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# Investigations on two-way coupling effects of particle-laden decaying isotropic turbulent flows

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PROJEKTARBEIT  
VON  
JULIAN STEMMERMAN, STEFFEN TRIENEKENS  
UND CHRISTIAN SOIKA

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Aerodynamisches Institut der RWTH Aachen

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Betreuer: Konstantin Fröhlich  
Erstprüfer: Univ.-Prof. Dr.-Ing. Wolfgang Schröder

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# 1 Nomenclature

$\eta$	Kolmogorov length scale	$\mathbf{f}_d$	Drag force
$\gamma$	Isentropic exponent	$\mathbf{f}_h$	History force
$\lambda$	Taylor microscale	$\mathbf{f}_l$	Lift force
$v$	Particle volume	$\mathbf{g}$	Gravity constant
$\boldsymbol{\tau}$	Stress tensor	$\mathbf{H}^i$	Stores the inviscid variables in the flux-vector included in the Navier-Stokes equations
$\mathbf{I}$	Identity tensor	$\mathbf{H}^v$	Stores the viscous variables in the flux-vector included in the Navier-Stokes equations
$\mathbf{S}$	Rate-of-strain-tensor	$\mathbf{H}$	Container for fluctuating variables in the Navier-Stokes equations
$\mu$	Dynamic viscosity	$\mathbf{Q}$	Container for conserved variables in the Navier-Stokes equations
$\nu$	Kinetic viscosity	$\mathbf{q}$	Heat conduction
$\nu$	Viscosity of the fluid	$\mathbf{u}$	Three-dimensional velocity
$\psi$	Couling rate	$\mathbf{v}$	particle velocity
$\rho$	Density	$\mathbf{v}_p$	Particle velocity
$\rho_p$	Particle density	$\mathbf{x}$	particle position
$\tau_\eta$	Kolmogorov time scale	$\mathbf{x}_p$	Particle position
$\tau_{ps}$	Stokes relaxation time	$\mathbf{x}_p^n$	Positions of the n-th particle
$\tau_p$	Particle relaxation time	$a_r$	Relative acceleration of particles
$\tau_p$	Particle response time	$c_p$	Specific isobaric heat capacity
$\nabla$	Nabla-operator	$c_v$	Specific isochoric heat capacity
$\mathbf{a}$	particle acceleration	$d$	Particle diameter
$\mathbf{F}$	Force per unit volume from particle on fluid		
$\mathbf{f}^n$	Sum of forces acting on the fluid		
$\mathbf{f}_a$	Added mass force		

$E$	Specific inner energy	$Pr$	Prandtl number
$e$	Specific internal energy	$R$	Universal gas constant
$f_D$	Drag correction	$r_p$	Particle radius
$k_t$	thermal conductivity	$Re$	Reynolds number
$L$	Integral length scale	$Re_p$	Particle Reynolds number
$m_c$	Number of particles in a particle cluster	$S$	Sutherland temperature
$m_c$	number of clustered particles	$T$	Temperature
$p$	Pressure	$t^*$	Eddy turnover time
		$U$	Characteristic velocity

## 2 Introduction

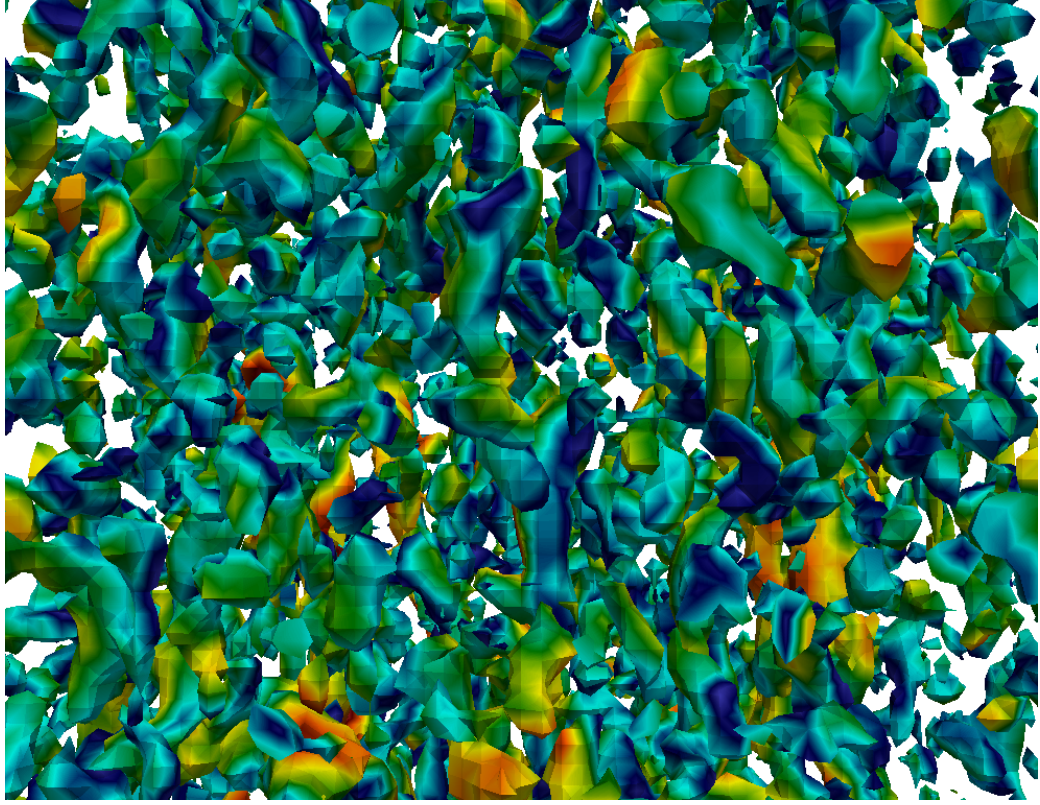


Figure 1:  $\lambda_2$ -contours colored by velocity magnitude: Tubular structures in an isotropic flow field

