

# Timing and Lensing of the Colliding Bullet Clusters: barely enough time and gravity to accelerate the bullet

HongSheng Zhao

University of St Andrews, Scottish University Physics Alliances, KY16 9SS, UK\*

We present semi-analytical constraint on the amount of dark matter in the merging bullet galaxy cluster using the classical Local Group timing arguments. We consider particle orbits in potential models which fit the lensing data. *Marginally consistent* CDM models in Newtonian gravity are found with a total mass  $M_{CDM} = 1 \times 10^{15} M_{\odot}$  of Cold DM: the bullet subhalo can move with  $V_{DM} = 3000 \text{ km s}^{-1}$ , and the "bullet" X-ray gas can move with  $V_{gas} = 4200 \text{ km s}^{-1}$ . These are nearly the maximum speeds that are accelerable by the gravity of two truncated CDM halos in a Hubble time even without the ram pressure. Consistency breaks down if one adopts higher end of the error bars for the bullet gas speed ( $5000 - 5400 \text{ km s}^{-1}$ ), and the bullet gas would not be bound by the sub-cluster halo for the Hubble time. Models with  $V_{DM} \sim 4500 \text{ km s}^{-1} \sim V_{gas}$  would invoke unrealistic large amount  $M_{CDM} = 7 \times 10^{15} M_{\odot}$  of CDM for a cluster containing only  $\sim 10^{14} M_{\odot}$  of gas. Our results are generalisable beyond General Relativity, e.g., a speed of  $4500 \text{ km s}^{-1}$  is easily obtained in the relativistic MONDian lensing model of Angus et al. (2007). However, MONDian model with hot dark matter  $M_{HDM} \leq 0.6 \times 10^{15} M_{\odot}$  and CDM model with a halo mass  $\leq 1 \times 10^{15} M_{\odot}$  are barely consistent with lensing and velocity data.

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## I. POTENTIAL FROM TIMING

Timing is a unique technique to establish the case for dark matter halos, first and most thoroughly explored in the context of the Local Group (Kahn & Woltjer 1959, Fich & Tremaine 1991, Peebles 1989, Inga & Saha 1998). In its simplest version the Local Group consists of the Milky Way and M31 as two isolated point masses, which formed close to each other, moved apart due to the Hubble expansion, and slowed down and moved towards each other upto their present velocity  $\sim 120 \text{ km s}^{-1}$  and separation (about 700 kpc) due to their mutual gravity. The age of the universe sets the upper limit on the period of this galaxy pair, hence the total mass of the pair through Kepler's 3rd law assuming Newtonian gravity.

Timing also finds a timely application in the pair of merging galaxy clusters 1E0657-56 at redshift  $z = 0.3$ , which is largely an extra-galactic grand analogy of the M31-MW system. The sub-cluster, called the "bullet", presently penetrates 400-700 kpc through the main cluster with an apparent speed of  $\sim 4750_{-550}^{+710} \text{ km s}^{-1}$  (Markevitch 2006). The X-ray gas of the bullet (amounts to  $2 \times 10^{13} M_{\odot}$ ) collides with the X-ray gas of the main cluster (with the total gas up to  $10^{14} M_{\odot}$ ) and forms a Mach-3 cone in front of the "bullet". The two clusters have at least four different centers, which are offset by 400 kpc between the pair of X-ray gas centers and by 700 kpc between the pair of star-light centers, which coincides with the gravitational lensing centers and (dark matter) potential centers (Clowe et al. 2006). The penetration speed is unusually high, hard for standard cosmology to explain statistically (Hayashi & White 2006), and modi-

fied force law has been suggested (Farrar & Rosen 2006, Angus et al. 2007).

The timing method applies in in MONDian gravity as well as Newtonian. Like lensing, timing is merely a method about constraining potential distribution, and is only indirectly related to the matter distribution. In this Letter we model the bullet clusters as a pair of mass concentrations formed at high redshift, and set constraint on their mutual force using the simple fact that their radial oscillation period must be close to the age of universe at  $z = 0.3$ . We check the consistency with the lensing signal of the cluster and give interpretations in terms of standard CDM and MOND.

