Image Processing e Machine Learning
Basi e Possibili Applicazioni ai Beni Culturali

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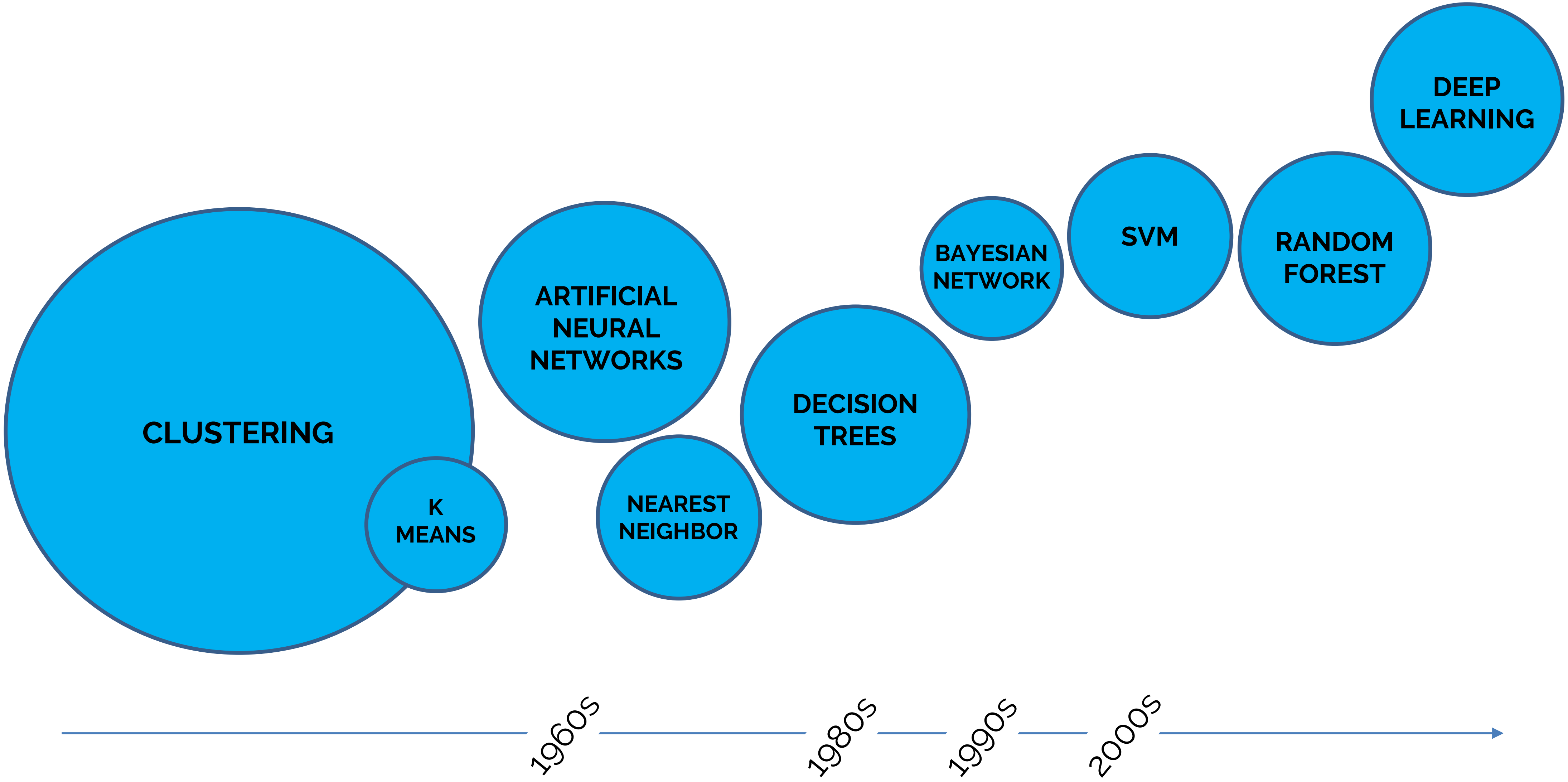
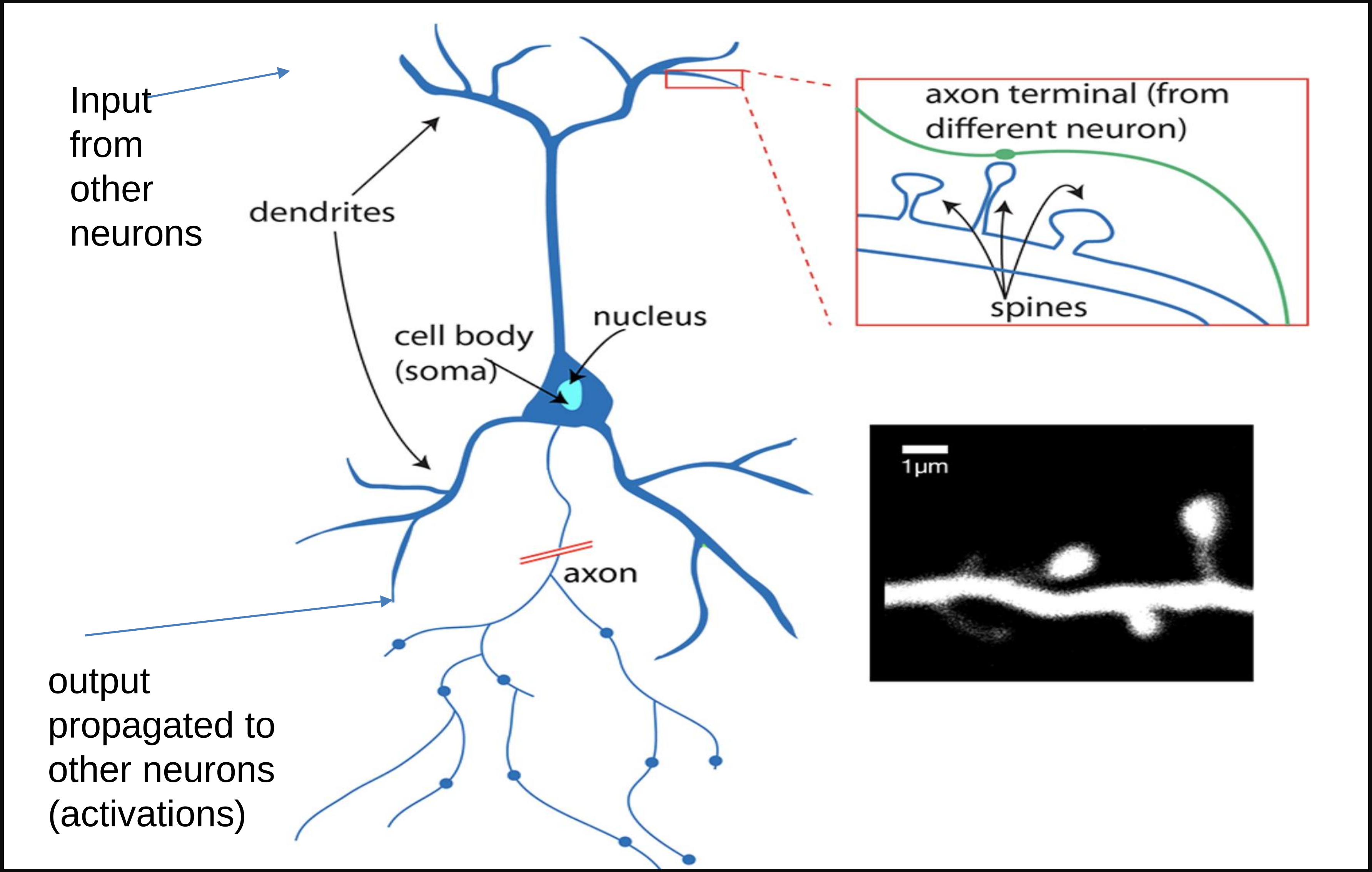
Machine learning



