Exercise 5.1 (homework)

Find the optimal control to move the following system

$$\begin{aligned}\dot{x}_1 &= -x_1 - u \\ \dot{x}_2 &= -3x_2 - 3u\end{aligned}$$

from an arbitrary initial state x_0 to the origin in minimal time. The control is constrained: $|u| \leq 1$. It is enough to define the optimal control w.r.t. p^*. Remember to check for singular intervals.

Exercise 5.2 (student presents)

We want to move the following system

$$\dot{x} = 2x + u$$

from an arbitrary initial state x_0 to the origin in minimal time, while the control is constrained $|u| \leq 1$. There are some initial states that can not be moved to the origin with any allowed control u. Find those initial states.

Exercise 5.3 (solved in class)

We want to move the following system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_1 + x_2 u\end{aligned}$$

from some initial state to the goal set $S(t)$, while minimizing the functional

$$J(u) = \int_0^{t_f} |u| dt.$$

The controls are constrained: $|u| \leq 1$.

Write the costate equations and form the optimal control as a function of x^* and p^*.

Exercise 5.4 (teacher demo)

The task is to build a road on a terrain. The road should be built in the interval $0 \leq s \leq S$. In this interval, the height of the terrain $y(s)$ is a differentiable function. We want to define the optimal road height $x(s)$ in each point s, when there is a constraint for the ascents and descents of the road

$$\left|\frac{dx}{ds}\right| \leq a$$

The costs for filling or digging one meter are directly proportional to the square of the difference of the height of the terrain and the road. Formulate the problem and analyze the solution.