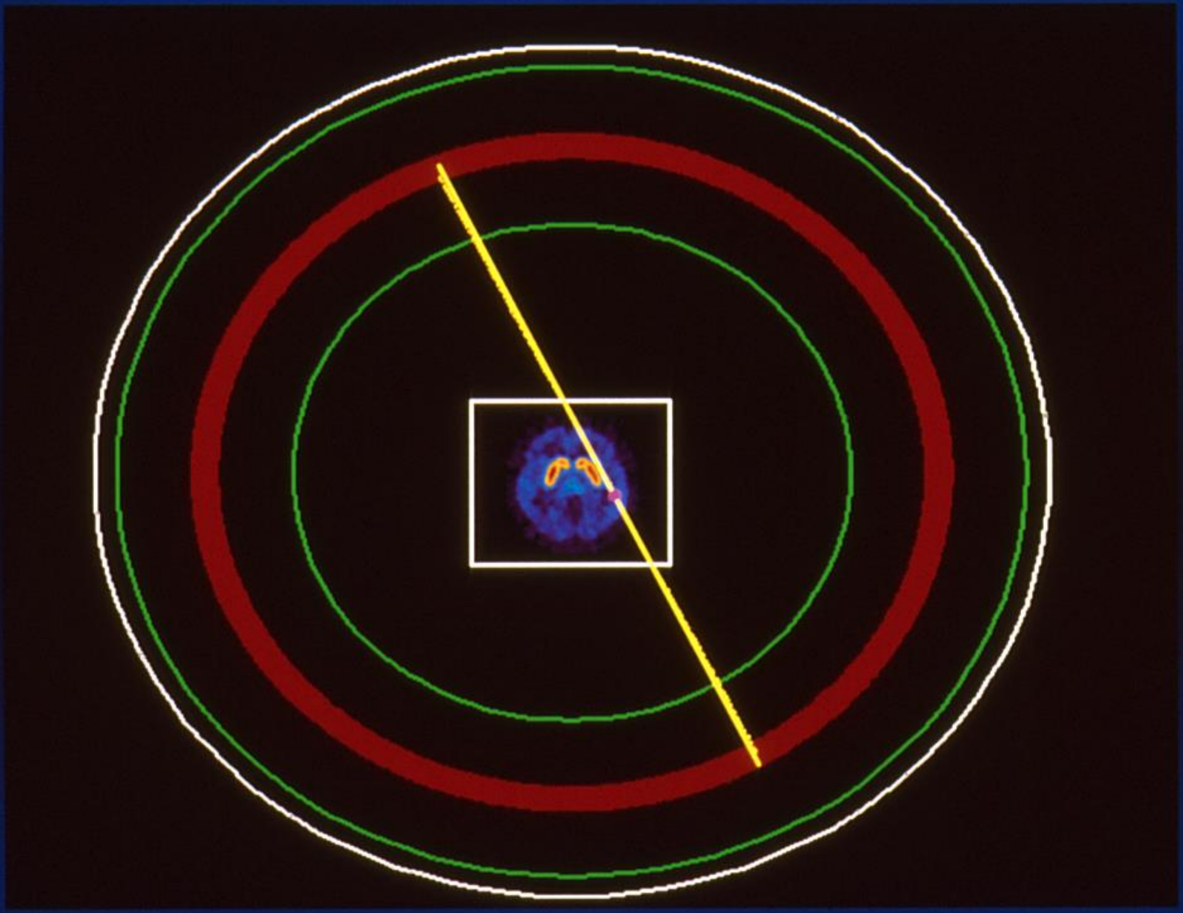
m_0 = classical electron mass ; $\frac{d\sigma}{d\Omega}$ = scattering cross section

Random coincidence → Event detection of two photons not from the same annihilation