NEURAL NETWORKs

A Biological Neuron



'healthy' vs overweighting men

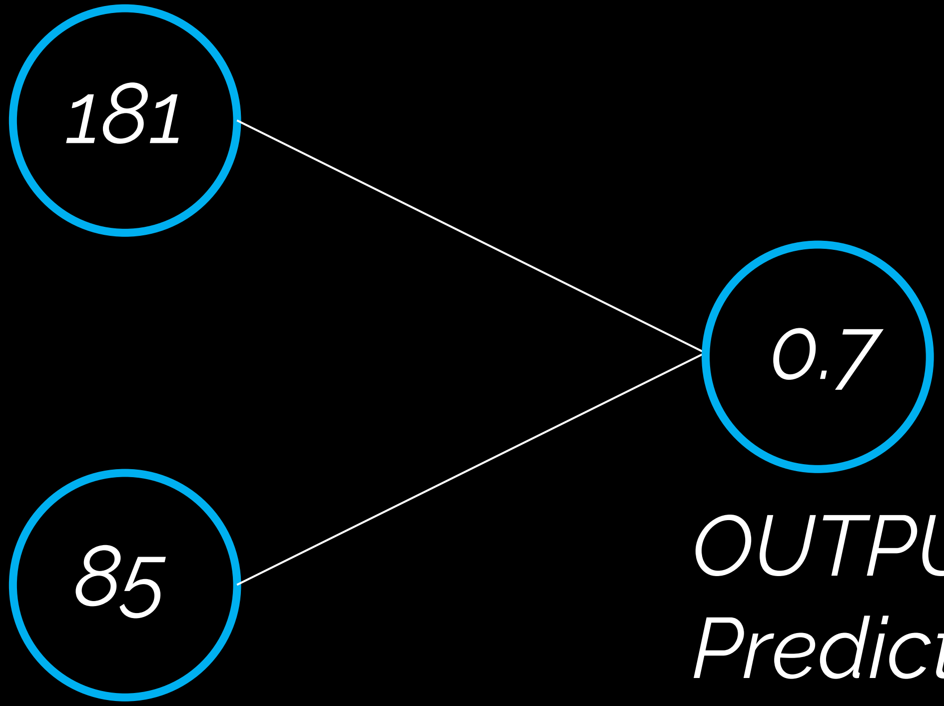
H (cm)	181	184	172	160	170	187	184	176	190
AC (cm)	85	94	102	80	98	110	116	77	84

