Exercise 8 – Solutions

23.11.2021

#1

a) The normalized attribute-specific utility function $u_i^N(x_i)$ is obtained by using the principle $u_i^N(x_i) = A * u_i(x_i) + B, A > 0, B \in \mathbb{R}$, where $u_i(x_i)$ is the unnormalized utility function, and the conditions $u_i^N(x_i^*) = 1$, $u_i^N(x_i^0) = 0$ for the best and worst attribute levels, x_i^* and x_i^0, respectively. The results are:

$$u_1^N(x_1) = \frac{e^{0.1} - e^{-x_1/50}}{e^{0.1} - e^{-1}} \qquad u_2^N(x_2) = \frac{e^{-0.3} - e^{-2x_2}}{e^{-0.3} - e^{-1.7}}$$

b) Preference statement results in:

$$w_1(u_1^N(50) - u_1^N(5)) = w_2(u_2^N(0.85) - u_2^N(0.15))$$
$$\Leftrightarrow w_1(1 - \frac{e^{0.1} - e^{-0.1}}{e^{0.1} - e^{-1}}) = w_2$$
$$\Leftrightarrow w_1(\frac{e^{-0.1} - e^{-1}}{e^{0.1} - e^{-1}}) = w_2$$

This together with the normalization constraint yields

$$w_1 = \frac{e^{-0.1} - e^{-1}}{e^{0.1} - 2e^{-1} + e^{-0.1}} \approx 0.58 \Rightarrow w_2 = 0.42.$$

Other approach (which is applied in excel): Equations for weights

$$w_1 + w_2 = 1$$

$w_1(u_1^N(50) - u_1^N(5)) = w_2(u_2^N(0.85) - u_2^N(0.15))$ can be rewritten as $w\_1\ C = \ w\_2$

These result in equation

$$w_1(1 + C) = 1$$

c) The attribute-specific utilities for different outcomes are shown below:

	Launch now		Development succeeds		Development fails	
	Win	Lose	Win	Lose	Win	Lose
$u_1^N(x_1)$	0.58980	0.38850	1	0.27172	0.38850	0
$u_2^N(x_2)$	0.83902	0.437588	1	0	0.83902	0
$w_1u_1^N(x_1) + w_2u_2^N(x_2)$	0.69482	0.40919	1	0.15722	0.57835	0

The corresponding decision tree is shown below. The optimal choice is to launch the smartwatch later and try to develop the additional feature. See also the related excel file.