MS-E2134 - Decision making and problem solving

Assignment 3

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Contents

1 Attribute-specific value functions - value functions

The value functions are as follows:

$$v_1(x_1) = \frac{1}{40}x_1 \tag{1}$$

$$v_2(x_2) = \begin{cases} 0, & 0 \le x_2 \le 2 \\ \frac{1}{12}x_2 - \frac{1}{6}, & 2 \le x_2 < 6 \\ \frac{1}{27}x_2 + \frac{1}{9}, & 6 \le x_2 < 15 \\ \frac{1}{45}x_2 + \frac{1}{3}, & 15 \le x_2 \le 30 \end{cases} \tag{2}$$

$$v_3(x_3) = \begin{cases} \frac{1}{14}x_3 - \frac{1}{7} & 2 \le x < 9 \\ \frac{1}{42}x_3 + \frac{2}{7} & 9 \le x \le 30 \end{cases} \tag{3}$$

$$v_4(x_4) = \frac{1}{18}x_4 - \frac{1}{9} \tag{4}$$

$$v_5(x_5) = \frac{1}{20}x_5 \tag{5}$$

$$v_6(x_6) = \frac{1}{20}x_6 \tag{6}$$

$$v_7(x_7) = \frac{1}{100}x_7 \tag{7}$$

$$v_8(x_8) = \begin{cases} \frac{1024}{1023}\left(1 - 2^{-x_8}\right) & 0 \le x_8 < 10 \\ 1 & 10 \le x_8 \end{cases} \tag{8}$$

$$v_9(x_9) = \begin{cases} \frac{1}{4}(1 - \sqrt{5})\left(1 - \left(\frac{1}{2}\left(1 + \sqrt{5}\right)\right)^{3-x_9}\right) & 0 \le x_9 < 3 \\ 0 & 3 \le x_9 \end{cases} \tag{9}$$

2 Attribute-specific value functions - values

The attribute values for all alternatives can be found from the attached excel.

3 Attribute-specific value functions - value plots

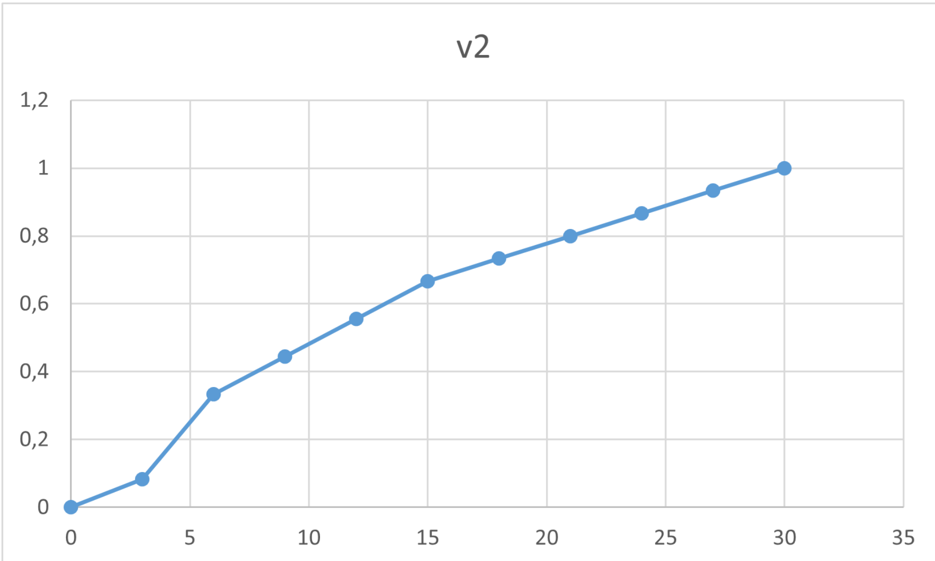

Figure 1: The plot for v_2.