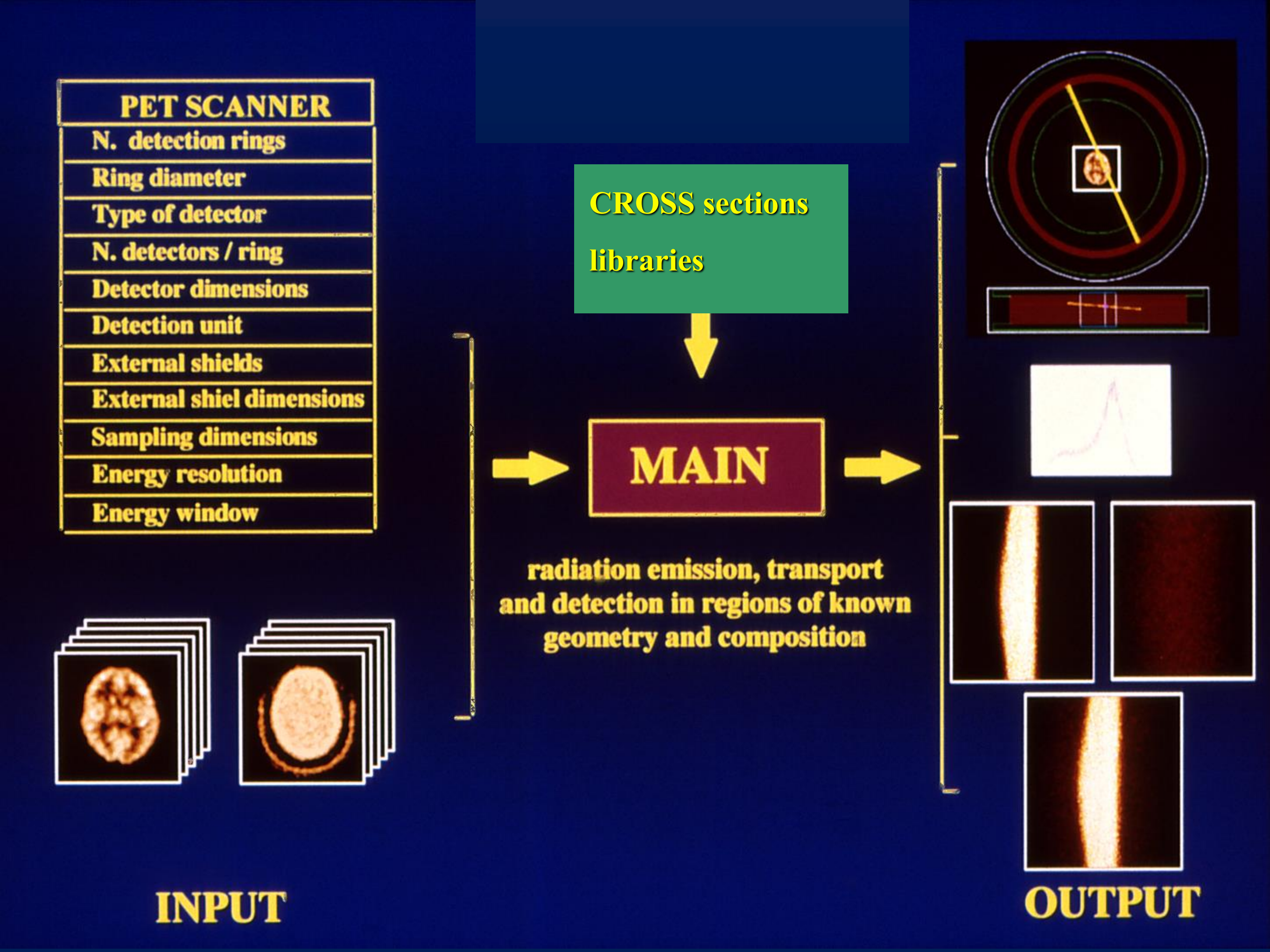
Distribution function of time intervals

$$P(t)\,dt = r\,e^{-rt}\,dt \qquad r = \text{countrate}$$

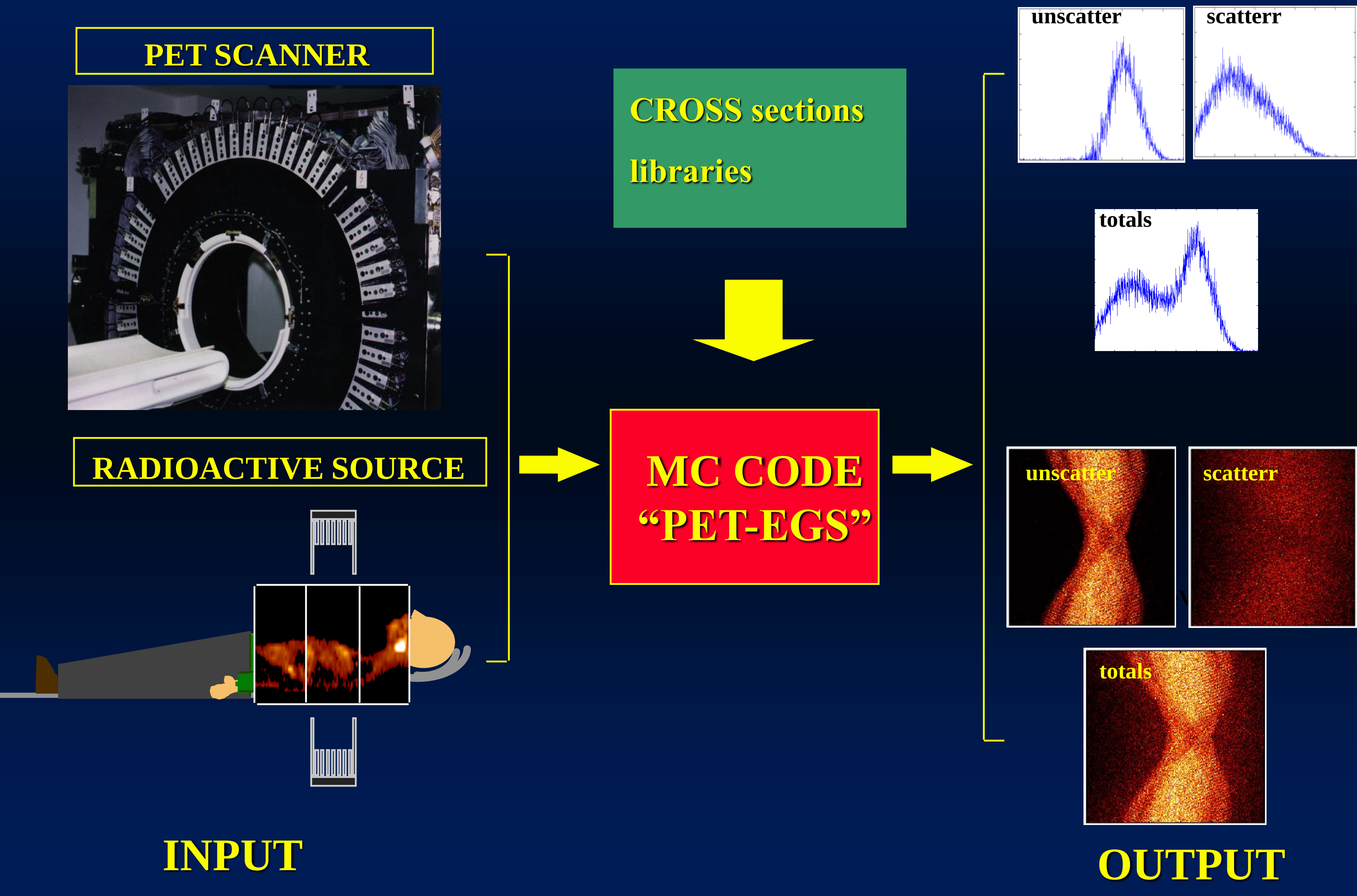
SIMULATION OF COINCIDENCE EVENTS



Simulated physical interactions:
- **photoelctric effect**
- **Compton scatter**
- **Rayleigh scatter**

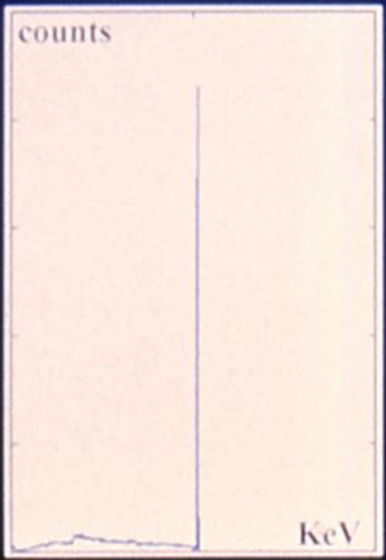

PET SCANNER
N. detection rings
Ring diameter
Type of detector
N. detectors / ring
Detector dimensions
Detection unit
External shields
External shiel dimensions
Sampling dimensions
Energy resolution
Energy window
CROSS sections
libraries
MAIN
radiation emission, transport and detection in regions of known geometry and composition
INPUT
OUTPUT

