Doubly charged vector tetraquark $Z_{\mathrm{V}}^{++}=[cu][\overline{s}\overline{d}]$

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We explore properties of the doubly charged vector tetraquark $Z_{\mathrm{V}}^{++}=[cu][\overline{s}\overline{d}]$ built of four quarks of different flavors using the QCD sum rule methods. The mass and current coupling of Z_{V}^{++} are computed in the framework of the QCD two-point sum rule approach by taking into account quark, gluon and mixed vacuum condensates up to dimension 10. The full width of this tetraquark is saturated by S-wave $Z_{\mathrm{V}}^{++}\to\pi^{+}D_{s1}(2460)^{+}$, $\rho^{+}D_{s0}^{*}(2317)^{+}$, and P-wave $Z_{\mathrm{V}}^{++}\to\pi^{+}D_{s}^{+}$, $K^{+}D^{+}$ decays. Strong couplings required to find partial widths of aforementioned decays are calculated in the context of the QCD light-cone sum rule method and a soft-meson approximation. Our predictions for the mass $m=(3340\pm 120)$ MeV and full width $\Gamma_{\mathrm{full}}=141_{-20}^{+30}$ MeV of Z_{V}^{++} are useful to search for this exotic meson in various processes. Recently, the LHCb collaboration discovered neutral states $X_{0(1)}(2900)$ as resonance-like peaks in $D^{-}K^{+}$ invariant mass distribution in the decay $B^{+}\to D^{+}D^{-}K^{+}$. We argue that mass distribution of $D^{+}K^{+}$ mesons in the same B decay can be used to observe the doubly charged scalar Z_{S}^{++} and vector Z_{V}^{++} tetraquarks.

I. INTRODUCTION

Exotic mesons containing four quarks of different flavors are interesting members in the family of XYZ mesons. Analyses of such structures were triggered by information of the $D0$ collaboration about evidence for the state $X(5568)$ composed presumably of four different quarks [1]. Later, other collaborations could not confirm existence of this state [2, 3], but knowledge gained due to theoretical studies of $X(5568)$, methods and schemes elaborated during this process, played an important role in our understanding of exotic mesons.

New stage in this field began recently by discovery of the LHCb collaboration, which reported about new resonance-like structures $X_{0(1)}(2900)$ in the process $B^{+}\to D^{+}D^{-}K^{+}$ [4, 5]. They were seen in the mass distribution of mesons $D^{-}K^{+}$, and may be considered as first strong evidence for exotic mesons composed of four quarks of different flavors. Indeed, from decay modes of these states, it is clear that they contain valence quarks $ud\overline{c}\overline{s}$, which makes them objects of intensive studies. But one should bear in mind that $X_{0(1)}(2900)$ may have alternative origin, and appear due to rescattering diagrams $\chi_{c1}D^{*-}K^{*+}$ and $D_{sJ}\overline{D}_{1}^{0}K^{0}$ in the decay $B^{+}\to D^{+}D^{-}K^{+}$ [6].

Masses and spin-parities of these structures were determined by LHCb and used in various models to explain internal organizations of $X_{0(1)}(2900)$. As usual, they were interpreted as hadronic molecules, diquark-antidiquark states, and rescattering effects (see, Refs. [7, 8] and references therein). In our articles [7, 8], we treated $X_{0}(2900)$ and $X_{1}(2900)$ as a scalar hadronic molecule $\overline{D}^{*0}K^{*0}$ and diquark-antidiquark vector states $[ud][\overline{c}\overline{s}]$, respectively. Predictions for their masses and widths extracted from sum rule analyses seem confirm assumptions made on their structures.

It is clear that resonances $X_{0(1)}(2900)$ are neutral states, and emerge in intermediate phase of the decay chain $B^{+}\to D^{+}X\to D^{+}D^{-}K^{+}$. In accordance with Ref. [9], these processes occur due to color-favored and color-suppressed transformations. But weak decays of B^{+} meson with the same topologies may trigger also processes $B^{+}\to D^{-}Z^{++}\to D^{-}D^{+}K^{+}$. In this chain an intermediate particle Z^{++} is an exotic meson with quark content $[uc][\overline{s}\overline{d}]$ carrying two units of electric charge. In our view, B meson weak decays which are already under detailed analysis, and collected experimental data may be used to uncover doubly charged tetraquarks $Z^{++}=[uc][\overline{s}\overline{d}]$ with different spin-parities. These new scalar and vector particles may appear as resonances in the $D^{+}K^{+}$ mass distribution: this fact is a main motivation for present studies.