First we can understand the speed of the bullet cluster analytically in simplified scenarios. Approximate the two clusters as points of fixed masses  $M_1$  and  $M_2$  on a head-on orbit, we can apply the usual MW-M31 timing argument. The total mass  $M_0 = M_1 + M_2$  is constant. The radial orbital period is computed from

$$T = 2 \int_0^{r_{max}} \frac{dr}{V(r)}, \quad (1)$$

$$= 2\pi \sqrt{\frac{r_{max}^3}{GM_0}}, \quad \text{Newtonian } p = 2 \quad (2)$$

$$= \frac{\sqrt{2\pi} r_{max}}{V_M}, \quad \text{deep-MONDian, } p = 1 \quad (3)$$

$$\propto K^{-n/2} r_{max}^{\frac{1+p}{2}}, \quad \text{for a } K/r^p \text{ gravity,} \quad (4)$$

where  $r_{max}$  is the apocenter and is related to the present relative velocity  $V(r)$  at separation  $r = 700 \text{ kpc}$  by energy

\*Electronic address: hz4@st-andrews.ac.uk

conservation

$$\frac{V(r)^2}{2} = -\frac{GM_0}{r_{max}} + \frac{GM_0}{r} \quad \text{Newtonian} \quad (5)$$

$$= V_M^2 (\ln r_{max} - \ln r) \quad \text{deep - MONDian} \quad (6)$$

$$\propto (r^{1-p} - r_{max}^{1-p}) K / (1-p) \quad \text{for a } K/r^p \text{ grav.} \quad (7)$$

where  $V_M = \sqrt{\xi}(GM_0 a_0)^{1/4}$  is the MOND circular velocity of two point masses,  $a_0$  equals one Angstrom per square second and is the MOND acceleration scale, and the dimensionless  $\xi \equiv \frac{2M_0^2}{3M_1 M_2} \left(1 - \left(\frac{M_1}{M_0}\right)^{3/2} - \left(\frac{M_2}{M_0}\right)^{3/2}\right) \sim 0.81 \sim 1$  (cf. Milgrom 1994, Zhao 2007, in preparation) for a typical mass ratio.

The predictions for simple Newtonian Keplerian gravity are given in Fig. 1; the more subtle case for a MONDian cluster is discussed in the final section. Setting the orbital period  $T = 10$  Gyrs, the age of the universe at the cluster redshift, yields presently  $V \sim 3200 \text{ km s}^{-1}$  in Newtonian for a normal combined mass of  $M_1 + M_2 = (0.7 - 1) \times 10^{15} M_\odot$  for the clusters, which is about 7-10 times their baryonic gas content ( $\sim 10^{14} M_\odot$ ) for Newtonian universe of  $\Omega = 0.3$  cold dark matter. In agreement with Farrar & Rosen and Hayashi & White, the simple timing argument suggests that dark halo velocities of  $4750 \text{ km s}^{-1}$ , as high as the "bullet" X-ray gas, would require halos with unrealistically larger masses of dark matter,  $\sim 10^{16} M_\odot$ , an order of magnitude more than what a universal baryon-dark ratio implies. As a sanity check, assuming a conventional  $3 \times 10^{12} M_\odot$  Local Group dark matter mass Fig.1 predicts the relative velocity of  $\sim 100 \text{ km/s}$  for the M31-MW system at separation 700 kpc after 14 Gyrs, consistent with observation (Binney & Tremaine 1987).

These analytical arguments, while straightforward, are not precise given its simplifying assumptions. For one, clusters do not form immediately at redshift infinity, and the cluster mass and size might grow with time gradually. More important is that point mass Newtonian halo models are far from fitting the weak lensing data of the 1E0657-56. A shallower Newtonian potential makes it even more difficult to accelerate the bullet. On the other hand, Angus, Shan, Zhao, Famaey (2007) show that there are MOND-inspired potentials that fits lensing. As commented in their conclusion, the same potential is deep enough that a  $V = 4750 \text{ km s}^{-1}$  "bullet" is bound in an orbit of apocenter  $r_{max}$  of a few Mpc, so the two clusters could be accelerated by mutual gravity from a zero velocity apocenter to  $4750 \text{ km/s}$  within the clusters' lifetime. This line of thought was further explored by the more systematic numerical study of Angus & McGaugh (2007).

