# Using PID and state controllers to stabalize the operations of an energy system

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## Contents

# 1 Introduction

In this assignment, the control of a thermal power plant is investigated. The power plant produces steam that is used by an industrial process, and the demand for steam can fluctuate greatly, depending on the consumption. The power plant can use multiple fuels to generate steam, however, all fuels do not have the same properties regarding how quickly they generate heat. Cheap fuels such as coal and peat are favored over the more expensive fuels oil and gas due to the monetary consequences. However, should the demand rise abruptly, it might be necessary to resort to the more costly fuels as only those are able to generate enough steam per unit of time. Should it be possible to predict peaks in steam consumption, a buffer of steam could be generated in advance by burning cheap fuels, to cover the demand and reduce the need for expensive fuel. The peaks in consumption can also be reduced, leading to a smaller demand for costly fuels, however, some necessary demands can not be dropped.

To control the power plant's operation, both PID controllers and state feedback controllers will be defined. To test the controllers, the power plant will be modelled and simulated in Simulink. The controls will be defined and tuned in such a way that they can keep the power plant stable, and minimize the amount of expensive fuel used as per the given instructions [1]. The suitability of PID and state feedback controllers for the power plant will also be examined and the preferable option will be chosen.

In Section 2 the power plant model is introduced. Section 3 examines the stability of the upper section of the power plant and the usage of a PID controller to control it. Section 4 continues on the earlier section by investigating the use of a state controller on the upper section to control it and Section 5 additionally adds an integrating component to the state controller. In Section 6 the two controllers are compared and discussed. Section 7 investigates the effect of changes in the counter pressure and tunes a controller responsible for it. The workings of the steam battery are introduced and investigated in Section 8. Section 9 investigates the model as a whole and tunes the parameters for the final model. Finally, Section 10 concludes the work.

# 2 The power plant

In Figures 1, 2, and 3, the power plant presented in voima.slx is visualized as Simulink diagrams. The power plant model consists of three main parts:

1. the upper section for steam production
2. the turbine flow
3. the lower section, which contain the counter pressure stock for steam distribution and the steam battery

Figure 1 gives an overview of the system. The inputs of the power plant are the fuel injection $u$ and the setting of the turbine control valve $z_1$. The steam consumption - the demand - of the power plant acts as the disturbance of the system.

The upper section where the steam is produced is shown in detail in Figure 2. In the diagram, from left to right, the steam is produced in the boiler and transferred to the high-pressure stock or out of the plant. The amount of steam generated depends on the

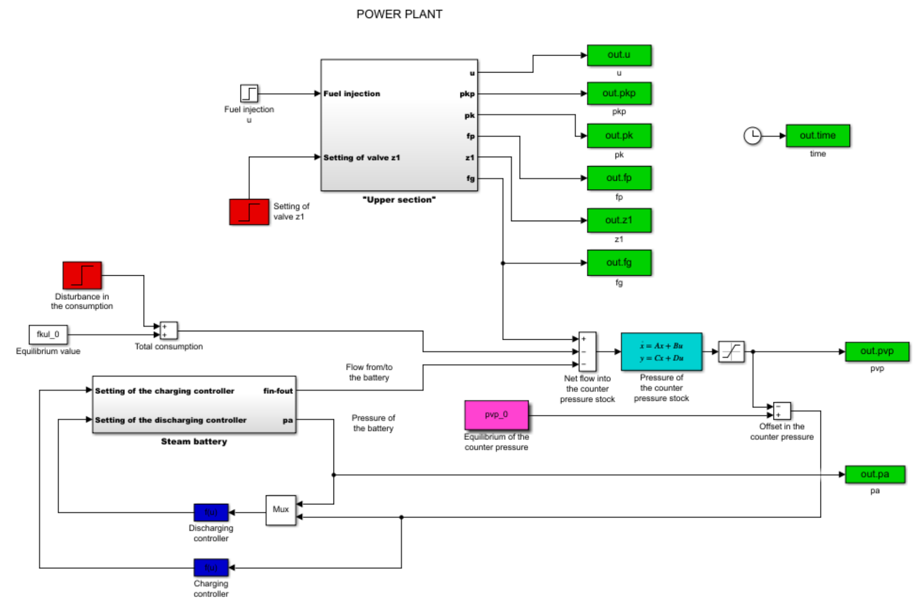



Figure 1: Diagram of the power plant in Simulink

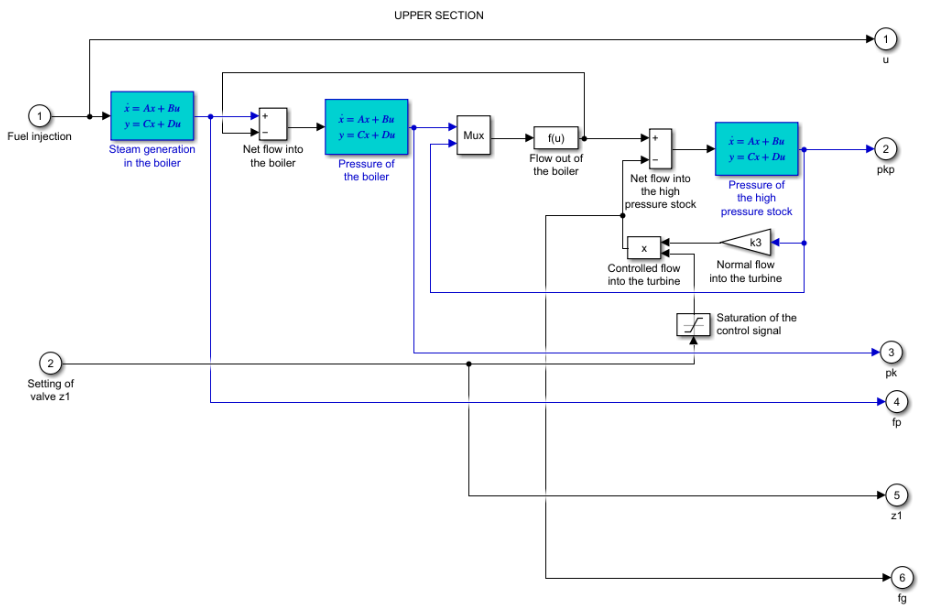



Figure 2: Diagram of the upper part of the power plant in Simulink

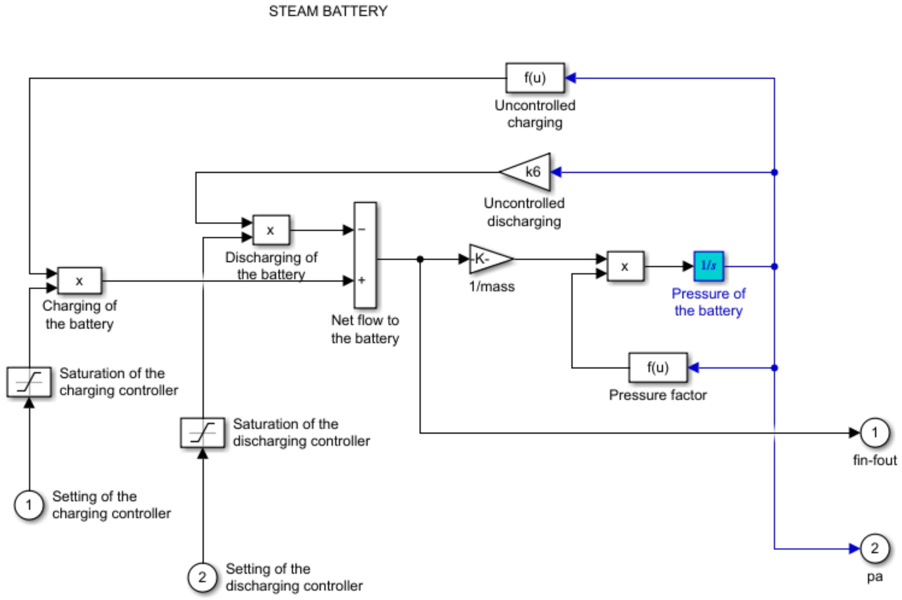



Figure 3: Diagram of the steam battery of the power plant in Simulink

amount of fuel that is injected. The pressure of the high-pressure stock is dependent on the flow out of the boiler and the valve $z$'s setting [1].

The steam battery part, shown in Figure 3, includes the flow in and out of the battery based on the charge and discharge controllers as well as uncontrolled charge and discharge of the battery. The flow in and out of the battery affects the pressure of the counter pressure stock.

A complete list of the variable and constant names used in the assignment is presented in Table 1.

| | |
|---|---|
| $f_p(t)$ | Steam generation in the boiler |
| $f_{ka}(t)$ | Flow of steam out of the boiler |
| $f_g(t)$ | Flow into the turbine |
| $f_{out}(t)$ | Flow out of the battery |
| $f_{in}(t)$ | Flow into the battery |
| $f_{kul}(t)$ | Flow to consumption |
| $u(t)$ | Fuel injection |
| $T_1$ | Parameter of the steam production in the boiler |
| $T_s$ | Storage capacity of the boiler |
| $k_0$ | Conversion factor from the fuel injection to the steam production |
| $k_1$ | Flow resistance from the boiler to the steam stock |
| $k_2$ | Storage capacity of the steam stock |
| $k_3$ | Flow resistance to the turbine |
| $k_4$ | Storage capacity of the counter pressure stock |
| $k_5$ | Flow resistance into the steam battery |
| $k_6$ | Flow resistance out of the steam battery |
| $p_k(t)$ | Pressure of the boiler |
| $p_{kp}(t)$ | Pressure of the high pressure stock |
| $p_{vp}(t)$ | Pressure of the counter pressure stock |
| $p_a(t)$ | Pressure of the battery |
| $z_1(t)$ | Output signal of controller 1 ($z_1 \in [0.8, 1.2]$) |
| $z_2(t)$ | Output signal of controller 2 ($z_2 \in [0, 1]$) |
| $z_3(t)$ | Output signal of controller 3 ($z_3 \in [0, 1]$) |
| $h_1$ | Parameter of the steam battery |
| $h_2$ | Parameter of the steam battery |
| $m_a$ | Mass of the water in the battery |

Table 1: List of variable and constant names.

# 3 Stability of the upper section

In this section, the stability of the plant's upper section is investigated. The upper section refers to what can be seen in Figure 2, i.e. the part of the plant that contains the turbine, the boiler, and the high-pressure stock.

## 3.1 Disturbance of a valve in the plant

The power plant is initially in a steady state, as all flows in and out are at an equilibrium. In an instant, a plant worker accidentally opens the valve $z_1$ enough to increase the flow into the turbine by 3kg/s.

To determine a value for the new setting for the valve $z_1$, it is assumed that the change in setting is instantaneous and that the pressure of the high-pressure stock does not change. Then, the new valve setting can be calculated as

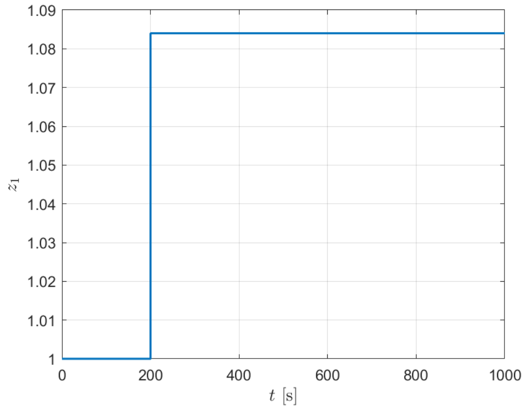
$$k_3 z_1' p_{kp0} = k_3 z_1 p_{kp0} + 3 \tag{1}$$

$$z_1' = \frac{k_3 z_1 p_{kp0} + 3}{k_3 p_{kp0}} \tag{2}$$

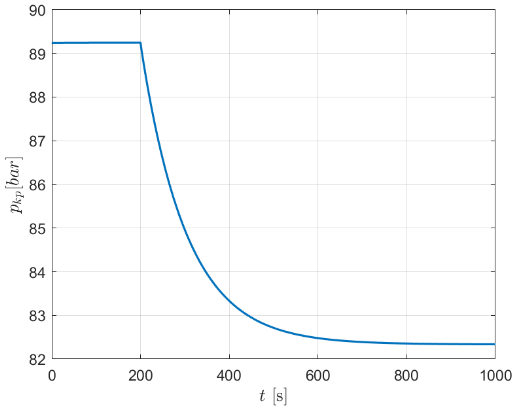
$$z_1' = 1.084, \tag{3}$$

where $z_1'$ is the new value of the valve. The valve setting thus changes by 0.084, and the new value is 1.084.

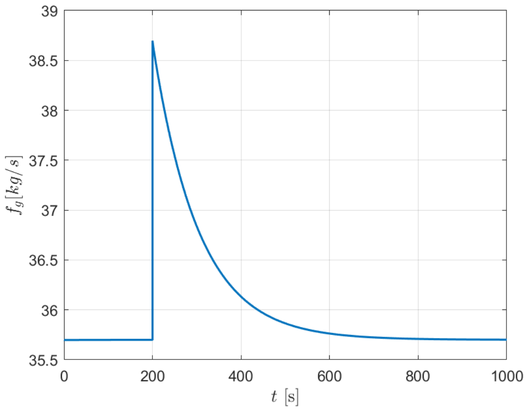
To find out what the consequences of such a change in valve position have, the change is simulated in the power plant's Simulink model. The change of valve setting happens at $t = 200$s and is illustrated in Figure 4a. As seen in Figure 4b, the pressure in the high-pressure stock goes down after the valve opens, until it reaches the new equilibrium value. This is logical since no additional fuel is injected, and the power plant can not sustain the initial pressure in the high-pressure stock. The flow into the turbine as seen in 4c increases as the valve is initially opened, but since the extra flow can not be sustained it also returns to the initial value, corresponding to the steam generated. The steam generation is shown in Figure 4d.



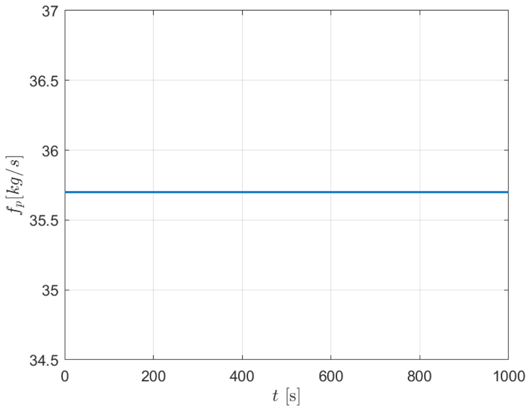

(a) The value of the valve of the upper section, $z_1$




(b) The pressure of the high-pressure stock, $p_{kp}$.




(c) The flow into the turbine, $f_g$.




(d) Steam generation in the boiler, $f_p$.

Figure 4: The state of the system when changing the setting of $z_1$ to $z_1'$ at $t = 200$.

## 3.2 PID-controller tuning

The goal is to operate the plant in a safe and efficient way. Large changes in the high-pressure Stock need to be avoided as well as rapid changes in the fuel injection. Large changes in pressure strain the stock, and the stock's maximum allowed pressure is 100 bar. Should the pressure exceed 100 bar, the entire plant will shut down as a safety precaution. Rapid changes in fuel injection are possible, but require the use of more expensive fuels, and should thus be avoided if possible. For this assignment, the goal is to keep the high-pressure stock offset from the steady state pressure within ±2% and the change of fuel to be less than 1 kg/s for a period of 100 s. However, should only one condition be feasible to fulfil, then it is more important to keep the pressure under control.