This work deals with the effects of particles on the flow conditions of isotropic turbulent flows. There are many examples of these particle laden turbulent flows in nature, for example in volcanic eruptions and in the "white water" of breaking waves.

Multiphase flows also occur in many technical applications. Spray atomization in fuel injectors, cyclonic particle separation in oil refineries and sediment accumulation in pipelines are just three of many cases, where it is of huge interest to predict the influence of particles on the turbulence of the mean flow. Of major industrial interest is the fuel dispersion and combustion in a petrol-powered engine. To get a homogeneous and fast combustion, both should take place at turbulent flow conditions. One could say it is an important question, in which way and intensity the particles influence the turbulence.

Another important example is a fluidized bed. This is an application to realize an effective suspension and combustion of fluid particles in a vessel. The particles, which are at the bottom of the vessel, are suspended by a fluid, in most cases a gas,

that streams from beyond the particles throw the porous bottom, if the pressure is larger than the gravity of the particles. While this happens the fluid suspends the particles and the contact between fluid and particles is very intimate, which facilitates for example the combustion of material with a low caloric content, the in situ absorption of the pollutants deriving from the combustion and the action of a catalyst. This process aims to produce a suspension with less heterogeneous regions, what means bubbles with a much smaller concentration of particles. Hence turbulence is very important requirement to produce a homogeneous fluid stream for effective and complete combustion.

In this work the influence of the particles on the streaming conditions is presented by using DNS and LES simulations. The focus lies on analyzing the energy transfer between particles and fluid as well as the intensity of the decrease of turbulent kinetic energy in the fluid. At the beginning some mathematical background information about modeling turbulent single phase and particle laden flows are given. Following the Navier-Stokes equations and the different scales of turbulent motion are presented as an approach to modelling turbulence. In addition to this for particle laden flows the equations of motion are expanded by a new term that describes the forces acting between fluid and particles. Also the coupling rate, that describes the exchanged energy is introduced. Then the computational basics of methods used in this work are introduced. First the simulation methods DNS and LES, how they work and in which case which one is used is explained. Then the used discretization tool to integrate the particle tracking equations, that works with a predictor-corrector-scheme, is described. Finally the computational method 'particle clustering' is introduced. This approach aims to decrease the computational effort of particle laden simulations. It works that way that the particles are coupled to cluster and each cluster is considered as one particle.

In the following the deviations caused by using this method are determined and the applicability is evaluated. Therefore the influence of the number of particles per cluster on the accuracy of the plots of the turbulent kinetic energy and the exchanged energy between particles and fluid per time is identified. Also the number of particles that is needed to get consistent average values for the flow characteristics is investigated.

### 3 Mathematical models

#### 3.1 Single-phase flow

In this section the mathematical basics for understanding and simulating turbulent flows are discussed. However, it should be pointed out that this is no complete treatise of the mathematical and physical basics. The reader can achieve further insight on this topic by looking at different books and papers, e.g. [10].

##### 3.1.1 The Navier-Stokes equations

The Navier-Stokes-Equations are of great importance for understanding turbulent phenomena. This set of equations exists in forms for compressible and incompressible fluids. For an infinitesimal small volume element  $d\tau$  and using the cartesian coordinate system, they can be written in the so-called 'divergence form':

$$\frac{\partial \mathbf{Q}}{\partial t} + \nabla \mathbf{H} = 0 \quad (3.1)$$

The vector  $\mathbf{Q}$  contains all the variables which are conserved, i.e. the density  $\rho$ , the velocity  $\mathbf{u}$  and the specific inner energy  $E$ :

$$\mathbf{Q} = \begin{pmatrix} \rho \\ \rho \mathbf{u} \\ \rho E \end{pmatrix} \quad (3.2)$$

$\mathbf{H}$  is the flux vector which stores all the floating variables and may be split up into two parts:

$$\mathbf{H} = \mathbf{H}^i + \mathbf{H}^v \quad (3.3)$$

The contents of the two vectors are displayed below:

$$\mathbf{H}^i = \begin{pmatrix} \rho \mathbf{u} \\ \rho \mathbf{u} \mathbf{u} + p \\ \mathbf{u}(\rho E + p) \end{pmatrix} \quad (3.4)$$

$$\mathbf{H}^v = -\frac{1}{Re} \begin{pmatrix} 0 \\ \boldsymbol{\tau} \\ \boldsymbol{\tau} \mathbf{u} + \mathbf{q} \end{pmatrix} \quad (3.5)$$