Doubly charged diquark-antidiquarks already attracted interests of researches, and some of these states was studied in a rather detailed form. Thus, spectroscopic parameters and strong decays of pseudoscalar tetraquarks $cc\overline{s}\overline{s}$ and $cc\overline{d}\overline{s}$ were calculated in Ref. [10]. Doubly charged scalar, pseudoscalar and axial-vector states $Z_{\overline{c}s}=[sd][\overline{u}\overline{c}]$ were considered in our article [11]. Results obtained for their masses and widths read

$$
\begin{aligned}
m_{\mathrm{Z_S}} &= 2628_{-153}^{+166}\ \mathrm{MeV},\ \Gamma_{\mathrm{Z_S}}=(66.89\pm 15.11)\ \mathrm{MeV},\\
m_{\mathrm{Z_{PS}}} &= 2719_{-156}^{+144}\ \mathrm{MeV},\ \Gamma_{\mathrm{Z_{PS}}}=(38.1\pm 7.1)\ \mathrm{MeV},\\
m_{\mathrm{Z_{AV}}} &= 2826_{-157}^{+134}\ \mathrm{MeV},\ \Gamma_{\mathrm{Z_{AV}}}=(47.3\pm 11.1)\ \mathrm{MeV},
\end{aligned}
\qquad (1)
$$

where subscripts Z_{S}, Z_{PS} and Z_{AV} refer to the scalar, pseudoscalar and axial-vector tetraquarks $Z_{\overline{c}s}$, respectively.

The positively charged counterparts of $Z_{\overline{c}s}$ are tetraquarks Z^{++} with the same characteristics and spin-parities. Therefore, one can safely use parameters of particles $Z_{\overline{c}s}$ to study exotic mesons Z^{++}. It is not difficult to see that scalar and vector states $Z_{\rm S}^{++}$ and $Z_{\rm V}^{++}$ decay to ordinary mesons $D_s^+\pi^+$ and D^+K^+. For $Z_{\rm S}^{++}$ these modes are S-wave decays, whereas for $Z_{\rm V}^{++}$ they are P-wave processes. In the present work, we are going to find parameters of the vector tetraquark $Z_{\rm V}^{++}$. The mass of $Z_{\rm V}^{++}$ should be around or larger than $m_{\rm Z_{AV}}$. Therefore, even without detailed analysis one sees that decays to final mesons $D_s^+\pi^+$ and D^+K^+ are among kinematically allowed modes of $Z_{\rm V}^{++}$. In other words, the states $Z_{\rm S}^{++}$ and $Z_{\rm V}^{++}$ may be seen in D^+K^+ mass distribution in the decay $B^+ \to D^-D^+K^+$.

We compute spectroscopic parameters and full width of the vector tetraquark $Z_{\rm V}^{++}$ using various versions of the QCD sum rule method. Thus, the mass and current coupling of $Z_{\rm V}^{++}$ are found by employing the QCD two-point sum rule approach [12, 13]. In our analysis, we take into account contributions of various vacuum condensates up to dimension 10. Prediction for the mass of $Z_{\rm V}^{++}$, as well as its quantum numbers $J^{\rm P}=1^-$ permit us to classify kinematically allowed decay modes of this tetraquark. The mass and coupling of $Z_{\rm V}^{++}$ are also input parameters necessary to calculate partial widths of the decays $Z_{\rm V}^{++} \to D_{s1}(2460)^+\pi^+$, $D_{s0}^*(2317)^+\rho^+$, D^+K^+, and $D_s^+\pi^+$. To this end, we explore vertices of $Z_{\rm V}^{++}$ with ordinary mesons (for example, $Z_{\rm V}^{++}D^+K^+$), and find corresponding strong couplings. Relevant investigations are carried out using the QCD light-cone sum rule (LCSR) method, which is one of effective nonperturbative tools to study conventional hadrons [14]. In the case of tetraquark-ordinary meson vertices the standard methods of LCSR have to be applied in conjunction with a soft-meson approximation [15, 16]. For analysis of the exotic mesons the light-cone sum rule method and soft approximation was suggested in Ref. [17], and used to investigate decays of various tetraquarks [18].

This paper is structured in the following manner: Section II is devoted to calculations of the mass and coupling of the vector tetraquark $Z_{\rm V}^{++}=[uc][\overline{s}\overline{d}]$. In Section III, we compute strong couplings in relevant tetraquark-meson vertices, and evaluate partial widths of $Z_{\rm V}^{++}$ decays. The full width of $Z_{\rm V}^{++}$ is found in this section as well. Section IV is reserved for our conclusions and final notes.

