Dynamic Optimization - Homework assignment 5

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5.1

We have the system

$$\dot{x}_1 = -x_1 - u \tag{1}$$
$$\dot{x}_2 = -3x_2 - 3u \tag{2}$$

with the arbitrary initial state x_0 and the final state $x_{t_f} = 0$. We know that the control is constrained to $|u| \leq 1$. The eigenvalues of this system is -1 and -3, which are both real and nonpositive so we know that there exists a optimal control that transfers the system from the x_0 to 0 in at most 1 switch.

From the system we get the Hamiltonian

$$\mathcal{H} = 1 + p_1(-x_1 - u) + p_2(-3x_2 - 3u). \tag{3}$$

The minimum principle tells us, based on the Hamiltonian, that

$$\mathcal{H}(x^*, u^*, p^*) \leq \mathcal{H}(x^*, u, p^*) \tag{4}$$
$$-p_1 u^* - 3p_2 u^* \leq -p_1 u - 3p_2 u \tag{5}$$
$$-(p_1 + 3p_2)u^* \leq -(p_1 + 3p_2)u, \tag{6}$$

from which we can see that the switching function is $s(t) = -(p_1(t)+3p_2(t))$. This gives us the optimal control

$$u^* = \begin{cases} -1, & s(t) > 0 \iff p_1(t) + 3p_2(t) < 0 \\ \text{undef} & , s(t) = 0 \iff p_1(t) = -3p_2(t) \\ 1, & s(t) < 0 \iff p_1(t) + 3p_2(t) > 0 \end{cases} \tag{7}$$