Notes and Comments on Lecture 2

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Euler-Lagrange equation (2).

- Partial derivatives $g_x, g_{\dot{x}}, g_t$ are taken with respect to the variables (or functions) denoted $x, \dot{x}, t$; and in this operation, g is understood as a mapping $g : \mathbb{R}^3 \to \mathbb{R}$.
- The notation $\frac{d}{dt} g_{\dot{x}}(x^*, \dot{x}^*, t)$ means $\frac{d}{dt} g_{\dot{x}}(x^*(t), \cdots)$, i.e., the total time derivative of a function $g_{\dot{x}}(\cdot, \cdots)$ of t. Applying the chain rule of differentiation:

  $$\frac{d}{dt} g_{\dot{x}}(\cdot) = g_{\dot{x}x}\dot{x}^* + g_{\dot{x}\dot{x}}\ddot{x}^* + g_{\dot{x}t}$$

  where $x^*, \dot{x}^*, \ddot{x}^*$ are functions of t.

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Show that the Euler equation holds in the free end points case, i.e., it is equal to zero for all t, along the extremum in the integrands in pages 7, 12, and so on.

Proof. Let x^* be a solution to a free end points case and let $x^*(t_0) = a$, $x^*(t_f) = b$. Then x^* is also the solution to the fixed end points problem with these end points and with variations $x^*(t) + \delta x(t)$, $\delta x(t_0) = \delta x(t_f) = 0$, since these variations form a subset of the original, free end point problem variations. $\square$

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Show that $\delta x_f = \delta x(t_f) + \dot{x}^*(t_f)\delta t_f$ to the first order.

$$x_f := x(t_f + \delta t_f) - x^*(t_f) \qquad \text{(by definition!)}$$

$$= x(t_f) - x^*(t_f) + \dot{x}(t_f)\delta t_f \qquad \text{(since: } x(t_f + \delta t_f) = x(t_f) + \dot{x}(t_f)\delta t_f\text{)}$$

$$= \delta x(t_f) + \left[\dot{x}(t_f) - \dot{x}^*(t_f)\right]\delta t_f + \dot{x}^*(t_f)\delta t_f \qquad \text{(since: } \delta x(t_f) = x(t_f) - x^*(t_f)\text{)}$$

$$= \delta x(t_f) + \dot{x}^*(t_f)\delta t_f \qquad \text{(since: } \left[\dot{x}(t_f) - \dot{x}^*(t_f)\right]\delta t_f = \text{ higher order term } = 0\text{)}$$

$\square$