Nonlinear Optimization - Homework 2

Christian Segercrantz 481056

November 17, 2021

3.1

$$\begin{aligned} \text{min.} \quad & -x_1 && (1) \\ \text{subject to:} \quad & x_2 \leq (1-x_1)^3 && (2) \\ & x_1 \geq 0 && (3) \\ & x_2 \geq 0 && (4) \end{aligned}$$

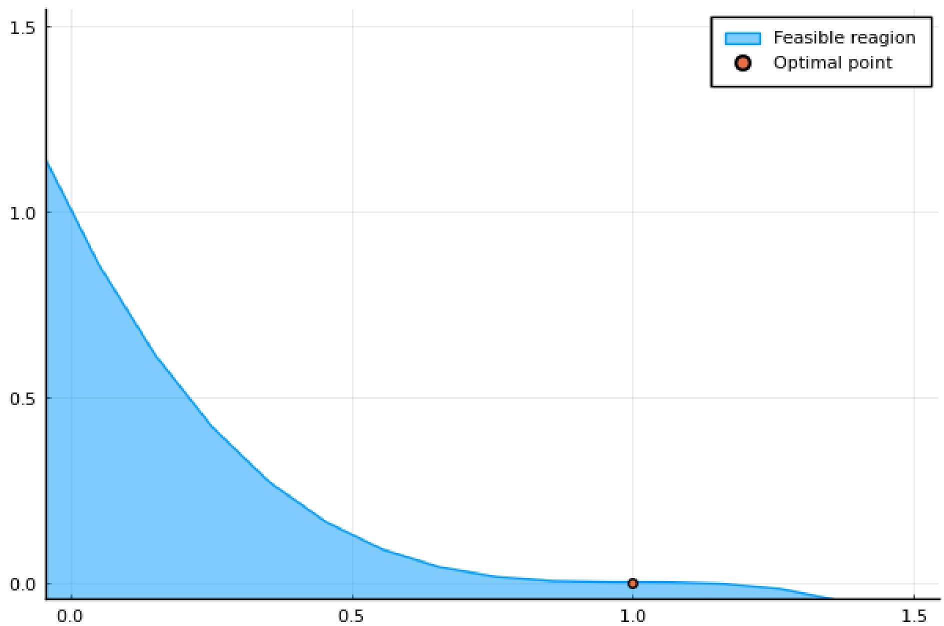

Figure 1: The feasible region of the problem of exercise 3.1. The condition of x_1,$x_2 \geq 0$ is implemented by the limits of the plot.

Figure 1 shows the feasible region for the problem above. Since minimizing $-x_1$ is the same as maximizing x_1, we can identify the optimal point as $\bar{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

We will change around Equation 2 to be $(1-x_1)^3 - x_2 \geq 0$ for it to fit into the FJ conditions.

$$0 = u_0 \nabla f(\bar{x}) + \sum_{i=1}^{m} u_i \nabla g_i(\bar{x}) \quad (5)$$

$$0 = u_0(-1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}) + u_1 \begin{bmatrix} 3(1-x_1)^2 \\ -1 \end{bmatrix} + u_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + u_3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (6)$$

$$0 = u_0(\begin{bmatrix} -1 \\ 0 \end{bmatrix}) + u_1 \begin{bmatrix} 0 \\ -1 \end{bmatrix} + u_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + u_3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (7)$$

$$0 = \begin{bmatrix} -u_0 + u_2 \\ -u_1 + u_3 \end{bmatrix} \quad (8)$$

$$\implies \begin{cases} u_0 = u_2 \\ u_1 = u_3 \end{cases} \quad (9)$$

3.2

a)

$$\begin{aligned}
\text{min.} \quad & (x_1 + \frac{9}{4})^2 + (x_2 - 2)^2 && (10)\\
\text{subject to:} \quad & x_2 - x_1^2 \geq 0 && (11)\\
& x_1 + x_2 \leq 6 && (12)\\
& x_1 \geq 0 && (13)\\
& x_2 \geq 0 && (14)
\end{aligned}$$

The KKT conditions for the problem is

$$\begin{aligned}
0 =& \nabla f(\bar{x}) + \sum_{i=1}^{m} u_i \nabla g_i(\bar{x}) && (15)\\
0 =& \begin{bmatrix} 2(\bar{x}_1 + \frac{9}{4}) \\ 2(\bar{x}_2 - 2) \end{bmatrix} + u_1 \begin{bmatrix} -2\bar{x}_1 \\ 1 \end{bmatrix} + u_2 \begin{bmatrix} -1 \\ -1 \end{bmatrix} + u_3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + u_4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} && (16)\\
0 =& \begin{bmatrix} 2(\frac{3}{2} + \frac{9}{4}) \\ 2(\frac{9}{4} - 2) \end{bmatrix} + u_1 \begin{bmatrix} -2\frac{3}{2} \\ 1 \end{bmatrix} + u_2 \begin{bmatrix} -1 \\ -1 \end{bmatrix} + u_3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + u_4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} && (17)\\
0 =& \begin{bmatrix} 2(\frac{3}{2} + \frac{9}{4}) - \frac{7}{2}u_1 - u_2 + u_3 \\ 2(\frac{9}{4} - 2) + u_1 - u_2 + u_4 \end{bmatrix} && (18)
\end{aligned}$$

b)

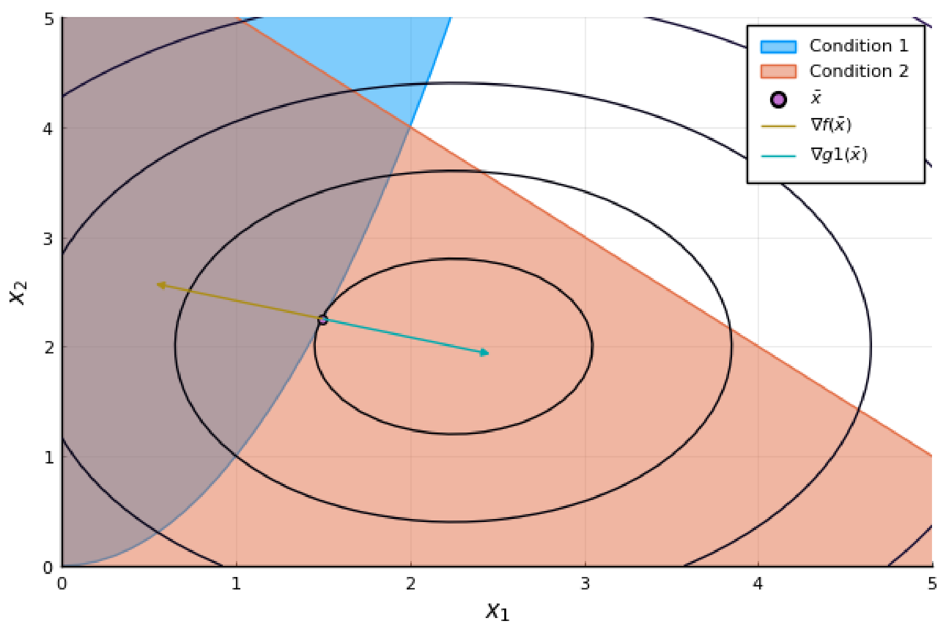

Figure 2