Our paper is a spin-off of these works and the works of Hayashi & White and Farrar & Rosen. We emphasize the unification of the semi-analytical timing perspective and the lensing perspective, and aim to derive robust constraints to the potential, without being limited to a

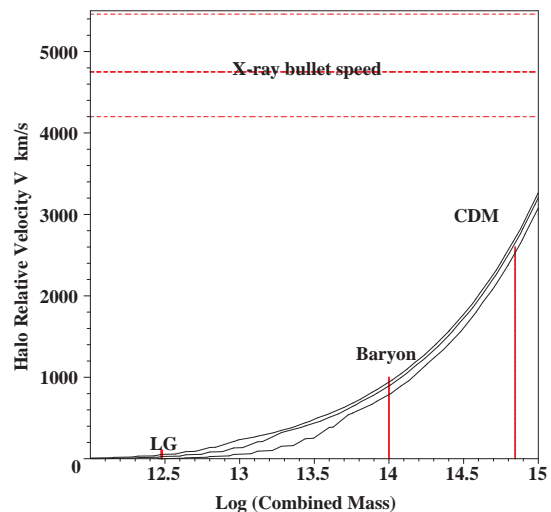


FIG. 1: Analytical timing-predicted dynamical mass vs. the relative speed of two objects separated by 700 kpc after  $10 \pm 4$  Gyrs (three lines in increasing order for increasing time) assuming Keplerian potential of point masses. Three vertical lines indicate typical Local Group Halo mass, Baryonic mass in galaxy clusters, and most massive CDM halo masses. Three horizontal lines indicate the error bar of the speed of the X-ray "bullet" gas.

specific gravity theory or dark matter candidate.

Towards the completion of this work, we are made aware by the preprint of Springel & Farrar (2007) that the unobserved bullet DM halo could be moving slower than its observed stripped X-ray gas. These authors, as well as the preprint of Milosovic et al. (2007), emphasized the effect of hydrodynamical pressure, which we will not be able to model realistically here. But to address the velocity differences, instead we treat the X-ray gas as a "bullet particle". We argue that *our hypothetical ballistic particle must move slow enough to be bound to vicinity of the subhalo before the collision, but moves somewhat faster than  $4700^{+700}_{-550} \text{ km s}^{-1}$  now, since it does not experience ram pressure of the gas.* This model follows the spirit of classical timing models of the separation of the Large and Small Magellanic Clouds and the Magellanic Stream (Lin & Lynden-Bell 1982).

## II. 3D POTENTIAL FROM LENSING

The weak lensing shear map of Clowe et al. (2006) has been fitted by Angus et al. (2007) using a four-component analytical potential each being spherical but on different centres. For our purpose we redistribute the minor components and simplify the potential into two components centred on the moving centroid of galaxy light of the main cluster with the present spatial coordinates  $r_1(t) = (-564, -176, 0) \text{ kpc}$  and subcluster galaxy centroid  $r_2(t) = (145, 0, 0) \text{ kpc}$ ; the coordinate origin is set at the present brightest point of the "bullet" X-ray

gas; presently the cluster is at  $z = 0.3$  or cosmic time  $t = 10\text{Gyrs}$ . We also apply a Keplerian truncation to the potential beyond the truncation radius  $r_t$ . So the following 3D potential is adopted for the cluster 1E0657-56 at time  $t$ ,

$$\Phi(X, Y, Z, t) = (1800 \text{ km s}^{-1})^2 \phi(|\mathbf{r} - \mathbf{r}_1|) + (1270 \text{ km s}^{-1})^2 \phi(|\mathbf{r} - \mathbf{r}_2|), \quad (8)$$

