Assignment 1

CS-E4830 Kernel Methods in Machine Learning 2022

The **deadline** for this assignment is **Monday 21.03.2022 at 4pm**. If you have questions about the assignment, you can ask them in the corresponding Zulip stream. We will have an exercise session regarding this assignment on 17.03.22 at 4:15 pm.

Please follow the **submission instructions** given in MyCourses: `https://mycourses.aalto.fi/course/view.php?id=32426&section=1` .

Pen & Paper (10 points)

Task 1 (2 points): Recall from Lecture 1, the form for the polynomial kernel

$$K_1(x, y) = (\langle x, y \rangle + c)^m$$

where $c \geq 0$, m is a positive integer and $x, y \in \mathbb{R}^d$. Prove that $K_1(x, y)$ as defined above is a valid kernel.

Task 2 (3 points): Recall from lecture 2, in the context of binary classification, the Parzen window classifier assigns a test instance x based on the distance to the centroids in the following way:

$$h(x) = \begin{cases} +1 \text{ if } ||\phi(x) - c_-||^2 > ||\phi(x) - c_+||^2 \\ -1 \text{ otherwise.} \end{cases}$$

where c_- and c_+ represent the centroids in the feature space of the negative and positive classes respectively. Show by deriving appropriate expressions for α_i and b, that the above decision function can be written in the following form $h(x) = \text{sgn}\left(\sum_{i=1}^{n} \alpha_i k(x, x_i) + b\right)$ such that $k(x, x_i) = \langle \phi(x), \phi(x_i) \rangle$. Here sgn(.) represents the sign function, and n is the total number of training samples.

Task 3 (3 points): For $x, y \in \mathbb{R}$, check if $K_2(x, y) = \cos(x + y)$ is a valid kernel.

Task 4 (2 points): For $x, y \in \{-1, 1\}$, prove that

$$K_3(x, y) = \frac{1}{1 - xy}$$

is a valid kernel.

Programming (8 points)

Solve the programming tasks in JupyterHub (`https://jupyter.cs.aalto.fi`). The instructions for that are given in MyCourses: `https://mycourses.aalto.fi/course/view.php?id=32426&section=4`.