MS-E2132 - Laboratory Assignments in Operations Research II

Assignment 2: Simulation and control of a dynamic system

1 Background

The assignment deals with the control of a thermal power plant that uses fuels such as peat, coal, oil and gas to produce steam and electricity. The schematics of the power plant is presented in Figure 1. The need for energy sources that are used by industrial processes, such as steam and electricity, fluctuates greatly according to consumption. For instance, the start-up of a paper machine causes a heavy increase in the demand for steam. Expensive fuels (oil and gas) generate heat more quickly than cheap fuels (peat, coal). If consumption peaks can be anticipated at the power plant, the production of steam can be increased ahead of time with the help of cheap fuels, while the need for more expensive fuels is reduced. On the other hand, by evening out the peaks in consumption, the cheaper fuels can be used as a base fuel and only minor adjustments are performed with the more expensive fuels.

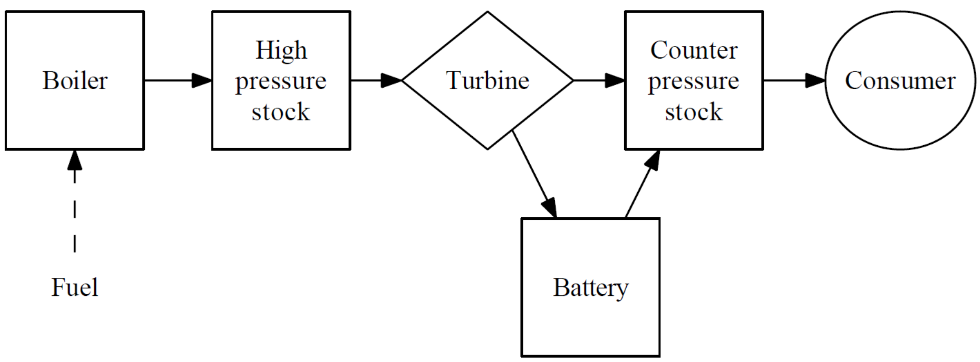

Figure 1: Schematics of the thermal power plant. The dashed arrow represents heating and the solid arrows represent the transport of steam.

2 Description of the thermal power plant model

The purpose of a thermal power plant is to produce the steam and electricity needed by consumers (e.g., a factory). In this assignment, the production of electricity is ignored and only control of the production of steam to match its consumption is investigated.

Examine a simplified model of the power plant. The connection between the injection of fuel and the generation of steam in the boiler is described by the first degree dynamics

$$T_1 \frac{df_p(t)}{dt} + f_p(t) = k_0 u(t), \tag{1}$$

where

$f_p(t)$	$=$	steam generation in the boiler
$u(t)$	$=$	fuel injection
T_1	$=$	parameter of the steam production in the boiler
k_0	$=$	conversion factor from the fuel injection to the steam production

The rate of change in the boiler's pressure, determined by the generation of steam and the flow of steam out of the boiler, is described by

$$\frac{dp_k(t)}{dt} = \frac{f_p(t) - f_{ka}(t)}{T_s}, \tag{2}$$

where

$p_k(t)$	$=$	pressure of the boiler
$f_{ka}(t)$	$=$	flow of steam out of the boiler
T_s	$=$	storage capacity of the boiler

The flow of steam from the boiler into the high pressure steam stock is assumed to be subcritical ($p_{kp}(t)/p_k(t) \leq 0.55$) at all times, which means that the flow is expressed as

$$f_{ka}(t) = k_1\sqrt{p_k^2(t) - p_{kp}^2(t)}, \tag{3}$$

where

$p_{kp}(t)$	$=$	pressure of the high pressure stock
k_1	$=$	flow resistance from the boiler to the steam stock

The rate of change in the high pressure steam stock's pressure, given by the flows of steam from the boiler and into the turbine, is

$$\frac{dp_{kp}(t)}{dt} = k_2\left(f_{ka}(t) - f_g(t)\right), \tag{4}$$

where

$f_g(t)$	$=$	flow into the turbine
k_2	$=$	storage capacity of the steam stock

The flow of steam from the high pressure steam stock to the turbine is proportional to the output signal of the control valve of the turbine. The flow into the turbine is otherwise dependent only on the pressure of the high pressure steam stock as the flow is assumed to be supercritical. The flow is thus

$$f_g(t) = k_3 z_1(t) p_{kp}(t), \tag{5}$$

where

$z_1(t)$	$=$	output signal of controller 1 ($z_1 \in [0.8, 1.2]$)
k_3	$=$	flow resistance to the turbine