II. THE SPECTROSCOPIC PARAMETERS OF $Z_{\rm V}$

The mass m and current coupling f are important parameters of the vector tetraquark $Z_{\rm V}^{++}=[uc][\overline{s}\overline{d}]$ (in what follows, we omit superscripts denoting charges of the tetraquark and various mesons). We use the QCD two-point sum rule method to evaluate values of these parameters.

We begin calculations from analysis of the two-point correlation function

$$\Pi_{\mu\nu}(p)=i\int d^4xe^{ipx}\langle 0|\mathcal{T}\{J_\mu(x)J_\nu^\dagger(0)\}|0\rangle, \qquad (2)$$

where $J_\mu(x)$ is the interpolating current for $Z_{\rm V}$, and $\mathcal{T}$ denotes the time-ordered product of two currents. The vector tetraquark $Z_{\rm V}$ can be modeled using a scalar diquark $uC\gamma_5 c$ and vector antidiquark $\overline{s}\gamma_\mu\gamma_5 C\overline{d}$. It is known that color-antitriplet diquarks and color-triplet antidiquarks are most stable two-quark structures [19]. Therefore, we chose as building blokes of $Z_{\rm V}$ structures $\varepsilon^{abc}u_bC\gamma_5c_c$ and $\varepsilon^{amn}\overline{s}_m\gamma_\mu\gamma_5C\overline{d}_n^T$, which belongs to $[\overline{\mathbf{3}}_c]$ and $[\mathbf{3}_c]$ representations of the color group $SU_c(3)$, respectively. Then, the colorless interpolating current $J_\mu(x)$ takes the form

$$J_\mu(x)=\varepsilon\widetilde{\varepsilon}[u_b^T(x)C\gamma_5c_c(x)][\overline{s}_m(x)\gamma_\mu\gamma_5C\overline{d}_n^T(x)], \qquad (3)$$

where $\varepsilon\widetilde{\varepsilon}=\varepsilon^{abc}\varepsilon^{amn}$, and a, b, c, m and n denote quark colors. In Eq. (3) $u(x)$, $c(x)$, $s(x)$ and $d(x)$ are the quark fields, and C stands for the charge-conjugation operator.

The sum rules for m and f can be derived by calculating $\Pi_{\mu\nu}(p)$ in terms of the physical parameters of $Z_{\rm V}$, as well as in the operator product expansion (OPE) with certain accuracy. Having equated these two expressions, applied the Borel transformation to suppress contributions of higher resonances and continuum states, and subtracted these contributions using the quark-hadron duality assumption [12, 13], it is possible to get required sum rules for the mass and coupling of the tetraquark $Z_{\rm V}$.

Expression of $\Pi_{\mu\nu}(p)$ in terms of parameters of $Z_{\rm V}$ is obtained by saturating the correlation function $\Pi_{\mu\nu}(p)$ with a complete set of $J^{\rm P}=1^-$ states and performing in Eq. (2) integration over x

$$\Pi_{\mu\nu}^{\rm Phys}(p)=\frac{\langle 0|J_\mu|Z_{\rm V}\rangle\langle Z_{\rm V}|J_\nu^\dagger|0\rangle}{m^2-p^2}+\cdots, \qquad (4)$$

where the dots stand for contributions of higher resonances and continuum states.

To proceed, it is convenient to introduce the matrix element

$$\langle 0|J_\mu|Z_{\rm V}\rangle=fm\epsilon_\mu, \qquad (5)$$

where ϵ_μ is the polarization vector of the tetraquark $Z_{\rm V}$. Then $\Pi_{\mu\nu}^{\rm Phys}(p)$ can be rewritten in the following form

$$\Pi_{\mu\nu}^{\rm Phys}(p)=\frac{m^2f^2}{m^2-p^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m^2}\right)+\cdots. \qquad (6)$$

The two-component Lorentz structure in the parentheses in Eq. (6) corresponds to a vector particle. The first component of this structure is proportional to $g_{\mu\nu}$ and does not contain effects of scalar particles: It is formed due to only vector states. Therefore, in our studies we utilize the structure $\sim g_{\mu\nu}$, and label corresponding invariant amplitude by $\Pi^{\rm Phys}(p^2)$.