For controlling the power plant's steam generation, a PID controller will be introduced into the upper section of the system. The PID controller will control the injection of fuel using as input the offset of the pressure of the high-pressure steam stock from its reference value, and thus compensate for disturbances in steam consumption.

A PID controller is defined as

$$u_{PID}(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de}{dt}, \tag{4}$$

where $u_{PID}(t)$ is the PID control variable, $e(t)$ is the error value, $K_p$ is the proportional gain, $K_i$ the integral gain, and $K_d$ the differential gain. The error value is the difference between the reference value and the current value.

Initial testing of the effect of the PID controller was conducted. The authors learned how the different components affect the pressure curve using a heuristic approach. The result of this is not reported here, since it is not used in the work other than through the authors' knowledge.

The PID controller for this assignment is tuned in steps, starting from a P controller. For finding appropriate PID parameters, a step response test is conducted, where the fuel injection is increased by 1 kg/s at the time t=200 s. The step response described is shown in Figure 5. Figure 5b illustrate how the fuel injection changes as described and Figure 5a the response of the high-pressure stock. The figures demonstrate the same effect seen in Figure 4.

The tuning of the PID controller will be done using the *Ziegler-Nichols* tuning rules [2]. Using the rules, we can characterize the step response using the two parameters $\alpha$ and $\tau$. These can be derived from Figure 5a using the tangent at the steepest point and setting origo to the moment the step response happens, i.e. $t = 200$. Then, $\alpha$ is the x-value where the tangent intercepts the y-axis and $\tau$ is the y-value where the tangent intercepts the x-axis.

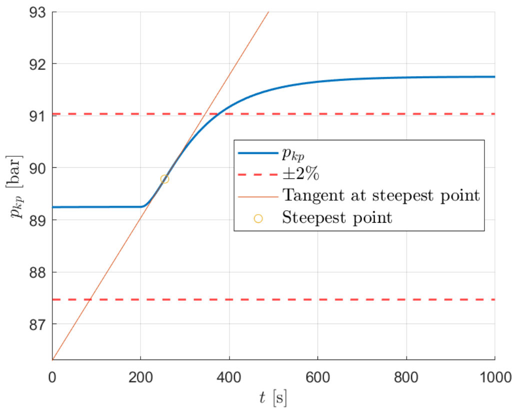
In order to obtain the gains $k_p$, $k_i$, and $k_d$ from the parameters $\alpha$ and $\tau$ the following formulas

$$T_i = k_i k_p \iff k_i = \frac{T_i}{k_p} \tag{5}$$

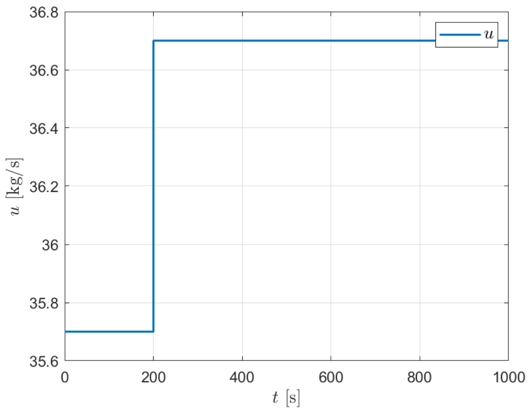
$$T_d = \frac{k_d}{k_p} \iff k_d = T_d k_p \tag{6}$$

Table 2: The table used to convert the parameters $\alpha$ and $\tau$ for the *Ziegler-Nichols* method[2].

| Type | $k_p$ | $T_i$ | $T_d$ |
|---|---|---|---|
| P | $1/\alpha$ | | |
| PI | $0.9/\alpha$ | $3\tau$ | |
| PID | $1.2/\alpha$ | $2\tau$ | $0.5\tau$ |




(a) The pressure of the high-pressure stock, $p_{kp}$, as a function of time $t$.




(b) The fuel injection $u$ as a function of time $t$.

Figure 5: The step response experiment.

and Table 2 are used depending on the type of the controller.

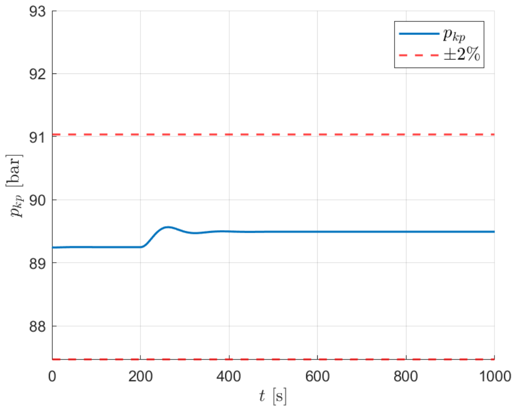
Tuning started with a P controller, as shown in Figures 6a and 6b. It can be seen that the controller manages to stabilize the pressure well within the safety margins of $\pm 2\%$, but it does not manage to return the pressure to the starting level. This is a feature of the P controller: as it requires an error to generate the proportional output response, it will always be offset.

Adding an I gain to the controller makes it a PI controller, and its operation is shown in Figures 6c and 6d. However, especially for a large I term, the system oscillates around the equilibrium during convergence, overshooting it several times. This might lead to additional fluctuation in fuel injection and unnecessary costs.

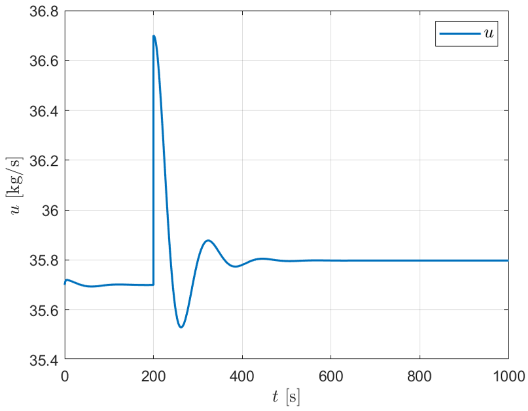
By adding a D gain, the system's behavior is further improved, as seen in Figures 6e and 6f. The D gain dampens the oscillations, as it depends on the current change of the error. During tuning, it was seen that higher D gains reduced the oscillations the most.

The complete PID controller is best suited for this system, as it returns the value to its reference value, keeps the pressure within limits at all times, and does not cause the need to use more expensive fuels. The PID gain coefficients used for tuning the P, PI, and PID controllers are presented in Table 3.

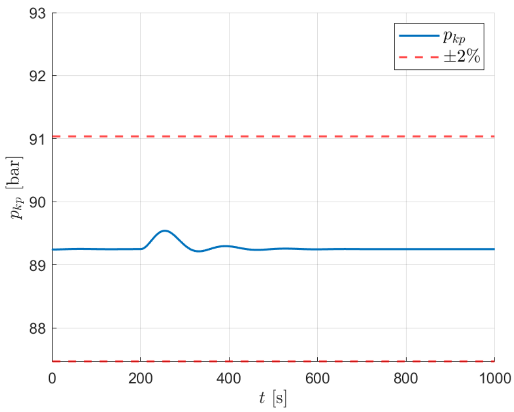
We now examine how the PID controller impacts the high-pressure stock when the change in the valve, discussed in Section 3.1, is applied. Figures 7 through 10 show the effects the controllers have on the high-pressure stock $p_{kp}$ and the fuel injection $u$. In Figures 7a and 7b we can see the state where no controller is applied at all. We can see that the



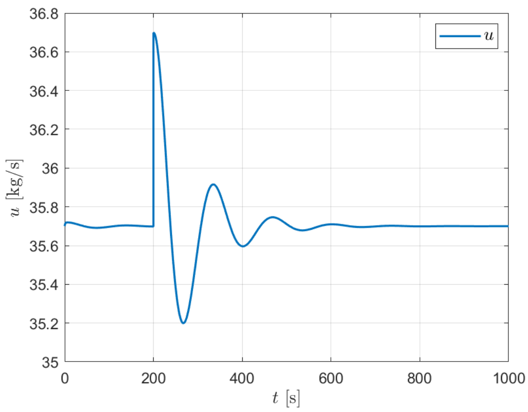

(a) High-stock pressure $P_{kp}$ with a **P** controller



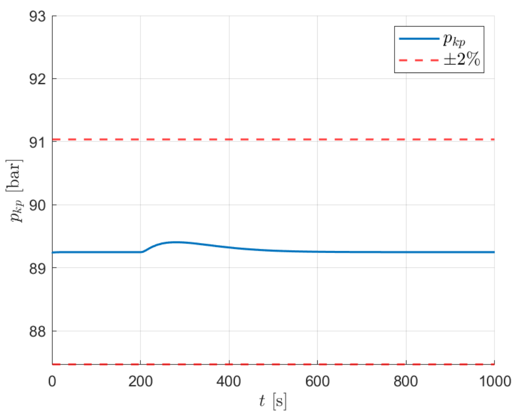

(b) The fuel injection $u$ with a **P** controller



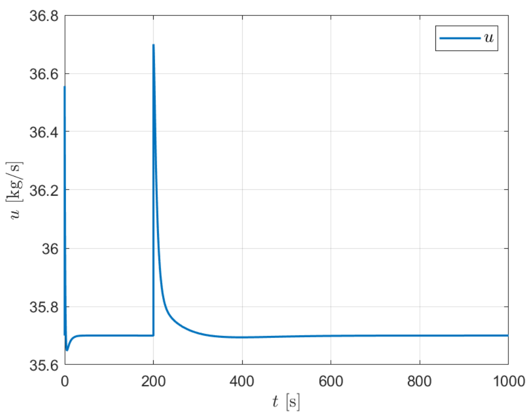

(c) High-stock pressure $P_{kp}$ with a **PI** controller




(d) The fuel injection $u$ with a **PI** controller




(e) High-stock pressure $P_{kp}$ with a **PID** controller

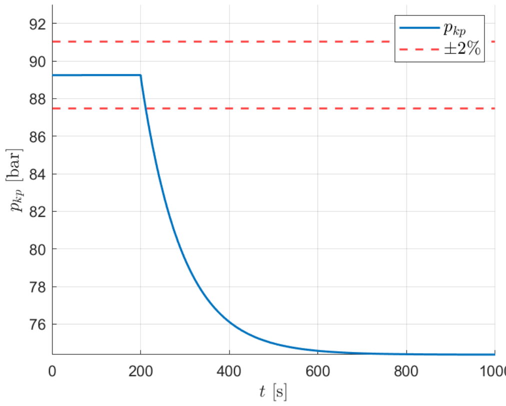



(f) The fuel injection $u$ with a **PID** controller

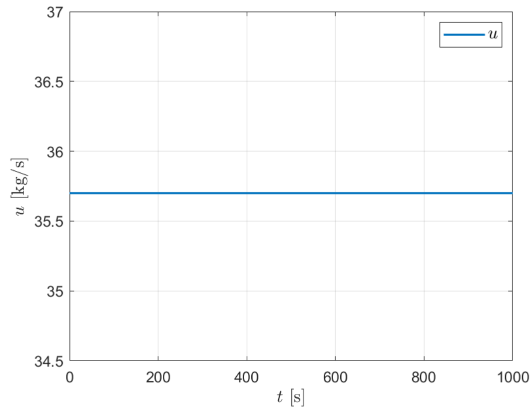
Figure 6: Figures illustrating the tuning of the PID controller to a step response of the fuel injection increasing by 1 kg/s.

Table 3: The gain coefficients chosen

| Gain | - | P | PI | PID |
|---|---|---|---|---|
| $\mathbf{K_p}$ | 0 | 4.72 | 4.25 | 5.66 |
| $\mathbf{K_i}$ | 0 | 0 | 0.15 | 0.10 |
| $\mathbf{K_d}$ | 0 | 0 | 0 | 37.77 |




(a) High-stock pressure $P_{kp}$ without a controller.




(b) The fuel injection $u$ without a controller.

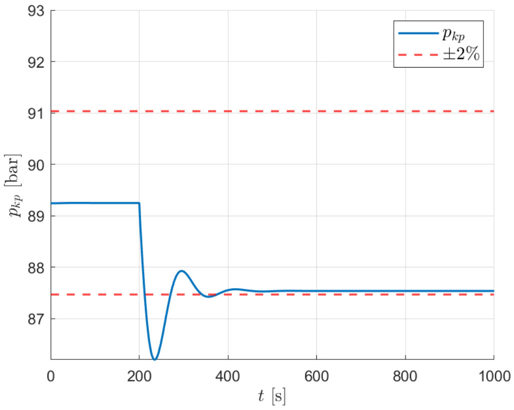
Figure 7: Figures illustrating the effect of the step-response on the fuel injection and the high-pressure stock when the valve is turned to $z_1 = 1.084$.

pressure drops far below the allowed 2% deviation.

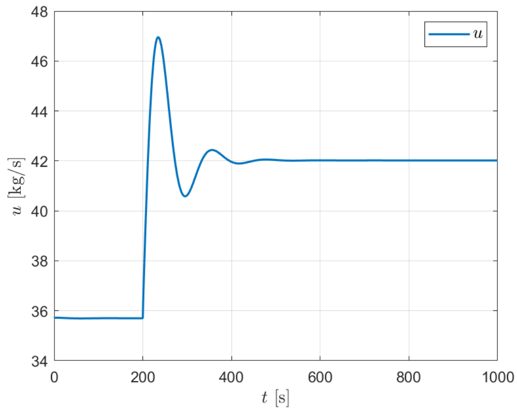
Let us now examine how the response affects the situation where the valve $z_1$ has been changed. In Figure 8 we can see the effect the P controller has on the system. We are able to stabilize the system inside the allowed margin, but the pressure initially dips below it. Additionally, the changes in the fuel injection are approx 12kg/100s after the initial change.

The effect of the I term is similar to that in the step response experiment, the value is able to return to the reference value as seen in Figure 9. However, the oscillations still occur and the required fuel per unit of time has increased furthermore.

Finally, by applying the PID controller to the system, visualized in Figure 10, we are able to keep the pressure within its given bounds over the course of the whole time span. The required fuel injection, however, has increased two folds in a matter of a second. However, based on the given guidelines of keeping the pressure within the 2% deviation over minimizing the use of costly fuels, the authors conclude that the result is acceptable and the values for the PID controller seen in Table 3 will be used in later parts of the study where the PID controller is present.

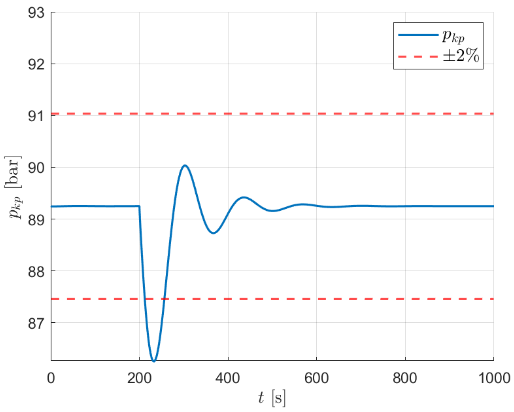



(a) High-stock pressure $P_{kp}$ with a **P** controller.

