

Project Work 1 is online from Monday 11.10.2021 <https://mycourses.aalto.fi/mod/assign/view.php?id=765881> and is due no later than **Monday 1.11.2021 23:55**. No homework this week.

Problem 5.1: Newton's Method for a Quadratic Problem

Consider the following quadratic optimization problem

$$\min_x f(x) = x^\top Ax \quad (1)$$

with variable vector $x \in \mathbb{R}^n$, and where $A \in \mathbb{R}^{n \times n}$ is a positive semidefinite (PSD) matrix.

- (a) Not all PSD matrices are symmetric. Show that if the PSD matrix A in (1) is not symmetric, we can always replace it with a symmetric PSD matrix B such that $x^\top Ax = x^\top Bx$.
- (b) Show that Newton's method converges in one iteration when applied to the problem (1).
- (c) Show that Newton's method converges in one iteration when applied to the following quadratic problem with an additional linear term:

$$\min_x f(x) = x^\top Ax - b^\top x \quad (2)$$

with variables $x \in \mathbb{R}^n$. Assume that $A \in \mathbb{R}^{n \times n}$ is a symmetric PSD matrix and $b \in \mathbb{R}^n$.

Problem 5.2: Affine Invariance of Newton's Method

Consider the following unconstrained optimization problem

$$\min_x f(x) \quad (3)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a twice differentiable function. Show that Newton's method applied to problem (3) is *affine invariant*, meaning that the progress of Newton's method is independent of affine transformations of the original problem (e.g., scaling, translation, and rotation).