When charging the steam battery, a portion of the flow from the turbine is directed to the battery. The majority of the flow is usually directed to the counter pressure steam stock. When discharging the battery, flow is directed from the battery to the counter pressure steam stock. The rate of change in the pressure of the counter pressure steam stock depends on the difference between the incoming and outgoing flows, i.e.,

$$\frac{dp_{vp}(t)}{dt} = k_4\left(f_g(t) + f_{out}(t) - f_{kul}(t) - f_{in}(t)\right), \tag{6}$$

where

$p_{vp}(t)$	$=$	pressure of the counter pressure stock
$f_{out}(t)$	$=$	flow out of the battery
$f_{in}(t)$	$=$	flow into the battery
$f_{kul}(t)$	$=$	flow to consumption
k_4	$=$	storage capacity of the counter pressure stock

It is assumed that the steam battery is charged from the turbine by a tap so that the charging pressure is constant (13 bar). The pressure in the battery is around 10 bar, so the charging flow is subcritical. The flow is additionally assumed to be proportional to the output signal of the charging controller's signal. The flow is thus

$$f_{in}(t) = k_5 z_2(t)\sqrt{13^2 - p_a^2(t)}, \tag{7}$$

where

$z_2(t)$	=	output signal of controller 2 ($z_2 \in [0, 1]$)
k_5	=	flow resistance into the steam battery
$p_a(t)$	=	pressure of the battery

The flow from the battery is supercritical, because the pressure of the counter pressure bar is around 3 bar. This flow is also controlled and is of the form

$$f_{out}(t) = k_6 z_3(t) p_a(t), \tag{8}$$

where

$z_3(t)$	=	output signal of controller 3 ($z_3 \in [0, 1]$)
k_6	=	flow resistance out of the steam battery

The rate of change in the pressure of the steam battery depends on the incoming and outgoing flows as well as its current pressure and the mass of the water in it. The rate of change in the pressure is approximated as

$$\frac{dp_a(t)}{dt} = -\left(h_1 p_a(t) + h_2\right)\frac{f_{out}(t) - f_{in}(t)}{m_a}, \tag{9}$$

where

h_1	=	parameter of the steam battery
h_2	=	parameter of the steam battery
m_a	=	mass of the water in the battery

In equilibrium the battery is not used, i.e. the flow into the battery f_{in} and the flow out of the battery f_{out} are zero.

3 Excercises

1. A SimuLink template (voima.slx) of the power plant described in the assignment is available on the course's MyCourses-page. The values of the parameters of the model are specified in param.m (also available on the MyCourses-page). This file needs to be run first in order to give the parameters of the model their correct values (you can also simulate voima.slx inside of the param.m, please see example in the end of the script). The dynamic system, i.e., the power plant, is initially in a steady state. The steady state values of the variables are given in the end of these instructions. Investigate the contents of voima.slx and describe the model briefly.

2. An overly eager plan worker accidentally opens the valve z_1 such that the flow to the turbine increases by 3 kg/s. How much does the setting of the valve change, and what is the new value? Simulate the situation in order to find out what the consequences of this disturbance are, in particular for the pressure of the high pressure stock.

3. Large changes in the pressure of the high pressure stock strain the stock and are challenging in the long run. In addition, a pressure of over 100 bar in the stock causes an emergency stop that shuts down the entire plant. Your goal is to keep the offset from the steady state pressure below $\pm$ 2%. On the other hand, rapid changes in the fuel injection are also bad. More expensive fuels have to be used when the injection changes by more than 1 kg/s over a period of 100 seconds. You therefore have a multiple objective optimization problem to solve. As a guideline, you are told that a stable pressure in the hight pressure stock is more important than minimizing the use of expensive fuels.

