BDA - Assignment 4

10/10/2021

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a)

```
mean = c(0,10)
corr = 0.6
a_std = 2
b_std = 10

cov = matrix( c(a_std^2, a_std*b_std*corr, a_std*b_std*corr, b_std^2),nrow = 2)
mean
```

```
## [1]  0 10
```

```
cov
```

```
##      [,1] [,2]
## [1,]    4   12
## [2,]   12  100
```

The mean vector and the covariance matrix can be seen printed by the last cell.

b)

```
alpha_mean = mean(bioassay_posterior$alpha)
beta_mean = mean(bioassay_posterior$beta)

alpha_interval = quantile(probs = c(0.05, 0.95), bioassay_posterior$alpha)
beta_interval = quantile(probs = c(0.05, 0.95), bioassay_posterior$beta)

alpha_mean_MSCE = sqrt(var(bioassay_posterior$alpha)/length(bioassay_posterior$alpha))
```

```
beta_mean_MSCE = sqrt(var(bioassay_posterior$beta)/length(bioassay_posterior$beta))

alpha_interval_MSCE = mcse_quantile(bioassay_posterior$alpha, 0.9)
beta_interval_MSCE = mcse_quantile(bioassay_posterior$beta, 0.9)
```

The mean alpha is 1.0 with MSCE 0.015 and beta 10.6 with MSCE 0.076.

The qunatile interval for alpha is [-0.5, 2.6] with a MSCE of 0.029 and for beta [4, 19] with a MSCE of 0.167.

c)

```
log_importance_weights = function(alpha, beta){
  bioassaylp(alpha, beta, bioassay$x, bioassay$y, bioassay$n)
}
```

```
liw = log_importance_weights(bioassay_posterior$alpha, bioassay_posterior$beta)
```

The some of the log importance weights are: -7.1657482, -7.2312935, -7.5522297, -7.1692339, -6.3182446, -6.2787463.

d

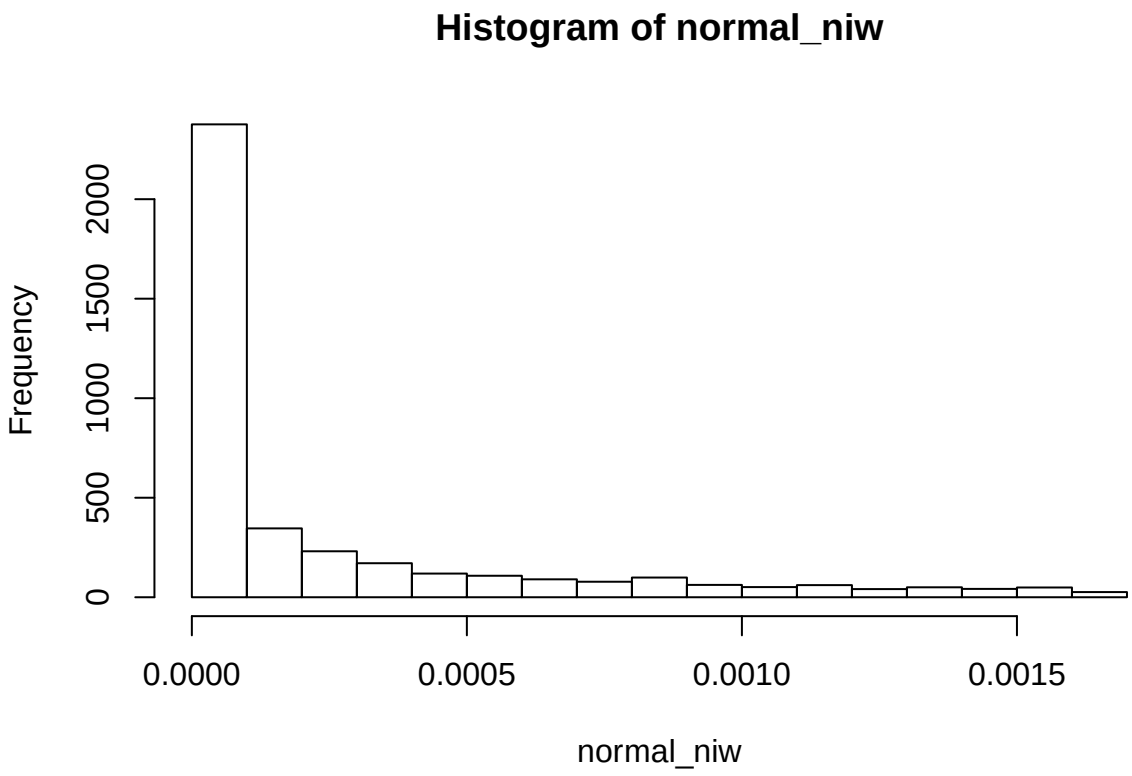
```
normalized_importance_weights = function(alpha, beta){
  liw = log_importance_weights(alpha,beta)
  exp_liw = exp(liw)
  norm_exp_liw = exp_liw/sum(exp_liw)
  return(norm_exp_liw)
}
```

```
niw = normalized_importance_weights(bioassay_posterior$alpha,bioassay_posterior$beta)
```

The some of the nromalized importance weights are: 1.3946567×10^{-4}, 1.3061751×10^{-4}, 9.4759021×10^{-5}, 1.3898039×10^{-4}, 3.2548689×10^{-4}, 3.3860032×10^{-4}.

e)

```
n=4000
draws = rmvnorm(n, mean, cov)
normal_niw = normalized_importance_weights(draws[,1], draws[,2])
hist(normal_niw)
```




```
  S_eff = function(alpha, beta){
  niw = normalized_importance_weights(alpha,beta)
  1/sum(niw^2)
}
```

```
isess = S_eff(alpha = draws[,1], beta = draws[,2])
```

The importance sampling effective sample size is 1138.599.

g)

h)

```
posterior_mean = function(alpha, beta){
  seff = S_eff(alpha,beta)

  niw = normalized_importance_weights(alpha,beta)

  alpha_mean = mean(niw*alpha)/mean(niw)
  beta_mean = mean(niw*beta)/mean(niw)

  alpha_var = mean(niw*alpha^2)/mean(niw) - alpha_mean^2
  beta_var = mean(niw*beta^2)/mean(niw) - beta_mean^2

  alpha_mean_MSCE = sqrt(alpha_var/seff)
```

```
  beta_mean_MSCE = sqrt(beta_var/seff)
  res = list(alpha_mean = alpha_mean, beta_mean = beta_mean, alpha_mean_MSCE = alpha_mean_MSCE, beta_mea
  return(res)
}
```

```
post_mean = posterior_mean(draws[,1],draws[,2])
```

The posterior mean for alpha is 0.9.0 with a MCSE of 0.026 and beta is 11 with MCSE of 0.138.