$$\begin{aligned} \phi(|\mathbf{r} - \mathbf{r}_i(t)|) &= \ln \sqrt{1 + \left( \frac{|r - r_i(t)|}{180 \text{ kpc}} \right)^2} + cst, \quad r < r_t \\ &= -\frac{\tilde{r}_t}{|\mathbf{r} - \mathbf{r}_i(t)|}, \quad r \geq r_t(t) = C \times t \end{aligned} \quad (9)$$

where  $\tilde{r}_t \equiv \frac{r_t^3}{r_t^2 + 180^2}$  is to ensure a continuous and smooth transition of the potential across the truncation radius  $r_t$ . The truncation  $r_t$  evolves with time, since a pre-cluster region collapses gradually after the big bang, and its boundary and total mass grows with time till it reaches the size of a cluster. In the interests of simplicity rather than rigour, we use a linear model  $r_t = C \times t$ , where  $C$  is a constant of the unit kpc/Gyr.

To check that the simplified potential is still consistent with weak lensing data, we recompute the 3D weak lensing convergence (Taylor et al. 2004) for sources at distance  $D(0, z_s)$  at the redshift  $z_s$ ,

$$\kappa(X, Y, z_s) = \sum_{i=X,Y} \frac{\partial_i}{2} \left[ \int_0^{D(0,z_s)} \frac{2D(z, z_s)}{c^2} (\partial_i \Phi) dZ \right] \quad (11)$$

where the integrations in square brackets are the deflection angles for a source at  $z_s$ , and the usual lensing effective distance is related to the comoving distances by  $D(z, z_s) = (1+z)^{-1} \tilde{D}(z) \left[ 1 - \frac{\tilde{D}(z)}{\tilde{D}(z_s)} \right] = 587 \text{ Mpc}$  is for the bullet cluster  $z = 0.3$  lensing sources at  $z_s = 1$ ; the distance increases by a factor 1.3 to 1.6 for source redshifts of 3 to infinity. Fig.2 shows the predicted  $\kappa$  along the line joining the two dark centers; the result is insensitive to the cluster truncation radius as long as  $r_t \geq 1000 \text{ kpc}$  presently. The lensing model predicts a signal in between that of the weak lensing data of Clowe et al., and strong lensing data of Bradac et al. It is known that these two data sets are somewhat discrepant to each other. So the fit here is reasonable. The method of deprojection is essentially similar to the decomposition method of Bradac et al. whose explicit assumption of Einsteinian gravity is however unnecessary.

The important thing here is that as far as deprojecting the above potential is concerned, no assumption is needed on the gravity theory as long as light rays follow geodesics, a feature built in most alternative gravity theory. Similarly orbits of massive particles are also (different) geodesics in these theories. The meaning of potential in such theories is that the potential (scaled by a factor  $2/c^2$ ) represents metric perturbations to the flat space-time, especially to the  $g_{00}(cdt)^2 = -(1 + \frac{2\Phi}{c^2})(cdt)^2$  term, so the Christoffel  $\Gamma_{00}^i \sim \frac{\partial}{\partial X^i} \Phi$ , it can be shown

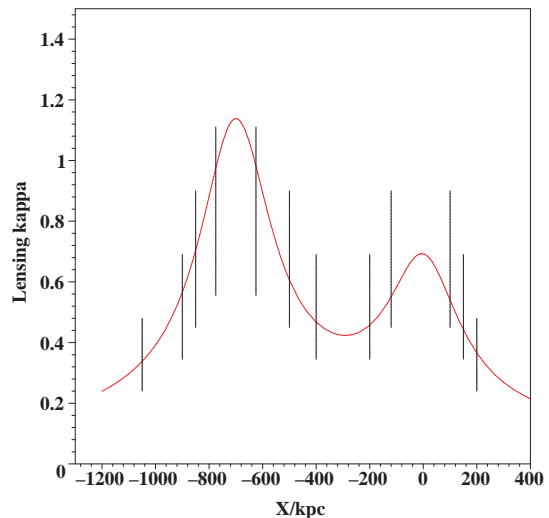


FIG. 2: Predicted bullet cluster convergence (rescaled for sources at infinity) along the line  $Y = 0.3X + cst$  connecting our two potential centroids. The model predicts a lensing signal in between that of observed weak lensing data from sources at  $z_s = 1$  (Clowe et al, lower end of error bars) and the united weak lensing and strong lensing ( $z_s = 3$ ) data (Bradac et al. upper part of error bars); the mismatch of these two datasets are presently unresolved.

