

# Assignment 1

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15.09.2021

## #1 Basic probability calculations

- a) False, they are not independent since mutually exclusive means  $P(A \cap B) = 0$  but we know  $P(A), P(B) > 0$ . Hence  $P(A \cap B) \neq P(A)P(B)$ .
- b) False, see (a)
- c) False, it follows from the mutual exclusivity:  $P(A) + P(B) = 1 \implies P(A) = 1 - P(B)$ .
- d) True, follows from the conditional probability  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ . We know the determinant equals 0 and  $P(B) > 0$  so the expression evaluates to 0.
- e) True, since it follows from (d) that also  $P(B|A) = \frac{P(B \cap A)}{P(A)} = 0$ . Since both are 0 they are equal in this case.
- f) False, since  $P(A|B^c) + P(B|A^c) = P(A|A) + P(B|B) = 1 + 1 = 2$  and  $P(A^c) + P(B^c) = P(B) + P(A) = 1$ .

## #2 Bayes' rule

For calculations, refer to Figure 1

- a) Using Bayes' rule we can calculate the probability:

$$P(dis = true|test = true) = \frac{P(test = true|dis = true)P(dis = true)}{P(test = true)} = 74.23\% \quad (1)$$

We get  $P(test = true)$  using the law of total probability.

- b) Using the new knowledge and recalculating the denominator for the value for diabetes we get a probability of 90.72%
- c) We can see that the results increase a) by nearly 11% and b) by 5%.

	Prob	Complement	Prob calculation
P(dis=true)	5.50 %	94.50 %	
P(dia=true)	7.00 %	93.00 %	
P(dis=true   dia=true)	16.50 %	83.50 %	
P(test=true   dis=true)	99.00 %	1.00 %	
P(test=false   dis=false)	98.00 %	2.00 %	
Alternative specificity	99.00 %	1.00 %	
P(test=true)	7.34 %	92.67 %	=B5*B2+C6*C2
P(dis=true   test=true)	74.23 %	25.77 %	=B5*B2/B9
P(test=true AND dia=true)	18.01 %	82.00 %	=B5*B2+C6*C2
P(dis=true   (test=true AND dia=true))	90.72 %	9.28 %	=B5*B4/B11
Alternative results:			
P(test=true)	6.39 %		=B5*B2+C7*C2
P(dis=true   test=true)	85.21 %		=B5*B2/B14
P(test=true AND dia=true)	17.17 %		=B5*B4+C7*C4
P(dis=true   (test=true AND dia=true))	95.14 %		=B5*B4/B17

Figure 1: The plotted PMF on the left and CDF on the right.

## 1 #3 Joint, conditional and marginal probabilities

For calculations, refer to Table 1.

- The probabilities can be found from the table. They can be calculated as taking the product of the probability that  $i$  numbers of systems are sold with the probability that weather  $j$  occurs. E.g. for 0 systems in cold weather we calculate  $0.1 \cdot 0.45 = 0.045 = 4.5\%$
- Since the product of the marginal probabilities are not the same as the joint probabilities, we can conclude that they are not independent.
- The marginal probabilities can be seen from the PMF column in the table
- The plots can be seen from Figure 2. The probability of at least three systems is 66.55%

Table 1: Table of joint probabilities, marginal probabilities (in column PMF) and the cumulative distribution function (CDF)

	Cold	Cool	Warm	Hot	PMF	CDF
0	4.50 %	2.50 %	2.00 %	0.00 %	9.00 %	9.00 %
1	2.50 %	7.50 %	4.00 %	1.25 %	15.25 %	24.25 %
2	1.50 %	7.50 %	10.00 %	2.50 %	21.50 %	45.75 %
3	0.80 %	3.75 %	10.00 %	6.25 %	20.80 %	66.55 %
4	0.50 %	2.50 %	10.00 %	11.25 %	24.25 %	90.80 %
5	0.20 %	1.25 %	4.00 %	3.75 %	9.20 %	100.00 %

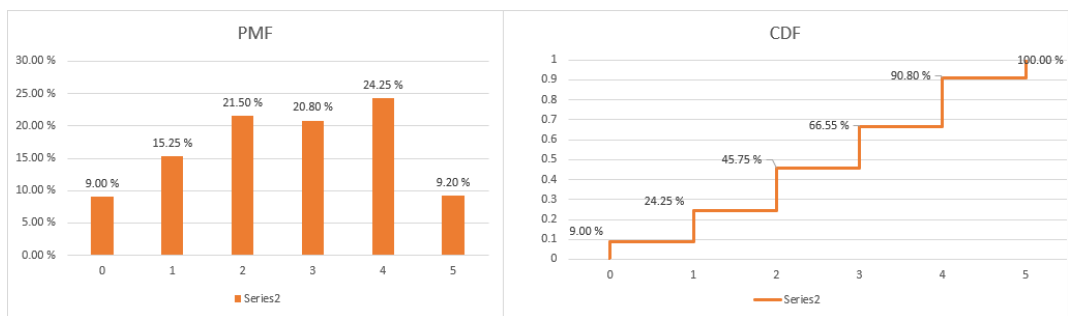


Figure 2: The plotted PMF on the left and CDF on the right.