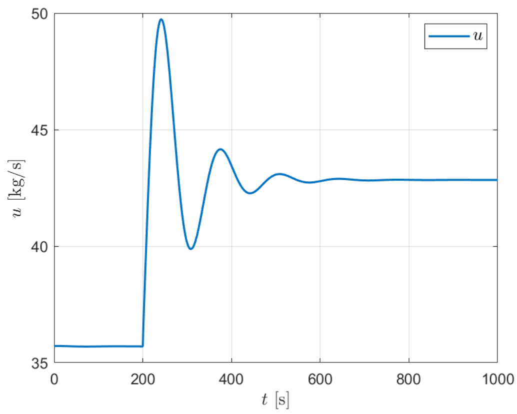



(b) The fuel injection $u$ with a **P** controller.

Figure 8: Figures illustrating the effect of a **P** controller on the fuel injection and the high-pressure stock when the valve is turned to $z_1 = 1.084$.

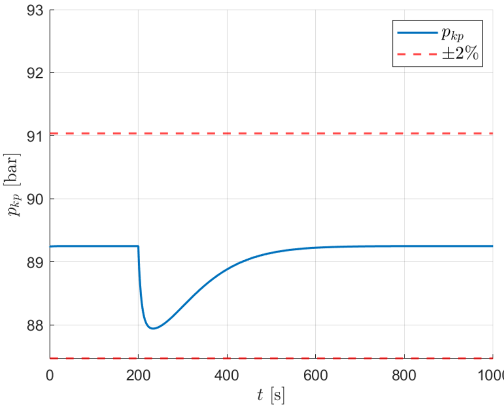



(a) High-stock pressure $P_{kp}$ with a **PI** controller.

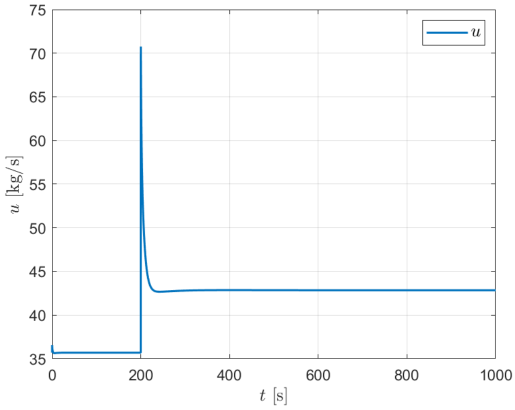



(b) The fuel injection $u$ with a **PI** controller.

Figure 9: Figures illustrating the effect of the **PI** controller on the fuel injection and the high-pressure stock when the valve is turned to $z_1 = 1.084$.




(a) High-stock pressure $P_{kp}$ with a **PID** controller.




(b) The fuel injection $u$ with a **PID** controller.

Figure 10: Figures illustrating the effect of the **PID** controller on the fuel injection and the high-pressure stock when the valve is turned to $z_1 = 1.084$.

# 4 State controller of the upper section

In the previous section, a PID controller was used to control the fuel injection. In this section, the usage of a state feedback controller is investigated as an alternative to the PID controller. The controller relies only on the states of the system itself and can thus be seen as a closed-loop system [1].

In the case of this (upper part of the) system, the state variables, which will be referred to as $x$, are $f_p$, $p_k$, and $p_{kp}$ [1]. Our controller is constructed as a linear function of these states, i.e. the system is defined as

$$\frac{dx}{dt} = Ax + Bu \tag{7}$$

Since our system is also time invariant we can formulate a Linear Quadratic Regulator (LQR) problem for the state feedback control. Thus, we want to minimize the cost

$$J = \int_0^\infty x^\top Qx + u^\top Rudt, \tag{8}$$

where Q is the cost state weighting matrix and R the control cost weighting matrix. Both Q and R are symmetric semi-positive definite matrices and R being not only semi-definite but definite positive[2]. The optimal input is then

$$u = -Kx \tag{9}$$

where

$$K = R^{-1}B^\top P[2]. \tag{10}$$

$P$ can be solved from the algebraic Riccatti equation

$$A^\top P + PA + PBR^{-1}B^\top P + Q = 0. \tag{11}$$

The derivatives, i.e. the linear approximations of the functions, of the function $f(x, u)$ can be solved from the equations defined in the assignment instructions, and are

$$f(x,u) = \begin{bmatrix} \frac{df_p}{dt} \\ \frac{dp_k}{dt} \\ \frac{dp_{kp}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{k_o u - f_p}{T_1} \\ \frac{f_p - k_1\sqrt{p_k^2 - p_{kp}^2}}{T_s} \\ k_2\left(k_1\sqrt{p_k^2 - p_{kp^2}} - k_3 z_1 p_{kp}\right) \end{bmatrix} \tag{12}$$

$$h(x,u) = p_{kp} \tag{13}$$

Since the reference values are not 0, we will move the system to the origin by defining a set of state variables such that

$$z = x - x_0 \tag{14}$$

$$v = u - u_0 \tag{15}$$

$$w = y - h(x,u) = y - p_{kp_0} \tag{16}$$

The Jacobian linearization of the system as presented in [2] is

$$\frac{dz}{dt} = Az + Bv \tag{17}$$

$$w = Cz + Dv \tag{18}$$

where

$$A = \frac{\delta f}{\delta x}(x_0,u_0), \quad B = \frac{\delta f}{\delta u}(x_0,u_0)$$
$$C = \frac{\delta h}{\delta x}(x_0,u_0), \quad D = \frac{\delta h}{\delta u}(x_0,u_0)$$

The matrices for the Jacobian linearization are calculated with Wolfram Mathematica

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & 0 \\ \frac{1}{T_s} & -\frac{k_1 p_{k0}}{T_s\sqrt{p_{k0}^2 - p_{\mathrm{kp0}}^2}} & \frac{k_1 p_{\mathrm{kp0}}}{T_s\sqrt{p_{k0}^2 - p_{\mathrm{kp0}}^2}} \\ 0 & \frac{k_1 k_2 p_{k_0}}{\sqrt{p_{k_0}^2 - p_{\mathrm{kp0}}^2}} & k_2\left(-\frac{k_1 p_{\mathrm{kp0}}}{\sqrt{p_{k_0}^2 - p_{\mathrm{kp0}}^2}} - k_3 z_1\right) \end{bmatrix}$$
$$B = \begin{bmatrix} \frac{k_0}{T_1} \\ 0 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$
$$D = 0$$

## 4.1 Controllability of the system

Next, the controllability of the system will be verified. As stated in [2], a linear system is reachable if its reachability matrix

$$W_r = \begin{bmatrix} B & AB & ... & A^{n-1}B \end{bmatrix}$$

is invertible. For a matrix to be invertible, its determinant must not be 0. The reachability matrix for the model is

$$\begin{aligned}
W_r &= \begin{bmatrix} B & AB & A^2B \end{bmatrix} && (19)\\
&= \begin{bmatrix} \frac{k_0}{T_1} & 0 & 0 \\ -\frac{k_0}{T_1} & \frac{k_0}{T_1 T_s} & 0 \\ \frac{k_0}{T_1} & \frac{k_0(-\frac{k_1 p_{k0}}{\sqrt{p_{k0}^2 - p_{kp_0}^2} T_s^2} - \frac{1}{T_1 T_s})}{T_1} & \frac{k_0 k_1 k_2 p_{k0}}{\sqrt{p_{k0}^2 - p_{kp_0}^2} T_1 T_s} \end{bmatrix} && (20)\\
det(W_R) &= \frac{k_0^3 k_1 k_2 p_{k0}}{\sqrt{p_{k0}^2 - p_{kp_0}^2} T_1^3 T_s^2} \neq 0. && (21)
\end{aligned}$$

As long as the pressure $p_k$ is not zero, the determinant is not zero. As the determinant is nonzero, the system is controllable.

## 4.2 State feedback model's performance with different weights

With the system linearized, it is now possible to calculate the LQR for the cost function, as seen in Equation 8. By using MatLab's lqr function and setting the cost weight matrices $Q$ and $R$, the optimal control law is then solved from the Riccati equation. The lqr function solves the optimal gain matrix K and the system's poles. The optimal control law is

$$v = -Kz. \tag{22}$$

As we previously moved the coordinates to start at origin, we must return to the original coordinates by substituting $v$ and $z$,

$$\begin{aligned}
v &= u - u_0 && (23)\\
z &= x - x_0 && (24)\\
\Rightarrow u - u_0 &= -K(x - x_0) \Rightarrow u = -K(x - x_0) + u_0 && (25)
\end{aligned}$$

Thus, the optimal control law is a sum of the equilibrium control $u_0$ and the state feedback control $-K(x-x_0)$, and can be implemented in Simulink as the sum of the existing signal for fuel injection and the state feedback. These controllers are visualized in Figure 11. The initial tuning of the weights $Q$ and $R$ was done by choosing the initial weights

| Test n | $Q_{11}$ | $Q_{22}$ | $Q_{33}$ | R | K | P |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0.7667<br>0.2803<br>0.5511 | -0.0317<br>-0.0361<br>-1.0292 |
| 2 | $10^{-5}$ | 1 | 1 | 0.06 | 1.7267<br>1.6086<br>3.1387 | -0.0499 + 0.0432i<br>-0.0499 - 0.0432i<br>-1.0292 + 0.0000i |
| 3 | $10^{-5}$ | 10 | 1 | 0.06 | 3.2310<br>4.2253<br>7.9445 | -0.0751 + 0.0686i<br>-0.0751 - 0.0686i<br>-1.0289 + 0.0000i |
| 4 | $10^{-6}$ | 1 | 0.0031 | 0.06 | 1.3405<br>1.1195<br>2.1674 | -0.0435 + 0.0355i<br>-0.0435 - 0.0355i<br>-1.0292 + 0.0000i |

Table 4: Poles and K's for different Q's and R's

according to Bryson's rule, where the diagonal elements are chosen according to the maximum acceptable value for costs,

$$Q_{ii} = \frac{1}{\text{maximum acceptable}\, z_i^2} \quad R_{jj} = \frac{1}{\text{maximum acceptable}\, u_j^2}$$

However, the results were not very good. Instead, tuning was done by iterating different values for the matrix elements, and choosing the ones that gave the best results. The values tried for the weighing matrices, as well as the resulting controls and poles are shown in Table 4. The performance of the state feedback control will be tested on the same scenario as in the previous section, where the plant worker opens the valve.

The values in the first experiment shown in Figure 12 stabilized the system, but did not keep $p_{kp}$ within the acceptable bounds, as it dropped to around 86 bar. Fuel consumption behaved smoothly, but the change went over the desired maximum of 1kg/100s.

The second values stabilized $p_{kp}$ within the bounds, as seen in Figure 13. It is however significantly below the equilibrium, and the peak in fuel consumption as seen in Figure 13d is above the desired maximum.

The third values gave a result as seen in Figure 14. The pressure again stabilized within the bounds, and as with the second values, the fuel consumption peaked higher than desired.

The fourth values provided a good balance of stabilizing the system well with only a small peak in fuel consumption, as can be seen in Figure 15. These values are chosen as the weighing matrices.

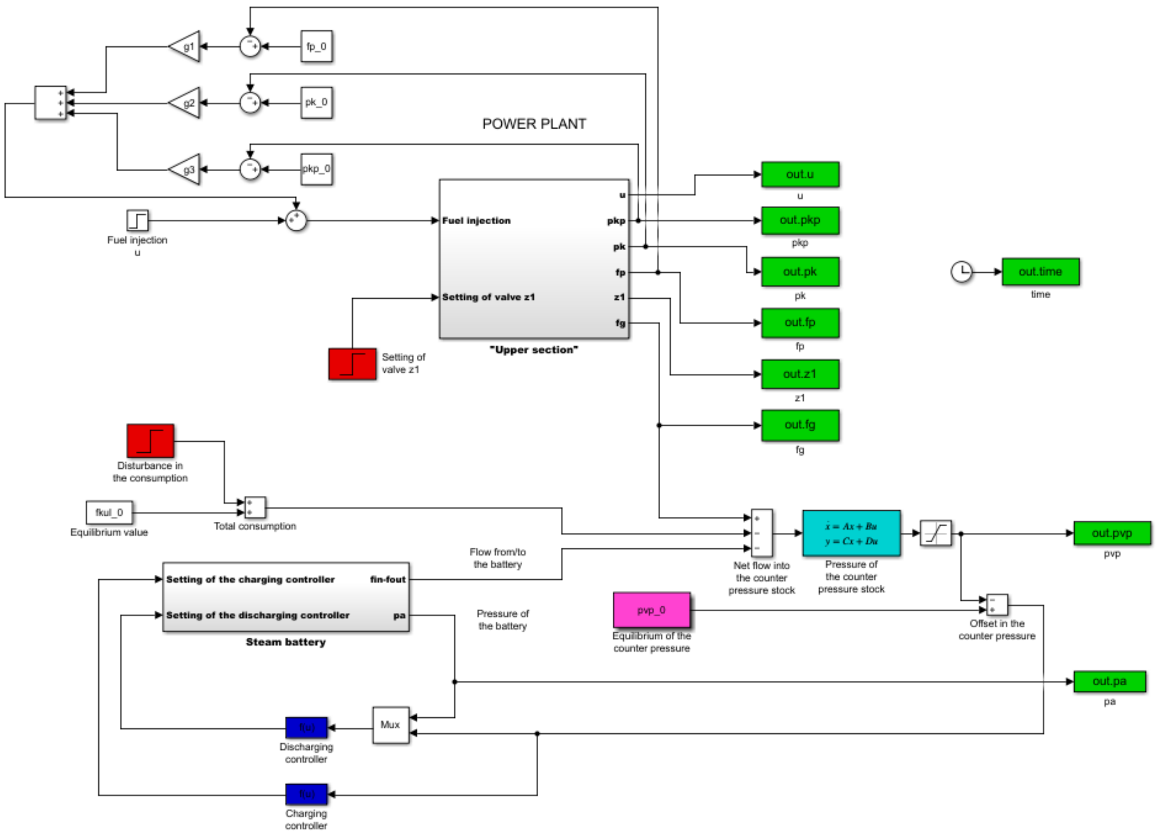
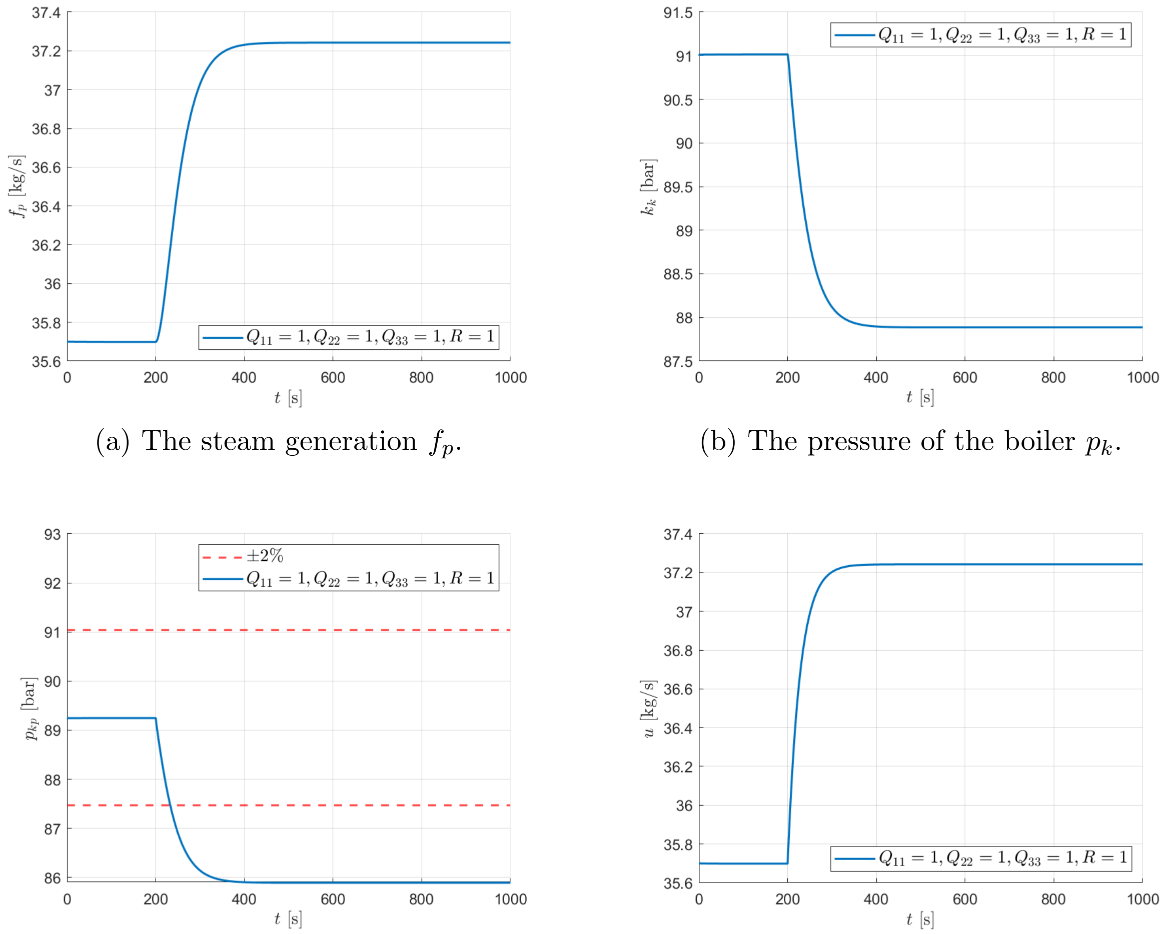



