MS-E2134 - Decision making and problem solving

Assignment 2

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Contents

1 Simulation

1.1 a)

	Own business									
	Year1	Year 1 total	Year2	Year2 total	Year3	Year3 total	Year4	Year4 total	Year5	Year5 total
Can not cover expensives		4 %		8 %		11 %		15 %		20 %
Average income		17 741		19 236		19 461		19 485		18 541
1	36 587	15 111	48 984	26 774	29 322	8 178	30 771	7 492	31 748	9 461
2	30 692	6 011	36 958	14 314	24 811	3 338	22 994	-1 064	13 074	-9 335
3	38 468	16 584	32 253	10 334	37 742	15 201	40 470	16 645	33 193	11 709
4	32 215	12 208	35 131	14 512	42 032	17 744	40 780	18 579	37 882	13 185
5	26 473	2 044	13 894	-10 253	-3 321	-26 612	-8 220	-29 931	-19 241	-42 459
6	53 857	30 798	63 557	42 555	61 534	38 028	63 648	39 368	64 831	43 809
7	49 641	27 903	54 971	34 083	67 017	44 949	80 162	56 928	68 098	47 137
8	27 352	4 839	35 635	15 398	41 180	16 736	38 949	14 667	26 484	3 830
9	46 697	23 034	27 803	6 852	46 209	25 238	60 452	39 082	59 094	36 155
10	50 698	27 413	51 089	29 599	68 425	47 194	68 523	44 638	82 517	59 534

	Steady job				
	Year1	Year2	Year3	Year4	Year5
Can not cover expensives	0 %	0 %	0 %	0 %	0 %
Average income	7 528	7 748	7 563	7 402	7 536
1	5 010	6 783	7 460	6 864	9 816
2	5 157	9 192	7 997	9 823	6 880
3	9 466	8 762	8 222	9 485	8 994
4	5 810	6 267	8 498	9 096	7 532
5	6 152	5 189	7 396	8 128	7 934
6	7 226	7 220	9 032	5 698	6 506
7	5 686	8 242	9 359	8 725	6 629
8	9 167	5 369	8 049	7 615	6 140
9	8 092	5 060	6 045	6 686	6 701
10	8 184	8 938	6 352	6 474	8 910

Figure 1: A table displaying ten iterations of the 5 first years for exercise 1.1 a)

1.2 b)

Table 1: Average cash flow for fives years for both the steady job and own business ventures.

	Year 1	Year 2	Year 3	Year 4	Year 5
Steady job	7 528	7 748	7 563	7 402	7 536
Own business	17 741	19 236	19 461	19 485	18 541

Based on the simulations, it seems that starting the own business venture would for sure be the more lucrative, and better, option. However, an average over 200 replications might give us an overly optimistic outlook.

1.3 c)

We can see the probabilities from Table 2. We can see that initially the probability is approximately 5% and rises to approximately 20% in year 5. For the steady job, as the maximum of the expenses is less than the constant income, the probability that he is not able to cover his expenses is 0% for all years.

Table 2: Probability that Dr. Cuckoo cannot cover his expenses each year for both the steady job and own business ventures.

	Year 1	Year 2	Year 3	Year 4	Year 5
Steady job	0 %	0 %	0 %	0 %	0 %
Own business	4 %	8 %	11 %	15 %	20 %

2 Decision trees

To make the numbers easier to handle, everything is handled in millions if not otherwise stated.

2.1 a)

The built decision tree can be seen in Figure 2. Based on the tree, I should not accept the offer, but instead give a counteroffer.

