#1 Basic probability calculations

a) False, they are not independent saince mutually exclusive means $P(A \cap B) = 0$ but we know $P(A), P(B) > 0$. Hence $P(A \cap B) \neq P(A)P(B)$.

b) False, see (a)

c) False, it follows from the mutual exclusivity: $P(A) + P(B) = 1 \implies P(A) = 1 - P(B)$.

d) True, follows from the conditional probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$. We know the determinant equals 0 and $P(B) > 0$) so the expression evaluates to 0.

e) True, since it follows from (d) that also $P(B|A) = \frac{P(B \cap A)}{P(A)} = 0$. Since both are 0 they are equal in this case.

f) False, since $P(A|B^c) + P(B|A^c) = P(A|A) + P(B|B) = 1 + 1 = 2$ and $P(A^c) + P(B^c) = P(B) + P(A) = 1$.

#2 Bayes' rule

For calculations, refer to Figure 1

a) Using Bayes' rule we can calculate the probability:

$$P(dis = true|test = true) = \frac{P(test = true|dis = true)P(dis = true)}{P(test = true)} = 74.23\% \tag{1}$$

We get $P(test = true)$ using the law of total probability.

b) Using the new knowledge and recalculating the denominator for the value for diabetes we get a probability of 90.72%

c) We can see that the results increase a) by nearly 11% and b) by 5%.

	Prob	Complement	Prob calculation
P(dis=true)	5.50 %	94.50 %	
P(dia=true)	7.00 %	93.00 %	
P(dis=true|dia=true)	16.50 %	83.50 %	
P(test=true|dis=true)	99.00 %	1.00 %	
P(test=false|dis=false)	98.00 %	2.00 %	
Aternative specificity	99.00 %	1.00 %	
P(test=true)	7.34 %	92.67 %	=B5*B2+C6*C2
P(dis=true|test=true)	74.23 %	25.77 %	=B5*B2/B9
P(test=true AND dia=true)	18.01 %	82.00 %	=B5*B2+C6*C2
P(dis=true|(test=true AND dia=true))	90.72 %	9.28 %	=B5*B4/B11
Alternative results:			
P(test=true)	6.39 %		=B5*B2+C7*C2
P(dis=true|test=true)	85.21 %		=B5*B2/B14
P(test=true AND dia=true)	17.17 %		=B5*B4+C7*C4
P(dis=true|(test=true AND dia=true))	95.14 %		=B5*B4/B17

Figure 1: The plotted PMF on the left and CDF on the right.

1 #3 Joint, conditional and marginal probabilities

For calculations, refer to Table 1.

a) The probabilities can be found from th table. They can be calculated as taking the product of the probability that i numbers of systems are sold with the probability that weather j occurs. E.g. for 0 systems in cold weather we calculate $0.1 \cdot 0.45 = 0.045 = 4.5\%$

b) Since the product of the marginal probabilities are not the same as the joint probabilites, we can conclude that they are not independent.

c) The marginal probabilities can be seen from the PMF column in the table

d) The plots can be seen from Figure 2. The probability of at least three systems is 66.55%

Table 1: Table of joint probabilities, marginal probabilities (in column PMF) and the cumulative distribution function (CDF)

	Cold	Cool	Warm	Hot	PMF	CDF
0	4.50 %	2.50 %	2.00 %	0.00 %	9.00 %	9.00 %
1	2.50 %	7.50 %	4.00 %	1.25 %	15.25 %	24.25 %
2	1.50 %	7.50 %	10.00 %	2.50 %	21.50 %	45.75 %
3	0.80 %	3.75 %	10.00 %	6.25 %	20.80 %	66.55 %
4	0.50 %	2.50 %	10.00 %	11.25 %	24.25 %	90.80 %
5	0.20 %	1.25 %	4.00 %	3.75 %	9.20 %	100.00 %

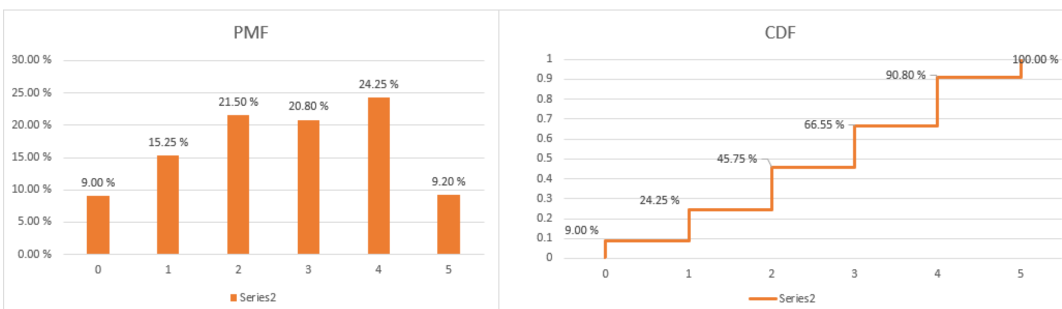

Figure 2: The plotted PMF on the left and CDF on the right.