H *OW*

0 *1*

Neural Network

$H = a_1$ $AC = a_2$



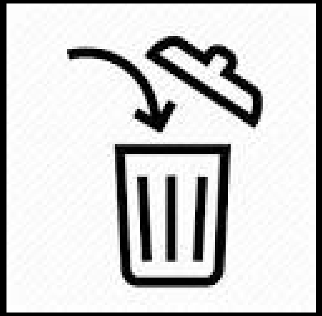
OUTPUT

Predicted

the activation of neuron

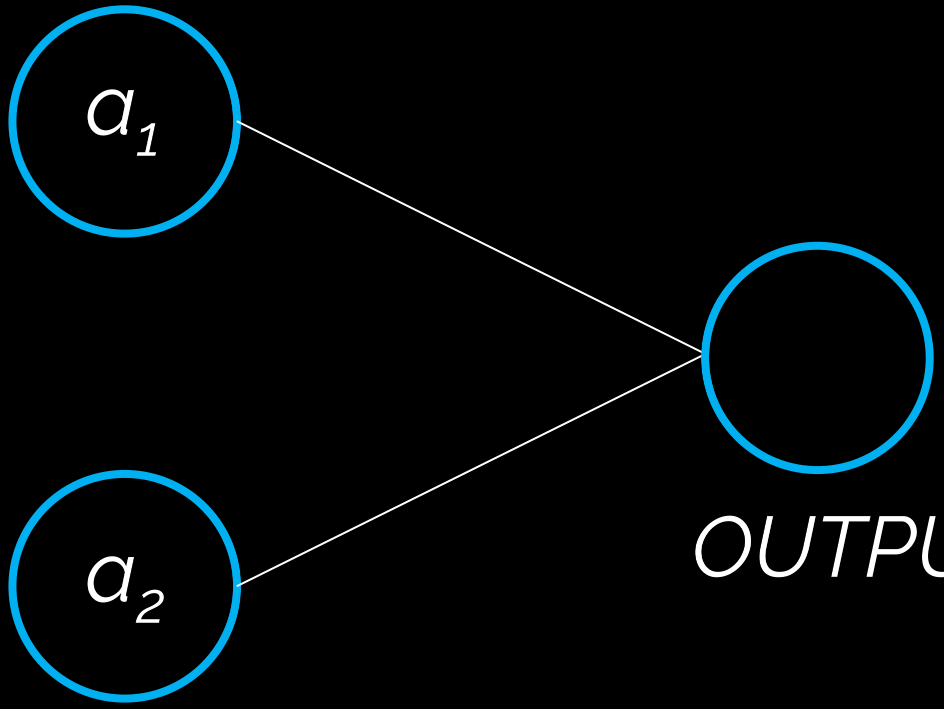
INPUT

H	OW
0	1



Neural Network

$H = a_1$ $AC = a_2$



OUTPUT

$OUTPUT = f\,(INPUT)$

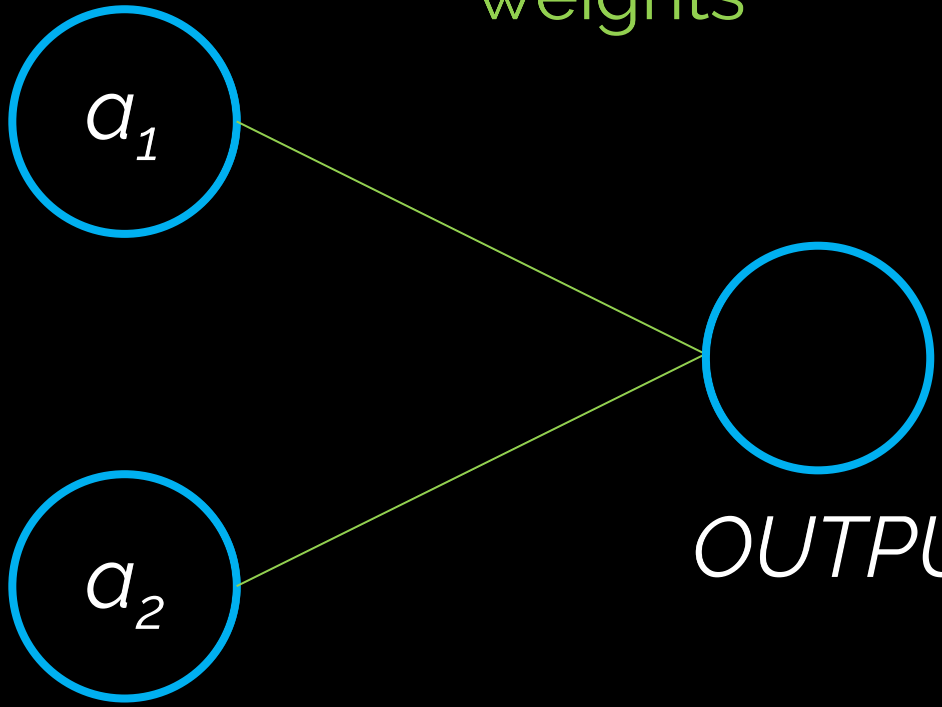
INPUT

Weights and Bias

Neural Network | Weights and Bias

$H = a_1$ $AC = a_2$

weights



Bias (for inactivity)