Figure 11: The diagram of the power plant with the state feedback control.




(a) The steam generation $f_p$.



(b) The pressure of the boiler $p_k$.

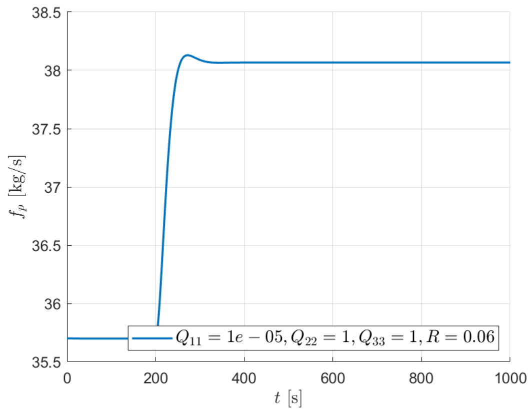


(c) The high-pressure stock pressure $p_{kp}$.

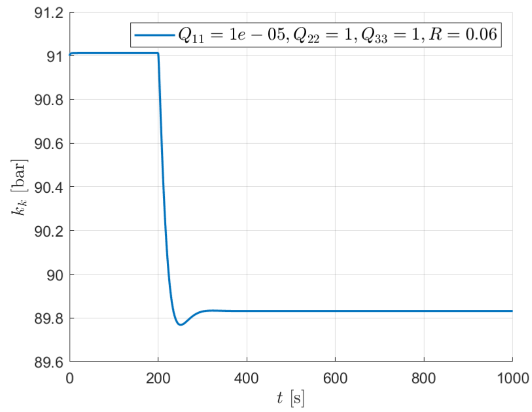


(d) The fuel injection $u$.

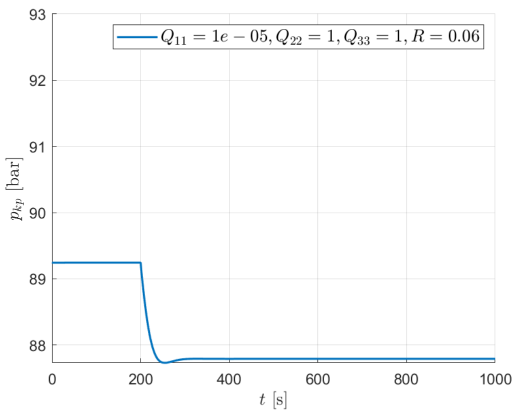
Figure 12: State feedback experiment 1



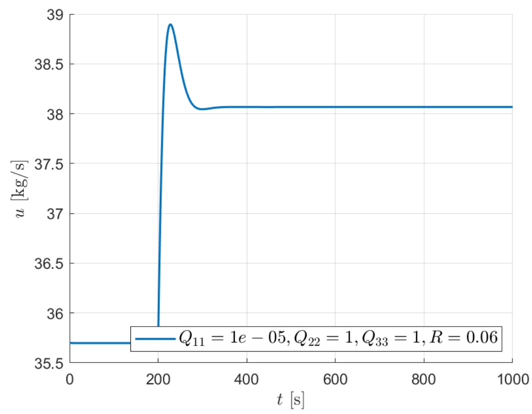

(a) The steam generation $f_p$.




(b) The pressure of the boiler $p_k$.

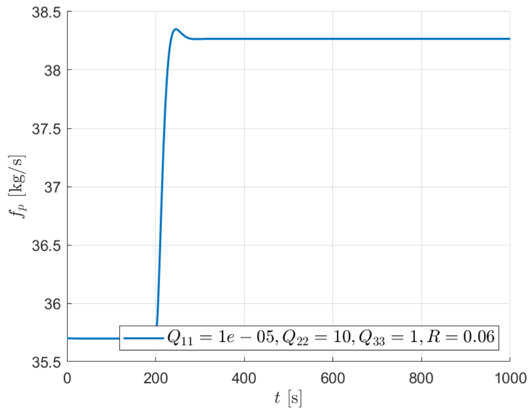



(c) The high-pressure stock pressure $p_{kp}$.

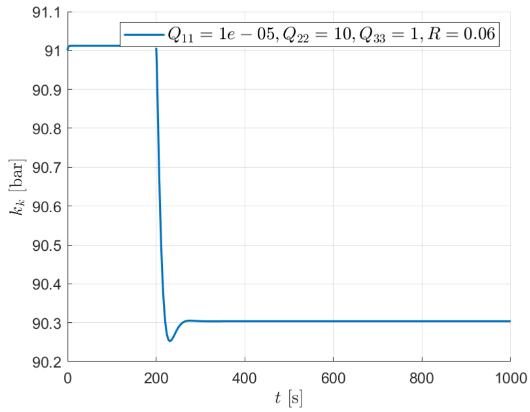



(d) The fuel injection $u$.

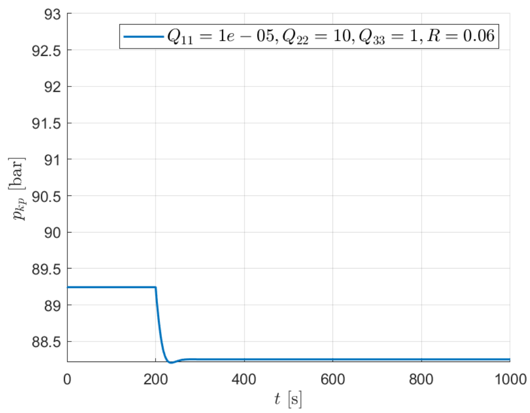
Figure 13: State feedback experiment 2



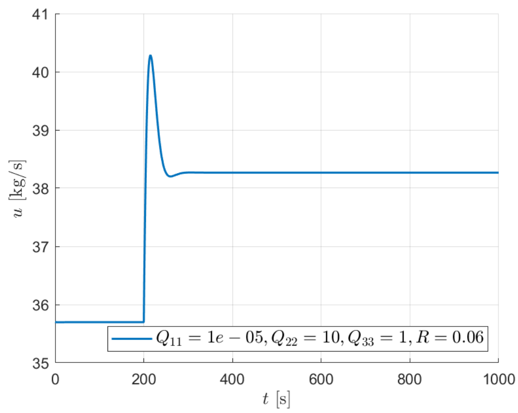

(a) The steam generation $f_p$.




(b) The pressure of the boiler $p_k$.

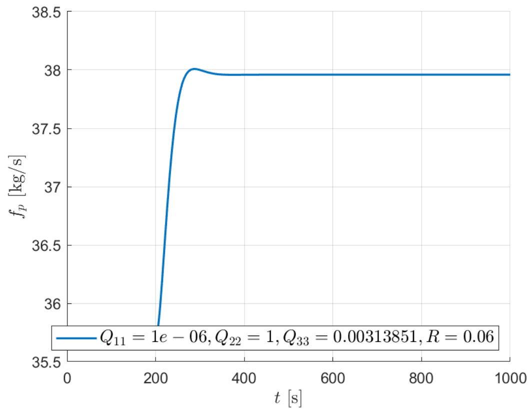



(c) The high-pressure stock pressure $p_{kp}$.

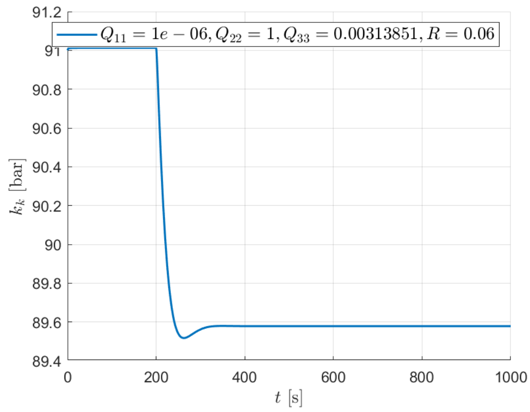



(d) The fuel injection $u$.

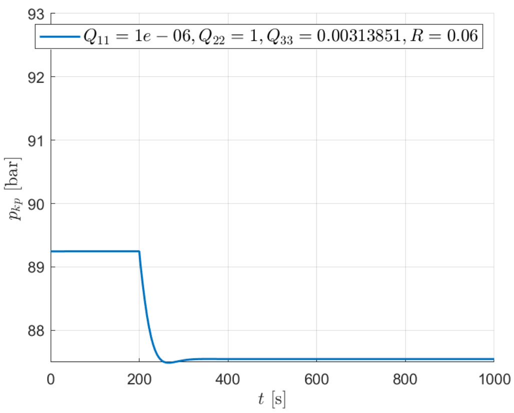
Figure 14: State feedback experiment 3



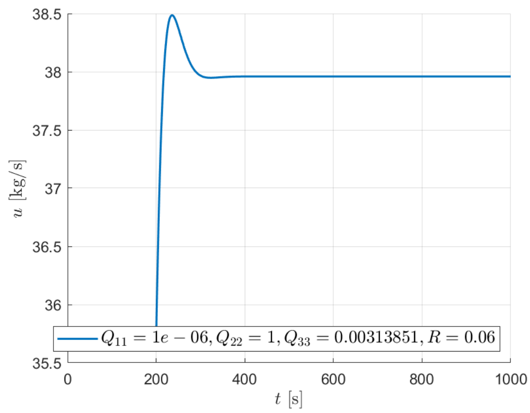

(a) The steam generation $f_p$.




(b) The pressure of the boiler $p_k$.




(c) The high-pressure stock pressure $p_{kp}$.




(d) The fuel injection $u$.

Figure 15: State feedback experiment 4

# 5 State controller with integration

The state feedback controller defined in the previous section was not sufficient in itself to compensate for the opening of the valve. This is clear from the experiments, as the pressure, $p_{kp}$ was stabilized but not returned to the equilibrium. The state feedback controller acts like a regular P controller. To improve the state feedback controller, an integrating part can be added by including a new artificial state variable $x_4$, with a state equation $\dot{x_4} = e = r - y = r - Cx$. As the output variable is $y = p_{kp}$, the resulting equation is

$$\dot{x_4} = r - Cx = p_{kp_0} - p_{kp}. \tag{26}$$

The new system is visualized in Figure 20.

After adding the new variable $x_4$, the Jacobians $A$, $B$, $C$ and $D$ can be calculated similarly as in the previous section:

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & 0 & 0 \\ \frac{1}{T_s} & -\frac{k_1 p_{k0}}{T_s\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} & \frac{k_1 p_{\mathrm{kp0}}}{T_s\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} & 0 \\ 0 & \frac{k_1 k_2 p_{kp}}{\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} & k_2\left(-\frac{k_1 p_{\mathrm{kp0}}}{\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} - k_3 z_1\right) & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{k_0}{T_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0.$$

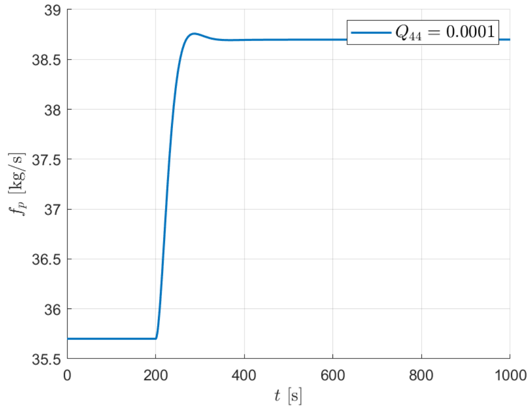
To see whether the system is reachable, the reachability matrix $W_r$ and its determinant is also calculated for this new system.