Start developing a control system for the power plant, where the injection of fuel is controlled by feedback from the pressure of the high pressure steam stock. First, tune by hand (i.e try out different values) a P-controller with the help of a step response test (u_0 + 1kg/s). What can you notice? Also apply and tune by hand a PI- and a PID-controller, so you understand the principal difference between the different controllers.

Next, explain the procedure for tuning a P-, PI- or PID-controller using the maximum value of the derivative in a step response (see e.g. Åström & Murray, Chapter 11). Try out how well the three different controllers can rectify the situation after the plant worker has opened the valve. Present all relevant figures (separately for the step response test and the disturbance) as well as the values of the controller parameters (k_p, k_i and k_d) and the formulas for calculating them. Also explain how the results obtained with the three controllers differ. In the end, choose the values for the PID-controller that you will use in the future.

4. An alternative to PID-control is state feedback. The state variables for the upper section of the power plant system are f_p, p_k, and p_{kp}, and the control is a linear function of these states. Construct a state feedback controller for the injection of fuel. Determine its parameters by setting suitable eigenvalues for the state matrix of the closed loop system. The eigenvalues are most conveniently determined by forming a linear-quadratic infinite horizon optimal control problem (see, e.g., "Optimal Control Theory" by Kirk) and solving the corresponding algebraic Riccati equation numerically using, e.g., the MATLAB functions lqr or lqr2. Note that finding a good control law by setting arbitrary eigenvalues is not recommended due to time-consumption.

   In order to construct the Riccati equation, the upper section of the model for the power plant has to be linearized at its equilibrium point. The linearization is **only** used for defining the control, i.e. do not replace the original model of the upper section with the linearized model. Mathematica or other appropriate software can be used for the symbolic calculations. Remember to verify that the system is controllable. (Hint: It should be). Investigate how well the state feedback controller can rectify the situation after the plant worker has opened the valve.

   Try out different weighing matrices for the Riccati equation. For each candidate, present the weighing matrices, the poles of the system, the resulting control law and figures of its operation. Select the weighing matrices that you consider most suitable.

5. State feedback as such is insufficient. Why is that? To fix the problem, include integration into the controller. Examine the system

$$\dot{x} = Ax + Bu \tag{10}$$

$$y = Cx, \tag{11}$$

   where u is the input variable, x the vector of state variables, and y the output variable. The error term is $e = r - y = r - Cx$, where r is the reference signal. In this assignment we are only investigating the behaviour of the pressure in the high pressure stock p_{kp}, and our reference signal is therefore $r = p_{kp_0}$. The integral of the error term is computed by including an additional state variable x_i, with the state equation $\dot{x_i} = e = r - Cx$, to the vector of state variables. The control is then calculated by the equation $u = -\tilde{K}\tilde{x} = (K\ k_i)(x\ x_i)^T$, so the integral of the error term is included with a coefficient of k_i.

   Formulate the extended state equations, included the integral in the controller and look for a suitable $\tilde{K}$ as in exercise 4.

6. Compare and discuss the application of PID-control (exercise 3) and state feedback (exercise 5) both on a general level and in this particular case. Select the more suitable one of these to control the injection of fuel from exercise 7 forward.

7. Let's examine the entire plant model. The pressure of the high pressure stock has been stabilized with the help of a controller. The upper section of the plant is working as it should (the plan worker has closed the valve back to $z_1 = 1.0$) . Just as everything is looking perfect, the flow of steam to consumption f_{kul} suddenly increases by 2 kg/s. How does this affect the counter pressure? The counter pressure is meant to stay within 10% of its equilibrium value.

8. Construct a controller to keep the counter pressure within acceptable limits. The controller operates the valve z_1 and therefore the flow of steam into the turbine. The goal is to only let the controller react to low frequency disturbances and let the steam battery deal with high frequency disturbances. On the other hand, the controller should also react quickly enough to disturbances to keep the counter pressure within its limits. Tune the controller so that it strives to keep the counter pressure around its equilibrium (3 bar). Try a P- and PI-controller as well as just I- or ID-control. What do you observe? Use only experiments - e.g., tuning of the controllers based on the step response is not required. Present relevant figures.