$\mathbf{H}^i$  is called inviscid flux and contains only the variables that are independent of the fluids viscosity, it describes the way a fluid with a viscosity of zero would behave. In contrast, the viscous flux  $\mathbf{H}^v$  represents the effects of viscosity. The

Reynolds number  $Re = \frac{\rho v d}{\eta}$  is defined to be the ratio of inertia to tenacity, which makes it very valuable for understanding turbulent flows. This is also due to the fact that two familiar objects with the same Reynolds number behave similar in turbulence. To solve the Navier-Stokes-Equations, more information regarding some variables is required. For Calculating the specific inner Energy  $E$  and the heat conduction  $\mathbf{q}$ , the following equations are used:

$$E = e \frac{1}{2} |\mathbf{u}|^2 \quad (3.6)$$

$$\mathbf{q} = -\frac{\mu}{Pr(\gamma - 1)} \nabla T \quad (3.7)$$

with

$$\gamma = \frac{c_p}{c_v} \quad (3.8)$$

and the Prandtl number

$$Pr = \frac{\mu_\infty c_p}{k_t} \quad (3.9)$$

using the specific heat capacities of the fluid  $c_v$  and  $c_p$ . If one could assume that the fluid is a newtonian fluid, the linear correlation between stress and the rate of strain results in:

$$\boldsymbol{\tau} = 2\mu \mathbf{S} - \frac{2}{3}\mu(\nabla * \mathbf{u})\mathbf{I} \quad (3.10)$$

in which  $\mathbf{S} = \frac{(\nabla \mathbf{u})(\nabla \mathbf{u})^T}{2}$  denotes the rate-of-strain-tensor. Additionally, the viscosity  $\mu$  can be approximated through Sutherland's law, which is based on the ideal gas-theory:

$$\mu(T) = \mu_\infty \left( \frac{T}{t_\infty} \right)^{3/2} \frac{T_\infty + S}{T + S} \quad (3.11)$$

$S$  is in this case the Sutherland temperature. To achieve closure the caloric state equation  $e = c_v T$  and the state equation for an ideal gas  $p = \rho R T$  are used. The specific gas constant is determined by  $R = c_p - c_v$ . These equations form a set of partial differential equations, as a consequence starting values are needed.



### 3.1.2 Turbulent motion across all scales

Turbulent flows contain eddies of all sizes and forms. Large-scale eddies bring energy to the flow which is then passed down to smaller-scale eddies and in the end dissipated into heat by viscous effects. This behavior is called the 'energy cascade' and was first described by Richardson in the year 1920 [?]. The theory then was developed further by Kolmogorov and published 1941 [?].

The first set of scales describe the large eddies. These scales are called *integral* scales and are determined by the physical boundaries of the flow. As said before, at these scales the energy is brought into the flow, creating the so-called 'energy-containing range'. The length scale is the called integral length scale  $L$ , and the corresponding timescale which is most times called 'eddy turnover time' is defined as:

$$\tau_L = \frac{L}{U} \quad (3.12)$$

where  $U$  denotes the characteristic velocity.

The smallest scales in a turbulent flow are the Kolmogorov length ( $\eta$ ) and time ( $\tau_\eta$ ) scale. At this scales, the effects of viscosity take place and the energy dissipates into heat. With the estimate  $\epsilon \sim \frac{U^3}{L}$  they can be written as

$$\eta = \left( \frac{\nu^3 L}{U^3} \right)^{1/4} \quad (3.13)$$

$$\tau_\eta = \left( \frac{\nu L}{U^3} \right) \quad (3.14)$$

Both these scales are coupled by the Reynolds number:

$$\frac{L}{\eta} = Re^{3/4} \quad (3.15)$$

$$\frac{\tau_L}{\tau_\eta} = Re_L^{1/2} \quad (3.16)$$

It can be seen from these to equations that the spacing between the scales increases for higher Reynolds numbers.

A scale between these two is the Taylor microscale, often referred to as 'turbulence length scale'. Although it lacks of a physical interpretation, it is often used to describe the intermediate range between integral and Kolmogorov scales. Its definition is:

$$\lambda = \sqrt{15 \frac{\nu}{\epsilon}} |\mathbf{v}'| \quad (3.17)$$

with  $|\mathbf{v}'|$  denoting the absolute value of the velocities fluctuation. This scale can be used to compute another Reynolds number  $Re_\lambda$ :

$$Re_\lambda = \frac{|\mathbf{v}'| \lambda}{\nu} \quad (3.18)$$

Together these scales form a powerful tool to understand, compute and describe turbulent flows.

### 3.2 Particle dynamics

This work deals with particle laden fluids. Therefore the impact of them on the flow behavior of the fluid needs to be described. This phenomenon is called two-way-coupling and the exchanged energy per time between fluid and particles is called coupling rate.

Here small and heavy, rigid particles, with a spherical shape are considered. Their radius  $r_p$  is even smaller than the Kolmogorov scale  $\eta$ , but also large enough to neglect the Brownian motion.

Due to the small particle concentration, the best and most common way to describe these flows is the point particle approach, which means that every particle is treated as an mathematical point source of mass, momentum and energy. In this case we focus on the momentum exchange. Effects like particle-particle interactions, particle-wall interactions are also neglected.