$a_1 w_1 + a_2 w_2 + b$

OUTPUT

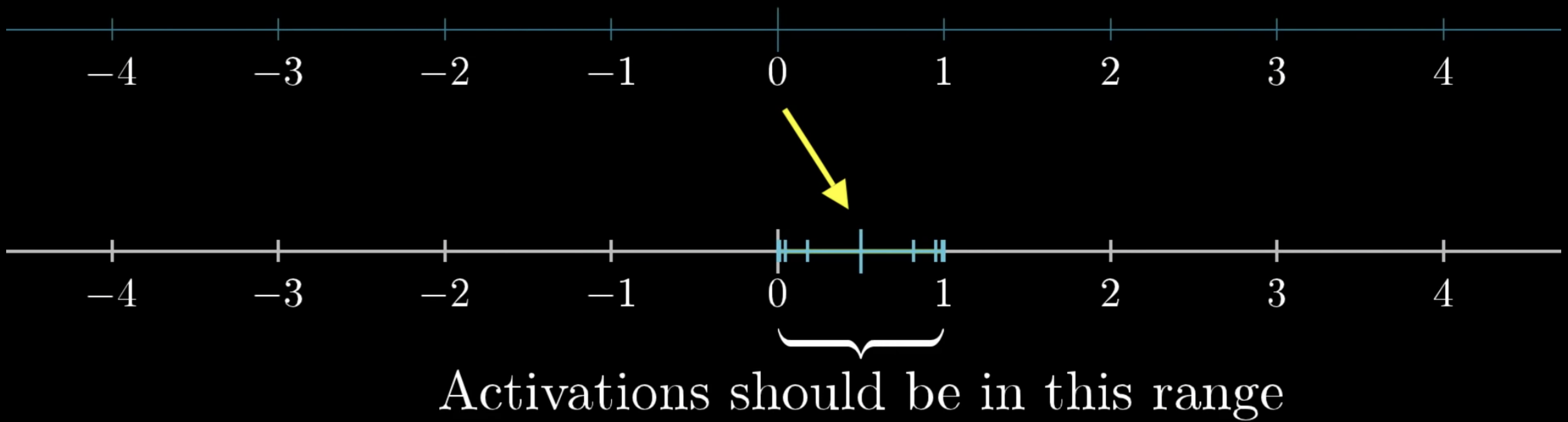
$OUTPUT = f\,(INPUT)$

INPUT

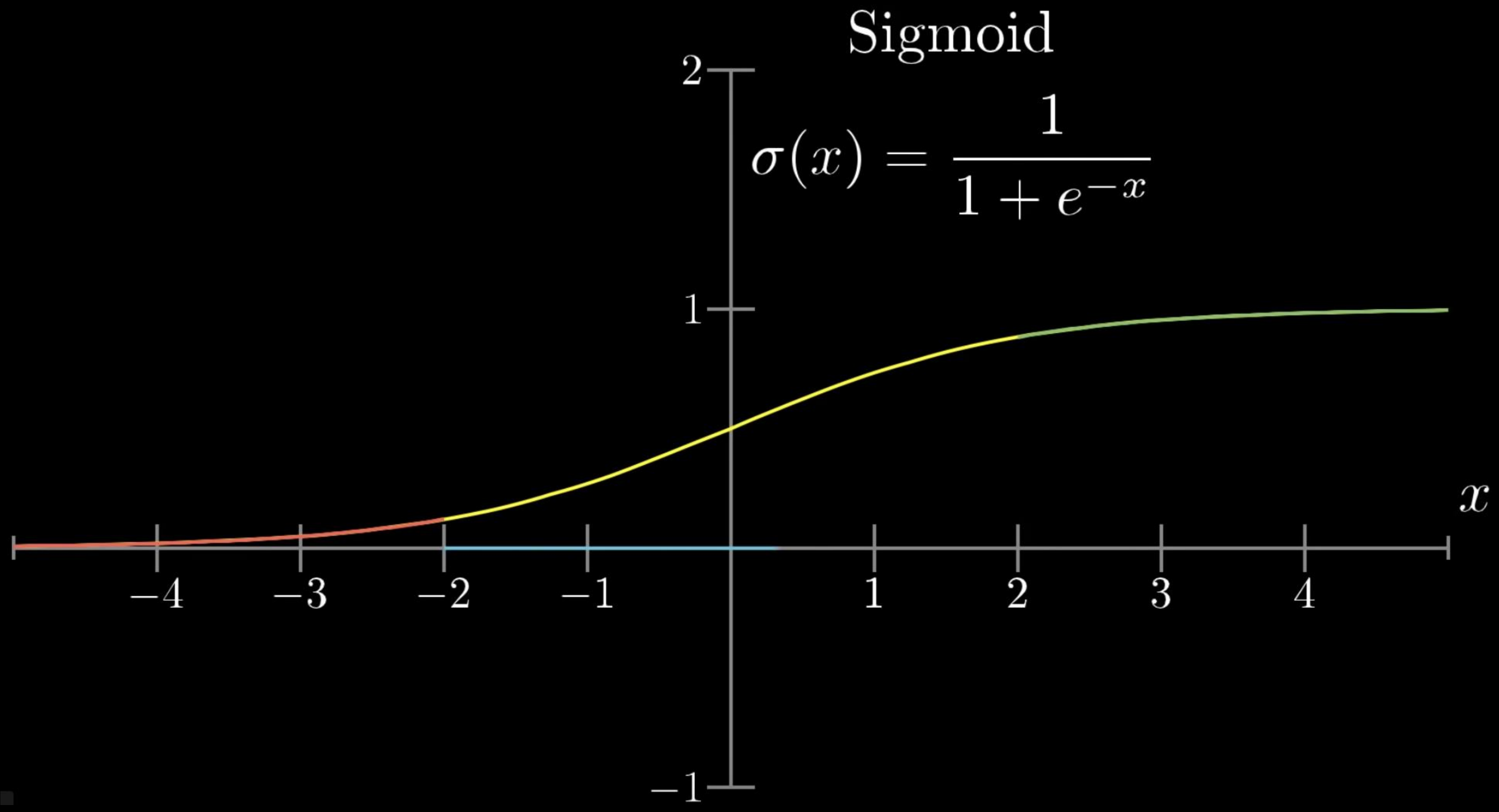
The Activation Function

Neural Network

$$w_1 a_1 + w_2 a_2$$



Neural Network

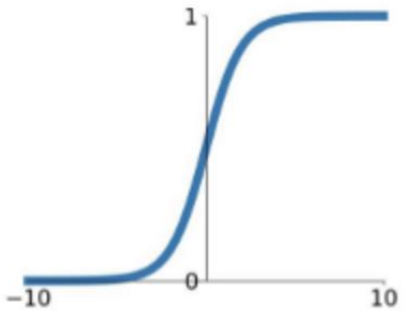


Neural Network

Other functions that progressively change from 0 to 1 with no discontinuity

Sigmoid

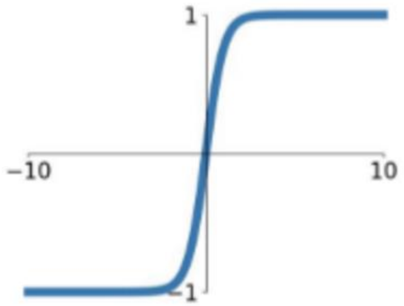
$$\sigma(x) = \frac{1}{1+e^{-x}}$$



Hyperbolic tangent funtion

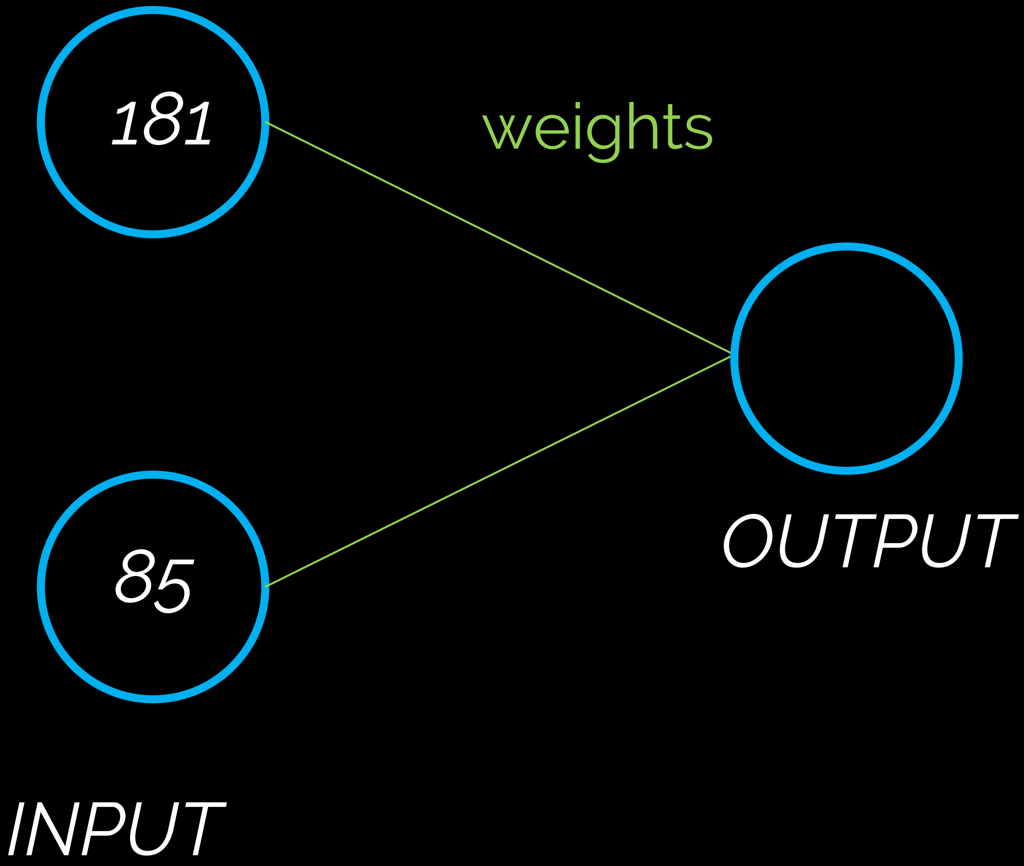
tanh

$$\tanh(x)$$



Neural Network

$H = a_1$ $AC = a_2$



Forward propagation

Activation function

$$Sigmoid\,(181w_1 + 85w_2 + b)$$

$$OUTPUT = f\,(INPUT)$$

w_1 ? w_2 ? b ?

INPUT

a_1 a_2

Cost Function?