that the geodesic equations have the same form as Einsteinian in the weak-field limit:  $\frac{d^2}{dt^2} \mathbf{R} \approx -(1 + \frac{v^2}{c^2}) \nabla \mathbf{R} \Phi$ , where  $\mathbf{R}$  is the pair of spatial coordinates perpendicular to the instantaneous velocity  $v$ ; the paths of light rays are deflected twice as much by the metric perturbation  $2\Phi/c^2$  as those of low-speed particles.

### III. ORBITS OF THE COLLIDING CLUSTERS

We now use this potential to predict the relative speed of the two clusters. This is possible using the classical timing argument, in the style of Kahn & Wolter (1959), Fich & Tremaine (1991) and Voltonen et al. (1998); we postpone most rigorous least action models (Peebles 1989, Schmoldt & Saha 1998) for later investigations since these require modeling a cosmological constant and other mass concentrations along the orbital path of the bullet clusters, which have technical issues in non-Newtonian gravity. We trace the orbits of the two centroids of the potentials according to the equation of motion  $\frac{d^2 \mathbf{r}_i}{dt^2} = -\nabla \Phi(\mathbf{r}_i)$ . We assign different relative velocities presently (at  $z = 0.3$ ), and integrate backward in time and require the two centroids of the potential be close together at a time 10 Gyrs ago. The motions are primarily in the sky plane, but we allow for 600 km/s relative velocity component in the line of sight. Clearly at earlier times when  $t$  is small, the two centroids are well-separated compared to their sizes, so they move in the growing Keplerian potential of each other. At later times the centroids came close and move in the cored

isothermal potential.

We shall consider models with a normal truncation  $r_t = C \times t = 1000 \text{ kpc}$  at time  $t = 10 \text{ Gyrs}$ . We also consider models with a very large truncation  $C \times t = 10000 \text{ kpc}$ . In the language of CDM, the truncation means the virial radius of the halo. The present instantaneous escape speed of the model can be computed by  $V_{esc} = \sqrt{-2\Phi(X, Y, Z, t)}$ . We find  $V_{esc} \sim 4200 - 4500 \text{ km s}^{-1}$  in the central region of the shallower potential model with a present truncation  $1000 \text{ kpc}$ . The escape speed increases to  $V_{esc} \sim 5700 \text{ km s}^{-1}$  for models with a present truncation  $10000 \text{ kpc}$ .

Fig. 3 shows the predicted orbits for different present relative velocities  $V_{DM} = |\frac{d\mathbf{r}_2}{dt} - \frac{d\mathbf{r}_1}{dt}|$ . Among models with a normal truncation, we find  $V_{DM} \sim 2950 \text{ km s}^{-1}$ ; a model with relative velocity  $V_{DM} < 2800 \text{ km s}^{-1}$  would predict an unphysical orbital crossing at high redshift, while models with  $V_{DM} > 3000 \text{ km s}^{-1}$  would predict that the two potential centroids were never close at high redshift.

Larger halo velocities are only possible in models with very large truncation. If the relative velocity is  $4200 \text{ km s}^{-1} < V_{DM} < 4750 \text{ km s}^{-1}$  between two cluster gravity centroids, then the truncation must be as big as  $10 \text{ Mpc}$  at  $z = 0.3$ .