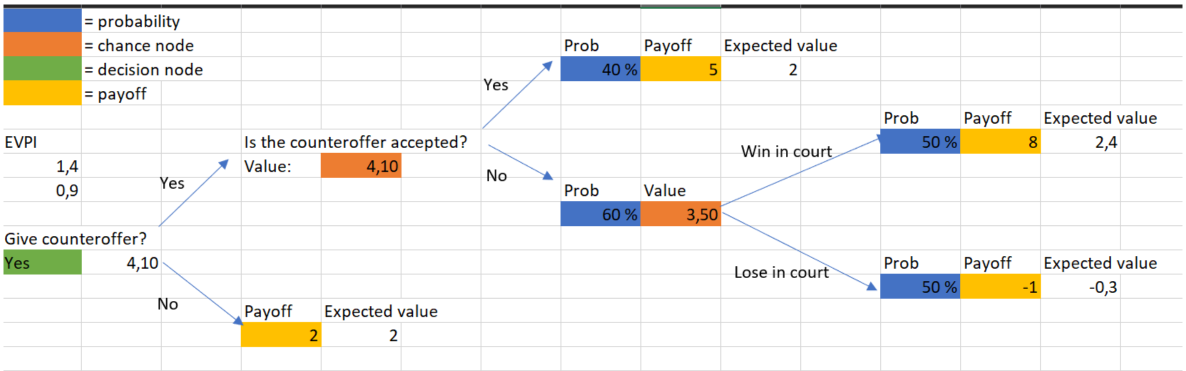

Figure 2: A figure displaying the decision tree 2.2 a)

2.2 b)

The value of prefect information (EVPI) can be calculated as the expected value of the possible outcomes times the probability of that outcome minus the expected value of the default decision tree, i.e. $EVPI = EV|PI - EMV$. We can build a table for our $EV|PI$-value, it can be seen in Table 3. The value is thus $EV|PI = 0.2 \cdot 5 + 0.2 \cdot 5 + 0.3 \cdot 8 + 0.3 \cdot 2 = 5$mil€which gives us the result

Table 3: Table displaying key values of prefect information.

Knowledge	Probability	Outcome
Accept deal + Win court	0.2	5mil€
Accept deal + Lose court	0.2	5mil€
Decline deal + Win court	0.3	8mil€
Decline deal + Lose court	0.3	2mil€

EVPI $= 5 - 4.1 = 0.9$mil€.

2.3 c)

The decision tree for this part can be seen in Figure 3. The lower part of the three, where consultation is not chosen, has been replaced by a reference to the original three from part a) which can bee seen in Figure 1. All relevant calculations can be seen in the left upper corner of the figure. The complements of $P(W|\text{Pred } W)$ and $P(W|\text{Pred } L)$ has been used directly in the needed probability nodes on the far right.

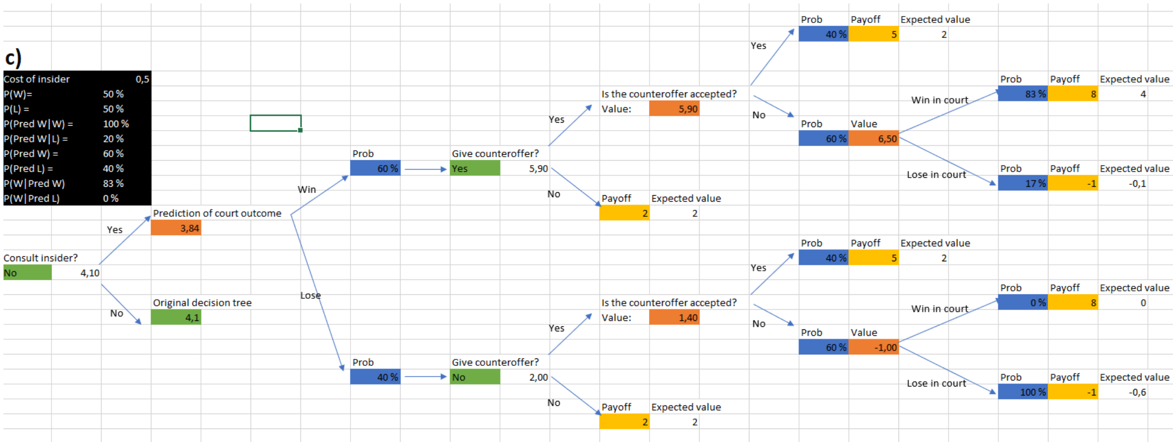

Figure 3: A figure displaying the decision tree 2.2 c)

2.4 d)

The optimal sequence of decisions is to not consult the justice system insider, as seen in Figure 3 and to give a counteroffer, as seen in Figure 2.

2.5 e)

The EVSI value can be calculated simply by taking the difference of the branches from the decision of the court insider: $4.1 - 3.84 = 0.24$. This means that we would at most be willing to pay 240k€for the information for it to be worth taking on. In other words, the cost would have to decrease by at least 260k€for us to be interested in it. The efficiency can be calculated as $\frac{\text{EVSI}}{\text{EVPI}} * 100 = \frac{0.24}{0.9} * 100 = 26.7\%$.

