Investigations on one-way coupling effects of particle-laden decaying isotropic turbulent flows

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Contents

1 Nomenclature

$\boldsymbol{\tau}$ Stress tensor

$\boldsymbol{I}$ Identity tensor

$\boldsymbol{S}$ Rate-of-strain-tensor

μ Dynamic viscosity

ρ Density

$\boldsymbol{\nabla}$ Nabla-operator

$\boldsymbol{Q}$ Container for conserved variables in the Navier-Stokes equations

$\boldsymbol{u}$ Three-dimensional velocity

$\boldsymbol{H^{i}}$ Stores the inviscid variables in the flux-vector included in the Navier-Stokes equations

$\boldsymbol{H^{v}}$ Stores the viscous variables in the flux-vector included in the Navier-Stokes equations

$\boldsymbol{H}$ Container for fluctuating variables in the Navier-Stokes equations

$\boldsymbol{q}$ Heat conduction

c_p Specific isobaric heat capacity

c_v Specific isochoric heat capacity

E Specific inner energy

e Specific internal energy

k_t thermal conductivity

p Pressure

Pr Prandtl number

R Universal gas constant

Re Reynolds number

S Sutherland temperature

T Temperature

2 Introduction

in Computational Methods for Multiphase Flow ist auf den Seiten 3-9 ein interessantes Beispiel.

3 Mathematical models

Sollen wir hier noch isotrope Turbulenz etc. erklaeren?

3.1 Single-phase flow

In this section the mathematical basics for understanding and simulating turbulent flows are discussed. However, it should be pointed out that this is no complete treatise of the mathematical and physical basics. The reader can achieve further insight on this topic by looking at different books and papers, e.g. [7].

3.1.1 The Navier-Stokes equations

The Navier-Stokes-Equations are of great importance for understanding turbulent phenomena. This set of equations exists in forms for compressible and incompressible fluids. For an infinitesimal small volume element $d\tau$ and using the cartesian coordinate system, they can be written in the so-called 'divergence form':

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \boldsymbol{\nabla} \boldsymbol{H} = 0 \tag{3.1}$$

It should be noted by the reader that this work only contains investigations about chemically inert fluids and particles, and that the simulation results only fit under this condition. The vector $\boldsymbol{Q}$ contains all the variables which are conserved, i.e. the density ρ, the velocity $\boldsymbol{u}$ and the specific inner energy E:

$$\boldsymbol{Q} = \begin{pmatrix} \rho \\ \rho \boldsymbol{u} \\ \rho E \end{pmatrix} \tag{3.2}$$

$\boldsymbol{H}$ is the flux vector which stores all the floating variables and may be split up into two parts:

$$\boldsymbol{H} = \boldsymbol{H}^{\mathbf{i}} + \boldsymbol{H}^{\mathbf{v}} \tag{3.3}$$

The contents of the two vectors are displayed below:

$$\boldsymbol{H}^{\mathbf{i}} = \begin{pmatrix} \rho \boldsymbol{u} \\ \rho \boldsymbol{u} \boldsymbol{u} + p \\ \boldsymbol{u}(\rho E + p) \end{pmatrix} \tag{3.4}$$

$$\boldsymbol{H}^{\mathbf{v}} = -\frac{1}{Re} \begin{pmatrix} 0 \\ \boldsymbol{\tau} \\ \boldsymbol{\tau} \boldsymbol{u} + \boldsymbol{q} \end{pmatrix} \tag{3.5}$$

$\boldsymbol{H}^{\mathbf{i}}$ is called inviscid flux and contains only the variables that are independent of the fluids viscosity, it describes the way a fluid with zero viscosity would behave. In contrast, the viscous flux $\boldsymbol{H}^{\mathbf{v}}$ represents the effects of viscosity. The Reynolds number $Re = \frac{\rho v d}{\eta}$ is defined to be the ratio of inertia to tenacity, which makes it very valuable for understanding turbulent flows. This is also due to the fact that two familiar objects with the same Reynolds number behave similar in turbulence. One can assume that flows with $Re \ll 1$ are laminar and