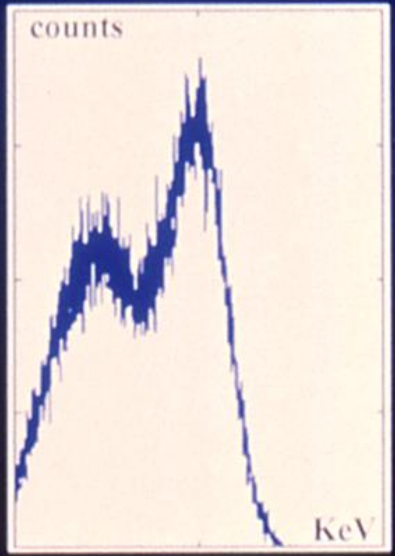
MONTE CARLO SIMULATION OF 3D PET STUDIES



POST-PROCESSING

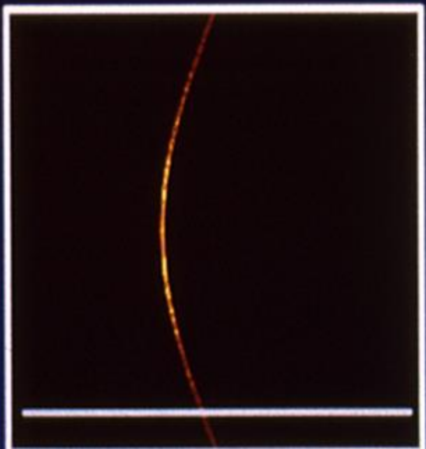
ENERGY RESOLUTION

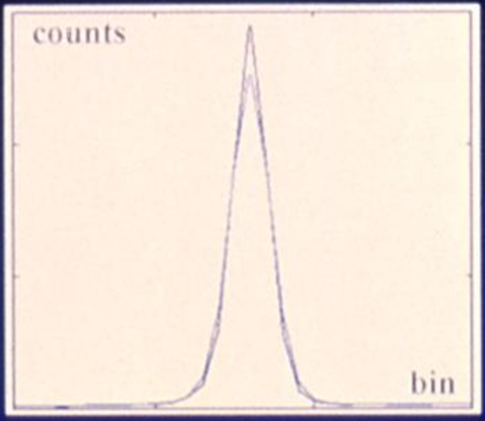




- Convolution with a gaussian function

SPATIAL RESOLUTION

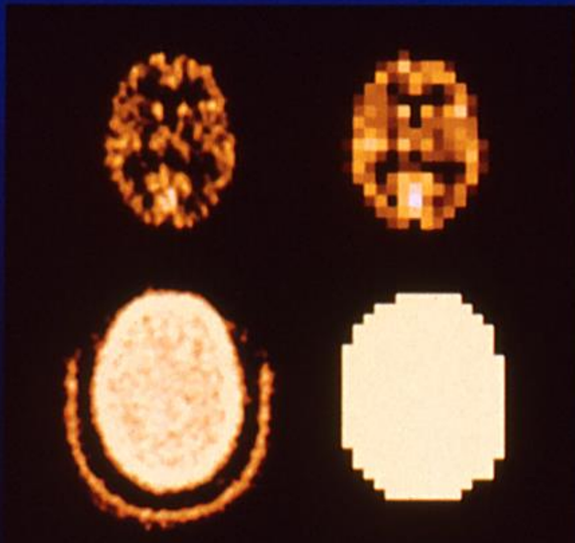




- Convolution with a gaussian function

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$$

σ_1 finite positron range
σ_2 noncolinearity effect
σ_3 block blurring effect



EM

TR

Brain

Thorax

STATISTICS OF SIMULATIONS

Analytical	UC	200 Mevents
	LS	200 Mevents
Voxelized	Brain 2 FOVs	400 Mevents
	Thorax 3 FOVs	600 Mevents

PET_EGS EFFICIENCY

Program languages : Fortran, C, matlab

Operating system : UNIX

Workstations: network of SUN SPARCstations

CPU time for:

- a uniform cylinder (d=20cm): 18 ms/event
  1 000 000 events/ h
- a brain study : 50 ms/event
  300 000 events/ h