2.6 f)

In order to calculated the sum we would be willing to pay at maximum to the justice insider we change the probability $P(\text{Pred } W|L) = 0$ i.e. for the insider to have perfect information on the outcome of the court. We can now perform the same calculation as in e) to get the value 300k€for the maximum amount we would be willing to pay.

3 Elicitation of utility functions

We start by calculating the different values for the utility functions. We normalize the utility function such that $u(10M) = 1$ and $u(-2M) = 0$. We can thus calculate:

$$
\begin{aligned}
u(1.5\text{M}) &= 0.5u(10\text{M}) + 0.5u(-2\text{M}) && (1)\\
u(1.5\text{M}) &= 0.5 \cdot 1 + 0.5 \cdot 0 && (2)\\
u(1.5\text{M}) &= 0.5 && (3)\\
u(4\text{M}) &= 0.5u(10\text{M}) + 0.5u(1.5\text{M}) && (4)\\
u(4\text{M}) &= 0.5 \cdot 1 + 0.5 \cdot 0.5 && (5)\\
u(4\text{M}) &= 0.75 && (6)\\
u(0.1\text{M}) &= 0.5u(-2\text{M}) + 0.5u(1.5\text{M}) && (7)\\
u(0.1\text{M}) &= 0.5 \cdot 0 + 0.5 \cdot 0.5 && (8)\\
u(0.1\text{M}) &= 0.25 && (9)\\
u(6\text{M}) &= 0.4u(10\text{M}) + 0.6u(4\text{M}) && (10)\\
u(6\text{M}) &= 0.4 \cdot 1 + 0.6 \cdot 0.75 && (11)\\
u(6\text{M}) &= 0.85 && (12)\\
& && (13)
\end{aligned}
$$

The plotted utility function can be seen in Figure 4. As we can see, the function is somewhat concave. According to EUT, Dr. Stoveo is, as he says, risk averse.

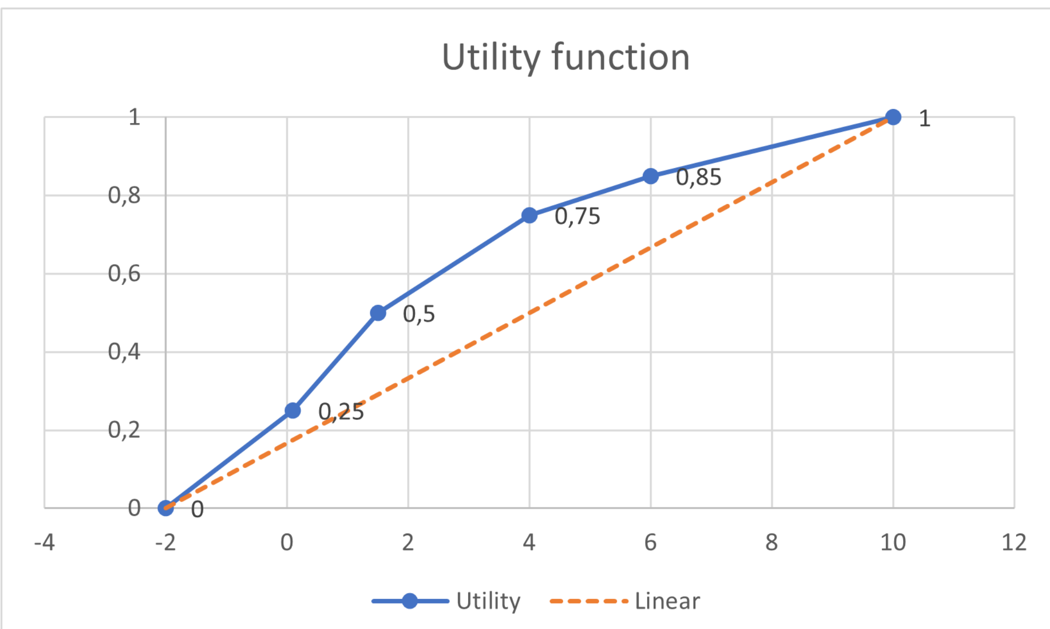

Figure 4: The plotted utility function based on Dr. Stoveos choices.

4 Expected utility, risk-attitude, and risk measures

4.1 a)

In order to find out what the investors risk attitude is, we will use first and second order derivatives.