At describing the motion of the particles in the following the fact, that we deal with gas-solid flows is an advantage, since we can make several simplifications. First the perspective is taken on the influence of the particles on the carrier fluid. Here the assumptions, that the fluid is incompressible and the mass exchange over the particle surface is zero are used. Hence the continuity equation becomes:

$$\nabla \cdot \mathbf{u}. \quad (3.19)$$

The Navier Stokes equation, described in chapter 1, becomes:

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) \right) = -\nabla p + \mu \nabla^2 \mathbf{u} - \mathbf{F}. \quad (3.20)$$

The influence of the particles on the fluid is represented by the new term  $\mathbf{F}$ , wich describes the force per unit volume on the fluid.  $\mathbf{F}$  could be approximated by a superposition of Dirac's delta functions over all particles, centered at the location  $\mathbf{x}_p^n$  of each particle:

$$\mathbf{F} = \sum_{n=1}^{N_p} \mathbf{f}^n(\mathbf{x}_p^n) \delta(\mathbf{x} - \mathbf{x}_p^n). \quad (3.21)$$

$\mathbf{x}_p^n$  is the position of the n-th particle and results from the kinematic equation

$$\frac{\partial \mathbf{x}_p}{\partial t} = \mathbf{v}_p^n \quad (3.22)$$

$\mathbf{v}_p$  and is the velocity of the n-th particle.  $\mathbf{f}^n$  is the sum of forces acting between fluid an particles. The delta function becomes one if  $\mathbf{x} - \mathbf{x}_p^n$  becomes zero. Otherwise its zero. In this way, the coupling forces act at the positions of the particles on the fluid. Also it plays also an important role in the equation of motion of the

particles:

$$v\rho_p \frac{\partial \mathbf{v}_p^n}{\partial t} = v\rho_p \mathbf{g} + \mathbf{f}^n(\mathbf{x}_p^n) \quad (3.23)$$

As already mentioned  $\mathbf{f}^n(\mathbf{x}_p^n)$  could be divided in several forces. Then the equation of particle motion looks like:

$$m_p \frac{\partial \mathbf{v}_p^n}{\partial t} = m_p \mathbf{g} + \rho v \left( \frac{D\mathbf{u}}{Dt} - \mathbf{g} \right) + \mathbf{f}_d + \mathbf{f}_l + \mathbf{f}_a + \mathbf{f}_h + \mathbf{f}_{\text{additional}} \quad (3.24)$$

Here  $\mathbf{f}_d$  represents the hydrodynamical drag force that is parallel to the undisturbed streamlines, which depends on an empirical drag coefficient  $C_d$ :

$$\mathbf{f}_d = -\frac{3}{4}\rho v \frac{C_d}{d} |\mathbf{v}_p - \mathbf{u}| (\mathbf{v}_p - \mathbf{u}) \quad (3.25)$$

It depends on the empirical drag coefficient defined after Schiller and Naumann as

$$C_d = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}). \quad (3.26)$$

$\mathbf{u}$  is the velocity of the uniform stream, which is enough away from the particle that it is undisturbed from the particle. The other partial hydrodynamical force, the lift force  $\mathbf{f}_l$  is perpendicular to the undisturbed streamlines. Furthermore the added mass force  $\mathbf{f}_a$  represents the influence of the inertia of the fluid that has an impact on the particle, if it has a different acceleration than the mean flow. Hence it could be determined by:

$$\mathbf{f}_a = \frac{1}{2}\rho v \left( \frac{D\mathbf{u}}{Dt} - \frac{d\mathbf{v}_p}{dt} \right) \quad (3.27)$$

The history force  $\mathbf{f}_h$  takes diffusion and convection, that results out of the vortices behind the particles, into account. Basselt's result, neglecting the finite size correction, is proportional to  $\nabla^2 \mathbf{u}$ :

$$\mathbf{f}_h = \frac{3}{2}d^2\rho\sqrt{\pi\nu} \int_{t_0}^t \frac{dt'}{(t-t')^{1/2}} \left( \frac{D\mathbf{u}}{Dt'} - \frac{d\mathbf{v}_p}{dt'} \right) \quad (3.28)$$

The last term  $\mathbf{f}_{\text{additional}}$  is attached for the case that we have to take other forces like electrostatic interactions into account. In this case of gas-solid suspensions there could be made several simplifications. In the following there is shown with an rough approximation that the added mass and history forces are negligible compared with the drag force. We use the approach that the relative velocity is of the same order as the terminal velocity  $v_t$ . The terminal velocity is the velocity of a particle in a resting fluid, when gravitation and drag force are in equilibrium. Then if the added mass and the drag force are compared, we get the following equation:

$$\frac{|\mathbf{f}_a|}{|\mathbf{f}_d|} \simeq \frac{\frac{1}{2}\rho v g}{\frac{3}{4}\rho v (C_d/d)v_t^2} \frac{a_r}{g} = \frac{1}{2} \frac{\rho}{\rho_p} \frac{a_r}{g} \quad (3.29)$$

with

$$a_r = \left( \frac{D\mathbf{u}}{dt} - \frac{d\mathbf{v}_p}{dt} \right) \quad (3.30)$$