Expression in Eq. (4) is obtained in the zero-width single-pole approximation. In general, the correlation function $\Pi_{\mu\nu}(p)$ receives contributions also from two-meson reducible terms, because the current J_μ couples not only to four-quark structures, but also to two mesons with relevant quantum numbers. Two-meson effects generate a finite width of the tetraquark $Z_{\rm V}$, and modify the quark propagator in Eq. (4). It was demonstrated that these effects do not change prediction for the mass of $Z_{\rm V}$. They rescale only the current coupling f, which is small and can be neglected [10, 20, 21].

The QCD side of the sum rules $\Pi_{\mu\nu}^{\rm OPE}(p)$ is found by inserting the interpolating current $J_\mu(x)$ into Eq. (2), and contracting relevant quark fields

$$\Pi_{\mu\nu}^{\rm OPE}(p) = i\int d^4x e^{ipx}\varepsilon\widetilde{\varepsilon}\varepsilon'\widetilde{\varepsilon}'{\rm Tr}\left[\gamma_5\widetilde{S}_u^{bb'}(x)\gamma_5 S_c^{cc'}(x)\right] \times{\rm Tr}\left[\gamma_\mu\gamma_5\widetilde{S}_d^{n'n}(-x)\gamma_5\gamma_\nu S_s^{m'm}(-x)\right], \tag{7}$$

where

$$\widetilde{S}_{c(q)}(x) = CS_{c(q)}^T(x)C, \tag{8}$$

with $S_c(x)$ and $S_{u(s,d)}(x)$ being the heavy c- and light $u(s,d)$-quark propagators, respectively (for explicit expressions, see Ref. [18]). The invariant amplitude in $\Pi_{\mu\nu}^{\rm OPE}(p)$ corresponding to the structure $g_{\mu\nu}$ in our following analysis will be denoted by $\Pi^{\rm OPE}(p^2)$.

Calculations performed in accordance with a scheme briefly explained above yield

$$m^2 = \frac{\Pi'(M^2,s_0)}{\Pi(M^2,s_0)}, \tag{9}$$

and

$$f^2 = \frac{e^{m^2/M^2}}{m^2}\Pi(M^2,s_0), \tag{10}$$

which are sum rules for m and f, respectively. Here, $\Pi(M^2,s_0)$ is the Borel transformed and subtracted amplitude $\Pi^{\rm OPE}(p^2)$, which depends on the Borel M^2 and continuum threshold s_0 parameters. In Eq. (9) $\Pi'(M^2,s_0)$ is the derivative of the amplitude over $d/d(-1/M^2)$.

It is clear that the amplitude $\Pi(M^2,s_0)$ is a key ingredient of the obtained sum rules. Calculations of this function give the following result

$$\Pi(M^2,s_0) = \int_{\mathcal{M}^2}^{s_0} ds\rho^{\rm OPE}(s)e^{-s/M^2} + \Pi(M^2), \tag{11}$$

where $\mathcal{M} = m_c + m_s$. Computations are carried out by taking into account vacuum condensates up to dimension 10. The amplitude $\Pi(M^2,s_0)$ has two components: First of them is expressed using the spectral density $\rho^{\rm OPE}(s)$, which is computed as an imaginary part of $\Pi_{\mu\nu}^{\rm OPE}(p)$. The Borel transformation of another terms are found directly from $\Pi_{\mu\nu}^{\rm OPE}(p)$ and included into $\Pi(M^2)$. The first component in Eq. (11) contains a main part of the amplitude, whereas $\Pi(M^2)$ is formed from higher dimensional terms. Analytical expressions of $\rho^{\rm OPE}(s)$ and $\Pi(M^2)$, are rather lengthy, therefore we do not write down them here.

The sum rules depend on vacuum condensates up to dimension 10. The basic condensates

$$\begin{aligned}
&\langle\overline{q}q\rangle = -(0.24\pm 0.01)^3~{\rm GeV}^3,~\langle\overline{s}s\rangle = (0.8\pm 0.1)\langle\overline{q}q\rangle,\\
&\langle\overline{q}g_s\sigma Gq\rangle = m_0^2\langle\overline{q}q\rangle,~\langle\overline{s}g_s\sigma Gs\rangle = m_0^2\langle\overline{s}s\rangle,\\
&m_0^2 = (0.8\pm 0.2)~{\rm GeV}^2\\
&\langle\frac{\alpha_s G^2}{\pi}\rangle = (0.012\pm 0.004)~{\rm GeV}^4,\\
&\langle g_s^3G^3\rangle = (0.57\pm 0.29)~{\rm GeV}^6.
\end{aligned} \tag{12}$$

were estimated from analysis of numerous hadronic processes [12, 13, 22, 23], and are known and universal parameters. Higher dimensional condensates are factorized and expressed in terms of basic ones: we assume that factorization does not lead to large uncertainties. The masses of s and c quarks are also among input parameters, for which we use $m_s = 93^{+11}_{-5}$ MeV, and $m_c = 1.27\pm 0.2$ GeV, respectively.

