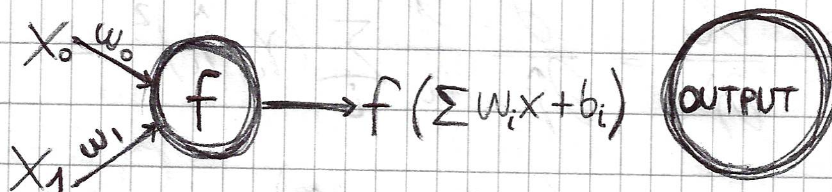
Neural Networks

NEURAL NETWORKS

SINGLE NEURON

ONE LAYER

FORWARD PROPAGATION



linear + activation

$wx+b \qquad f \to \sigma$

IN PRACTICE we have INPUT + OUTPUT Layers

$$\hat{y} = \text{OUTPUT} = \sigma\left(\Sigma w_i x_i + b_i\right)$$

$$= \frac{1}{1+e^{-\Sigma w x + b}} \qquad \sigma = 1/_{1+e^{-x}}$$

Qual è l'errore?

Loss $\longleftrightarrow$ Cost function

CALCULATE THE ERROR

... usiamo la mean-squared error

$$C = (y-\hat{y})^2$$

Come cambia la Cost function rispetto alle variazioni di W e b?

BACKWARD PROPAGATION

$$\frac{\partial C}{\partial w_i} = \frac{\partial C}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \cdot} \cdot \frac{\partial \cdot}{\partial w_i}$$

argomento della f

Ⓐ $$\frac{\partial C}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} \frac{1}{n} \sum_i (y_i - \hat{y}_i)^2 =$$

$$= \frac{2}{n} \sum_i (y_i - \hat{y}_i)$$

Ⓑ $$\frac{\partial \sigma(\cdot)}{\partial \cdot} = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{(1+e^{-x}) - 1}{1+e^{-x}} \cdot \frac{1}{1+e^{-x}} =$$

$$= (1 - \sigma(x)) \cdot \sigma(x)$$

Ⓒ $$\frac{\partial \cdot}{\partial w_i} = \frac{\partial(\sum wx + b)}{\partial w_i} = x_i$$

$$\Longrightarrow \boxed{\frac{\partial C}{\partial w_i} = \frac{2}{n} \times \sum (y - \hat{y}) \times \sigma(\cdot) \times (1 - \sigma(\cdot)) \times x_i}$$

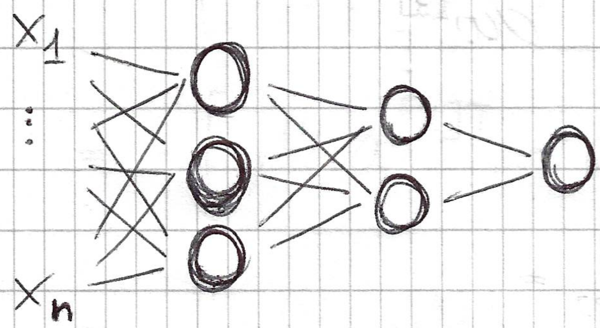
What about Bias?

$$\frac{\partial C}{\partial B} =$$

$$\frac{2}{n} \times \sum (y - \hat{y}) \times \sigma(\cdot) \times (1 - \sigma(\cdot))$$

$\Longrightarrow$ Adjustment $\rightarrow$ Weights = weight + adjustment
Bias = Bias + adjustment

+ Hidden Layers = Deep Learning



FORWARD PROPAGATION

$$z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$
$$z^{[3]} = w^{[3]}a^{[2]} + b^{[3]}$$
$$a^{[3]} = \sigma(z^{[3]})$$

Define the loss function ~~for the Logistics~~

$$\boxed{\mathcal{L}_{TOT}} = \frac{1}{m}\sum \mathcal{L}^{(i)}$$ → if we have multiple (batch) samples

$$\mathcal{L}^{(i)} = -\left[y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)}\log(1-\hat{y}^{(i)})\right]$$

LOGISTIC LOSS

$$\frac{\partial L}{\partial w^{[3]}} \rightarrow \frac{\partial L}{\partial w^{[2]}} \rightarrow \frac{\partial L}{\partial w^{[3]}}$$

Backward...

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial L}{\partial ?} \cdot \frac{\partial ?}{\partial w^{[3]}} =$$

$$= \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial w^{[3]}}$$

$$\frac{\partial L}{\partial w^{[2]}} = \boxed{\boxed{\frac{\partial L}{\partial z^{[3]}}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial w^{[1]}} = \frac{\partial L}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} \cdot \frac{\partial a^{[1]}}{\partial z^{[1]}} \cdot \frac{\partial z^{[1]}}{\partial w^{[1]}}$$

$a^{[3]}$

$$\frac{\partial \mathcal{L}}{\partial w^{[3]}} = -\Big[y^{(i)} \frac{\partial}{\partial w^{[3]}} \left(\log \sigma (w^{[3]} a^{[2]} + b^{[3]}) \right) +$$

$$+ (1-y^{(i)}) \frac{\partial}{\partial w^{[3]}} \left(\log(1 - \sigma(w^{[3]} a^{[2]} + b^{[3]})) \right) \Big]$$

$$= -\Big[y^{(i)} \frac{1}{a^{[3]}} \cdot a^{[3]} \cdot (1-a^{[3]}) \cdot a^{[2]T} +$$

$$+ (1-y^{(i)}) \frac{1}{1-a^{[3]}} \cdot (-1) \cdot a^{[3]} \cdot (1-a^{[3]}) \cdot a^{[2]T}$$

$$= -\left[y^{(i)} \left(1 - \cancel{a^{[3]}}\right) a^{[2]T} - \left(1 - \cancel{y^{(i)}}\right) a^{[3]} \cdot a^{[2]T} \right]$$

$$= -\left[y^{(i)} a^{[2]T} - a^{[3]} a^{[2]T} \right]$$

$$= -\left[y^{(i)} - a^{[3]} \right] a^{[2]T}$$

$$\hookrightarrow \frac{\partial L}{\partial w^{[3]}} = -\frac{1}{m} \sum_{i=1}^{m} \left(y^{(i)} - a^{[3]} \right) a^{[2]T}$$

$$\frac{\partial L}{\partial w^{[2]}} = \underbrace{\frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}}}_{(a^{[3]} - y)} \cdot \underbrace{\frac{\partial z^{[3]}}{\partial a^{[2]}}}_{w^{[3]T}} \cdot \underbrace{\frac{\partial a^{[2]}}{\partial z^{[2]}}}_{\text{Sigmoid}\ a^{[2]}(1 - a^{[2]})} \cdot \underbrace{\frac{\partial z^{[2]}}{\partial w^{[2]}}}_{a^{[1]T}}$$

$$\frac{\partial L}{\partial w^{[2]}} = \left(a^{[3]} - y\right) \cdot w^{[3]T} \times a^{[2]} \left(1 - a^{[2]}\right) a^{[1]T}$$

$$= w^{[3]T} \times a^{[2]} \left(1 - a^{[2]}\right) \cdot \left(a^{[3]} - y\right) \cdot a^{[1]T}$$

$$\frac{\partial L}{\partial w^{[2]}} = \frac{1}{m} \sum_{i} \frac{\partial L}{\partial w^{[2]}}$$