Advanced probabilistic methods - Sketch

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$$\log(q^*(\tau)) = \mathbb{E}_{\mathbf{z},\theta}\left[\log(p(\mathbf{x},\mathbf{z},\tau,\theta))\right] \tag{1}$$
$$p(\mathbf{x},\mathbf{z},\tau,\theta) = p(\tau)p(\theta)p(\mathbf{z}\mid\tau)p(\mathbf{x}\mid\mathbf{z},\theta) \tag{2}$$
$$\log(q^*(\tau)) = \mathbb{E}_{\mathbf{z},\theta}\left[\log(p(\tau)p(\theta)p(\mathbf{z}\mid\tau)p(\mathbf{x}\mid\mathbf{z},\theta))\right] \tag{3}$$
$$\propto \log(p(\tau)) + \mathbb{E}_{\mathbf{z}}\left[\log(p(\mathbf{z}\mid\tau))\right] \tag{4}$$
$$p(\mathbf{z}\mid\tau) = \prod_{n=1}^{N} \tau^{z_{n2}}(1-\tau)^{z_{n1}} \tag{5}$$
$$\log(p(\mathbf{z}\mid\tau)) = \sum_{n=1}^{N} z_{n2}\log(\tau) + z_{n1}\log(1-\tau) \tag{6}$$
$$\log(q^*(\tau)) = \log(p(\tau)) + \mathbb{E}_{\mathbf{z}}\left[\sum_{n=1}^{N}\{z_{n2}\log(\tau) + z_{n1}\log(1-\tau)\}\right] \tag{7}$$
$$= \log(p(\tau)) + \sum_{n=1}^{N}\{\log(\tau)\mathbb{E}_{\mathbf{z}}\left[z_{n2}\right] + \log(1-\tau)\mathbb{E}_{\mathbf{z}}\left[z_{n1}\right]\} \tag{8}$$
$$= \log(p(\tau)) + \sum_{n=1}^{N}\{\log(\tau)r_{n2} + \log(1-\tau)r_{n1}\} \tag{9}$$
$$= \log(p(\tau)) + \log(\tau)\sum_{n=1}^{N} r_{n2} + \log(1-\tau)\sum_{n=1}^{N} r_{n1} \tag{10}$$
$$q^*(\tau) = p(\tau)\tau^{\sum_{n=1}^{N} r_{n2}}(1-\tau)^{\sum_{n=1}^{N} r_{n1}} \tag{11}$$
$$\sum_{n=1}^{N} r_{nk} = N_k \tag{12}$$
$$p(\tau) = Beta(\tau\mid\alpha_0,\alpha_0) \tag{13}$$
$$q^*(\tau) = Beta(\tau\mid\alpha_0,\alpha_0)\cdot\tau^{N_2}(1-\tau)^{N_1} \tag{14}$$
$$\tau^{N_2}(1-\tau)^{N_1} \propto Beta(\tau\mid N_2,N_1) \tag{15}$$
$$q^*(\tau) \propto Beta(\tau\mid\alpha_0,\alpha_0)\cdot Beta(\tau\mid N_2,N_1) \tag{16}$$
$$= Beta(\tau\mid\alpha_0+N_2,\alpha_0+N_1) \tag{17}$$