Since in gas-solid suspensions the particle density is of an factor of 1000 higher than the fluid density, it shows that, if it's not the unlikely case that the relative acceleration is of the same order higher than the gravity, the added mass force is negligible compared to the drag force. The relation of the history force and the added mass force with the assumption of comparable accelerations, with the relaxation time of the order of  $\tau$  results in

$$\frac{|\mathbf{f}_h|}{|\mathbf{f}_a|} \simeq 18 \sqrt{\frac{\nu\tau}{d^2}}. \quad (3.31)$$

Hence it is of the order of the ratio of the diffusion length to the particle diameter, which might be of the order of one, if  $\tau \sim \frac{d^2}{\nu}$ . Hence the history force is also negligible compared to the drag force. For the lift force the situation is the same, because it is also proportional to the fluid density and the length scale of the ambient flow vorticity might be much greater than the particle size. It shows that the particle motion is only depending on the hydrodynamical drag force which results from the actual difference between the fluid velocity  $\mathbf{u}(\mathbf{x}_p)$  at particle position  $\mathbf{x}_p$  and the particle velocity  $\mathbf{v}_p$ . Due to this assumptions equation (3.17) becomes

$$\rho_p \frac{d\mathbf{v}_p^n}{dt} = \rho_p \mathbf{g} + \mathbf{f}_d^n \quad (3.32)$$

This equation can be rewritten with help of the relaxation time  $\tau_p$ , which physically represents the time scale over which the drag force decreases the particle relative velocity to zero. At one point in the fluid with the proportionality

$$\frac{\rho_p v_p}{\tau_p^n} \sim f_d \quad (3.33)$$

it follows the equation for the relaxation time

$$\tau_p^n = \frac{4}{3} \frac{\rho_p}{\rho} \frac{d}{C_d^n} \frac{1}{v_r^n} \quad (3.34)$$

where  $v_r^n$  is the relative velocity. With equation (3.26) and the Reynoldsnumber in relation to the particles  $Re_p^n = \frac{v_r^n d}{\nu}$ , with the particle diameter  $d$  it follows

$$\tau_p^n = \frac{\rho_p}{\rho} \frac{d^2}{18\nu} \frac{1}{1 + 0.15 Re_p^{0.687}}. \quad (3.35)$$

Then the equation of motion of the particles becomes

$$\rho_p \frac{d\mathbf{v}^n}{dt} = \rho_p \mathbf{g} - \rho_p \frac{\mathbf{v}_p^n - \mathbf{u}^n}{\tau_p^n}. \quad (3.36)$$

Using the Stokes response time  $\tau_{ps} = \frac{\rho_p d^2}{18\nu\rho}$ , that is the particle response time at resting fluid conditons, there finally exists another form of this equaion

$$\rho_p \frac{d\mathbf{v}_p^n}{dt} = \rho_p \mathbf{g} - \rho_p \frac{\mathbf{v}_p^n - \mathbf{u}^n}{\tau_{ps}} f_d \quad (3.37)$$

with  $f_d = 1 + 0.15 Re_p^{n^{0.687}}$ , which is not the drag force.

The coupling rate  $\psi$ , that describes the transfered Energy per time between particles and fluid, is defined as

$$\psi = \mathbf{F} \mathbf{u}. \quad (3.38)$$

Hence it results from the force per unit volume exerted between the fluid and the particles and the main velocity of the fluid. It is an useful variable to describe, how much the main fluid and the particles influence each other.

## 4 Numerical methods

To simulate flows like those described above we have two options. The direct numerical simulation (DNS) is the easier one to understand, although it is numerically very expensive. The Large-eddy simulation (LES) is numerically more capable, still we must accept certain inaccuracies. These two numerical methods are now discussed in the following chapter.

### 4.1 Direct numerical simulation

The basis of the direct numerical simulation (DNS) are the Navier-Stokes equations as described above. The idea is that the computer is very good at calculating and solves these equations completely. This provides a very accurate result, as all scales of motion are being resolved. Still it requires an immense level of computational resources which increases rapidly with the Reynolds number. These computational resources were not available until the 1970s. Even though it is not advisable to resolve every scale of motion, if only the contained energy is of greatest interest. With the large-eddy simulation, as described below, the computational effort is 99.98 % less compared to DNS, which indeed is the fraction of the dissipative scale. This leaves 0.02 % of the flow, which is correlative with the fraction of the energy-containing larger-scale [10].

### 4.2 Large-eddy simulation

Due to the fact that DNS is effortful and wasting resources if a fully resolved resolution is not required, large-eddy simulation (LES) was created to save time and resources. This is especially efficient if mainly the temporal energy trend is considered, because the energy containing larger-scale motion is completely resolved and the indeed small effects of the expensive smaller-scale motion are just modelled. Otherwise in DNS resolving the small dissipative scale would require most of the computational resources.

Simulating only the larger-scale motions is also called filtering, which means that the smaller-scale motions are filtered out. To model the filtered smaller-scale motions usually a subgrid-scale (SGS) model is used. According to Hickel (2007) the interference between explicit SGS and the truncation error can be exploited, i.e. the truncation error can serve as model of the effects of unresolved scales, which is therefore an implicit SGS model. Thus we call it implicit LES (ILES) [6].

