Dynamic Optimization - Homework assignment 5

Christian Segercrantz 481056

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7.6

We know that on the 0th and 1st day the probabilities of the demand being 0, 1, 2, or 3 is $20\%, 40\%, 30\%$ and 10% and after that $10\%, 20\%, 30\%,$ and 40% respectively. We can sell a package at 5€á piece. We additionally know that we can manufacture 0-3 packages per day at the cost of 0€, 4€, 7€, or 10€. We also assume that unsold packages can transfer to the following day. The mean of the demand of the 0th and 1st day is 1.3 and for the remaining days 2. We assume that $x_0 = 0$ and that since the demand is 0 in the end $J_4(x_4) = 0$

We can model the packages per day as

$$x_{k+1} = x_k + u_k - w_k, \qquad x_{k+1} \geq 0 \tag{1}$$

where x is the amount of items per day, u is the amount of new packages fabricated and w is the stochastic demand. Here we assume that the packs can be stored from one day to the next.

The cost can be modeled as

$$g_k(x_k, u_k, w_k) = cu_k - 5\min(w_k, x_k) = \begin{cases} cu_k - 5x_k, & w_k \geq x_k \\ cu_k - 5w_k, & w_k < x_k \end{cases} \tag{2}$$

where c describes the cost per fabricated package.

The control policy $u_k = \mu_k^*(x_k)$ has the following constrains

$$u \in \begin{cases} \{0,1,2,3\}, & x_k = 0 \\ \{0,1,2\}, & x_k = 1 \\ \{0,1\}, & x_k = 2 \\ \{0\}, & x_k = 3 \end{cases} \tag{3}$$

We want to minimize the cost according to the formula

$$\sum_{n=1}^{3} g_k(x_k, u_k, w_k) \tag{4}$$

where c is the cost of production.

$$\begin{aligned} J_k(x_k) &= \max_{u_k} \mathbb{E}_{w_k}\{g_k(x_k, u_k, w_k) + J_{k+1}(x_{k+1})\} && (5)\\ &= \max_{u_k} \mathbb{E}_{w_k}\{g_k(x_k, u_k, w_k) + J_{k+1}(x_k + u_k - w_k)\} && (6) \end{aligned}$$

a)

We start with k=3

$$J_3(x_3) = \begin{cases} cu_3 - 5x_3 + J_4(x_4), & w_3 \geq x_3 \\ cu_3 - 5w_3 + J_4(x_4), & w_3 < x_3 \end{cases} \tag{7}$$

$$\begin{aligned} \mathbb{E}_{w_k}\{J_3(x_3)\} =& cu_3 - 5(0.1\min(0, \max(x_3 + u_3, 3)) + 0.2\min(1, \max(x_3 + u_3, 3)) + && (8)\\ & 0.3\min(2, \max(x_3 + u_3, 3)) + 0.4\min(3, \max(x_3 + u_3, 3))) && (9) \end{aligned}$$

We can write a table to find the optimal control policy which can be seen in Table 1. An example of the calculation is e.g. for $x_3 = 2, u_3 = 1$: $4 - 5(0.1 \cdot 0 + 0.2 \cdot 1 + 0.3 \cdot 2 + 0.4 \cdot 3) = -6$

For k=2 we then get,

Table 1: $k = 3$, the minimi per row is marked with bold.

k=3	$u_3 = 0$	$u_3 = 1$	$u_3 = 2$	$u_3 = 3$	μ_3^*
$x_3 = 0$	0	-0.5	**-1**	0	2
$x_3 = 1$	**-4.5**	-4	-3	-	0
$x_3 = 2$	**-8**	-6	-	-	0
$x_3 = 3$	**-10**	-	-	-	0

$$J_2(x_2) = \begin{cases} cu_2 - 5x_2 + J_3(x_3), & w_2 \geq x_2 \\ cu_2 - 5w_2 + J_3(x_3), & w_2 < x_2 \end{cases} \tag{10}$$

$$= \begin{cases} cu_2 - 5x_2 + J_2(x_2 + u_2 - w_2), & w_2 \geq x_2 \\ cu_2 - 5w_2 + J_2(x_2 + u_2 - w_2), & w_2 < x_2 \end{cases} \tag{11}$$

$$J_2(x_2) = \mathbb{E}_{w_k}\{g_2(x_2, u_2, w_2)\} + \mathbb{E}_{w_k}\{J_3(x_2 + u_2 - w_2)\} \tag{12}$$

Table 2: k=2, , the minimi per row is marked with bold

k=2	$u_2 = 0$	$u_2 = 1$	$u_2 = 2$	$u_2 = 3$	μ_2^*
$x_2 = 0$	0	-1.85	-3.4	**-4.35**	3
$x_2 = 1$	-5.85	-6.4	**-7.35**	-	2
$x_2 = 2$	**-10.4**	-10.35	-	-	0
$x_2 = 3$	**-14.35**	-	-	-	0