The Borel and continuum threshold parameters M^2 and s_0 are auxiliary parameters of computations and their choice should meet usual requirements imposed on $\Pi(M^2,s_0)$ by the pole contribution (PC) and convergence of the operator product expansion. Technical side of such analysis was explained numerously in the literature (see, for instance, Refs. [8, 17]), for this reason we skip here further details. Computations of $\Pi(M^2,s_0)$ demonstrate that working regions for the parameters M^2 and s_0

$$M^2\in[4,6]~{\rm GeV}^2,~s_0\in[13,14]~{\rm GeV}^2. \tag{13}$$

satisfy these constraints. In fact, in these regions PC changes within limits

$$0.78\leq {\rm PC}\leq 0.26. \tag{14}$$

At the minimum $M^2 = 4~{\rm GeV}^2$, a contribution to $\Pi(M^2,s_0)$ arising from a sum of last three terms in OPE does not exceed 1% of the full result, which proves convergence of the sum rules. Central values of the mass m and coupling f are evaluated at the middle point of regions (13), i.e., at $M^2 = 5~{\rm GeV}^2$ and $s_0 = 13.5~{\rm GeV}^2$. At this point the pole contribution is ${\rm PC}\approx 0.53$, which ensures the ground-state nature of $Z_{\rm V}$.

Results for m and f read

$$\begin{aligned}
m &= (3340~\pm 120)~{\rm MeV},\\
f &= (3.7\pm 0.5)\times 10^{-3}~{\rm GeV}^4.
\end{aligned} \tag{15}$$

The mass and coupling of the tetraquark $Z_{\rm V}$ are depicted in Figs. 1 and 2 as functions of M^2 and s_0. Because M^2 is the auxiliary parameter, prediction for m and f should be stable under its variations. But in a real situation there is residual dependence of m and f on the Borel parameter. In our case, we observe a mild variation of the tetraquark's mass on M^2, whereas the coupling f is almost stable in the explored range of the Borel parameter.

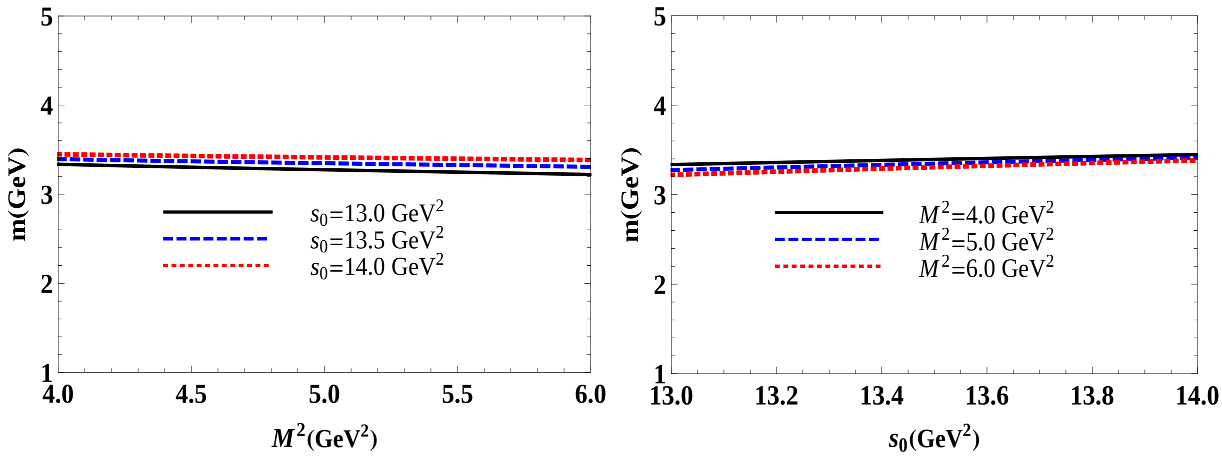

FIG. 1: Dependence of the $Z_{\rm V}$ tetraquark's mass m on the Borel parameter M^2 (left panel), and on the continuum threshold parameter s_0 (right panel).

