Dynamic Optimization - Homework assignment 2

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1.5

a)

We have the functional

$$J(x,t) = \int_0^s \frac{1}{2}\dot{x}^2 - x + \frac{3}{2} dt \tag{1}$$

with the end points $x(0) = 0$ and $x(s)$ is free.

To find the extremals fo the functional, we use Eulers equation

$$g_x - \frac{d}{dt} g_{\dot{x}} = 0. \tag{2}$$

where

$$g(x,\dot{x},t) = g = \frac{1}{2}\dot{x}^2 - x + \frac{3}{2} \tag{3}$$

$$\frac{\partial}{\partial x} g = g_x = -1 \tag{4}$$

$$\frac{\partial}{\partial \dot{x}} g = g_{\dot{x}} = \dot{x} \tag{5}$$

$$\implies -1 - \frac{d}{dt}\dot{x} = -1 - \ddot{x} = 0 \iff \ddot{x} + 1 = 0. \tag{6}$$

We can solve the second order DE by integrating twice:

$$\int \left(\int \ddot{x} dt \right) dt = \int \left(\int -1 dtd \right) dt \tag{7}$$

$$\int (\dot{x})\, dt = \int (-t + c_1)\, dt \tag{8}$$

$$x = -\frac{1}{2} t^2 + c_1 t + c_2. \tag{9}$$

From the start point condition we get

$$x(0) = 0 = -\frac{1}{2} 0^2 + c_1 \cdot 0 + c_2 \implies c_2 = 0 \tag{10}$$

and $x = -\frac{1}{2}t^2 + c_1 t$. Since the end state is free, we need the transversality condition

$$g_{\dot{x}}(x(t_f), \dot{x}(t_f), t_f) = 0 \tag{11}$$

$$\dot{x}(s) = 0, \qquad | \; \dot{x} = -t + c_1 \tag{12}$$

$$-s + c_1 = 0 \iff s = c_1 \tag{13}$$

Which gives us the solution

$$x^*(t) = -\frac{1}{2} t^2 + st \tag{14}$$