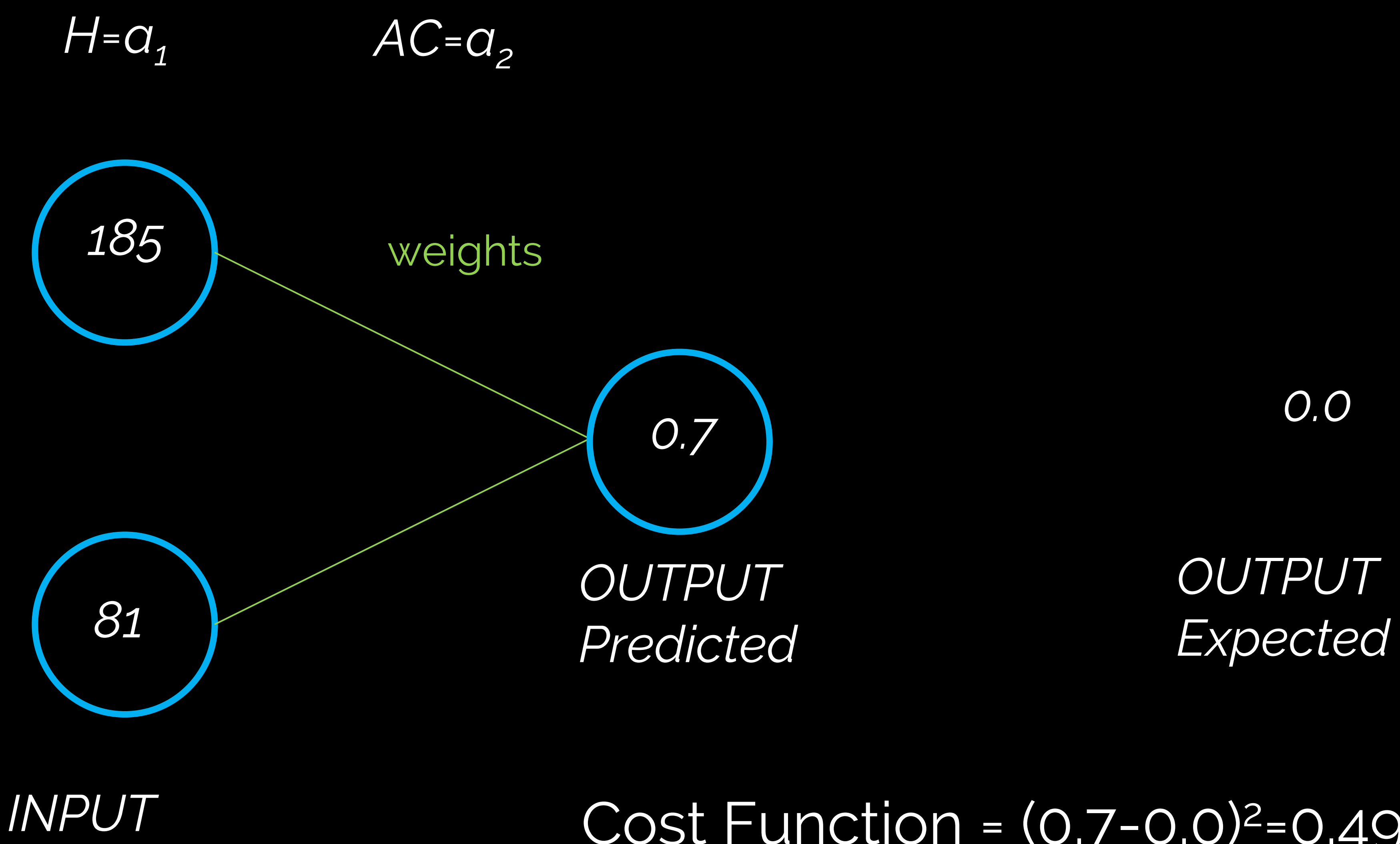
$$Sigmoid\,(a_1 w_1 + a_2 w_2 + b)$$

OUTPUT

Cost Function = $(\text{Predicted}-\text{Expected})^2$

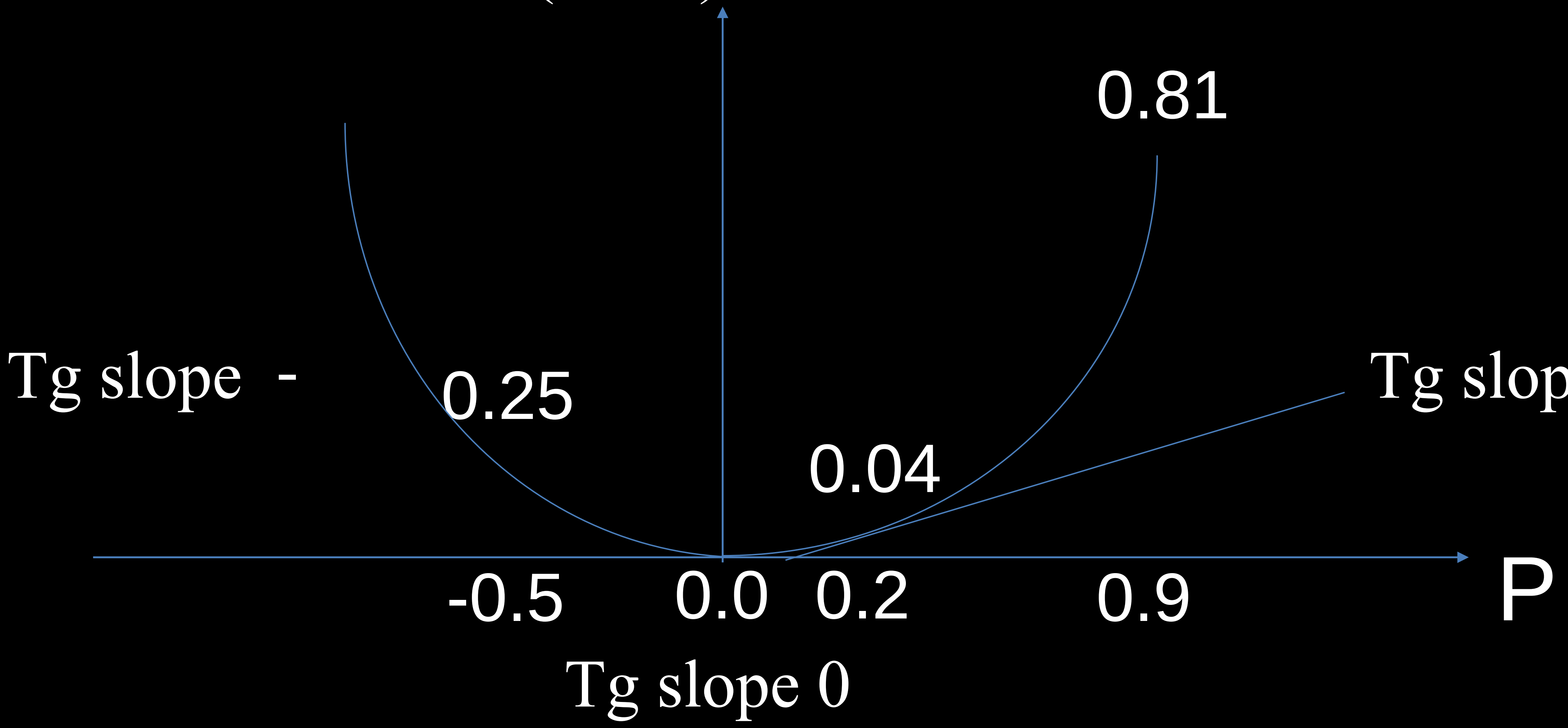
Squared error cost

Neural Network



Cost Function = $(0.7-0.0)^2=0.49$

The Cost Function = $(P-0.0)^2$



Neural Network

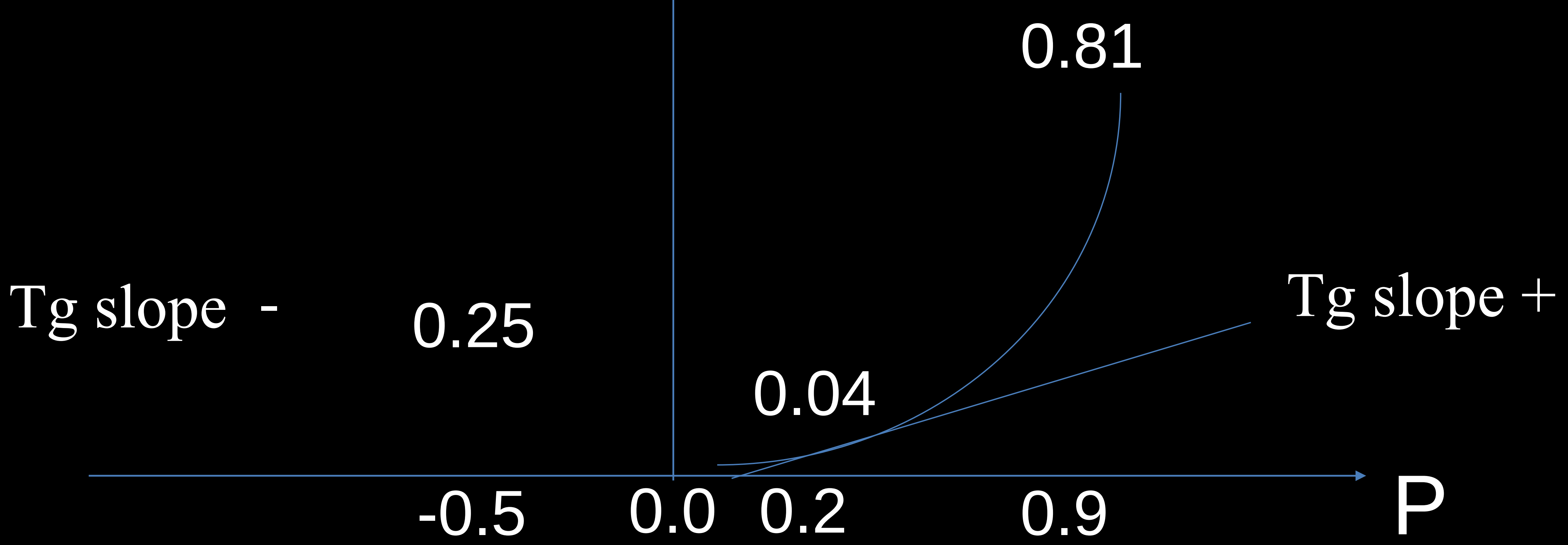
If the slope is + we must decrease P of a fraction of the slope

If the slope is - we must increase P of a fraction of the slope

If the slope is 0 we have the solution

The Learning Rate

The Cost Function = $(P-0.0)^2$



Next P = P – LR*(Tg slope)P   LR*=Learning Rate

Neural Network

Next P = P – LR* (slope Tg)P

Slope Tg= derivative of the Cost Function vs P= 2(P-E)

In our case

Neural Network

LR determines how much weights are changed every time
Too high→ output wanders around the expected solutions
Too low → output fails to converge to acceptable solution

The Training

Neural Network

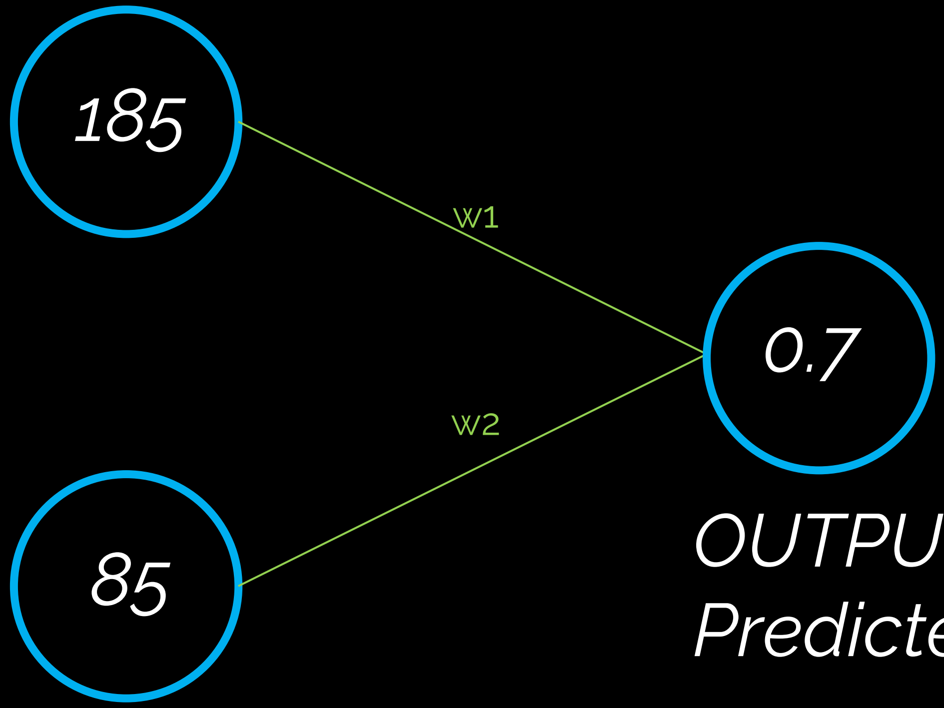
Different training methods

Supervised

learning rule that trains the neural network on already known correct output

Neural Network

$H = a_1$ $CA = a_2$



OUTPUT Predicted

0.0

OUTPUT Expected

INPUT

Neural Network

w1 = 0.0006

w2 = 0.0002
b=1

0.7 =Sigmoid (185x0.0006+ 85x0.0002+1) = Sigmoid (0.111+0.281+1)= Sigmoid (1.281)

the back propagation

Computation of the error (STEP 2)