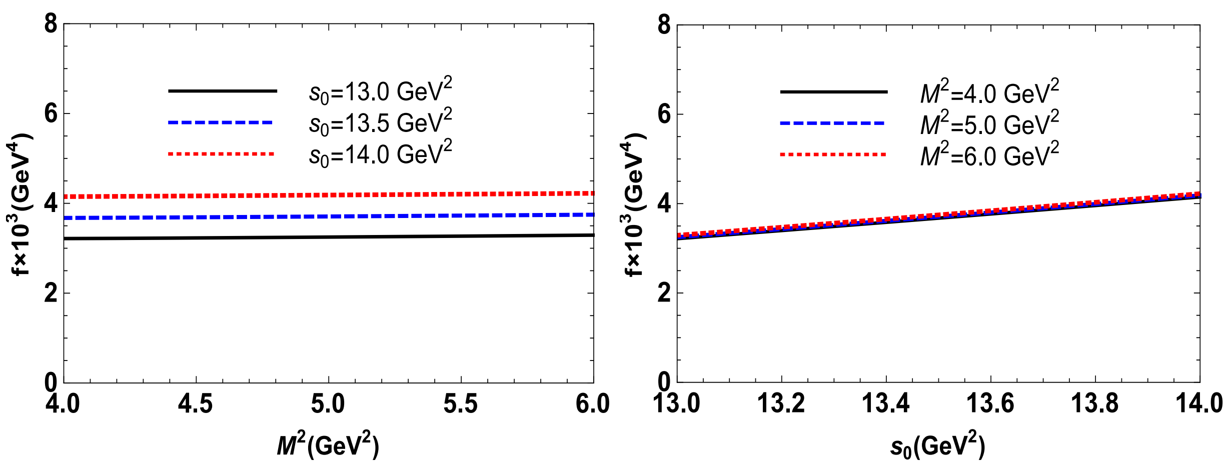

FIG. 2: The same as in Fig. 1, but for the coupling f of the tetraquark $Z_{\rm V}$.

The second source of theoretical errors is connected with a choice of the continuum threshold parameter s_0. Despite M^2, it bears physical information about first excitation of the tetraquark $Z_{\rm V}$. The self-consistent analysis implies that $\sqrt{s_0}$ has to be smaller than mass of such state. There are only a few samples when two observed resonances were assumed to be ground and radially excited states of the same tetraquark. The resonances $Z_c(3900)$ and $Z_c(4430)$ may form one of such pairs [24]. The difference between masses of $Z_c(3900)$ and $Z_c(4430)$ is equal to ≈ 530 MeV, therefore a mass gap $\sqrt{s_0} - m \approx (400 - 600)$ MeV can be considered as a reasonable estimate for tetraquarks. Here, we get $\sqrt{s_0} - m \approx 400$ MeV which is in accord with this general analysis.

Effects connected with a choice of parameters M^2 and s_0 are two main sources of theoretical uncertainties in sum rule computations. In the case of the mass m these ambiguities equal to $\pm 3.6\%$, whereas for the coupling f they are $\pm 14\%$ of the full result. Theoretical uncertainties for f are larger than for the mass, nevertheless, they do not overshoot accepted limits.

It is interesting to analyze a gap between masses of axial-vector and vector tetraquarks with the same content. Our present calculations demonstrate that for tetraquarks $[uc][\overline{s}\overline{d}]$ the difference between masses of the axial-vector and vector particles $Z_{\rm AV}$ and $Z_{\rm V}$ is $m - m_{Z_{\rm AV}} \approx 500$ MeV. This result can be compared with similar prediction for other tetraquarks. Thus, resonances $Y(4660)$ and $X(4140)$ with quantum numbers $J^{\rm PC} = 1^{--}$ and 1^{++}, quark content $[cs][\overline{cs}]$ and color structure $[\overline{\mathbf{3}}_c] \otimes [\mathbf{3}_c]$ were investigated in Refs. [25, 26], respectively. An estimate for $m_Y - m_{X_1}$ extracted from these studies is approximately equal to 500 MeV as well.

In other words, radially excited axial-vector and ground-state vector tetraquarks, in this special case, are very close in mass. This fact maybe useful to explain numerous charged and neutral resonances from XYZ family in the mass range of $4-5$ GeV.

