# Investigations on one-way coupling effects of particle-laden decaying isotropic turbulent flows

Julian Stemmermann, Steffen Trienekens, Christian Soika

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# 1 Nomenclature

## 2 Introduction

# 3 Mathematical models

Sollen wir hier noch isotrope Turbulenz etc. erklaeren?

## 3.1 Single-phase flow

In this section the mathematical basics for understanding and simulating turbulent flows are discussed. However, it should be pointed out that this is no complete treatise of the mathematical and physical basics. The reader can achieve further insight on this topic by looking at different books and papers, e.g. Pope (2000).

### 3.1.1 The Navier-Stokes equations

The Navier-Stokes-Equations are of great importance for understanding turbulent phenomena. This set of equations exists in forms for compressible and incompressible fluids. For an infinitesimal small volume element $d\boldsymbol{\tau}$ and using the cartesian coordinate system, they can be written in the so-called 'divergence form':

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \boldsymbol{\nabla H} = 0 \tag{1}$$

It should be noted by the reader that this work only contains investigations about chemically inert fluids and particles, and that the simulation results only fit under this condition. The vector $\boldsymbol{Q}$ contains all the variables which are conserved, i.e. the density $\rho$, the velocity $\boldsymbol{u}$ and the specific inner energy $E$:

$$\boldsymbol{Q} = \begin{pmatrix} \rho \\ \rho \boldsymbol{u} \\ \rho E \end{pmatrix} \tag{2}$$

$\boldsymbol{H}$ is the flux vector which stores all the floating variables and may be split up into two parts:

$$\boldsymbol{H} = \boldsymbol{H}^{\mathbf{i}} + \boldsymbol{H}^{\mathbf{v}} \tag{3}$$

The contents of the two vectors are displayed below:

$$\boldsymbol{H}^{\mathbf{i}} = \begin{pmatrix} \rho \boldsymbol{u} \\ \rho \boldsymbol{u} \boldsymbol{u} + p \\ \boldsymbol{u}(\rho E + p) \end{pmatrix} \tag{4}$$

$$\boldsymbol{H}^{\mathbf{v}} = -\frac{1}{Re} \begin{pmatrix} 0 \\ \boldsymbol{\tau} \\ \boldsymbol{\tau u} + \boldsymbol{q} \end{pmatrix} \tag{5}$$

$\boldsymbol{H}^{\mathbf{i}}$ is called inviscid flux and contains only the variables that are independent of the fluids viscosity, it describes the way a fluid with zero viscosity would behave. In contrast, the viscous flux $\boldsymbol{H}^{\mathbf{v}}$ represents the effects of viscosity. The Reynolds number $Re = \frac{\rho v d}{\eta}$ is defined to be the ratio of inertia to tenacity, which makes it very valuable for understanding turbulent flows. This is also due to the fact that two familiar objects with the same Reynolds number behave similar in turbulence. One can assume that flows with $Re << 1$ are laminar and flows with $Re >> 1$ are turbulent. To solve the Navier-Stokes-Equations, more information regarding some variables is required. For Calculating the specific inner Energy $E$ and the heat conduction $\boldsymbol{q}$, the following equations are used:

$$E = e\frac{1}{2}|\boldsymbol{u}|^2 \tag{6}$$

$$\boldsymbol{q} = -\frac{\mu}{Pr(\gamma - 1)}\boldsymbol{\nabla} T \tag{7}$$

with

$$\gamma = \frac{c_p}{c_v} \tag{8}$$

and the Prandtl number

$$Pr = \frac{\mu_\infty c_p}{k_t} \tag{9}$$

using the specific heat capacities of the fluid $c_v$ and $c_p$. If one could assume that the fluid is a newtonian fluid, the linear correlation between stress and the rate of strain results in:

$$\boldsymbol{\tau} = 2\mu\boldsymbol{S} - \frac{2}{3}\mu(\boldsymbol{\nabla} * \boldsymbol{u})\boldsymbol{I} \tag{10}$$

in which $\boldsymbol{S} = \frac{(\boldsymbol{\nabla u})(\boldsymbol{\nabla u})^T}{2}$ denotes the rate-of-strain-tensor. Additionally, the viscosity $\mu$ can be aprroximated through Sutherland's law, which is based on the ideal gas-theory:

$$\mu(T) = \mu_\infty(\frac{T}{t_\infty})^{3/2}\frac{T_\infty + S}{T + S} \tag{11}$$

S is in this case the Sutherland temperature. To achieve closure the caloric state equation $e = c_v T$ and the state eqaution for an ideal gas $p = \rho RT$ are used. The specific gas constant is determined by $R = c_p - c_v$. These equations form a set of partial differential equations, so for solving them starting values are needed. These are initialized at the first timestep of the simulation. To achieve physical solutions, 150 timesteps are computed before the particles are initialized, so the turbulence can evolve from the synthetic values to a natural flow field.
Christoph Siewert: -2.1 bis 2.6 Stephan Fritz: -Navier-Stokes-Gleichungen (Anhang B) Randbedingungen?

## 3.2 Particle dynamics

Siewert: -3.1a-3.14 (spherical particles) OHNE GRAVITATION Stokes Drag/Stokes Coefficient Filterung (Fritz) -¿Viskositaet durch numerischen Fehler, Smagorinksy nicht benutzen

# 4 Numerical methods

Pojektion (noComputationalParticles) implizite LES (Motivation fuer LES -¿ Pope Chapter 9, Bild 9.4), DNS

## 5 Results

Graphen (particleFree rot, Laden gruen)

## 6 Conclusion

# 7 Bibliography

## References

[1] S. Elghobashi. Particle-laden turbulent flows: direct simulation and closure models. *Applied Scientific Research*, 1991.

[2] S. Fritz. Simulation isotroper turbulenz, 2003.

[3] M. Meinke K. Fröhlich, L. Schneiders and W. Schröder. Validation of particle-laden large-eddy simulation using hpc systems. In *Sustained Simulation Performance 2017*, 2017. unpublished conference document.

[4] S. B. Pope. *Turbulent Flows*. Cambridge University Press, 2010.

[5] W. Schröder. Fluidmechanik, 2010.