$$W_r = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$$

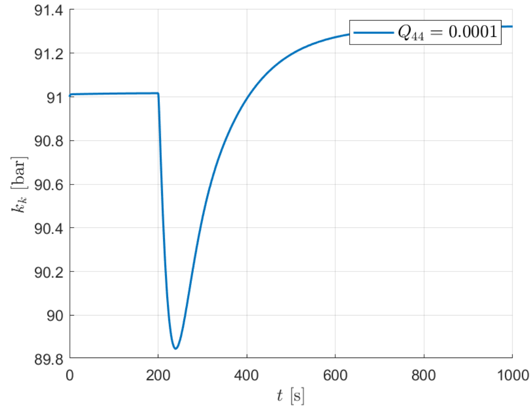
$$W_r = \begin{bmatrix} \frac{k_0}{T_1} & 0 & 0 & 0 \\ -\frac{k_0}{T_1^2} & \frac{k_0}{T_1 T_s} & 0 & 0 \\ \frac{k_0}{T_1^3} & \frac{k_0\left(-\frac{k_1 p_{kp}}{T_s^2\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} - \frac{1}{T_1 T_s}\right)}{T_1} & \frac{k_0 k_1 k_2 p_k}{T_1 T_s\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} & 0 \\ -\frac{k_0}{T_1^4} & \frac{k_0\left(\frac{\frac{k_2 k_1^2 p_{kp} p_{\mathrm{kp0}}}{T_s\left(p_{kp}^2 - p_{\mathrm{kp0}}^2\right)} + \frac{k_1^2 p_{kp}^2}{T_s^2\left(p_{kp}^2 - p_{\mathrm{kp0}}^2\right)}}{T_s} - \frac{-\frac{k_1 p_{kp}}{T_s^2\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} - \frac{1}{T_1 T_s}}{T_1}\right)}{T_1} & \frac{k_0\left(\frac{\frac{k_1 k_2^2 p_{kp}\left(-\frac{k_1 p_{\mathrm{kp0}}}{\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} - k_3 z_1\right)}{\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} - \frac{k_1^2 k_2 p_{kp}^2}{T_s\left(p_{kp}^2 - p_{\mathrm{kp0}}^2\right)}}{T_s} - \frac{k_1 k_2 p_{kp}}{T_1 T_s\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}}\right)}{T_1} & -\frac{k_0 k_1 k_2 p_{kp}}{T_1 T_s\sqrt{p_{kp}^2 - p_{\mathrm{kp0}}^2}} \end{bmatrix}$$

$$\det(W_r) = -\frac{k_0^4 k_1^2 k_2^2 p_{kp}^2}{T_1^4 T_s^3\left(p_{kp}^2 - p_{kp0}^2\right)} \neq 0$$

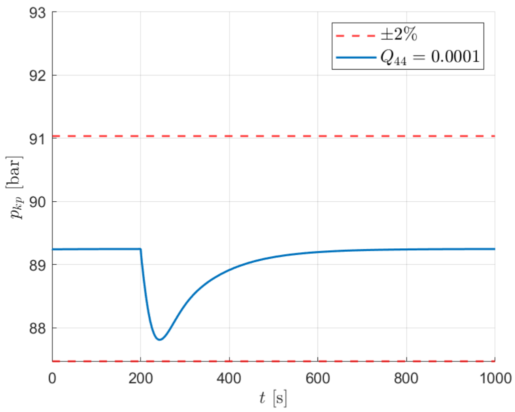
Again, the determinant is nonzero, and the system is reachable.



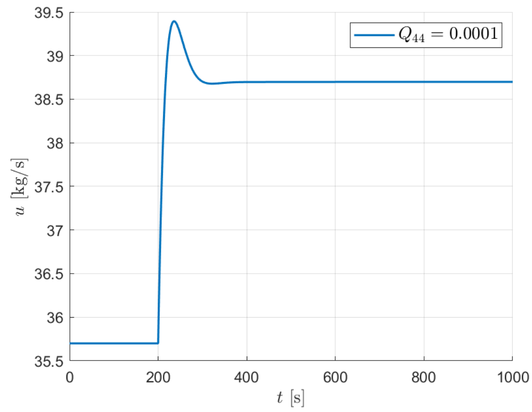

(a) $f_p$ for the first experiment of state feedback with integrating part.




(b) $p_k$ for the first experiment of state feedback with integrating part.




(c) $p_{kp}$ for the first experiment of state feedback with integrating part.



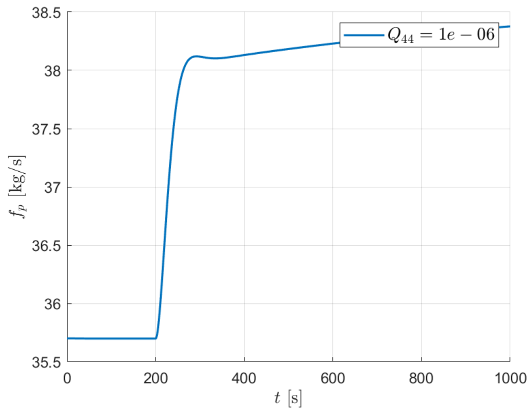

(d) $u$ for the first experiment of state feedback with integrating part.

Figure 16: State feedback experiment with integral 1

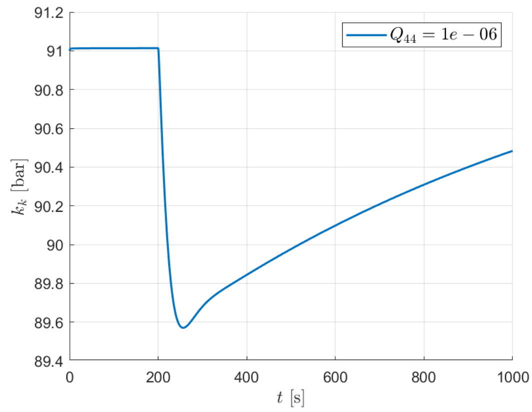
## 5.1 State feedback model with integral's performance with different weights

The values for the weighing matrices from the previous section are used, and different values for $Q_{44}$ are tried. The values tried for the weighing matrices, as well as the resulting controls and poles are shown in Table 4.

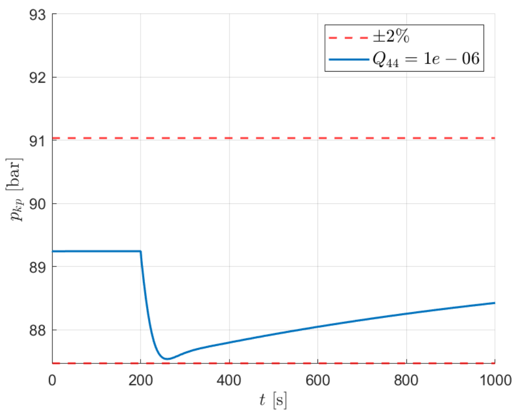
In Figure 16, the first value for $Q_{44}$ is tried. Now, $p_{kp}$ returns to equilibrium. Trying a smaller value for $Q_{44}$ in 17 decreased the jump in fuel consumption, but barely kept $p_{kp}$ within the bounds. Trying the higher values 1 and 10 for $Q_{44}$ leads to a very high jump in fuel consumption as seen in Figure 18 and 19, and a very quick correction in $p_{kp}$. The peak in fuel consumption is so high that they are not good candidates. In the end, the value $Q_{44} = 10^{-4}$ is chosen.



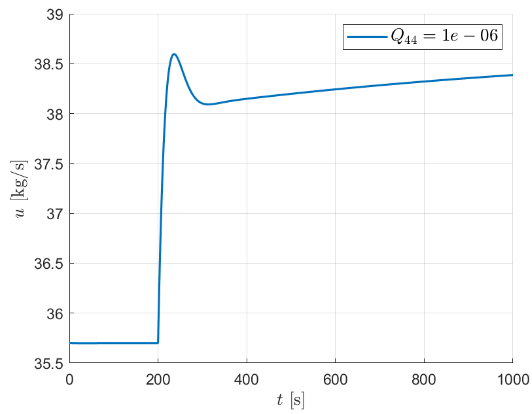

(a) $f_p$ for the second experiment of state feedback with integrating part.




(b) $p_k$ for the second experiment of state feedback with integrating part.

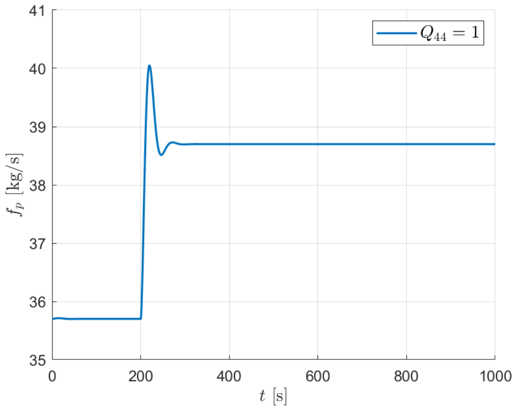



(c) $p_{kp}$ for the second experiment of state feedback with integrating part.

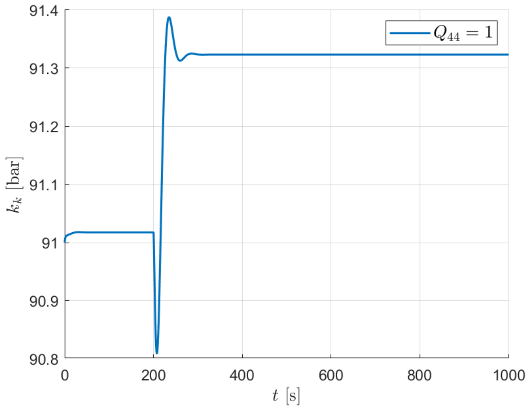



(d) $u$ for the second experiment of state feedback with integrating part.

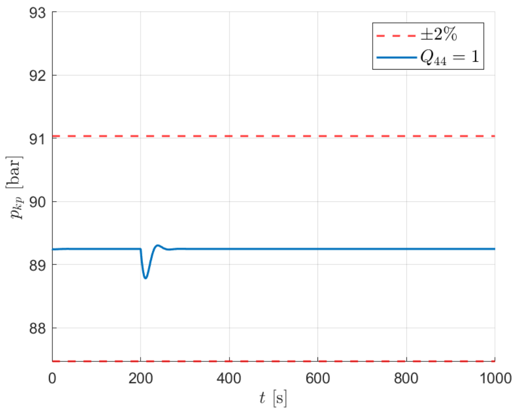
Figure 17: State feedback experiment with integral 2



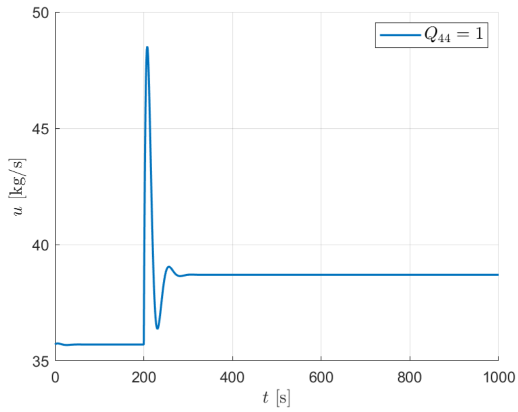

(a) $f_p$ for the third experiment of state feedback with integrating part.




(b) $p_k$ for the third experiment of state feedback with integrating part.

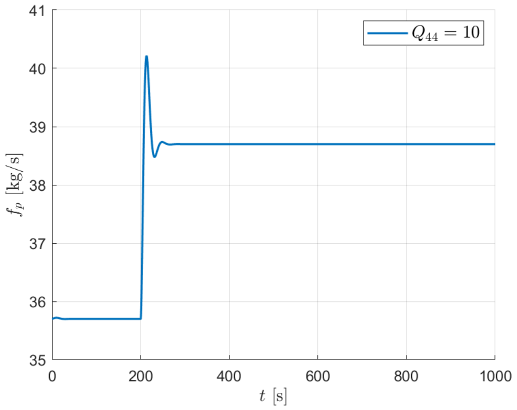



(c) $p_{kp}$ for the third experiment of state feedback with integrating part.

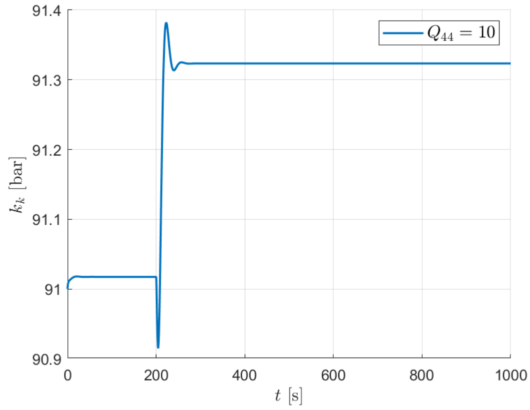



(d) $u$ for the third experiment of state feedback with integrating part.

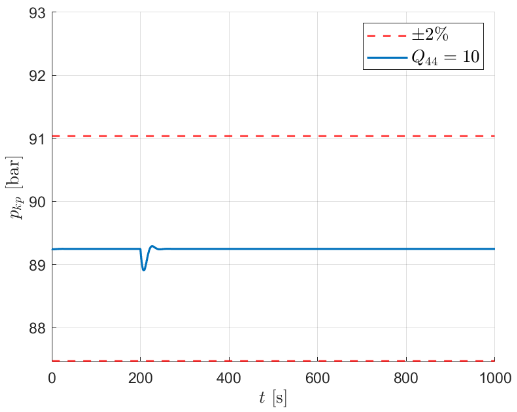
Figure 18: State feedback experiment with integral 3



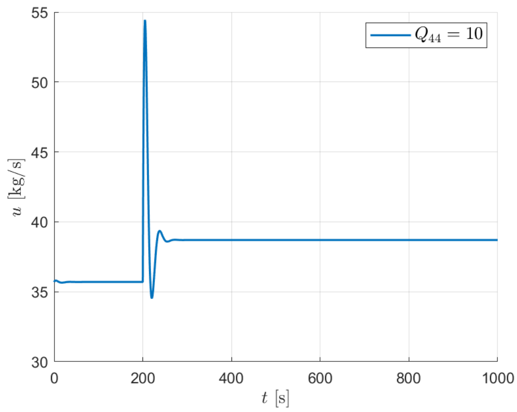

(a) $f_p$ for the fourth experiment of state feedback with integrating part.




(b) $p_k$ for the fourth experiment of state feedback with integrating part.




(c) $p_{kp}$ for the fourth experiment of state feedback with integrating part.




(d) $u$ for the fourth experiment of state feedback with integrating part.

Figure 19: State feedback experiment with integral 4

| Test $n$ | $Q_{11}$ | $Q_{22}$ | $Q_{33}$ | $Q_{44}$ | $R$ | $K$ | $P$ |
|---|---|---|---|---|---|---|---|
| 1 | $10^{-6}$ | 1 | 0.0031 | $10^{-4}$ | 0.06 | 1.6059<br>1.4477<br>2.8552<br>-0.0408 | -0.0096 + 0.0000i<br>-0.0431 + 0.0357i<br>-0.0431 - 0.0357i<br>-1.0292 + 0.0000i |
| 2 | $10^{-6}$ | 1 | 0.0031 | $10^{-6}$ | 0.06 | 1.3675<br>1.1513<br>2.2340<br>-0.0041 | -0.0010 + 0.0000i<br>-0.0434 + 0.0355i<br>-0.0434 - 0.0355i<br>-1.0292 + 0.0000i |
| 3 | $10^{-6}$ | 1 | 0.0031 | 1 | 0.06 | 7.4866<br>17.7556<br>38.9786<br>-4.0825 | -0.0730 + 0.1238i<br>-0.0730 - 0.1238i<br>-0.1458 + 0.0000i<br>-1.0292 + 0.0000i |
| 4 | $10^{-6}$ | 1 | 0.0031 | 10 | 0.06 | 11.4729<br>38.6432<br>88.7613<br>-12.9099 | -0.1052 + 0.1827i<br>-0.1052 - 0.1827i<br>-0.2143 + 0.0000i<br>-1.0292 + 0.0000i |

Table 5: Poles and K's for different Q's and R's (with integral).

