

#1 Basic probability calculations

- a) False, they are not independent since mutually exclusive means $P(A \cap B) = 0$ but we know $P(A), P(B) > 0$. Hence $P(A \cap B) \neq P(A)P(B)$.
- b) False, see (a)
- c) False, it follows from the mutual exclusivity: $P(A) + P(B) = 1 \implies P(A) = 1 - P(B)$.
- d) True, follows from the conditional probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$. We know the determinant equals 0 and $P(B) > 0$ so the expression evaluates to 0.
- e) True, since it follows from (d) that also $P(B|A) = \frac{P(B \cap A)}{P(A)} = 0$. Since both are 0 they are equal in this case.
- f) False, since $P(A|B^c) + P(B|A^c) = P(A|A) + P(B|B) = 1 + 1 = 2$ and $P(A^c) + P(B^c) = P(B) + P(A) = 1$.

#2 Bayes' rule

For calculations, refer to Figure 1

- a) Using Bayes' rule we can calculate the probability:

$$P(dis = true|test = true) = \frac{P(test = true|dis = true)P(dis = true)}{P(test = true)} = 74.23\% \quad (1)$$

We get $P(test = true)$ using the law of total probability.

- b) Using the new knowledge and recalculating the denominator for the value for diabetes we get a probability of 90.72%
- c) We can see that the results increase a) by nearly 11% and b) by 5%.

| | Prob | Complement | Prob calculation |
|--|---------|------------|------------------|
| P(dis=true) | 5.50 % | 94.50 % | |
| P(dia=true) | 7.00 % | 93.00 % | |
| P(dis=true dia=true) | 16.50 % | 83.50 % | |
| P(test=true dis=true) | 99.00 % | 1.00 % | |
| P(test=false dis=false) | 98.00 % | 2.00 % | |
| Alternative specificity | 99.00 % | 1.00 % | |
| | | | |
| P(test=true) | 7.34 % | 92.67 % | =B5*B2+C6*C2 |
| P(dis=true test=true) | 74.23 % | 25.77 % | =B5*B2/B9 |
| P(test=true AND dia=true) | 18.01 % | 82.00 % | =B5*B2+C6*C2 |
| P(dis=true (test=true AND dia=true)) | 90.72 % | 9.28 % | =B5*B4/B11 |
| | | | |
| Alternative results: | | | |
| P(test=true) | 6.39 % | | =B5*B2+C7*C2 |
| P(dis=true test=true) | 85.21 % | | =B5*B2/B14 |
| P(test=true AND dia=true) | 17.17 % | | =B5*B4+C7*C4 |
| P(dis=true (test=true AND dia=true)) | 95.14 % | | =B5*B4/B17 |

Figure 1: The plotted PMF on the left and CDF on the right.

1 #3 Joint, conditional and marginal probabilities

For calculations, refer to Table 1.

- The probabilities can be found from the table. They can be calculated as taking the product of the probability that i numbers of systems are sold with the probability that weather j occurs. E.g. for 0 systems in cold weather we calculate $0.1 \cdot 0.45 = 0.045 = 4.5\%$
- Since the product of the marginal probabilities are not the same as the joint probabilities, we can conclude that they are not independent.
- The marginal probabilities can be seen from the PMF column in the table
- The plots can be seen from Figure 2. The probability of at least three systems is 66.55%

Table 1: Table of joint probabilities, marginal probabilities (in column PMF) and the cumulative distribution function (CDF)

| | Cold | Cool | Warm | Hot | PMF | CDF |
|---|--------|--------|---------|---------|---------|----------|
| 0 | 4.50 % | 2.50 % | 2.00 % | 0.00 % | 9.00 % | 9.00 % |
| 1 | 2.50 % | 7.50 % | 4.00 % | 1.25 % | 15.25 % | 24.25 % |
| 2 | 1.50 % | 7.50 % | 10.00 % | 2.50 % | 21.50 % | 45.75 % |
| 3 | 0.80 % | 3.75 % | 10.00 % | 6.25 % | 20.80 % | 66.55 % |
| 4 | 0.50 % | 2.50 % | 10.00 % | 11.25 % | 24.25 % | 90.80 % |
| 5 | 0.20 % | 1.25 % | 4.00 % | 3.75 % | 9.20 % | 100.00 % |

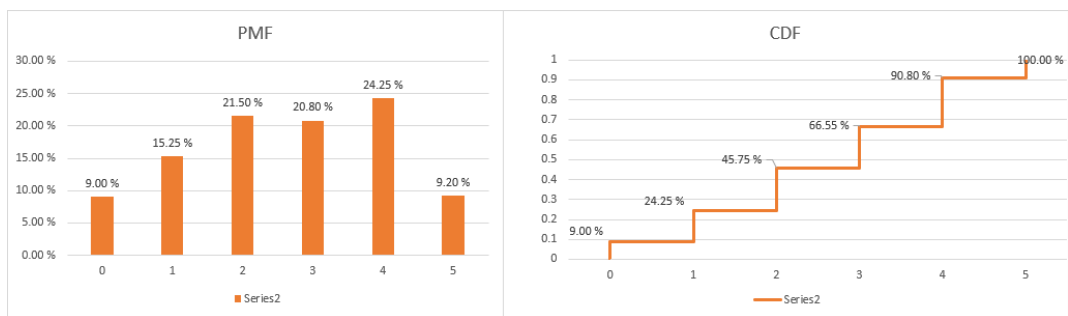


Figure 2: The plotted PMF on the left and CDF on the right.