### 4.3 Discretisation

To integrate the Lagrangian particle tracking equations, discussed above, a predictor-corrector scheme based on the trapezoidal rule for numerical integration

$$f(t + \delta t) \approx f(t) + \frac{\delta t}{2} \left[ \frac{\partial f(t)}{\partial t} + \frac{\partial f(t + \delta t)}{\partial t} \right] \quad (4.1)$$

is used.

The first step is the prediction of the new particle position  $\mathbf{x}_{n+1}^{(p)}$  using a Taylor expansion for a small time step  $\delta t$

$$\mathbf{x}_{n+1}^{(p)} = \mathbf{x}_n + \delta t \mathbf{v}_n + \frac{1}{2} \delta t^2 \mathbf{a}_n. \quad (4.2)$$

Due to the computational effort we will put  $\mathbf{u}_{n+1}^{(p)}$  on the level of the nearest cell fluid velocity.

The updated velocity and acceleration are calculated as

$$\mathbf{v}_{n+1} = \frac{\mathbf{v}_n + \frac{1}{2} \delta t \left( \mathbf{a}_n + \frac{f_D}{\tau_p} \mathbf{u}_{n+1}^{(p)} + \mathbf{g} \right)}{1 + \frac{1}{2} \frac{f_D}{\tau_p} \delta t}, \quad (4.3)$$

$$\mathbf{a}_{n+1} = \frac{\frac{f_D}{\tau_p} \left( \mathbf{u}_{n+1}^{(p)} - \mathbf{v}_n - \frac{1}{2} \delta t \mathbf{a}_n \right) + \mathbf{g}}{1 + \frac{1}{2} \frac{f_D}{\tau_p} \delta t}. \quad (4.4)$$

The updated particle position must be corrected by an additional term according to the trapezoidal rule

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \frac{1}{2} \delta t (\mathbf{v}_{n+1} + \mathbf{v}_n) + \frac{1}{12} \delta t^2 (\mathbf{a}_{n+1} - \mathbf{a}_n). \quad (4.5)$$

### 4.4 Particle clustering

The high number of point particles require even more computational resources for the particle-laden simulation. The main idea to reduce this requirement is to create clusters of point particles, meaning that a new variable  $m_c$  is introduced. We consider a cluster of  $m_c$  point particles as one larger point particle, i.e. the program has less particles to simulate. To compensate this lack of particles, the coupling force is multiplied by  $m_c$ , due to the  $m_c$ -fold mass of the (cluster-)particles. In chapter 5 (results) we evaluate the legitimacy of particle clustering and the maximum acceptable value of  $m_c$ .

Projektion (noComputationalParticles), Diskretisierung implizite LES (Motivation fuer LES - Pope Chapter 9, Bild 9.4), DNS



## 5 Results

The simulations were carried out using ZFS, the simulation tool developed and implemented at the Institute of Aerodynamics at RWTH Aachen University [1] [2]. The tool is capable of simulating finite-volume flows of compressible fluids. In the simulations, which results the reader can see at hand, the Mach-number  $Ma$  was set to 0.1 to simulate an almost incompressible fluid. The turbulence was simulated on a cubic grid using  $64^3$ ,  $96^3$ ,  $128^3$  and  $256^3$  grid points. As a consequence of grid refinement the first three cases were simulated using LES, the case in which  $256^3$  cells were used is carried out as DNS. Further information can be gained by looking at [10, p.344-357 for DNS and p. 558-639 for LES].

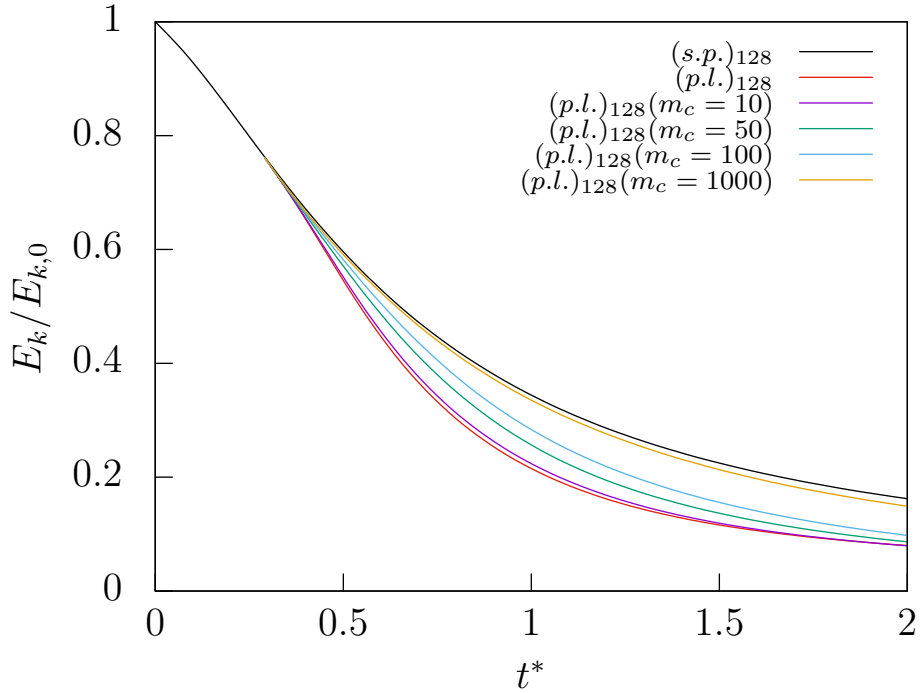


Figure 2: Kinetic Energy over eddy turnover times for different numbers of clustered particles with fixed all-over particle number