9. Next, include the steam battery in the control of the counter pressure. It is meant for compensating high frequency disturbances in the consumption and for softening quick changes so that the input of fuel does not oscillate. The control strategy for the battery is based on two P-controllers:

$$z_2 = k_{in}(p_{vp} - p_{vp_0}) \tag{12}$$

$$z_3 = k_{out}(p_{vp_0} - p_{vp})/p_a, \tag{13}$$

where k_{in} and k_{out} are amplifications and p_{vp0} is the equilibrium pressure of the counter pressure stock. Why is it justified not to include an I-component in the controllers? Modify the saturation limits of the charging and discharging signals of the battery to appropriate values (they have so far been 0) and tune the controllers. Use $k_{in} = 25.5$ and $k_{out} = 210.0$ as a starting point and find values that together with the control of the flow to the turbine provides the best behavior of the plant. It might be useful to also consider the frequency response of a PID-controller.

Does the proposed strategy properly address the recharging of the battery? How could one ensure that the battery is never emptied completely? Which kind of disturbances are best suited for testing the cooperation of the battery and the counter pressure controller? Present relevant figures.

10. Adjust the parameters of all the controllers in the model together so that the counter pressure stays within its limits, the fluctuations in the pressure of the high steam stock are as small as possible, and the use of expensive fuels in the heating of the boiler is minimized. Test the operation of the plant as a whole. Experiment with different step and ramp disturbance in the consumption. In order to check the frequency properties of the model, also use consumptions of the form

$$f_{kul} = f_{kul0} + B\sin(\omega_1 t) + C\sin(\omega_2 t), \tag{14}$$

where ω_1 is a low frequency of the magnitude of 0.005 rad/s and ω_2 a high frequency of the magnitude of 0.1 rad/s. The amplitudes are assumed to be approximately inversely proportional to the frequencies and a 15% deviation from the equilibrium is considered large. Based on your experiments, give a general recommendation to the consumer of steam regarding the amplitudes of disturbances that are allowed at high and low frequencies, as well as the size of step disturbances that are compatible with the limits of the counter pressure. Remember to also keep in mind the objectives of maintaining the pressure of the high pressure stock within its limits and minimizing the use of expensive fuels. You can assume that disturbances with frequencies over 0.2 rad/s are not encountered. Present relevant figures.

4 Reporting

In addition to answers to all the questions in the exercises, the report should contain the following:

- An introduction including information about the background and purpose of the assignment.
- For each simulated disturbance, figures of both the input and output of the controllers.
- Reflection on the methods and models used, as well as the assignment as a whole. Suggestions for improving the assignment are welcome.

Values of constants:

T_1	$=$	30.0 time constant of the steam production in the boiler [s]
T_s	$=$	15.0 storage capacity of the boiler [kg/bar]
k_0	$=$	1.0 conversion factor from the fuel injection to the steam production
k_1	$=$	2.0 flow resistance from the boiler to the steam stock [kg/(s*bar)]
k_2	$=$	1/30 storage capacity of the steam stock [kg/bar]
k_3	$=$	0.4 flow resistance to the turbine [kg/(s*bar)]
k_4	$=$	1/300 storage capacity of the counter pressure stock [kg/bar]
k_5	$=$	0.6 flow resistance into the steam battery [kg/(s*bar)]
k_6	$=$	0.6 flow resistance out of the steam battery [kg/(s*bar)]
h_1	$=$	8.3 parameter of the steam battery
h_2	$=$	21.0 parameter of the steam battery [bar]

Initial values for equilibrium:

u_0	$=$	35.7 fuel injection [kg/s]
z_{1_0}	$=$	1.00 output signal of controller 1
f_{p_0}	$=$	35.7 steam generation in the boiler [kg/s]
p_{k_0}	$=$	91.0 pressure of the boiler [bar]
p_{kp_0}	$=$	89.25 pressure of the high pressure stock [bar]
f_{in_0}	$=$	0.0 flow into the battery [kg/s]
f_{out_0}	$=$	0.0 flow out of the battery [kg/s]
p_{a_0}	$=$	10.0 pressure of the battery [bar]
m_{a_0}	$=$	150 000.0 mass of the water in the battery [kg]
f_{kul_0}	$=$	35.7 flow to consumption [kg/s]
p_{vp_0}	$=$	3.0 pressure of the counter pressure stock [bar]