The Cost Function = $(0.7-0.0)^2=0.49$

$= (\,\mathit{Sigmoid}\,(185 \times 0.0006 + 85 \times 0.0002 + 1) - 0.0)^2$

Weights and Bias Adjustment: the Back Propagation

Neural Network

Adjust weights and b to reduce the error (STEP 3)

$0.0005 = 0.0006 - 0.0001$

$0.0001 = 0.0002 - 0.0001$

$0.0008 = 1 - 0.0002$

$w_1 = w_1 - LR^* \text{ slope} = w_1 - LR^* \text{ derivative}_{w1} \text{ of the Cost Function}$

$w_2 = w_2 - LR^* \text{ slope} = w_2 - LR^* \text{ derivative}_{w2} \text{ of the Cost Function}$

$b = b - LR^* \text{ slope} = b - LR^* \text{ derivative}_b \text{ of the Cost Function}$

Neural Network

$$w1 = w1 - LR^* \frac{\partial costo}{\partial w1}$$

$$w2 = w2 - LR^* \frac{\partial costo}{\partial w2}$$

$$b = b - LR^* \frac{\partial costo}{\partial b}$$

Neural Network

$$\frac{\partial \text{costo}}{\partial \text{w1}} = \frac{\partial \text{costo}}{\partial \text{p}} \times \frac{\partial \text{p}}{\partial \text{t}}$$

$$\frac{\partial}{\partial \text{t}} \text{sigmoide(t)} = \text{sigmoide(t)(1 - sigmoide(t))}$$

$$\frac{\partial \text{costo}}{\partial \text{w1}} = \frac{\partial \text{costo}}{\partial \text{p}} \times \frac{\partial \text{p}}{\partial \text{t}} \times \frac{\partial \text{t}}{\partial \text{w1}}$$

$$\frac{\partial \text{costo}}{\partial \text{w1}} = \text{2(sigmoide(2w1+5w2+b) - 1) x sigmoide(2w1+5w2+b)(1-sigmoide(2w1+5w2+b)) x 2}$$