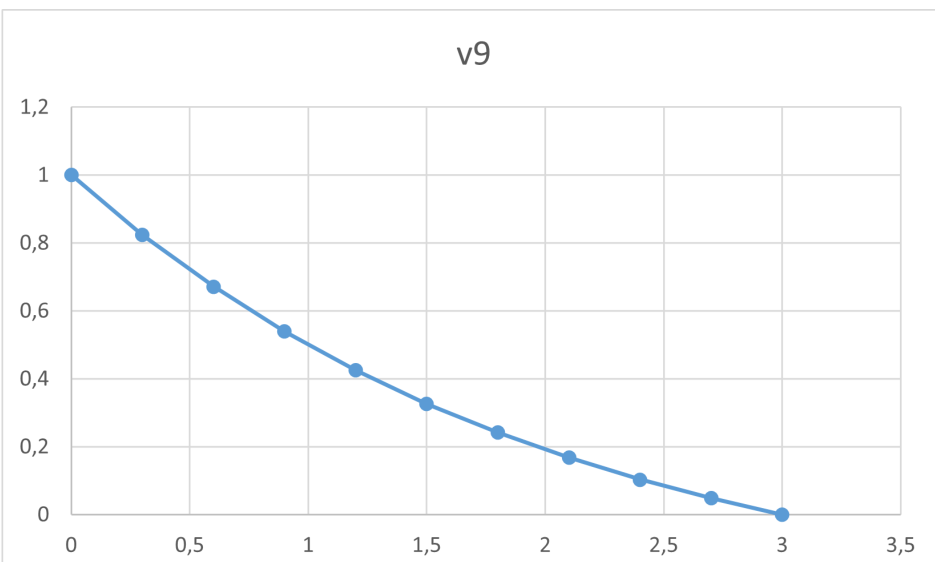

Figure 2: The plot for v_9.

4 Attribute weights

From 1. , and knowing that 20 is the max and 0 the min for both value functions, we get the equation

$$\frac{w_5 v_5(20) - w_5 v_5(0)}{w_6 v_6(20) - w_6 v_6(0)} = \frac{40}{45} \tag{10}$$

$$w_5 = \frac{8}{9} w_6. \tag{11}$$

From 2., and knowing that the max value is 20 and min value is 2, we get

$$\frac{w_6 v_6(20) - w_6 v_6(0)}{w_4 v_4(20) - w_4 v_4(2)} = \frac{45}{30}. \tag{12}$$

$$w_4 = \frac{2}{3} w_6. \tag{13}$$

From 3. we get information about two equally preferred preferences, and again remembering the min value of the value functions,

$$w_8 v_8(1) - w_8 v_8(0) = w_4 v_4(10) - w_4 v_4(2) \tag{14}$$

$$w_8 = \frac{v_4(10)}{v_8(1)} w_4. \tag{15}$$

From 4. we get a similar equation as above

$$w_7 v_7(1) - w_7 v_7(0) = w_4 v_4(3) - w_4 v_4(2) \tag{16}$$

$$w_7 = \frac{v_4(3)}{v_7(1)} w_4. \tag{17}$$

From 5. get multiple the following equality

$$w_1 v_1(40) - w_1 v_1(0) = w_2 v_2(30) - w_2 v_2(0) + w_3 v_3(20) - w_3 v_3(0) \tag{18}$$

$$w_1 = w_2 + w_3 v_3(20). \tag{19}$$

In 6., we know that all but two values do not change, hence we can omit them from the start

$$w_1 v_1(40) + w_9 v_9(1.2) = w_1 v_1(10) + w_9 v_9(0) \tag{20}$$

$$w_1 + w_9 v_9(1.2) = w_1 v_1(10) + w_9 \tag{21}$$

$$w_1(1 - v_1(10)) = w_9(1 - v_9(1.2)) \tag{22}$$

$$w_9 = \frac{1 - v_1(10)}{1 - v_9(1.2)} w_1. \tag{23}$$

In 7., we know that the two changes from the minimum x^0 are equally preferred, i.e

$$w_4 v_4(10) + w_9 v_9(1.2) - w_4 v_4(2) - w_9 v_9(100) = w_4 v_4(18) + w_9 v_9(3) - w_4 v_4(2) - w_9 v_9(100) \tag{24}$$