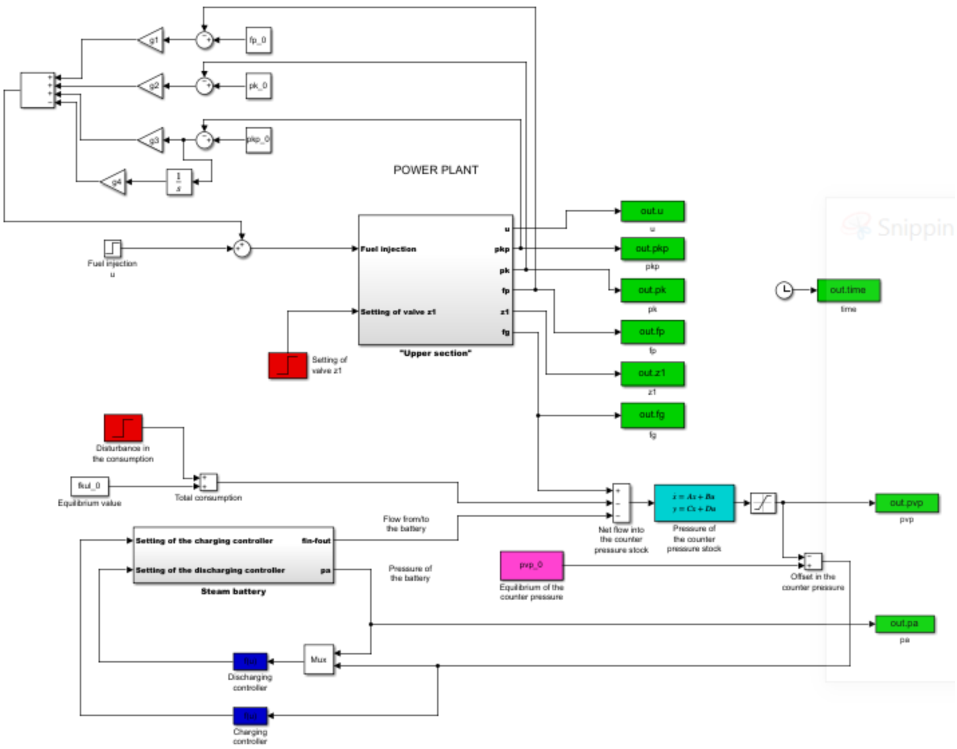
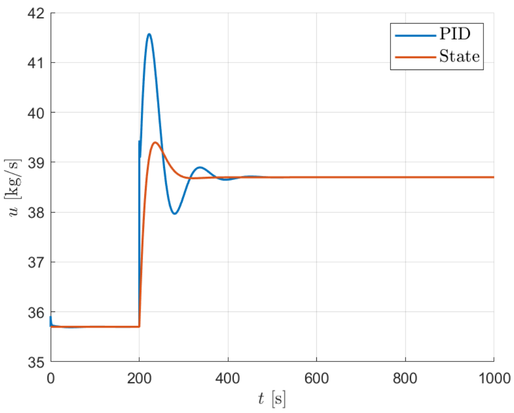



Figure 20: The diagram of the powerplant with the state feedback control with integrating part

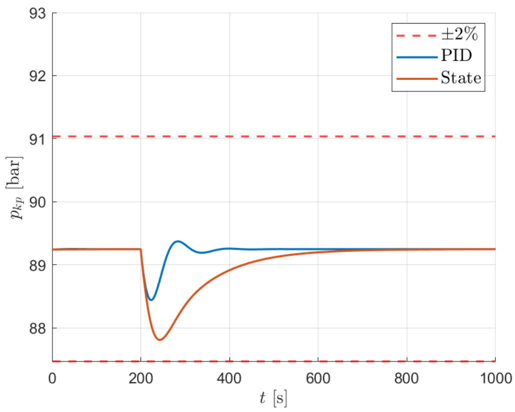
# 6 PID-control vs. state feedback control

PID and state feedback controllers are both used for selecting the control signal based on the difference between the output and reference signal. They do however differ in some aspects, mainly what they use to determine the current state of the system. A key difference is that a PID controller can be defined without knowing everything about the system's dependencies, whereas a state feedback controller takes all the system's dependencies into account. A PID controller can be tuned by using the Ziegler-Nichols method for a step response experiment. In this method, only a single input and a single output are examined. Tuning a state feedback controller involves solving the optimal control from the cost function with the Riccati equation. This takes into account the multiple inputs and also their magnitude.

For the system described in this assignment, the PID controller forces the high-pressure stock pressure faster back to the equilibrium, due to its tuning. A comparison of the results of the two controllers can be seen in Figure 21. The PID controller will be chosen to move forward with, even though the Figure shows that the PID controller's performance is worse than that of the state control. This is due to the point that it is easier to tune the complete system using the PID controller than the state controller. Additionally, later we will find that the PID controller is superior with the correct tuning.




(a) The fuel injection $u$.




(b) The high pressure stock $p_{kp}$.q

Figure 21: A comparison of the performance of state feedback and PID controllers.

# 7 Counter-pressure tuning

## 7.1 Increase in steam consumption

Steam flow $f_{kul}$ suddenly increases with 2kg/s. This is simulated in Simulink, and the result of the occurrence is shown in Figure 22. As seen in the figure, the counter pressure drops at a constant rate and goes to zero as the outflow becomes larger than the inflow. This is not acceptable, as the pressure is meant to stay within 10% of its equilibrium value. The acceptable limits are indicated by the red dotted line. To remedy this, a PID controller for the counter pressure will be defined.

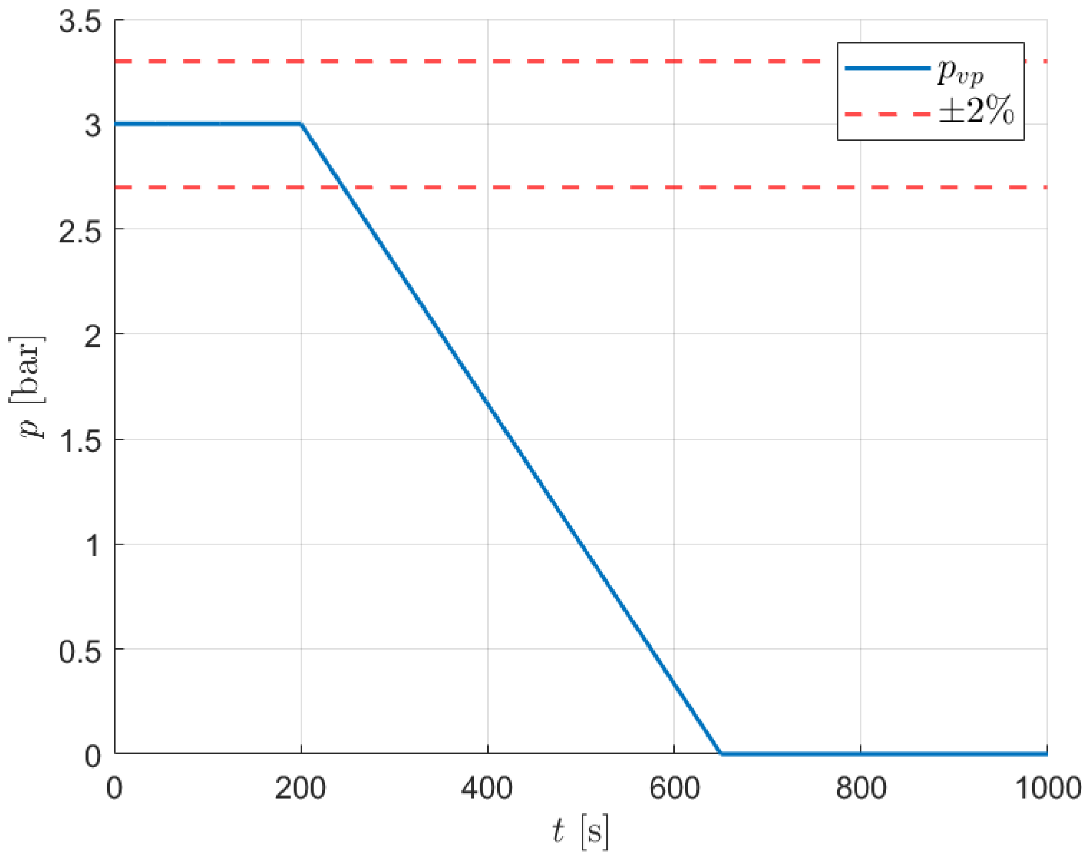
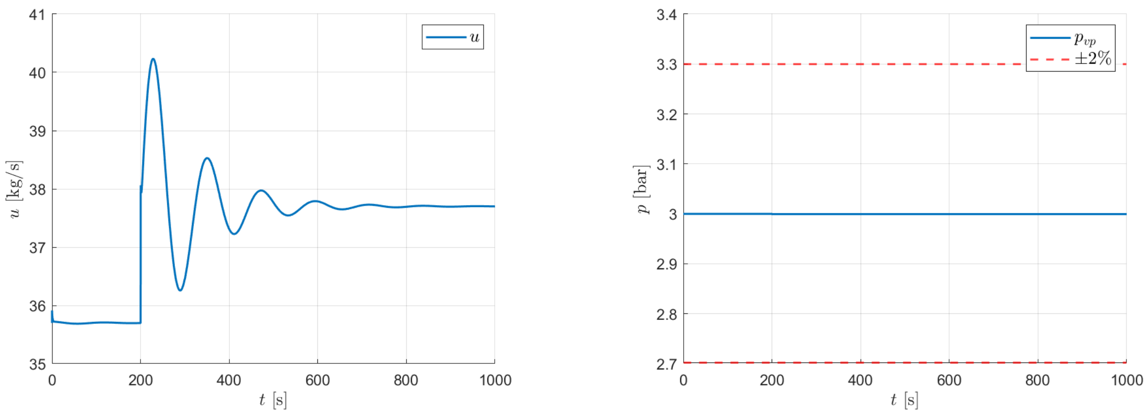



Figure 22: The counter pressure $p_{vp}$ as steam consumption $f_{kul}$ increases by 2 kg/s.

## 7.2 Counter pressure controller tuning

To keep the counter pressure within limits, a PID controller is defined. This PID controller operates the valve $z_1$, and thus the flow of steam into the turbine. To start, a simple P control is tried. The goal is to keep the counter pressure around its equilibrium of 3 bar. The operation of the P controller is shown in Figure 23. The P controller stabilizes the counter pressure, but does not return it to its equilibrium. A PI controller is tried and the result is shown in Figure 24. Even though the result seems identical to the P controller, when examining the later parts of the time spectrum it can be seen that the PI controller manages to return the counter pressure back to equilibrium when the P controller does not.

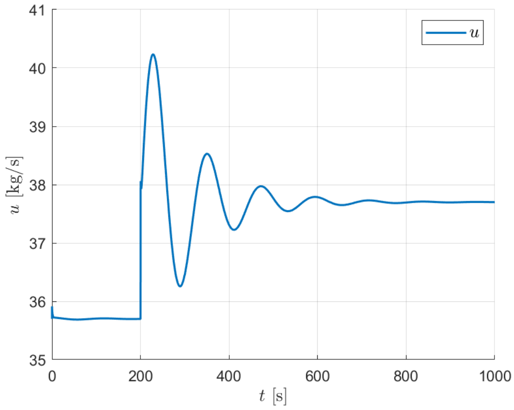
When only using I or ID controller as seen in Figures 25 and 26, the system starts to oscillate and never returns to equilibrium. The PI-controller is found to be the best choice, with values $K_p = 100$ and $K_i = 0.01$, as then the return is quick enough. Smaller I's lead to a longer time until returning to equilibrium, and smaller P's tend to lead to a larger disturbance in $p_{vp}$. Further increasing the values of the parameters does not make the system perform better.



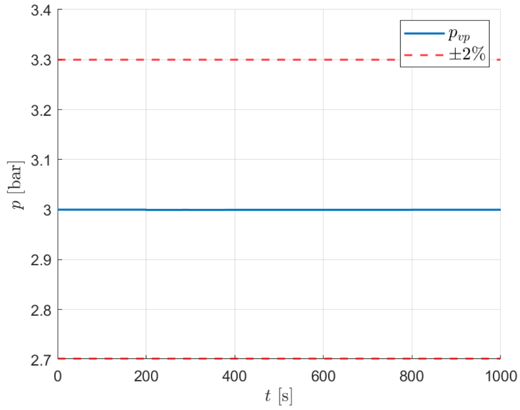

(a) Fuel injection $u$.

(b) Counter-pressure stock $p_{vp}$.

Figure 23: Plots related to the tuning of the **P** controller of the counter pressure stock.

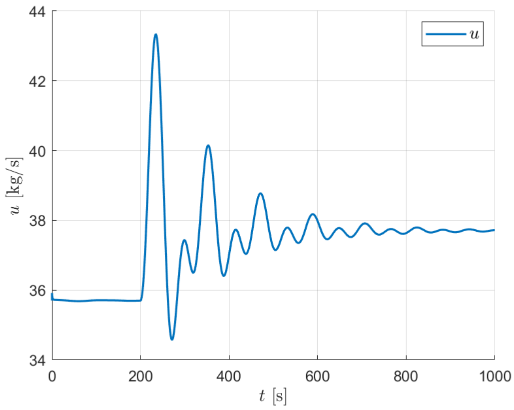



(a) Fuel injection $u$.

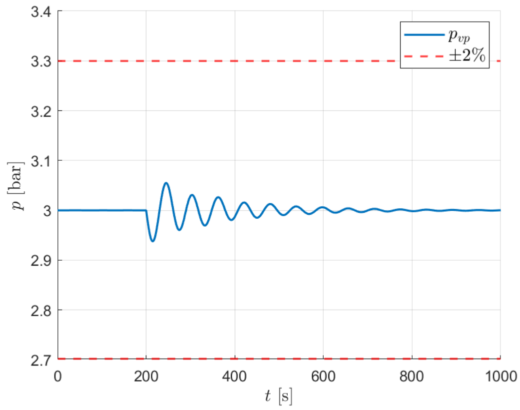



(b) Counter pressure stock $p_{vp}$.

Figure 24: Plots related to the tuning of the **PI** controller of the counter pressure stock.

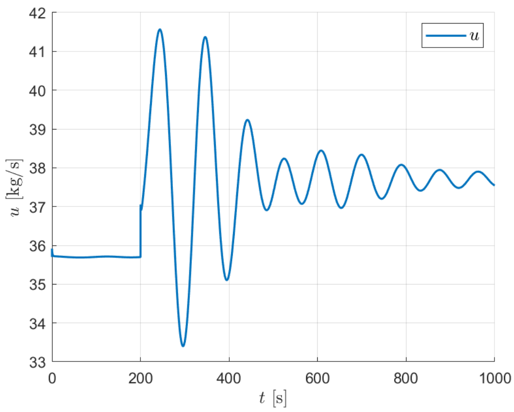



(a) Fuel injection $u$.

