Exercise 4 – Solutions

12.10.2021

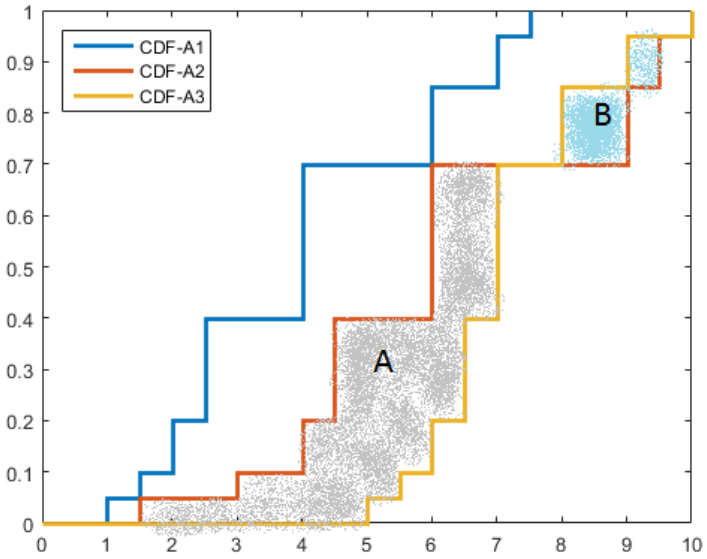
#1 Risk measures

VaR-10% is *t* such that F(t)=0.1. Thus, the VaR-10% values of the alternatives can be deduced based on the PMF table of the exercise sheet or read directly from the CDF plot formed already in Exercise 3 (see the plot again below). Now,

- A1: VaR-10%=1.5
- A2: VaR-10%=3
- A3: VaR-10%=5.5

CVaR-10% is E[X|X≤ VaR-10%] so that

- A1: $\mathrm{CVaR}_{10\%} = \frac{0.05(1)+0.05(1.5)}{0.1} = 1.25$
- A2: $\mathrm{CVaR}_{10\%} = \frac{0.05(1.5)+0.05(3)}{0.1} = 2.25$
- A3: $\mathrm{CVaR}_{10\%} = \frac{0.05(5)+0.05(5.5)}{0.1} = 5.25$



In Exercise 3, it was concluded that A1 is FSD dominated by both A2 and A3. This is now reflected by the VaR-10% value of A1 being smaller than those of A2 and A3. The fact that A1 is also SSD dominated by A2 and A3 is reflected by the $\mathrm{CVaR}_{10\%}$ value of A1 being smaller than those of A2 and A3. The SSD dominance of A3 over A2 is reflected by the $\mathrm{CVaR}_{10\%}$ value of the former being larger than that of the latter.

#2 Risk measures with Matlab

a) See the model solution file “exercise4task2solution.m”

b) See Figure 2.1. The code for the figure can be seen from the model solution file “exercise4task2solution.m”.

c) The results are saved in vectors where each component represent one investment opportunity. When looking at the “exercise4task2solution.m” code and after running it the values of expected value are in variable “EV”, the three VaR values are in variables “Var1”, “Var5” and “Var10” and the three CVaR values are in variables “CVar1”, “CVar5” and “CVar10”

d) See Figure 2.2. The code for the figure can be seen from the model solution file.

e) When looking at the options we can see that some options give both the better expected value and risk measure values that others (look at pair A7 and A10 for example) so A10 should be preferred over A7. Also, other such combinations exist. However, there is a tradeoff between expected value and risk meaning that some options are better in terms of risk and some in terms of expected value. The DM needs to choose between different alternatives.

f) The CVaR values of A6, A7, A8, and A9 are always less than of A5 so A5 has a second degree stochastic dominance over these options so risk averse DM should avoid opportunities A6, A7, A8, and A9. Notice here that the CVar 100% is equal to the expected value of the investment opportunity so this makes it easier to identify different plots.

g) See Figure 2.3. It now shows the SSD between the investment opportunities.