$$w_4 v_4(10) + w_9 v_9(1.2) = w_4 v_4(18) + w_9 v_9(3) \tag{25}$$

$$w_4 v_4(10) + w_9 v_9(1.2) = w_4 v_4(18) \tag{26}$$

$$w_9 = \frac{v_4(18) - v_4(10)}{v_9(1.2)} w_4. \tag{27}$$

From 8. we get

$$w_2 v_2(15) - w_2 v_2(0) = w_3 v_3(30) - w_3 v_3(2) \tag{28}$$

$$w_2 v_2(15) = w_3. \tag{29}$$

Additionally we know that the sum of weights equals to one

$$\sum_{i=1}^{9} w_i = 1. \tag{30}$$

By solving these equations, we get the following values

$$w_1 = 0.062131... \approx 0.06, \tag{31}$$
$$w_2 = 0.0412027... \approx 0.04, \tag{32}$$
$$w_3 = 0.0274685... \approx 0.03, \tag{33}$$
$$w_4 = 0.076779... \approx 0.08, \tag{34}$$
$$w_5 = 0.102372... \approx 0.10, \tag{35}$$
$$w_6 = 0.115168.. \approx 0.12, \tag{36}$$
$$w_7 = 0.42655... \approx 0.43, \tag{37}$$
$$w_8 = 0.0681814... \approx 0.07, \tag{38}$$
$$w_9 = 0.080147... \approx 0.08. \tag{39}$$

5 Overall values

Table 1 displays the normalized values of the sites times the area.

Table 1: Normalized vlaue functions of the sites times the area.

Site	Area	Normalizied value
1	1,2	0,26
2	3	0,88
3	2,1	0,71
4	3	0,64
5	0,8	0,25
6	2	0,70
7	3	0,70
8	0,9	0,23
9	1,1	0,37
10	2,4	0,48

6 Recalculated overall values

Since a_7 most preferred level changes from 100 to 40, the value function becomes $v_7 = \frac{1}{40}x_7$. Thus, the only value in our system of equations for solving the weights that change is $v_7(1)$. The recalculated

Table 2: Normalized value functions of the sites times the area.

Site	Area	Normalizied value
1	1,2	0,34
2	3	1,18
3	2,1	0,95
4	3	0,86
5	0,8	0,34
6	2	0,94
7	3	0,94
8	0,9	0,31
9	1,1	0,50
10	2,4	0,65

weights are

$$w_1 = 0.0835016... \approx 0.08, \quad (40)$$
$$w_2 = 0.0553747... \approx 0.06, \quad (41)$$
$$w_3 = 0.0369165... \approx 0.04, \quad (42)$$
$$w_4 = 0.103188... \approx 0.10, \quad (43)$$
$$w_5 = 0.137584... \approx 0.14, \quad (44)$$
$$w_6 = 0.154782.. \approx 0.15, \quad (45)$$
$$w_7 = 0.229306... \approx 0.23, \quad (46)$$
$$w_8 = 0.091633... \approx 0.09, \quad (47)$$
$$w_9 = 0.107714... \approx 0.11. \quad (48)$$

and the scores can be sen in Table 2.

If we calculate $\frac{V_i'(x_i)}{V_i(x_i)}$ we get a constant $\alpha \approx 1.34$ for all i. This suggest that $V_i'(x_i) = \alpha V_i(x_i)$ for all i, i.e. the V' is a affine transformation of V.

7 Site combination selection

The binary linear optimization problem we need to solve is

$$\max_y \sum_{j=1}^{10} V(x^j) y_j \quad (49)$$
$$\text{subject to} \sum_{j=1}^{10} c_j y_j \leq 25000 \quad (50)$$
$$y_j \in \mathbb{B}, \quad (51)$$

where b is a binary variable indicating if a site is to be acquired.

The calculations for this part is done in the attached excel. The optimal solution are the following sites: 2, 3, 5, 6, 7, 9. This gives a optimal value of approximately $4,85$ at a cost of 24494€.

8 Multi-objective optimization applied to site combination selection - optimization