Another issue to be addressed here, is a mass gap between vector tetraquarks $Z_{\rm V}$ and $X_1(2900)$, which amounts to ≈ 450 MeV and is quite large. It has been noted in section I, that X_1 can be modeled as a vector tetraquark $[ud][\overline{cs}]$, and hence both $Z_{\rm V}$ and X_1 are composed of same quarks. But there are two reasons, which may lead to aforementioned mass effect. First of them is internal organizations of these states: The tetraquark $Z_{\rm V}$ is composed of a heavy diquark $[uc]$ and relatively heavy antidiquark $[\overline{s}\overline{d}]$, whereas X_1 is strongly heavy-light compound. The latter is more tightly connected structure and should be lighter than $Z_{\rm V}$. Besides, diquarks in $Z_{\rm V}$ have fractional positive electric charges which generate repulsive forces between them. In the case of X_1, between $[ud]$ and $[\overline{cs}]$ exists attractive electromagnetic interaction. Whether these features of $Z_{\rm V}$ and X_1 are enough to explain a mass gap between them or there are other sources of this effect, requires additional studies.

III. STRONG DECAYS OF THE TETRAQUARK $Z_{\rm V}$

The sum of partial widths of $Z_{\rm V}$ tetraquark's different decay channels constitutes its full width. The result for m obtained in the previous section is necessary to fix kinematically allowed strong decay modes of $Z_{\rm V}$. Decays to final states $D_{s1}(2460)\pi$ and $D_{s0}^{*}(2317)\rho$ (below, simply $D_{s1}\pi$ and $D_{s0}^{*}\rho$) are among allowed S-wave process for the tetraquark $Z_{\rm V}$. The P-wave processes, which will be explored, are decays $Z_{\rm V}\to DK$ and $Z_{\rm V}\to D_s\pi$.

A. Processes $Z_{\rm V}\to D_{s1}\pi$ and $Z_{\rm V}\to D_{s0}^{*}\rho$

Here, we study the decays $Z_{\rm V}\to D_{s1}\pi$ and $Z_{\rm V}\to D_{s0}^{*}\rho$, and compute their partial widths. We provide details of calculations for the first process and write down only essential formulas and final results for the second decay. It should be noted that the mass $m=(3340\ \pm 120)$ MeV makes possible S-wave decays of $Z_{\rm V}$ to final states $DK_1(1270)$, $D_sb_1(1235)$ and $D_sa_1(1260)$ as well. But widths of these processes, as it will be explained below, are suppressed relative to aforementioned two decays due to kinematical factors. Therefore, we restrict ourselves by investigation of two dominant S-wave decays.

The width of the process $Z_{\rm V}\to D_{s1}\pi$ can be found using the coupling g_1 that describes strong interactions of particles $Z_{\rm V}$, D_{s1}, and π at the vertex $Z_{\rm V}D_{s1}\pi$. In order to evaluate g_1, we use the QCD light-cone sum rule method [14] and a soft-meson approximation [15, 16].

Starting point in LCSR method is the correlation function

$$\Pi_{\mu\nu}(p,q)=i\int d^4xe^{ipx}\langle\pi(q)|\mathcal{T}\{J_{\mu}^{D_1}(x)J_{\nu}^{\dagger}(0)\}|0\rangle, \quad (16)$$

where by D_1 in the current $J_{\mu}^{D_1}$ we denote the meson D_{s1}. The current $J_{\nu}(0)$ in the correlation function $\Pi_{\mu\nu}(p,q)$ is introduced in Eq. (3). As interpolating current $J_{\mu}^{D_1}(x)$ for the axial-vector meson D_{s1}, we use the expression

$$J_{\mu}^{D_1}(x)=\overline{s}_l(x)i\gamma_5\gamma_{\mu}c_l(x), \quad (17)$$

with l being the color index.

The function $\Pi_{\mu\nu}(p,q)$ has to be rewritten in terms of the physical parameters of the initial and final particles involved into the decay. By taking into account the ground states in the D_{s1} and $Z_{\rm V}$ channels, we get

$$\Pi_{\mu\nu}^{\rm Phys}(p,q)=\frac{\langle 0|J_{\mu}^{D_1}|D_{s1}\,(p)\rangle}{p^2-m_1^2}\langle D_{s1}\,(p)\,\pi(q)|Z_{\rm V}(p')\rangle \times\frac{\langle Z_{\rm V}(p')|J_{\nu}^{\dagger}|0\rangle}{p'^2-m^2}+\cdots, \quad (18)$$

where p, q and $p'=p+q$ are the momenta of D_{s1}, π , and $Z_{\rm V}$, respectively. In Eq. (18) m_1 is the mass of the meson D_{s1}, and the ellipses stand for contributions of higher resonances and continuum states in the D_{s1} and $Z_{\rm V}$ channels.