For simplification, the special case of isotropic turbulence was used. For this idealised flow form the statistical velocities are invariant in all directions of the grid. It follows that the flows velocity are also invariant for rotations and reflections. The turbulence was initialized using a seed-based random generator. To achieve physical results, the simulation of the particle-free flow was carried out a reasonable amount of time, at which a restart file was written out. This procedure ensures a fully developed turbulent flow, whom has emancipated from the initialization. In this flow field, a specific number of spherical particles were injected.

To minimize computational effort for simulating the particle-laden flow while still achieving high quality results, a clustering of particles was integrated into the simulation tool. To investigate the differences in accuracy for different sizes of particles sizes, the variable  $m_c$  was introduced to the code describing the number of particles in one cluster. The simulations were then set up with the overall same number of particles ( $10^6$ ), altering just the number of particles in one cluster. Then the simulations were carried out normally and result in the graphs at hand (Figure 2, 3 and 4). It can be seen in figure 2 that the decay in kinetic energy from the starting point depends highly on the number of clustered particles. The kinetic energy in the particle-laden cases drops more rapidly in the time-period after the injection than the particle-free case due to a higher dissipation rate induced by the particles.

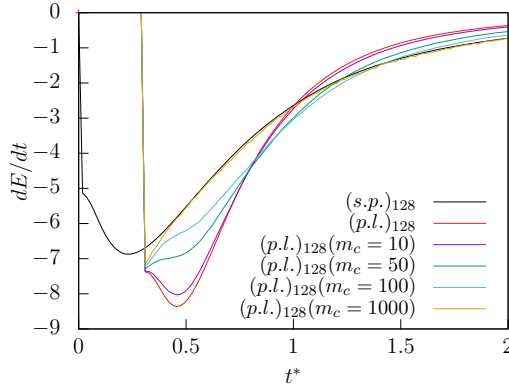


Figure 3: Change in kinetic Energy over eddy turnover times

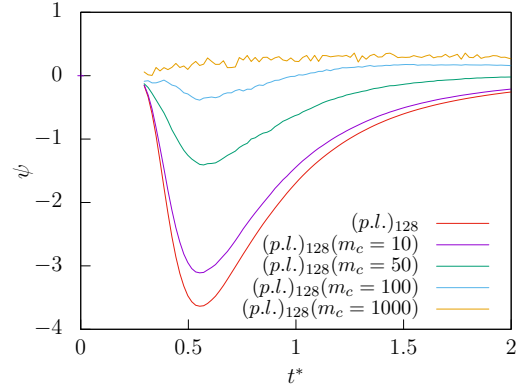


Figure 4: Coupling Rate over eddy turnover times

Fitting to this first results, the Graphs of figure 3 and 4 clearly show that for simulations with highly clustered particles the results differ a lot from unclustered simulations. Looking at the results for the change in kinetic Energy, the difference becomes evident: The lower  $m_c$  is, the higher is the drop in rate of change of the kinetic energy. A similar result was found by Schneiders et al [9]. The particles therefore have two effects on the fluid: Firstly, the viscous dissipation rises in the case of particle-load. Secondly, there is always an energy transfer from the particles to the surrounding fluid. Both of these effects influence the kinetic energy and a change in the direction of transferred energy can be observed at about one eddy turnover time for the unclustered simulation. At this point in time the rate of kinetic energy flowing from the particles to the fluid matches the additional rate of dissipated energy induced by the particles.

Schneiders' results show, that for particles with the same diameter this point is

reached later in time for heavier/more dense particles due to their inertia. One could observe a similar effect with the simulation at hand: The more particles are summarized in one cluster, the later the balanced state of energy flow is achieved. From this point on, the kinetic energy drops slower than in the particle-free case. In difference to the others, the case in which 1000 particles were clustered shows a strange behavior. (It catches up to the particle-free case very fast, which leads to the conclusion that the amount of clusters is so small that the flow almost behaves like one without particles.) Additionally the change in kinetic Energy shows inconstancy which can also be traced back to the small number of clusters. The amount is just too small to achieve high-quality information in the statistical variables.

The same impression can be achieved by looking at the graphs describing the coupling rate (figure 4). The particle-laden-case makes the biggest jump into negative coupling rate. The higher the amount of coupled particles, the lower is the negative coupling rate. This evolution continues until the physicality vanishes and the randomness starts to show at the results for  $m_c$  higher than 50.