We also track the orbit of the bullet X-ray gas centroid as a tracer particle in the above bi-centric potential. We look for orbits where the bullet X-ray gas will always be bound to one member of the binary system since the ram pressure in a hydrodynamical collision is unlikely to be so efficient to eject the X-ray gas out of potential wells of both the main and sub-clusters. This means that the bullet speed must not exceed greatly the present instantaneous escape speed of the model, which is  $\sim 4200 - 4500 \text{ km s}^{-1}$  in the central region of the shallow potential of a model with a present truncation  $1000 \text{ kpc}$ . The escape speed increases to  $\sim 5700 \text{ km s}^{-1}$  for models with a present truncation  $10000 \text{ kpc}$ . The model with normal truncation is *marginally consistent* with the observed gas speed  $V_{gas} \sim 4750^{+710}_{-550} \text{ km s}^{-1}$ . The problem would become more severe if the potential were made shallower by an even smaller truncation. The gas speed is less an issue in models with larger truncation.

In short the present velocity and lensing data are easier explained with potential models of very large truncation. Models with normal truncation have smaller gravitational power, can only accelerate the subhalo to  $3000 \text{ km s}^{-1}$  in  $10 \text{ Gyrs}$ . Models with normal CDM truncation can only accelerate the bullet X-ray gas cloud to  $\sim 4200 - 4400 \text{ km s}^{-1}$ , the escape speed, *marginally consistent* with observations.

Above simulation results are sensitive to the present cluster separation, but insensitive to the present direction of the velocity vector. Unmodeled effects such as dynamical friction associated with a live halo will reduce the predicted  $V_{DM}$  for the same potential, but the effect is mild since the actual collision is brief  $\sim 0.1 - 0.3 \text{ Gyrs}$  and the factor  $\exp(-M^2/2)$  in Chandrasekhar's formulae