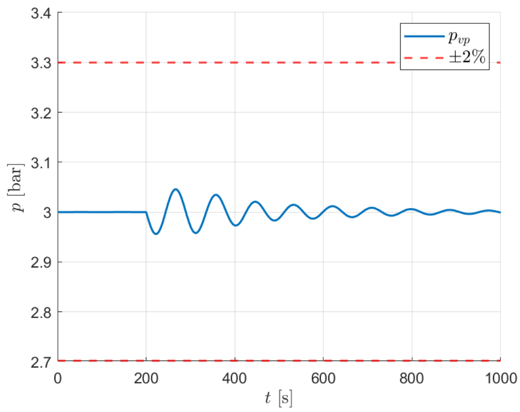



(b) Counter pressure stock $p_{vp}$.

Figure 25: Plots related to the tuning of the **I** controller of the counter pressure stock




(a) Fuel injection $u$.




(b) Counter-pressure stock $p_{vp}$.

Figure 26: Plots related to the tuning of the **ID** controller of the counter pressure stock

# 8 Utilizing the steam battery to control the counter pressure

To control the steam battery, two valves are used. These two valves, which can be seen as P-controllers of the system, work according to

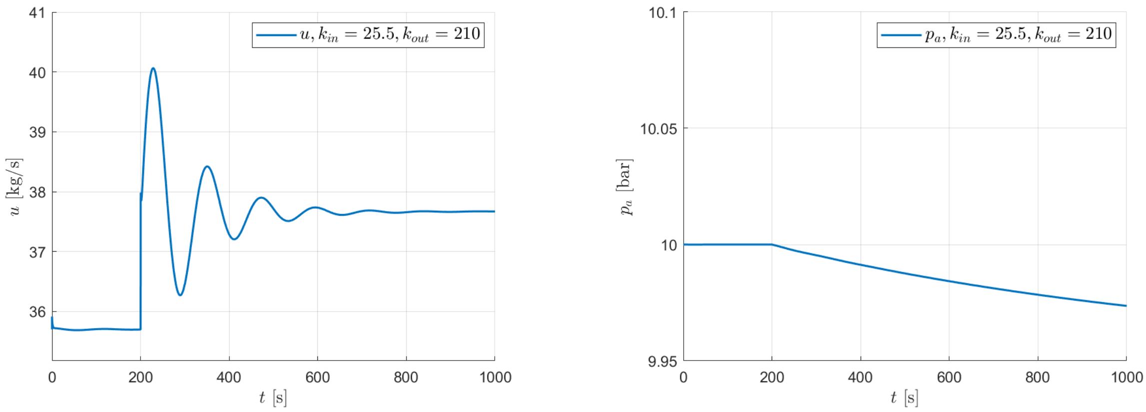
$$z_2 = k_{in}(p_{vp} - p_{vp0}) \tag{27}$$
$$z_3 = k_{out}(p_{vp0} - p_{vp})/p_a \tag{28}$$

The steam battery is a storage facility that empties and fills, and thus it is in essence an integrating component. Furthermore, the system does not need to force the battery to be at equilibrium by using more fuel. If the system had an integrating component, it would burn extra fuel to fill the battery even if the pressure of the high-pressure stock would be at its reference value. Instead, the system can fill the steam battery when there is a surplus of pressure flowing from the turbine, i.e. when the consumption goes down.

In Figure 27, the fuel consumption $u$ and pressure of the steam battery $p_a$ are visualized without changing the provided values for $k_{in}$ and $k_{out}$. Different values for $k_{in}$ and $k_{out}$ are evaluated to find out how the two parameters affect the fuel injection and the steam battery pressure. Values ranging from 0.1 to 10 times the given starting values give similar results in the step response experiment. Starting with the given value times 0.1 for $k_{in}$, as seen in Figure 28, the steam battery is drained very conservatively, and hardly reduces the fuel consumption. Trying a, relatively speaking, large value, 10 times the given value, for $k_{in}$ as seen in Figure 29, the result seems to be practically identical as when $k_{in}$ was small. It seems as if $k_{in}$ has no impact on the behavior of the system with the current response. Reducing $k_{out}$ to a tenth, as seen in Figure 30, completely stopped the steam battery from discharging. Increasing $k_{out}$ ten folds compared to the reference value, as seen in Figure 31, lead to the steam battery discharging more and evened out the oscillation in fuel consumption by some. Further increasing $k_{out}$ made the oscillations of fuel consumption almost completely disappear, and the jump in fuel consumption $u$ was drawn out over an extended period. However, the battery's charge does not return to equilibrium, so this strategy does not address recharging. This effect can be seen in Figure 32, where $k_{out}$ is made 250 times larger than the original value. Note also that the simulation is made over 35000 time units compared to the normal 1000 to show the effect of the change.

To ensure that the battery is never emptied completely some method to restrict the outflow of the battery could be used. For example, the outflow could be bound to the battery's pressure and even have a cutoff pressure at which the battery does not allow a flow out of it at all.

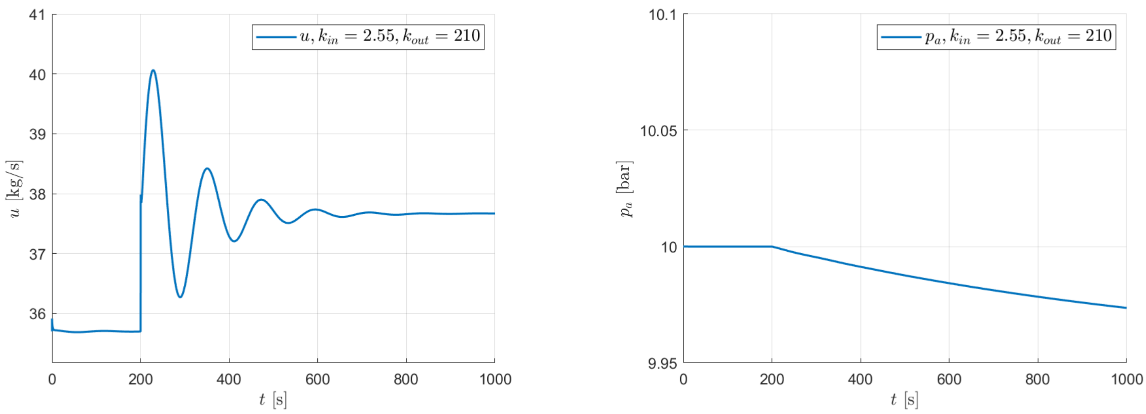
The kind of disturbances needed to properly test how the battery works is a disturbance where the consumption fluctuates around the equilibrium instead of simply being an increase or decrease. When the consumption is lower it leads to the flow into the high-pressure stock increase and in turn, this pressure can be captured by the battery and vice versa. This is seen when we tune the full model using a sin-waved consumption in Section 9.




(a) The fuel injection $u$.

(b) The steam pressure in the battery $p_a$.

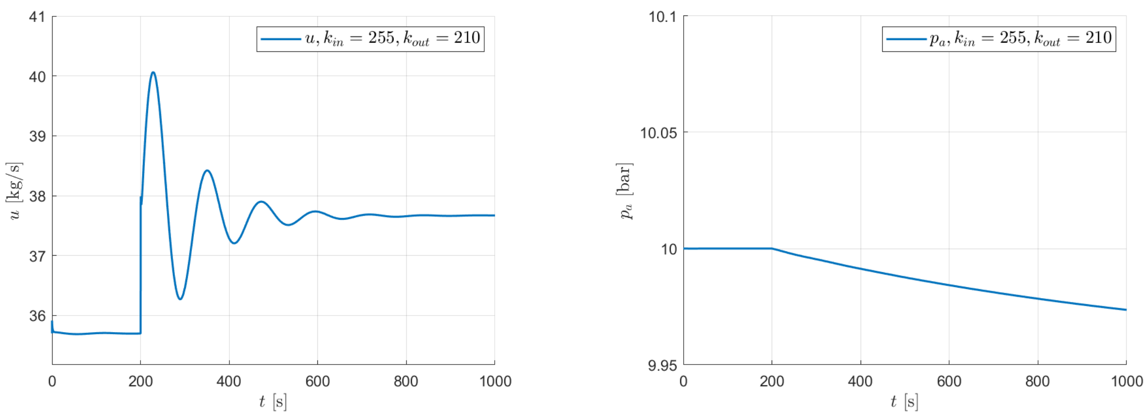
Figure 27: The steam battery when the parameters $k_{in}$ and $k_{out}$ are unmodified from the ones given in the instructions [1].




(a) The fuel injection $u$.

(b) The steam pressure in the battery $p_a$.

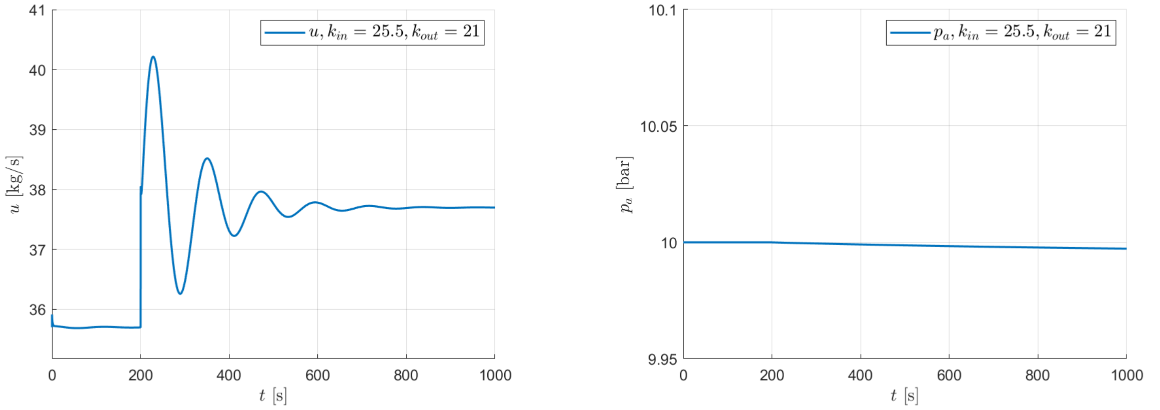
Figure 28: Steam battery when $k_{in}$ is 0.1 times the reference value [1].




(a) The fuel injection $u$.

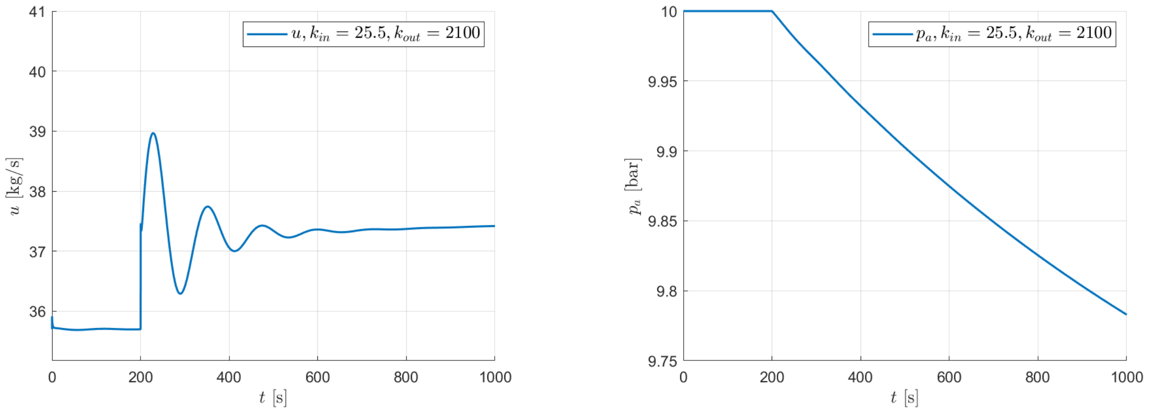
(b) The steam pressure in the battery $p_a$.

Figure 29: Steam battery when $k_{in}$ is 10 times the reference value [1].



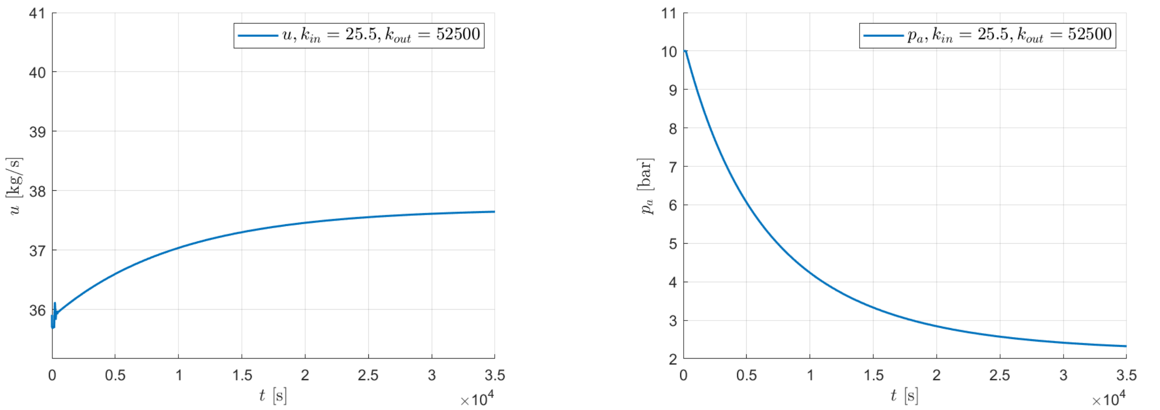

(a) The fuel injection $u$. (b) The steam pressure in the battery $p_a$.

Figure 30: Steam battery when $k_{out}$ is 0.1 times the reference value [1].




(a) The fuel injection $u$. (b) The steam pressure in the battery $p_a$.

Figure 31: Steam battery when $k_{out}$ is 10 times the reference value [1].




(a) Fuel injection $u$. (b) The steam pressure in the battery $p_a$.

Figure 32: Steam battery when $k_{out}$ is 250 times the reference value [1].

# 9 Testing the complete power plant system

In this section, all the previously defined controls of the power plant are in use for managing disturbances. The stability will be tested on two types of disturbances, the step response and also a periodic disturbance of the form.

$$f_{kul} = f_{kul0} + B\sin(\omega_1 t) + C\sin(\omega_2 t) \tag{29}$$

where $\omega_1 = 0.005$ rad/s is a low frequency and $\omega_2 = 0.1$ rad/s is a high frequency disturbance. The amplitudes $B$ and $C$ are approximately inversely proportional to the frequencies and chosen as $B = 3.57$kg/s and $C = 0.1785$kg/s. The wave thus leads to a 10.5% deviation from the reference value at its extreme points.

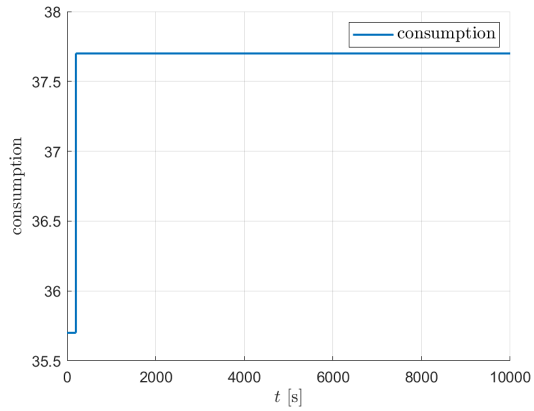
The benefit of applying all the controls is that no single control needs to be as aggressive as when used individually. This is due to the synergy of the individual controls working to balance the same problem. Thus, the gains for the controls have all been scaled down, and chosen as seen in Table 6. The values were found using a heuristic iterative process of tuning each of the parameters alone and in combination with the other parameters. Figure 33 displays how the system performance with to a step change in consumption and Figure 34 to the sine waved consumption.

