Exercise 5 – Solutions

19.10.2021

#1 Elicitation of attribute-specific value functions

The statements about equally preferred distances give equations:

$$v(900) - v(600) = v(1250) - v(900) = v(1650) - v(1250) = v(2100) - v(1650).$$

These three equations together with the normalization constraints $v(600) = 0$ and $v(2000) = 1$ could be used to determine the value function at the given points. The problem is that $x_N = 2100 > 2000 = a^*$. There are two possible approaches to overcome this problem.

Approach 1:

1. Change a^* to 2100.
   - Now v(2100)-v(600)=v(2100)-v(1650)+v(1650)-v(1250)+v(1250)-v(900)+v(900)-v(600)=4[v(900)-v(600)].
2. Normalize the value function such that v(2100)=1, v(600)=0.
   - Then, 4v(900)=1⇒v(900)=0.25, v(1250)=0.5, v(1650)=0.75.
3. Interpolate to get v(1200) and v(1700).
   - v(1200)-v(900)=[1200-900]/[1250-900]*[v(1250)-v(900)] ⇒ v(1200)=0.46
   - v(1700)-v(1650)=[1700-1650]/[2100-1650]*[v(2100)-v(1650)] ⇒ v(1700)=0.78

Values: (A)=0.25, (B)=0.46, (C)=0.78

Approach 2:

1. Interpolate to get v(2000)-v(600)
   =v(2000)-v(1650)+v(1650)-v(600)
   =v(2000)-v(1650)+3[v(900)-v(600)]
   =(3+[2000-1650]/[2100-1650])*[v(900)-v(600)]=
   (34/9)*[v(900)-v(600)]

2. Normalize such that v(2000)=1, v(600)=0
   - Then, v(900)=9/34, v(1250)=18/34, v(1650)=27/34.
3. Interpolate to get v(1200) and v(1700)
   - v(1200)-v(900)=[1200-900]/[1250-900]*[v(1250)-v(900)] ⇒ v(1200)=0.49
   - v(1700)-v(1650)=[1700-1650]/[2100-1650]*[v(2100)-v(1650)] ⇒ v(1700)=0.82

Values: (A)=0.26, (B)=0.49, (C)=0.82

#2 Difference independence

Let x1 denote the time since last pavement and x2 denote the average hourly number of cars that use the road. The fact that the value increases with respect to both attributes can be represented as follows:

I. $x1 > x1' \Rightarrow V(x1,x2) \geq V(x1',x2)$

II. $x2 > x2' \Rightarrow V(x1,x2) \geq V(x1,x2')$

The fact that the engineer rather has the road (10 years,100 cars) paved than the road (13 years, 10 cars) can be represented as follows:

III. $(0,100) \leftarrow (10,100) \succ_d (0,10) \leftarrow (13,10) \Leftrightarrow V(10,100) - V(0,100) > V(13,10) - V(0,10)$

Based on I. and III., it applies:

IV. $V(13,100) - V(0,100) \geq V(10,100) - V(0,100) > V(13,10) - V(0,10)$

From IV., one can see that

V. $V(13,100) - V(0,100) > V(13,10) - V(0,10)$.

Now V. indicates that a change from $x1=0$ to $x1=13$ is more valuable when $x2=100$ than when $x2=10$. Thus, x1 is not difference independent of x2.

By rearranging V., one obtains

VI. $V(13,100) - V(13,10) > V(0,100) - V(0,10)$.

Now VI. Indicates that a change from $x2=10$ to $x2=100$ is more valuable when $x1=13$ than when $x1=0$. Thus, x2 is not difference independent of x1.