Neural Network

$$\frac{\partial costo}{\partial w1} = 2(\text{sigmoide}(2w1+5w2+b) - 1) \times \text{sigmoide}(2w1+5w2+b)(1-\text{sigmoide}(2w1+5w2+b)) \times 2$$

$$\frac{\partial costo}{\partial w2} = 2(\text{sigmoide}(2w1+5w2+b) - 1) \times \text{sigmoide}(2w1+5w2+b)(1-\text{sigmoide}(2w1+5w2+b)) \times 5$$

$$\frac{\partial costo}{\partial b} = 2(\text{sigmoide}(2w1+5w2+b) - 1) \times \text{sigmoide}(2w1+5w2+b)(1-\text{sigmoide}(2w1+5w2+b)) \times 1$$

Neural Network

$$\frac{\partial costo}{\partial w1} = \frac{\partial costo}{\partial p} \times \frac{\partial p}{\partial t} \times \frac{\partial t}{\partial w1}$$

$$\frac{\partial costo}{\partial w1} = 2(\text{sigmoide}(2w1+5w2+b) - 1) \times \text{sigmoide}(2w1+5w2+b)(1-\text{sigmoide}(2w1+5w2+b)) \times 2$$

$$\frac{\partial costo}{\partial w2} = \frac{\partial costo}{\partial p} \times \frac{\partial p}{\partial t} \times \frac{\partial t}{\partial w2}$$

$$\frac{\partial costo}{\partial w2} = 2(\text{sigmoide}(2w1+5w2+b) - 1) \times \text{sigmoide}(2w1+5w2+b)(1-\text{sigmoide}(2w1+5w2+b)) \times 5$$

$$\frac{\partial costo}{\partial wb} = \frac{\partial costo}{\partial p} \times \frac{\partial p}{\partial t} \times \frac{\partial t}{\partial b}$$

$$\frac{\partial costo}{\partial b} = 2(\text{sigmoide}(2w1+5w2+b) - 1) \times \text{sigmoide}(2w1+5w2+b)(1-\text{sigmoide}(2w1+5w2+b)) \times 1$$

Neural Network

w_1 = 0.0006 w_2= 0.0002 b= +1

The Cost Function ($w_1\ w_2\ b$) = (Sigmoid-0.0)^2=0.49

Back propagation

Weights and bias adjustment

w_1 = 0.0005 w_2= 0.0001 b=0.0008

The Cost Function ($w_1\ w_2\ b$) = (Sigmoid-0.0)^2=0.43

Neural Network

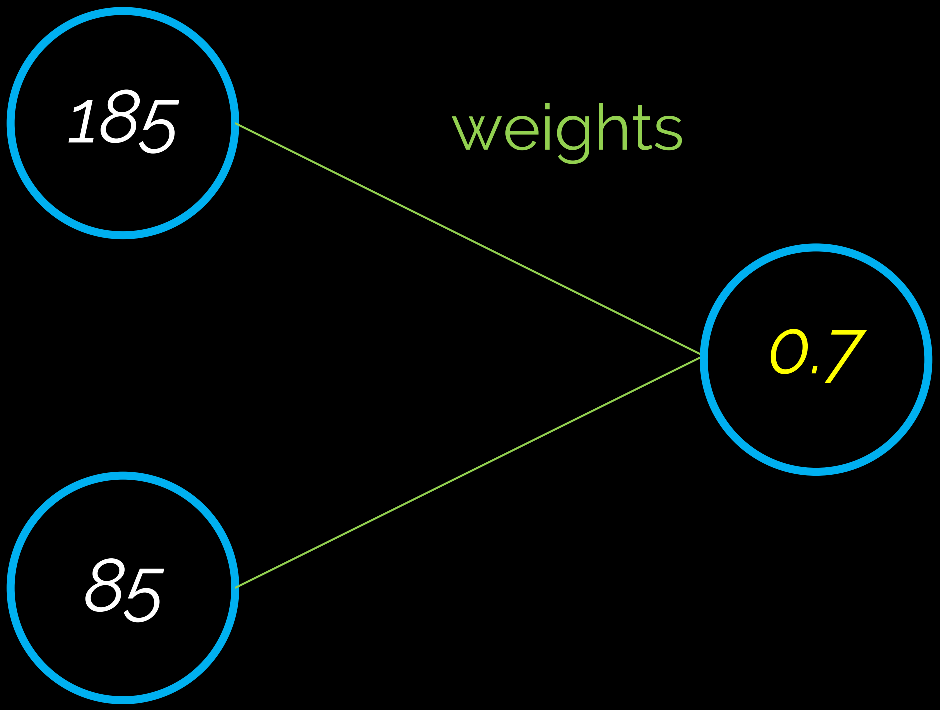
$$Sigmoid\ (185 \times 0.0005 + 85 \times 0.0001 + 1) = Sigmoid\ (0.0925 + 0.085 + 0.008) = Sigmoid\ (1.178) = 0.65$$