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2}x^{-\frac{1}{2}} \tag{14}$$

$$\frac{d^2}{dx^2}\sqrt{x} = -\frac{1}{4}x^{-\frac{3}{2}} \tag{15}$$

$$\frac{d}{dx}x^3 = 3x^2 \tag{16}$$

$$\frac{d^2}{dx^2}x^3 = 6x \tag{17}$$

From these we can see that Rick Averell's utility functions first order derivative is positive for all $x > 0$ and hence is the utility function increasing. The second order derivative is negative for all $x > 0$ the growth is decreasing. From this we can conclude that the function is concave and Rick Averell is risk averse. We can draw conclusions in the same way for Ricki Seeck. Since both order derivatives are positive for all $x > 0$, we know that the function is increasingly positive and thus convex. Mr. Seeck's utility function is thus risk seeking.

4.2 b)

4.2.1 i

We can calculate the expected utility as $E[u(X)] = \int f_x(x)u(x)dx$.

$$f_x(t) = \begin{cases} \frac{1}{50}, & 50 \leq x \leq 100 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

By integrating over the whole space we get,:

$$\int f_x(x)u(x)dt = \tag{19}$$

$$\int_{50}^{100} \frac{1}{50}u(x)dt \tag{20}$$

We count the expected utility for Rick Averell:

$$\int_{50}^{100} \frac{1}{50}\sqrt{x}dx = \tag{21}$$

$$\Big/_{50}^{100} \frac{1}{50}\frac{3}{2}x^{\frac{3}{2}} = \frac{3}{2}x^{\frac{3}{2}} = \tag{22}$$

$$\Big/_{50}^{100} \frac{3}{100}x^{\frac{3}{2}} \approx 8.62 \tag{23}$$

Using the same logic we calculate the expected utility for Ricki Seeck:

$$\int_{50}^{100} \frac{1}{50}x^3dx = 468750. \tag{24}$$

To summarize, the expected utilities are $E[u(X)]_{RickAverall} \approx 8.62$ and $E[u(X)]_{RickiSeeck} = 468750$

The certainty equivalent is calculated by $CE[x] = u^{-1}(E[u(X)])$. For Rick Averall this means

$$CE[x]_{RickAverall} = u^{-1}(E[u(X)]) = E[u(X)]^2_{RickAverall} \approx 74.3 \tag{25}$$

and for Ricki Seeck

$$CE[x]_{RickiSeeck} = u^{-1}(E[u(X)]) = \sqrt[3]{E[u(X)]_{RickiSeeck}} \approx 77.7. \tag{26}$$

The risk premia is calculated as $RP[X] = E[X] - CE[X]$. The expected vlaue is $E(X) = 75$ For Rick Averall this becomes:

$$E[X] - CE[x]_{RickAverall} \approx 0.7, \tag{27}$$

and for Ricki Seeck:

$$E[X] - CE[x]_{RickiSeeck} \approx -2.7. \tag{28}$$

Since Rick Averall's risk premia is postivie and Ricki Seeck's is negative, they are inline with our earlier interpretation of the risk attitudes.

4.2.2 ii

The calculations can be seen in the attached .mlx file. The values we received were approximately the following:
$E[u(X)]_{RickAverall} = 5.1$
$E[u(X)]_{RickiSeeck} = 6 \cdot 10^5$
$CE[x]_{RickAverall} = 26.3$
$CE[x]_{RickiSeeck} = 84.7$
$RP[X]_{RickAverall} = 7.2$
$RP[X]_{RickiSeeck} = -51.1$
These numbers are inline with our conclusions from part a) about the risk attitudes of the DM's.

4.2.3 iii

The calculations can be seen in the attached .mlx file. The values we received were approximately the following:
$E[u(X)]_{RickAverall} = 9.49$
$E[u(X)]_{RickiSeeck} = 1.64 \cdot 10^6$
$CE[x]_{RickAverall} = 90.00$
$CE[x]_{RickiSeeck} = 117.83$
$RP[X]_{RickAverall} = 8.25$
$RP[X]_{RickiSeeck} = -19.58$
These numbers are inline with our conclusions from part a) about the risk attitudes of the DM's.