To test the robustness of the controls, the step response and the sin-wave response were increased, until the system could no longer cope and the counter pressure went beyond the allowed limits. The highest feasible step increase was 12.5bar, and the result is seen in Figure 35 which highlights that $p_{kp}$ is barely within the $\pm 2\%$ margins. However, already around the 10bar mark, the more expensive fuels are required to be able to keep up with the rapid changes required. The highest feasible amplitude of the sin-wave disturbance was a combined extreme of 31.5% the reference value, and as seen in Figure 36, the high-pressure increases in oscillation but is stabilized at the feasible limit.

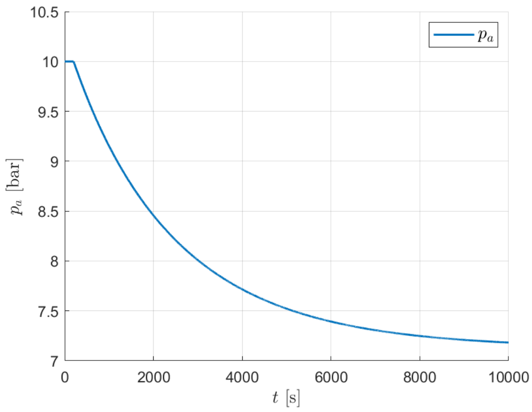
As a recommendation, the maximum amplitudes in sin disturbances should not exceed 30% of the equilibrium, and step disturbances should not exceed it by more than 10 bar.

Table 6: The values of the parameters in the final setup of the system. The subscripts *fuel* refers to the PID controller's parameters related to the fuel injection and *valve* to the one controlling the valve $z_1$. The battery's parameters are compared to the given values in [1].

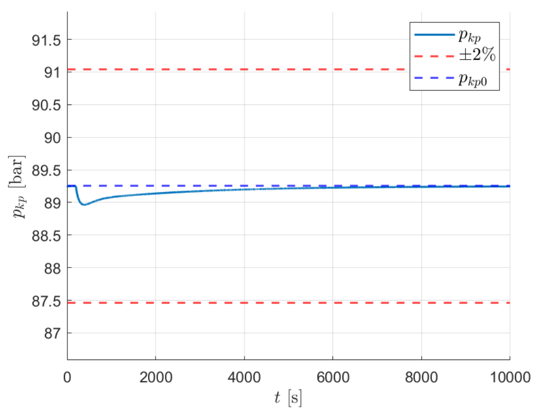
| Parameter | Final value | Ratio to previous |
|---|---|---|
| $k_{p_{fuel}}$ | 0.5665 | 0.12 |
| $k_{i_{fuel}}$ | 0.0033 | 0.0225 |
| $k_{d_{fuel}}$ | 22.6593 | 0.6 |
| $k_{p_{valve}}$ | 10 | 0.1 |
| $k_{i_{valve}}$ | 0.03 | 0.3 |
| $k_{d_{valve}}$ | 0 | - |
| $k_{in}$ | 510 | 20 |
| $k_{out}$ | 4200 | 20 |



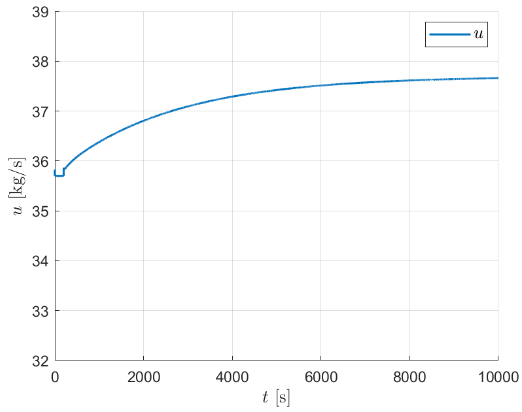

(a) The fuel consumption as a function of time $t$.




(b) The steam battery pressure $p_a$ as a function of time $t$.

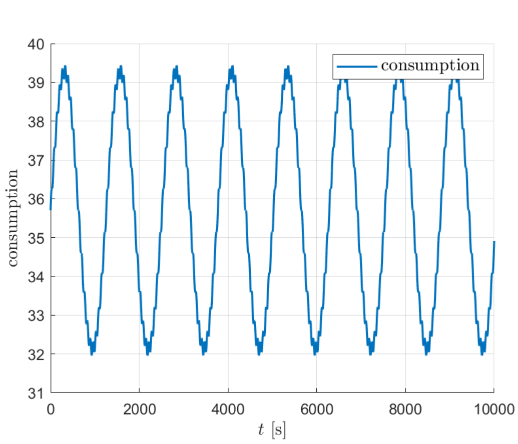



(c) The high-stock pressure $p_{kp}$ as a function of time $t$.

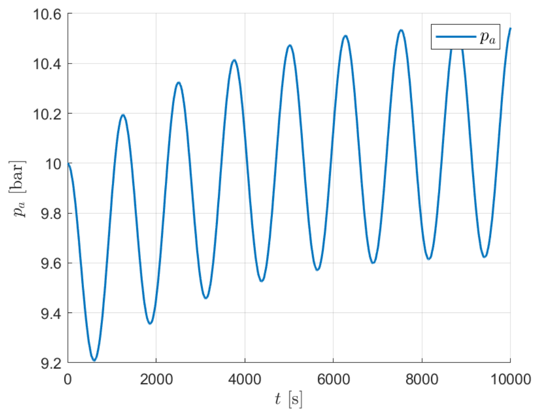



(d) The fuel injection $u$ as a function of time $t$.

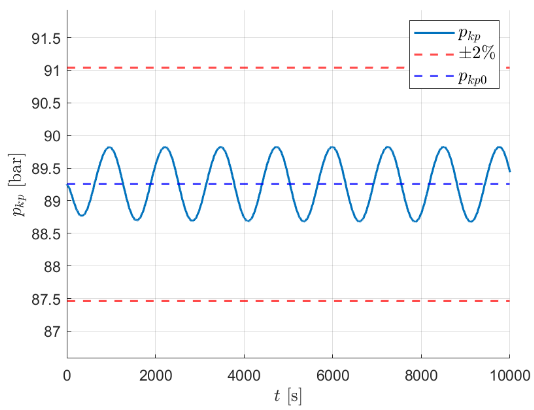
Figure 33: The model with the final parameters for the step response.



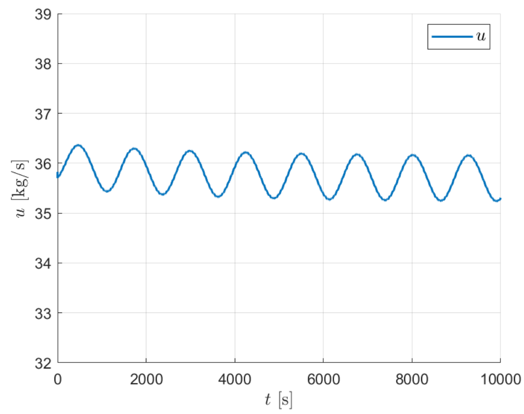

(a) The consumption as a function of time $t$.




(b) The steam battery pressure $p_a$ as a function of time $t$.

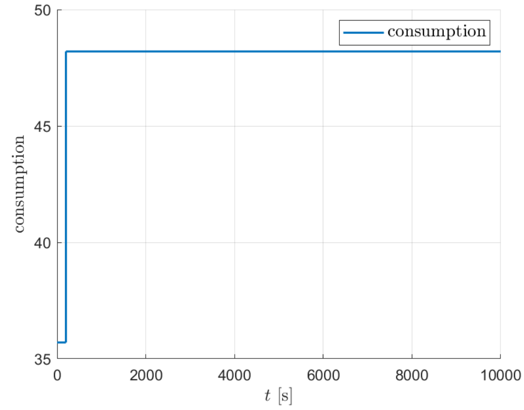



(c) The high-stock pressure $p_{kp}$ as a function of time $t$.

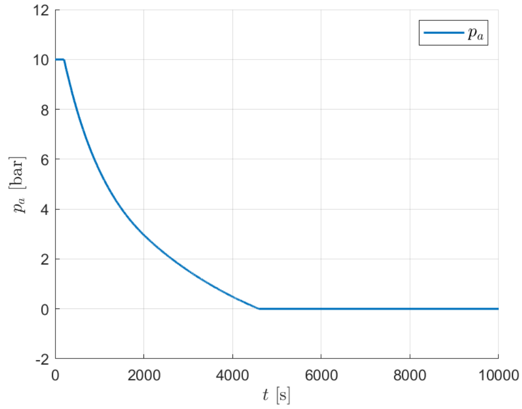



(d) The fuel injection $u$ as a function of time $t$.

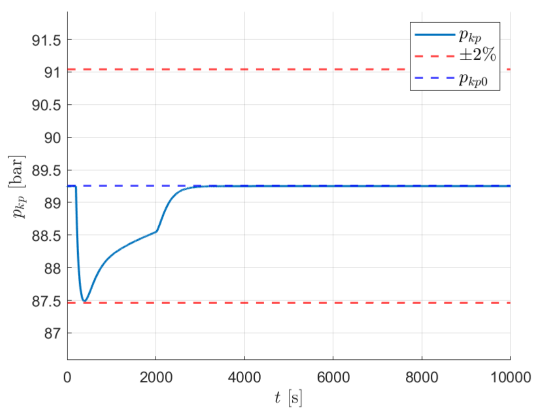
Figure 34: The model with the final parameters for the sin-wave formed response.



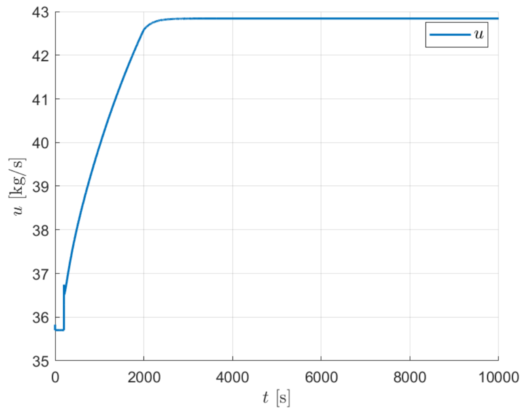

(a) The fuel consumption as a function of time $t$.




(b) The steam battery pressure $p_a$ as a function of time $t$.

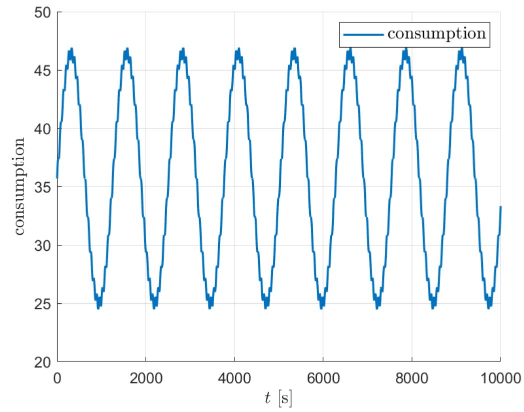



(c) The high-stock pressure $p_{kp}$ as a function of time $t$.

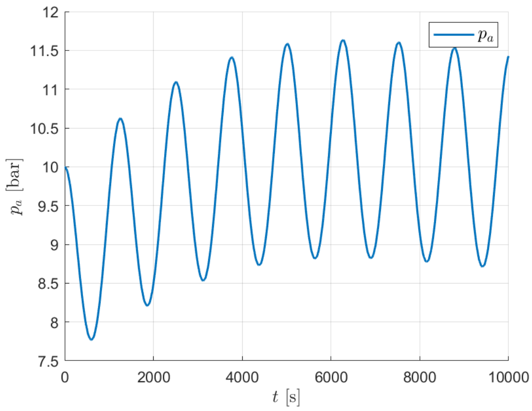



(d) The fuel injection $u$ as a function of time $t$.

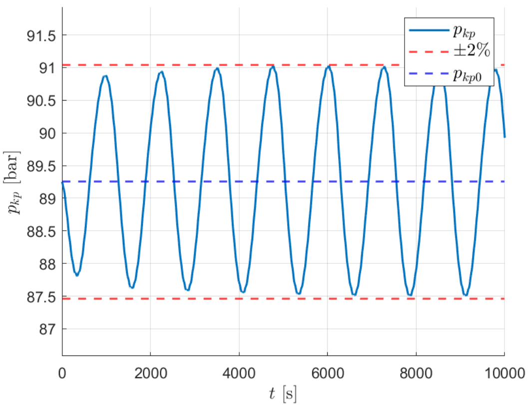
Figure 35: The model with the final parameters for the extreme step response.



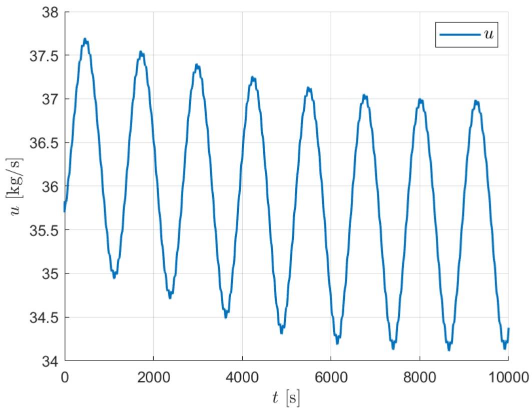

(a) The consumption as a function of time $t$.




(b) The steam battery pressure $p_a$ as a function of time $t$.




(c) The high-stock pressure $p_{kp}$ as a function of time $t$.




(d) The fuel injection $u$ as a function of time $t$.

Figure 36: The model with the final parameters for the extreme sin-wave formed response.

# 10 Conclusions

In this assignment, both PID and state feedback controllers were tested as methods of controlling a steam power plant. Both types of controllers stabilized the system, but the PID controller was more efficient and gave good results. The PID controller's ease of definition, as there need not be knowledge of the inner dependencies of the system when defining a PID controller, is also good.

The defined PID controllers kept the power plant operating at an acceptable state also when more intricate periodic disturbances were tried. All magnitudes of disturbances are not possible to rectify while keeping the high-pressure stock within limits, and thus it is also key to time the demand in such a way as to support the operation of the plant. The upper limits suggested in section 9 were however unrealistically high, so the system should be able to cope with most disturbances.

## 10.1 Suggestions for improvement

The power plant featured in the assignment seems quite old and not as contemporary. Some newer applications of PID and state feedback could be nice. Also, the assignment's nature was mainly looking at very repetitive plots which was dull and didn't feel like we learned anything outside this specific assignment. To find minor differences in some abstract pressures is not very interesting. In Task 9, the instructions for the saturation limits were very vague and it was unclear even to find the blocks to change. Additionally, no instructions were provided on how to iterate or how the saturation blocks affect the outcome.

# References

[1] Course Staff 2022. Assignment 2: Simulation and control of a dynamic system. https://mycourses.aalto.fi/. [Online; posted August 2022].

[2] K. J. Åström and R. M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton University Press, 2008. Available online at `http://press.princeton.edu/titles/8701.html`.