Exercise 1 – Solutions

21.9.2021

#1 Decision trees

a) See the model Excel sheet for the implementation of the calculations.

From the data:

P(NS) = 46/100

P(OS) = 34/100

P(CS) = 20/100

P(dry) = 50/100

P(intermediate) = 30/100

P(rich) = 20/100

P(dry|NS)=35/46

P(int.|NS)=9/46

P(rich|NS)=2/46

P(dry|OS)=12/34

P(int.|OS)=16/34

P(rich|OS)=6/34

P(dry|CS)=3/20

P(int.|CS)=5/20

P(rich|CS)=12/20

Let: TNS: test says no structure

TOS: test says open-type structure

TCS: test says closed-type structure

P(TNS|NS) = 0.8, P(TOS|NS) = P(TCS|NS) = 0.1

P(TOS|OS) = 0.85, P(TNS|OS) = P(TCS|OS) = 0.075

P(TCS|CS) = 0.9, P(TOS|CS) = P(TNS|CS) = 0.05.

P(TNS) = P(TNS|NS)P(NS) + P(TNS|OS)P(OS) + P(TNS|CS)P(CS) = 0.8*0.46 + 0.075*0.34 + 0.05 * 0.20 = **0.4035**. Similarly

P(TOS) = 0.85*0.34+0.1*0.46+0.05*0.20=**0.345**

P(TCS) = 0.2515

Bayes' rule:

P(NS|TNS)=P(TNS|NS)P(NS)/P(TNS) = 0.8*0.46/0.4035 = **0.91202**. Similarly we get the probabilities

	NS	OS	CS
P(* | TNS)	0.912	0.063	0.025
P(* | TOS)	0.133	0.838	0.029
P(* | TCS)	0.183	0.101	0.716

We still need conditional probabilities **P(oil amount class | test result)**

For example **P(dry|TNS)** =P(dry|NS,TNS)P(NS|TNS)+P(dry|OS,TNS)P(OS|TNS) + P(dry|CS,TNS)P(CS|TNS)

=P(dry|NS)P(NS|TNS)+P(dry|OS)P(OS|TNS) + P(dry|CS)P(CS|TNS)

=35/46*0.91202 + 12/34*0.063+3/20*0.025 = **0.720**

Above, P(dry|NS,TNS)=P(dry|NS) independently of the test result.

	dry	intermediate	rich
P(* | TNS)	0.72	0.21	0.07
P(* | TOS)	0.40	0.43	0.17
P(* | TCS)	0.28	0.26	0.46

b) Filling the needed cells in the decision tree in the Excel file reveals that the expected payoff when taking the test is 30710 NKr, whereas the expected profit for straight drilling is 20000 NKr and 0 NKr for not drilling without even taking the test. Thus, the expected value of sample information is 30710 NKr – 20000 NKr=10710 NKr.

c) In order for the test cost not to offset the benefit gained from it, Ivarsen should not pay more than 10710 NKr for the test.

d)

a. If the drilling cost would reduce by 1759.60 NKr or more, then the straight drilling would be the best choice. How to calculate this? Let c be the decrease in the drilling cost. Then the expected value of the decision drill is $20000 + c$. Also, the expected value of decision “take the test” will increase but only if the test result is open or closed type structure, because then we drill. More precisely we have

$$0.345(17478 + c) + 0.2515(74413.519 + c) + 0.4035(-10000)$$
$$= 20710 + 0.5965c$$

Now by setting $20000 + c = 20710 + 0.5965c$ we can obtain $c = 1759.60$

b. For example, if P(TNS | NS) = 0.7, then straight drilling becomes the best choice.