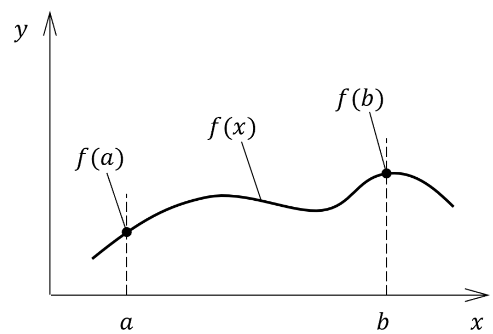
Notes and Comments on Lecture 4

About Lecture 4, equation (2) and the minimum principle. The analogy in ordinary calculus is shown in the figure.



$f(a), f(b)$ local minima at the end points, $f'(a) > 0$, $f'(b) < 0$. At a, the feasible increment is $\Delta x \geq 0$, at b it is $\Delta x \leq 0$. Hence, for the differentials, it holds that $\overset{\geq 0}{f'(a)}\overset{\geq 0}{\Delta x} \geq 0$ and $\overset{\leq 0}{f'(b)}\overset{\geq 0}{\Delta x} \geq 0$. Hence,

$$f(a + \overbrace{\Delta x}^{\geq 0}) - f(a) = \underbrace{\Delta f(a, \Delta x)}_{\text{difference} \geq 0 \text{ for } |\Delta x| \to 0} = \overbrace{f'(a)\Delta x}^{\geq 0} + \text{higher order terms}$$

$$f(b + \overbrace{\Delta x}^{\geq 0}) - f(a) = \underbrace{\Delta f(a, \Delta x)}_{\geq 0 \text{ for } |\Delta x| \to 0} = \overbrace{f'(b)\Delta x}^{\geq 0} + \text{higher order terms} \quad \square$$