## Exercise 8.1 (student presents)

A system without a disturbance is

$$x_{k+1} = x_k + u_k, \qquad x_k, u_k \in \mathbb{R}$$

and its initial state $x_0$. The cost to be minimized over all periods is

$$x_2^2 + u_0^2 + u_1^2.$$

There are no constraints for the control

a) What is the cost function $g_k(x_k, u_k)$ in periods $k = 0, 1, 2$?

b) Find the optimal control policy $u_k^* = \mu_k^*(x_k)$, $k = 0, 1$ with the DP-algorithm.

## Exercise 8.2 (solved in class)

Lets restrict the control $u_k$ in Exercise 8.1 to attain only the values $-1$ and 1. Find the optimal control policy $u_k^* = \mu_k^*(x_k)$, $k = 0, 1$ with the DP-algorithm. Hint: the solution is piecewise.

## Exercise 8.3 (teacher demo)

According to the application example in Lecture 7 your task is to to find the optimal alignment for the two DNA-sequences:

(1) G A A T T C A G T T A

(2) G G A T C G A

Lets assume that, in evolution, there are two types of mutations that happen more than others; C $\rightarrow$ T and T $\rightarrow$ C. Unlike om the example in the lecture slides, $S_{i,j} = 1$, if in sequence 1 in the place $i$ is the structural unit T or C and in sequence 2 in the place $j$ is the structural unit C or T, otherwise in a non-matching case $S_{i,j} = 0$.

Create a scoring matrix and using the DP-algorithm find the optimal alignment for sequences (1) and (2).

You can use the Matlab-codes *scoring.m* and *nwmatrix.m* and edit them.

## Exercise 8.4 (self-study, answer only revealed after homework deadline)

Your task is to hire a handyman out of $N$ candidates. The handymen arrive one by one for an interview to your office in a random order. In the interview you will find out how good the handyman is compared to those you have already interviewed (better/worse, not the size of the skill difference). In the end of the interview, you can choose either to hire or reject the candidate. If you reject him, the next candidate will step into your office. If you hire him, you won't interview any more candidates. If you rejected all previous candidates, so that the last candidate is in your office, you have to hire him.

Lets assume, that the handymen can be ordered by competence, so that no two handymen can be equally good. You will score a point, if you hire the best handyman, otherwise your utility is zero. Solve the optimal hiring policy with the DP-algorithm, when $N = 2, 3, 4$.

## Exercise 8.5 (homework)

The Ombudsman for Equality has decided, that the problem of hiring a handyman should be changed into the problem of hiring a handyperson. Now a company faces the task to hire a handyperson. The pipes in the offices of the company are leaking so terribly that it doesn't matter to them whether the hired handyperson is the best or second best of the candidates. Thus, the problem is the same as in Exercise 8.4, expect for that now you score a point for hiring the best or the second-best handyperson.

Solve the optimal hiring policy with the DP-algorithm, for $N = 2, 3, 4$. Additionally, as a bonus problem (which doesn't affect the grading of your solution) solve what happens when $N \to \infty$.