flows with $Re >> 1$ are turbulent. To solve the Navier-Stokes-Equations, more information regarding some variables is required. For Calculating the specific inner Energy E and the heat conduction $\boldsymbol{q}$, the following equations are used:

$$E = e \frac{1}{2} |\boldsymbol{u}|^2 \tag{3.6}$$

$$\boldsymbol{q} = -\frac{\mu}{Pr(\gamma - 1)} \boldsymbol{\nabla} T \tag{3.7}$$

with

$$\gamma = \frac{c_p}{c_v} \tag{3.8}$$

and the Prandtl number

$$Pr = \frac{\mu_\infty c_p}{k_t} \tag{3.9}$$

using the specific heat capacities of the fluid c_v and c_p. If one could assume that the fluid is a newtonian fluid, the linear correlation between stress and the rate of strain results in:

$$\boldsymbol{\tau} = 2\mu \boldsymbol{S} - \frac{2}{3}\mu(\boldsymbol{\nabla} * \boldsymbol{u})\boldsymbol{I} \tag{3.10}$$

in which $\boldsymbol{S} = \frac{(\boldsymbol{\nabla u})(\boldsymbol{\nabla u})^T}{2}$ denotes the rate-of-strain-tensor. Additionally, the viscosity μ can be aprroximated through Sutherland's law, which is based on the ideal gas-theory:

$$\mu(T) = \mu_\infty (\frac{T}{t_\infty})^{3/2} \frac{T_\infty + S}{T + S} \tag{3.11}$$

S is in this case the Sutherland temperature. To achieve closure the caloric state equation $e = c_v T$ and the state eqaution for an ideal gas $p = \rho R T$ are used. The specific gas constant is determined by $R = c_p - c_v$. These equations form a set of partial differential equations, so for solving them starting values are needed. These are initialized at the first timestep of the simulation. To achieve physical solutions, 150 timesteps are computed before the particles are initialized, so the turbulence can evolve from the synthetic values to a natural flow field.

Christoph Siewert: -2.1 bis 2.6 Stephan Fritz: -Navier-Stokes-Gleichungen (Anhang B) Randbedingungen?

3.2 Particle dynamics

Siewert: -3.1a-3.14 (spherical particles) OHNE GRAVITATION Stokes Drag/Stokes Coefficient Filterung (Fritz) -¿Viskositaet durch numerischen Fehler, Smagorinksy nicht benutzen

4   Numerical methods

To simulate flows like those described above we have two options. The direct numerical simulation (DNS) is the easier one to understand, although it is numerically very expensive. The Large-eddy simulation (LES) is numerically more capable, still we must accept certain inaccuracies. These two numerical methods are now discussed in the following chapter.

4.1   Direct numerical simulation

The basis of the direct numerical simulation (DNS) are the Navier-Stokes equations as described above. The idea is that the computer is very good at calculating and solves these equations completely. This provides a very accurate result, as all scales of motion are being resolved. Still it requires an immense level of comptational resources which increases rapidly with the Reynolds number. These computational resources were not available until the 1970s [7]. ++zum Uebergang zu LES siehe Pope S. 352 und Fig. 9.4++

4.2   Large-eddy simulation

Due to the fact that DNS is effortful and wasting resources if a fully resolved resolution is not required, large-eddy simulation (LES) was created to save time and resources. This is especially efficient if mainly the temproal energy trend is considered, because the energy containing larger-scale motion is completely resolved and the indeed small effects of the expensive smaller-scale motion are just modelled. Otherwise in DNS resolving the small dissipative scale would require most of the computational resources.
Simulating only the larger-scale motions is also called filtering, which means that the smaller-scale motions are filtered out. To model the filtered smaller-scale motions ususally a subgrid-scale (SGS) model is used and this method is also called an implicit LES (ILES) [7].

4.3   Particle clustering

As we use the LES already

$$\mu(T) = \mu_\infty \left(\frac{T}{t_\infty}\right)^{3/2} \frac{T_\infty + S}{T + S} \tag{4.1}$$

Projektion (noComputationalParticles) implizite LES (Motivation fuer LES - Pope Chapter 9, Bild 9.4), DNS

5   Results

Graphen (particleFree rot, Laden gruen)

6 Conclusion

7 References

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