Dynamic Optimization - Presentation exercise 1

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1.1

b)

$$g_{x_1} = 2x_1 + x_2 \tag{1}$$
$$g_{x_2} = x_1 + 2x_2 \tag{2}$$
$$g_{\dot{x}_1} = 2x_2 \tag{3}$$
$$g_{\dot{x}_2} = 2x_1 \tag{4}$$

We use equation 10 from the lecture slides

$$\delta J(x, \delta x) = \int_{t_0}^{t_f} \left[g_{x_1}\delta x_1 + g_{x_2}\delta x_2 + g_{\dot{x}_1}\delta \dot{x}_1 + g_{\dot{x}_2}\delta \dot{x}_2 \right] dt \tag{5}$$

Integration by parts

$$\int_{t_0}^{t_f} 2\dot{x}_1 \delta \dot{x}_2 dt = 2\left(\dot{x}_1(t_f)\underbrace{\delta x_1(t_f)}_{0} - \dot{x}_1(t_0)\underbrace{\delta x_2(t_0)}_{0} \right) - \int_{t_0}^{t_f} 2\ddot{x}_1 \delta x_2 dt \tag{6}$$

Subsitution

$$\delta J(x, \delta x) = \int_{t_0}^{t_f} \left[(2x_1 + x_2)\delta x_1 + (x_1 + 2x_2)\delta x_2 \right] dt - \int_{t_0}^{t_f} 2\ddot{x}_2 \delta x_1 - \int_{t_0}^{t_f} 2\ddot{x}_1 \delta x_2 \tag{7}$$
$$\delta J(x, \delta x) = \int_{t_0}^{t_f} \left[(2x_1 + x_2)\delta x_1 + (x_1 + 2x_2)\delta x_2 - 2\ddot{x}_2 \delta x_1 - 2\ddot{x}_1 \delta x_2 \right] dt \tag{8}$$

c)

We use equation 10 from the lecture slides

$$J(x) = \int_{t_0}^{t_f} e^{x(t)} dt \tag{9}$$

$$\delta J(x, \delta x) = \int_{t_0}^{t_f} \left[\frac{\partial g}{\partial x} \delta x + \underbrace{\frac{\partial g}{\partial \dot{x}} \delta \dot{x}}_{0} \right] \tag{10}$$

$$\delta J(x, \delta x) = \int_{t_0}^{t_f} \left[\frac{\partial e^x}{\partial x} \delta x \right] \tag{11}$$

$$\delta J(x, \delta x) = \int_{t_0}^{t_f} e^x \delta x \tag{12}$$

1.3

a)

$$x^2 + \dot{x}^2 \tag{13}$$

Eulers:

$$\begin{aligned} g_x - \frac{d}{dt} g_{\dot{x}} &= 0 && (14) \\ g_x &= 2x && (15) \\ g_{\dot{x}} &= 2\dot{x} && (16) \\ \implies 2x - \frac{d}{dt} 2\dot{x} &= 0 && (17) \\ 2x - 2\ddot{x} = 0 & && (18) \\ x - \ddot{x} = 0 & && (19) \end{aligned}$$

We solve the DE by using the guess $x = e^{zt}$:

$$\begin{aligned} x - \ddot{x} = 0 \implies e^{zt} - z^2 e^{zt} = 0 & && (20) \\ e^{zt}(1 - z^2) = 0 & && (21) \\ 1 - z^2 = 0 & && (22) \end{aligned}$$

General solution: $x(t) = c_1 e^{z_1 t} - c_2 e^{z_2 t}$

b)

$$J(x) = \int_2^0 x^2 + 2x\dot{x} + \dot{x}^2 dt, x(0) = 1, x(2) = -3 \tag{23}$$
$$g_x = 2x + 2\dot{x} \tag{24}$$
$$g_{\dot{x}} = 2x + 2\dot{x} \tag{25}$$

Eulers:

$$g_x - \frac{d}{dt} g_{\dot{x}} = 0 \tag{26}$$
$$2x + 2\dot{x} - \frac{d}{dt} 2x + 2\dot{x} = 0 \tag{27}$$
$$2x + 2\dot{x} - 2\dot{x} + 2\ddot{x} = 0 \tag{28}$$
$$2x + 2\ddot{x} = 0 \iff x + \ddot{x} = 0 \tag{29}$$

Same as in the pervious exercise:
We solve the DE by using the guess $x = e^{zt}$:

$$x - \ddot{x} = 0 \implies e^{zt} - z^2 e^{zt} = 0 \tag{30}$$
$$e^{zt}(1 - z^2) = 0 \tag{31}$$
$$1 - z^2 = 0 \tag{32}$$
$$z = \pm 1 \tag{33}$$

General solution: $x(t) = c_1 e^{z_1 t} + c_2 e^{z_2 t}$ and using the roots we get the solution below. By thend point

$$x = c_1 e^t + c_2 e^{-t} \tag{34}$$
$$x(0) = c_1 e^0 + c_2 e^0 = c_1 + c_2 = 1 \iff c_1 = 1 - c_2 \tag{35}$$
$$x(2) = c_1 e^2 + c_2 e^{-2} = (1 - c_2) e^2 + c_2 e^{-2} \tag{36}$$
$$= e^2 - c_2 e^2 + c_2 e^{-2} \tag{37}$$
$$= c_2(-e^2 + e^{-2}) + e^2 = -3 \tag{38}$$
$$c_2(-e^2 + e^{-2}) = -3 - e^2 \tag{39}$$
$$c_2 = \frac{-3 - e^2}{-e^2 + e^{-2}} = -\frac{3 + e^2}{e^{-2} - e^2} \tag{40}$$
$$\implies c_1 = 1 + \frac{3 + e^2}{e^{-2} - e^2} = \frac{e^{-2} - e^2 + 3 + e^2}{e^{-2} - e^2} \tag{41}$$
$$\implies x = \frac{e^{-2} - e^2 + 3 + e^2}{e^{-2} - e^2} e^t - \frac{3 + e^2}{e^{-2} - e^2} e^{-t} \tag{42}$$
$$\tag{43}$$