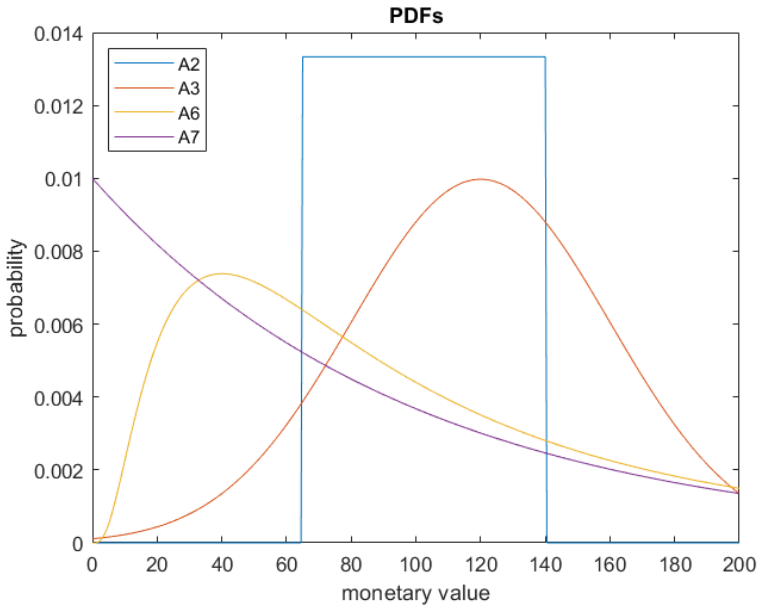


Figure 2.1: PDFs

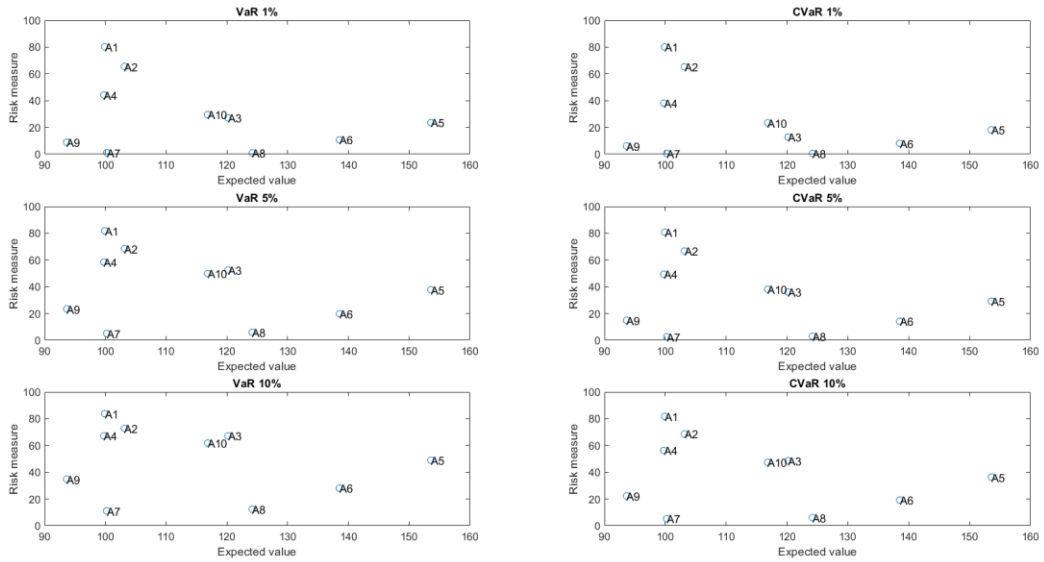


Figure 2.2: Risk measures plotted for each investment opportunity

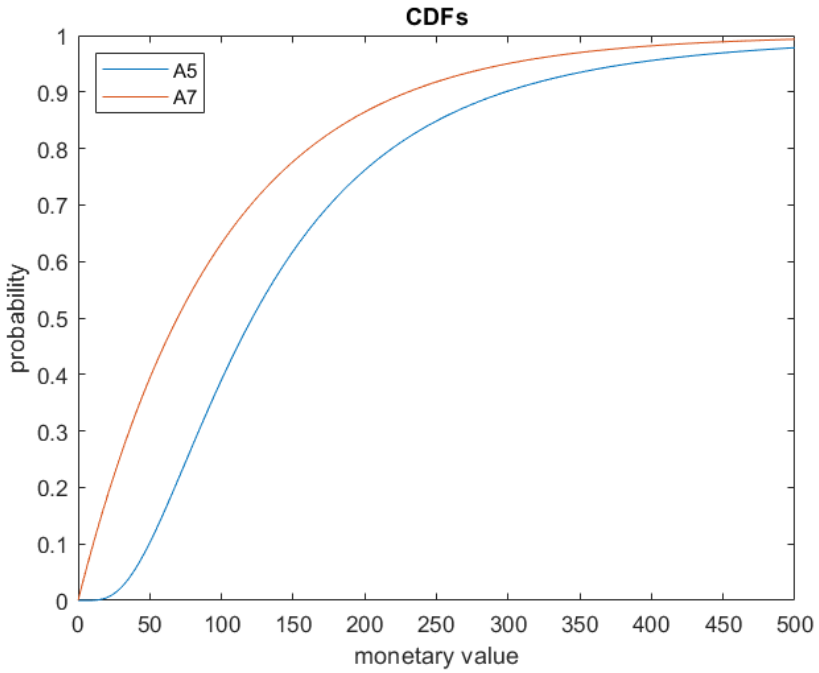


Figure 2.3: CDFs