Exercise 9

30.11.2021

#1 Finding Pareto-optimal solutions with Excel

Riikka is at an amusement park that offers 2 different rides: Tickets to ride 1 cost 2 € and each ticket lets you take the ride twice. Tickets to ride 2 are for one ride and cost 3 €. Riikka has 20 euros to spend on tickets to ride 1 and ride 2. Each time Riikka takes ride 2, her grandfather cheers for her. Riikka wants to get as many rides and cheers as possible.

a) Formulate Riikka's desire to maximize the number of rides and cheers as a MOO problem.
b) Use the Excel template provided to solve PO solutions of the MOO problem using weighted sum and weighted max-norm approaches: Execute loops of the approaches manually. On each loop, solve the related (M)ILP problem by using Excel's Solver tool.
c) Suppose Riikka states that getting one additional ride is as nice as getting two additional cheers from the grandfather. Study the situation now through MAVT so that the first attribute y_1 is the number of rides and the second attribute y_2 is the number of cheers. Let $y_1^0 = 0$, $y_1^* = 20$, $y_2^0 = 0$, and $y_2^* = 6$. Assume that the attribute-specific value functions are linear. What is the best strategy for buying the tickets?

#2 Finding Pareto-optimal solutions with MATLAB

Consider a company with an R&D budget of 80M€. A team of managers is currently considering 40 R&D project proposals with costs c_j, j=1,…,40. Each proposal is presented by one of the company's three research divisions. Let r_{kj}=1, if the j^{th} project is proposed by the k^{th} division, and r_{kj}=0 otherwise. The projects can only be carried out by the proposing division.

The team wants you, the company's decision analyst, to present them the Pareto-optimal project portfolios with regard to the objectives of minimizing risk (variance of profit) and maximization of expected profit of the portfolio. The profits of the projects are assumed independent and normally distributed, with expected profits μ_j and standard deviations σ_j. The team requires that at least 20 % of the total R&D budget is allocated to each division (in terms of funding proposals).

a) Formulate a multi-objective integer linear programming model of the form (here natural numbers include 0)

$$\underset{x\in\mathbb{N}^m}{\text{v-max}}\{Cx | Ax \leq B\},\ \mathrm{C} \in \mathbb{R}^{\mathrm{n}\times\mathrm{m}}, \mathrm{A} \in \mathbb{R}^{\mathrm{q}\times\mathrm{m}}, B \in \mathbb{R}^{\mathrm{q}}$$

b) Assume you have a mixed integer linear programming (MILP) solver that solves problems of the form

$$\max_{\substack{x\in\mathbb{N}^m \\ y\in\mathbb{R}^{m'}}} \{c\begin{bmatrix}x\\y\end{bmatrix} | A\begin{bmatrix}x\\y\end{bmatrix} \leq B\},\ \mathrm{c} \in \mathbb{R}^{1\times(\mathrm{m}+\mathrm{m}')}, \mathrm{A} \in \mathbb{R}^{\mathrm{q}\times(\mathrm{m}+\mathrm{m}')}, B \in \mathbb{R}^{\mathrm{q}}$$

Formulate i) weighted sum and ii) weighted max-norm algorithms that use this solver to find Pareto-optimal portfolios.

c) The function “intlinprog” included in the Optimization Toolbox of Matlab is a MILP solver of the kind described in b).

1. Install the Optimization Toolbox to your computer (if you do not have it yet). See the installation help guide from the exercise folder if needed.
2. Write “help intlinprog” to Matlab’s Command Window and familiarize yourself with the syntax of the function.
3. Open exercise9_Task2_template.m. Complete the missing parts of the code (4 instances) and use it to find Pareto-optimal solutions to the problem.
   i. How many Pareto optimal portfolios can you find?
   ii. Which projects are included in all Pareto-optimal portfolios?