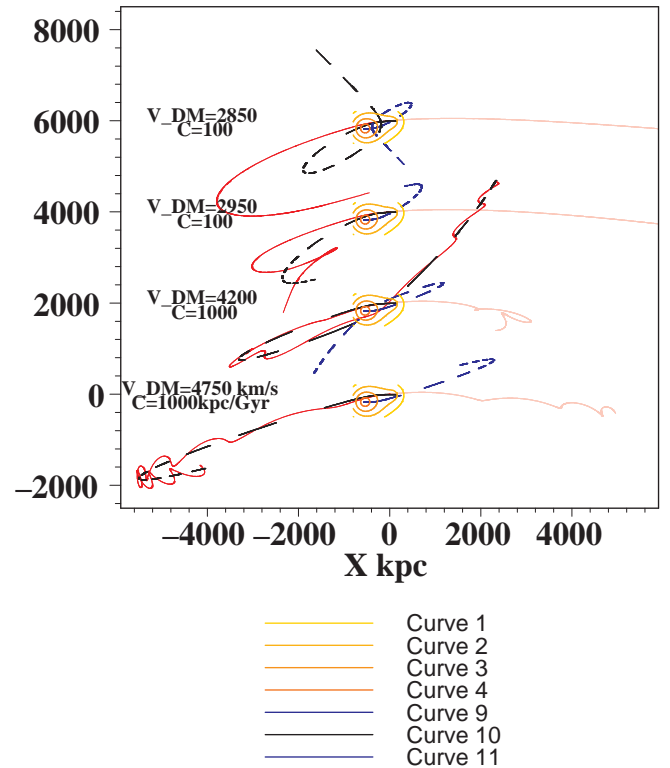


FIG. 3: The orbit of the bullet subcluster X-ray gas (red, with present  $V_{gas} = 5400 \text{ km s}^{-1}$  for the  $10 \text{ Gyrs}$  in the past, and pink: for the future  $4 \text{ Gyrs}$ ), and the orbits of the colliding main cluster halo (blue dashes) and subhalo (black dashes) in the potential (eqs. 8-10) determined by lensing data; dashes indicate length traveled in  $0.5 \text{ Gyrs}$  steps. No explicit assumption of gravity is needed for these calculations. Orbits with different present halo relative velocity  $V_{DM}$  and halo growth rate  $C$  are shown after a vertical shift for clarity. Timing requires the present cluster relative velocity in between  $2800 \text{ km s}^{-1} < V_{DM} < 3000 \text{ km s}^{-1}$  for potentials of normal truncation (lowest panels where the cluster truncation grows from zero to  $C \times 10 \text{ Gyr} = 1000 \text{ kpc}$ ), and  $4200 \text{ km s}^{-1} < V_{DM} < 4750 \text{ km s}^{-1}$  for potentials with large truncation (two upper panels where the cluster truncation grows from zero to  $C \times 10 \text{ Gyr} = 10000 \text{ kpc}$ ).

sharply reduces dynamical friction for a supersonic body, where  $M \sim 2 - 3$  is the Mach number for the bullet.

#### IV. NEWTONIAN AND MONDIAN MEANINGS OF THE POTENTIAL MODEL

Assuming Newtonian gravity the models with normal truncation  $r_t = 1 \text{ Mpc}$  at  $t = 10 \text{ Gyrs}$  correspond to cluster (dark) masses of  $M_1 = 0.745 \times 10^{15} M_\odot$  and  $M_2 = 0.345 \times 10^{15} M_\odot$ ; the larger truncation  $r_t = 10 \text{ Mpc}$  corresponds to  $M_1 = 7.45 \times 10^{15} M_\odot$  and  $M_2 = 3.45 \times 10^{15} M_\odot$  in Newtonian. All these models fit lensing.

Interpreted in the MONDian gravity, the truncation is due to external field effect and cosmic background

so to make the MOND potential finite hence escapable (Famaey, Bruneton, Zhao 2007). Beyond the truncation radius, MOND potential becomes nearly Keplerian. The MONDian models, insensitive to truncation, would have masses only  $M_1 = 0.66 \times 10^{15} M_\odot$  and  $M_2 = 0.16 \times 10^{15} M_\odot$ . These masses are still higher than their baryonic content  $\sim 10^{14} M_\odot$ , implying the need for, e.g., massive neutrinos; the neutrino density is too low in galaxies to affect normal MONDian fits to galaxy rotation curves, but is high enough to bend light and orbits significantly on 1Mpc scale. The neutrino-to-baryon ratio, approximately 7:1 in the bullet cluster, would be a reasonable assumption for a MONDian universe with  $\Omega_b \sim 0.04$  plus 2eV neutrinos hot dark matter  $\Omega_{HDM} \sim 0.25 \sim 7 \times \Omega_b$  (Sanders 2003, Pointecoute & Silk 2005, Skordis et al. 2006, Angus et al. 2007). The amount of hot dark matter inferred here is the same as Angus et al. (2007) since their potential parameters are fixed by the same lensing data.

## V. CONCLUSION

In short a consistent set of simple lensing and dynamical model of the bullet cluster is found. The present

relative speeds between galaxies of the two clusters is predicted to be  $V_{DM} \sim 2900 \text{ km s}^{-1}$  in CDM and  $V_{DM} \sim 4500 \text{ km s}^{-1}$  in  $\mu\text{HDM}$  (MOND + Hot Dark Matter) if the two clusters were born close to each other 10 Gyrs ago; both models assume close to universal gas-DM ratio in clusters, i.e., about  $(0.6 - 1) \times 10^{15} M_\odot$  Hot or Cold DM. Modeling the bullet X-ray gas as ballistic particle, we find the gas particle with speed of  $V_{gas} = 4200 \text{ km/s}$  (at the lower end of observed speed) is bound to the potential of the subcluster for most part of the Hubble time for both above models, insensitive to the preference of the law of gravity. But if future relative proper motion measurements of the subcluster galaxy speed is as high as  $V_{DM} = 4500 \text{ km/s}$ , or the gas speed is as high as  $V_{gas} \sim 5400 \text{ km s}^{-1}$ , then Newtonian models would need to invoke unlikely  $7 \times 10^{15} M_\odot$  DM halos around  $10^{14} M_\odot$  gas.

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