Being very similar in the time shortly after the injection, the flow statistics diverge more and more when time passes by. The variables of these simulations one turnover time after the injection can be found in table 1 on page 18. The ratio of the densities for this simulations was set to  $\frac{\rho_p}{\rho} = 1000$ , the particle diameter was 0.0794048628, so that  $d_p \approx 0.6\eta$ . At the timestep of injection the particle response time  $\tau_p = \frac{d^2}{18\nu} \frac{\rho_p}{\rho}$  was 0.03497 and  $Re_\lambda$  was 57.9757. In all of these simulations the volume and mass fractions  $\phi_v$  and  $\phi_m$  were constant ( $\phi_v = ???$ ,  $\phi_m = ???$ ).

$m_c$	$u_0$	$\epsilon \frac{u_0^3}{L}$	$\frac{\lambda}{L}$	$\frac{\eta}{L}$	$Re_\lambda$	$Pr$
1	0.21496	0.79150	0.0432	0.00333	43.360	
10	0.22364	1.02918	0.03866	0.00312	39.5653	
50	0.25654	1.78788	0.03144	0.00272	34.4702	
100	0.28357	2.26144	0.02940	0.00257	33.8869	
1000	0.33543	2.2855	0.03184	0.00256	39.8993	

Table 1: Variables of the first set of simulations one turnover time after injection for the  $128^3$ -case

To find out at which number of particles the results are sufficiently exact, a second set of simulations was carried out. As mathematics of turbulent flows are based on averaged variables, small numbers of particles can lead to false and even physically questionable results. In these simulations, no particle clustering was

used, just different numbers of particles were injected into the same flow. For these simulations similar properties to the ones from the first set of simulations were used, just the mentioned number of particles was changed. The results can be seen on page 19 on figure 5. The normalized difference in turbulent kinetic

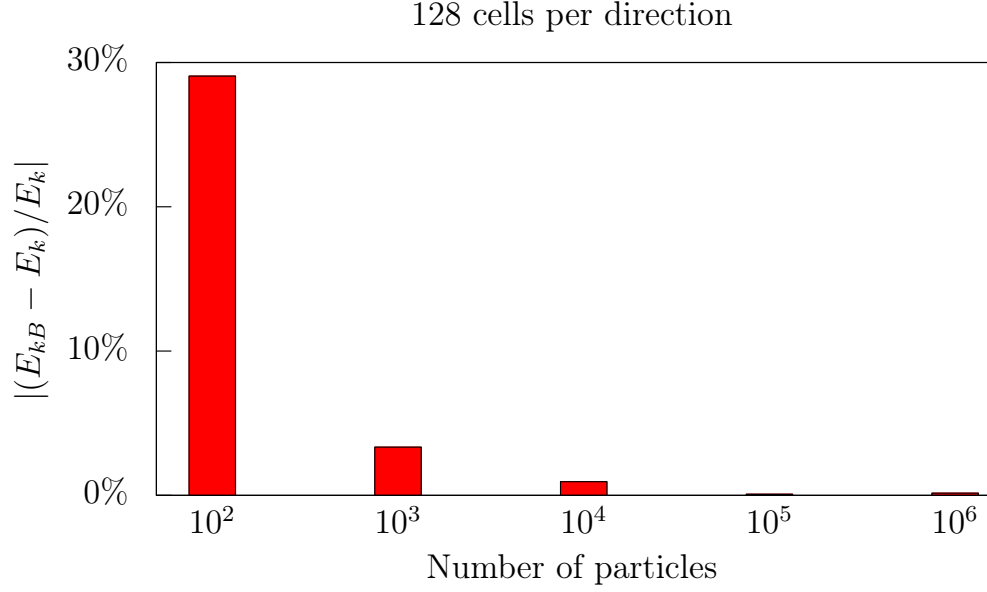


Figure 5: Results for initializing different numbers of particles

Energy shows in this one-time simulation a correlation between particle number and accuracy in the simulation. Although this was just a single initialization of particles in a flow, it can be stated that simulations using only  $10^2$ ,  $10^3$  or even up to  $10^4$  particles are not accurate enough for technical or scientific use of the data. One-time simulations in other gridsizes show similar results.

## 6 Conclusion and outlook

In this work two sets of simulations were carried out to evaluate a method for lowering computational effort and to find a limit of particles at which a simulation becomes inaccurate.

To achieve this goals, first the mathematical basics for understanding particle-free and particle-laden flows were introduced. The reader achieved knowledge about the Navier-Stokes equations and their enhancement for computing interactions between fluid and particles. In the following the reader got information about the numeric basics and methods for simulating such complicated phenomena.

The results presented in this work show, that for a sufficiently exact simulation, particle clustering has to be treated with caution. Depending on the application maybe small numbers of clustered particles could be used, but the savings in computing time would not balance the loss in accuracy out. This is the case in particular high numbers of clustered particles at which the results get highly inaccurate. Maybe investigations on smaller numbers of clustered particles could follow, the space between 2 and 10 could be closed by simulations in the future to determine the real border for inaccurate results.

Further investigation is needed regarding the second set of simulation, in which the goal was to find out which number of particles is necessary to get accurate results for the turbulent kinetic energy of the particles. The deviation from of the kinetic energy of the particles and the fluid should be zero for high numbers of particles, so the averaged difference of the energies should be zero as well. The distribution of the experiment's outcomes should match the well-known normal distribution, which leads to analyzing the standard deviation. Concluding, further simulations should be carried out until a sufficient standard deviation can be computed, from which one could make an assumption about the accuracy of the initialization.

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