the back propagation

Neural Network

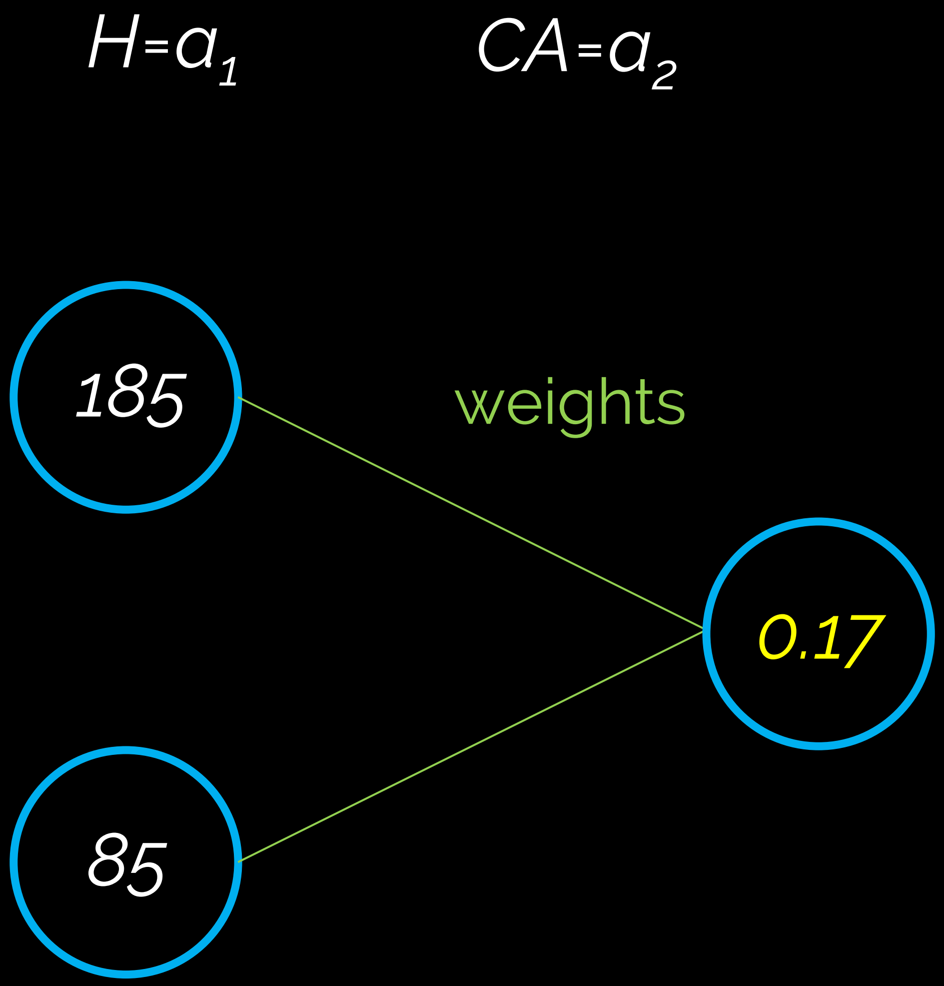
$H = a_1$ $CA = a_2$

Forward propagation



Activation function

$$Sigmoid\,(185w_1 + 85w_2 + b)$$



Back propagation

Cost function

$$\left(Sigmoid\left(185w_1 + 85w_2 + b\right) - 0.0\right)^2$$

Training set: ‘healthy’ vs overweighting men

H	185	178	184	190	169	188	185	192	175	H	168	179	170	180	169	189	188	190	178	H	185	192	175	185	192	168	190	186	168
C	85	94	102	80	98	110	116	77	84	C	88	90	68	120	112	109	89	92	94	C	77	103	119	98	99	100	98	96	78

H	188	190	178	169	188	185	192	190	188	H	185	192	168	179	188	185	192	175	180	H	169	189	188	190	169	188	185	192	188
C	90	79	68	120	98	99	102	104	89	C	87	90	102	98	96	78	90	103	110	C	90	89	90	79	68	120	96	99	79

H	168	179	170	182	168	178	190	188	177	H	175	188	178	169	176	168	179	170	180	H	178	193	190	169	188	185	177	186	165
C	113	90	90	79	68	120	98	96	78	C	119	98	106	116	77	103	119	98	77	C	90	79	68	120	112	109	89	92	78

H	190	170	179	170	180	179	177	185	159	H	176	168	179	188	185	177	168	179	188	H	176	181	174	185	192	175	185	192	169
C	68	120	98	96	77	103	119	98	89	C	104	77	98	96	78	90	79	68	120	C	102	98	96	78	82	95	99	100	96

Neural Network

Calculate the error (STEP 2)

Adjust weights and b to reduce the error (STEP 3)

Repeat step 2 and step 3 for all training data until the error is within acceptable level

These steps are similar to supervised machine learning (model adjusting vs learning rules adjusting) (model adjusting vs weights and bias adjusting)

EPOCH (1 Trainig iteration)

Neural Network

$Sigmoid\ (185x0.0006 + 85x0.0002 + 1) = 0.7$

$Sigmoid\ (178x0.0004 + 94x0.0003 + 0.008) = 0.65$

$Sigmoid\ (184x0.0003 + 102x0.0001 + 0.007) = 0.64$

$Sigmoid\ (100x0.0005 + 80x0.0002 + 0.008) = 0.55$

$(Sigmoid - 0.0)^2 = 0.49$

$(Sigmoid - 0.0)^2 = 0.43$

$(Sigmoid - 0.0)^2 = 0.41$

$(Sigmoid - 0.0)^2 = 0.30$

Generalized delta rule

SGD (Stochastic Gradient Descent) method (error is calculated for each training data, weights updated immediately)

Batch method (error is calculated for all training data, each of the weight update are calculated but the avarage of all weight updates are used only once in each epoch)

Mini-Batch method (mix) →

- Small values give a learning process that converges quickly at the cost of noise in the training process.
- Large values give a learning process that converges slowly with accurate estimates of the error gradient.

Mini-Batch method

Training data 1

Training data 2

Training data 3

Training data N

Training using batch method

Part of the training data is selected

Mini-Batch method

1-10

11-20

21-40

41-60

61-80

Batch method applied

5 weight update will be performed to complete the training process

Robusteness of SGD
Efficiency of batch

SOFTWARE CODES

Matlab: essential functions and scripts

Matlab: simple examples

Neural Network

Sigmoid.m

```
function y = Sigmoid(x)
y = 1/(1+exp(-x));
end
```

Neural Network

Sigmoid.m SGD_method.m

```
function Weight = SGD_method(Weight, input, correct_Output)
alpha = 0.9;

N = 4;
for k = 1:N
    transposed_Input = input(k, :)';
    d = correct_Output(k);
weighted_Sum = Weight*transposed_Input;
output = Sigmoid(weighted_Sum);

error     = d - output;
delta = output*(1-output)*error;

dWeight = alpha*delta*transposed_Input;

Weight(1) = Weight(1) + dWeight(1);
Weight(2) = Weight(2) + dWeight(2);
```

Neural Network

Sigmoid.m | SGD_method.m*

```

N = 4;
for k = 1:N
    transposed_Input = input(k, :)';
    d = correct_Output(k);
weighted_Sum = Weight*transposed_Input;
output = Sigmoid(weighted_Sum);

error     = d - output;
delta = output*(1-output)*error;

dWeight = alpha*delta*transposed_Input;

Weight(1) = Weight(1) + dWeight(1);
Weight(2) = Weight(2) + dWeight(2);
Weight(3) = Weight(3) + dWeight(3);
end
end

```

Neural Network

Sigmoid.m | SGD_method.m | Training.m

```
input = [ 0 0 1;
          0 1 1;
          1 0 1;
          1 1 1;
        ];
correct_Output = [0
                  0
                  1
                  1
                  ];
Weight = 2*rand(1, 3) - 1;
for epoch = 1:10000
    Weight = SGD_method(Weight, input, correct_Output);
end

save('Trained_Network.mat')
```

Neural Network

Sigmoid.m | SGD_method.m | Training.m | testing.m*

```
load('Trained_Network.mat');
input = [ 0 0 1;
          0 1 1;
          1 0 1;
          1 1 1;
        ];
N = 4;
for k = 1:N
    transposed_Input = input(k, :)';
    weighted_Sum = Weight*transposed_Input;
    output = Sigmoid(weighted_Sum)
end
```

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https://christiansalvatore.github.io/machinelearning-culturalheritage-iusspavia/