Exercise 6.3 (student presents)

You want to plant tomatoes, potatoes and peas on a 180 cm wide allotment. A row of tomatoes or potatoes takes up 40 cm of space, and a row of peas 20 cm. The utility is 10 for a row of tomatoes, 7 for a row of potatoes and 3 for a row of peas. Additionally, according to an EU-directive the maximum number of allowed rows of tomatoes is two. Give a dynamic programming algorithm that could be used to solve this problem.

Solution

Let l be the remaining width of the allotment, k the number of tomato rows planted, x_1, x_2, x_3 the required spaces and u_1, u_2, u_3 the utilities associated with peas, potatoes and tomatoes, respectively. Let $J(l, k)$ be the maximal utility one can get from an allotment, where there is l amount of remaining space, and k number of tomato rows planted. With the following algorithm the maximal utility of the allotment can be calculated.

$$
J(l, k) = \begin{cases} 0 & \text{if } l < x_1 \\ u_1 + J(l - x_1, k) & \text{if } x_1 \leq l < x_2 \\ \max_{i=1,2} \{u_i + J(l - x_i, k)\} & \text{if } k \geq 2 \text{ and } l \geq x_2 \\ \max \begin{cases} \max_{i=1,2} \{u_i + J(l - x_i, k)\} \\ u_3 + J(l - x_3, k + 1) \end{cases} & \text{else.} \end{cases}
$$

Initially, $l = 180$ and $k = 0$.