4.3 c)

4.3.1 i

We start by analytically calculating the 10% Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) for the uniform distribution. The VaR is calculated as

$$\int_{-\infty}^{\mathrm{VaR}_\alpha[X]} f_X(t)dt = \alpha = F_X(\mathrm{VaR}_\alpha[X]) \iff \mathrm{VaR}_\alpha[X] = F_X^{-1}(\alpha). \tag{29}$$

Let's compute the inverse of the CDF of our function.

$$F_X = \begin{cases} 0, & x < 50 \\ \frac{x-50}{50}, & 50 \le x \le 100 \\ 1, & 100 < x \end{cases} \tag{30}$$

$$\implies F_X^{-1} = 50x + 50, 50 \le x \le 100 \tag{31}$$

$$\tag{32}$$

Using this we can the substitute x for $\alpha = 0.1$

$$\mathrm{VaR}_{0.1}[X] = F_X^{-1}(0.1) = 55. \tag{33}$$

We can now calculate the CVar as

$$E[X|X \le \mathrm{VaR}_\alpha[X]] = \int_{-\infty}^{\mathrm{VaR}_\alpha[X]} x\frac{f_x(x)}{\alpha}dx. \tag{34}$$

For our unifrom function this becomes

$$\mathrm{CVaR}_{10\%} = \int_{50}^{55} x\frac{1/50}{0.1}dx \tag{35}$$

$$= \Big/_{50}^{55} \frac{x^2}{10} \tag{36}$$

$$= 52.5. \tag{37}$$

To summarize $\mathrm{VaR}_{10\%} = 55$ and $\mathrm{CVaR}_{10\%} = 52.5$.

4.3.2 ii

The calculations can be seen in the attached .mlx file. We receive the values $\mathrm{VaR}_{10\%} = 5.6$ and $\mathrm{CVaR}_{10\%} = 3.6$.

4.3.3 iii

The calculations can be seen in the attached .mlx file. We receive the values $\mathrm{VaR}_{10\%} = 10$ and $\mathrm{CVaR}_{10\%} = 7.50$.

5 Stochastic dominance

5.1 a)

From Figure 5 we can see that discrete distribution nearly first order stochastic dominates the Log-normal distribution but since the Log-normal distribution never actually reaches 1 and the discrete distribution does at 200, there is no first order stochastic domination

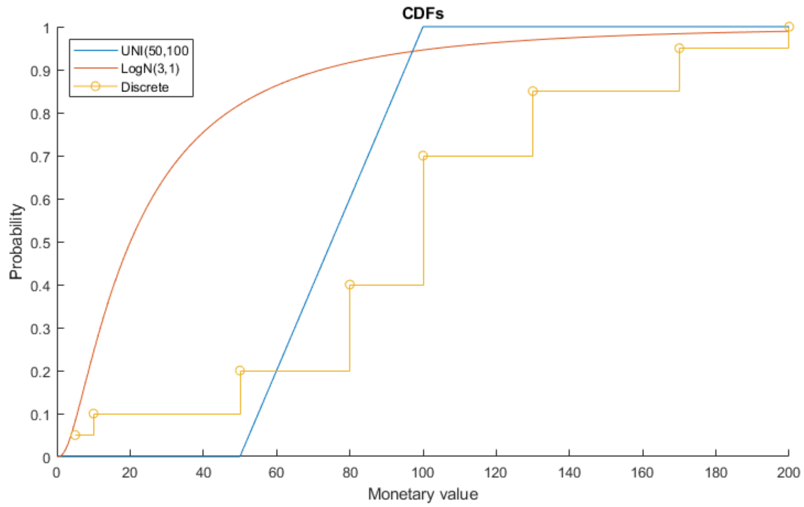

Figure 5: The Cumulative distribution functions three distributions

5.2 b)

We can identify three different second order stochastic dominations from the CDF's

1. The discretre distribution dominates the Log-normal distrubtion as a result of it's nearly first order stochastic dominance.
2. By examining the plot, we can see that the area below the discrete and above the uniform is less than that which is below the uniform and above the discrete. I.e. the integral of the discrete function will be larger than that of the uniform.
3. The same reason from the point above can be applied to the uniform and log-normal distribution. The integral of the uniform distrubtion is larger than that of the log-normal.

We can thus conclude that we have

$$\text{Discrete} \succcurlyeq_{SSD} \text{UNI}(50,100) \succcurlyeq_{SSD} \text{LogN}(3,1^2). \tag{38}$$

5.3 c)

For a DM that is risk-averse, any option that is strictly dominated in the sense of FSD should not be chosen. I.e. we should not recommend the log-normal distribution to them. Since the discrete distribution dominates the uniform in the sense of SSD we would recommend it to the risk averse DM.

5.4 d)

As in part c), any option that is strictly dominated in the sense of FSD should not be chosen. Hence, I would recommend the discrete distribution to the risk neutral DM.