To get more detailed expression for $\Pi_{\mu\nu}^{\rm Phys}(p,q)$, we utilize the matrix elements

$$\langle 0|J_{\mu}^{D_1}|D_{s1}\rangle=f_1m_1\epsilon_{\mu},\ \langle Z_{\rm V}(p')|J_{\nu}^{\dagger}|0\rangle=fm\epsilon_{\nu}^{\prime *}, \quad (19)$$

and model the vertex $\langle D_{s1}\,(p)\,K(q)|Z_{\rm V}(p')\rangle$ in the following form

$$\langle D_{s1}\,(p)\,\pi(q)|Z_{\rm V}(p')\rangle=g_1\left[(p\cdot p')\,(\epsilon^{*}\cdot\epsilon') -(p\cdot\epsilon')\,(p'\cdot\epsilon^{*})\right]. \quad (20)$$

In Eqs. (19) and (20) f_1 and ϵ_{μ} are the decay constant and polarization vector of the meson D_{s1}, respectively: Polarization vector of the tetraquark $Z_{\rm V}$ in this section is denoted by ϵ_{ν}'. Using these matrix elements in Eq. (18), it is not difficult to find

$$\Pi_{\mu\nu}^{\rm Phys}(p,q)=g_1\frac{m_1f_1mf}{(p^2-m_1^2)\,(p'^2-m^2)} \times\left(\frac{m^2+m_1^2}{2}g_{\mu\nu}-p_{\mu}p_{\nu}'\right). \quad (21)$$

The function $\Pi_{\mu\nu}^{\rm Phys}(p,q)$ contains two structures proportional to $g_{\mu\nu}$ and $p_{\mu}p_{\nu}'$, respectively. These structures and relevant invariant amplitudes can be employed to extract the sum rule for the strong coupling g_1. We choose to work with the term $\sim g_{\mu\nu}$ and corresponding invariant amplitude.

The QCD side of the sum rule can be obtained using explicit expressions of the correlation function $\Pi_{\mu\nu}(p,q)$

and interpolating currents. Having contracted quark and antiquark fields in the correlation function, we get

$$\Pi_{\mu\nu}^{\mathrm{OPE}}(p,q)=\int d^4xe^{ipx}\varepsilon\widetilde{\varepsilon}\left[\gamma_5\widetilde{S}_c^{lc}(x)\gamma_\mu\gamma_5\right.$$
$$\left.\times\widetilde{S}_s^{ml}(-x)\gamma_\nu\gamma_5\right]_{\alpha\beta}\langle\pi(q)|\overline{u}_\alpha^b(0)d_\beta^n(0)|0\rangle, \qquad (22)$$

where α and β are the spinor indexes.

The function $\Pi^{\mathrm{OPE}}(p,q)$ contains local matrix elements of the quark operator $\overline{u}d$ sandwiched between the vacuum and pion. Simple transformations allow one to express $\langle\pi(q)|\overline{u}_\alpha^b(0)d_\beta^n(0)|0\rangle$ in terms of the pion's known local matrix elements. To this end, we expand $\overline{u}(0)d(0)$ by employing full set of Dirac matrices $\Gamma^J=\mathbf{1},\ \gamma_5,\ \gamma_\mu,\ i\gamma_5\gamma_\mu,\ \sigma_{\mu\nu}/\sqrt{2}$, and project obtained operators onto the color-singlet states. These manipulations lead to replacement

$$\overline{u}_\alpha^b(0)d_\beta^n(0)\to\frac{1}{12}\delta^{bn}\Gamma_{\beta\alpha}^J\left[\overline{u}(0)\Gamma^Jd(0)\right], \qquad (23)$$

which should be fulfilled in $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,q)$. The operators $\overline{u}(0)\Gamma^Jd(0)$ placed between the vacuum and pion are matrix elements of the π meson.

The QCD expression of the correlation function (22) contains hard-scattering and soft parts. The hard-scattering part of $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,q)$ is expressed in terms of quark propagators, whereas matrix elements of the pion form its soft component. For vertices built of conventional mesons correlation functions depend on non-local matrix elements of a final meson which are expressible in terms of its distribution amplitudes (DAs). In the case under analysis, due to four-quark nature of Z_{V}, $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,q)$ contains only local matrix elements of the pion. As a result, integrals over DAs which are typical for LCSR method, reduce to overall normalization factors. In this case, the correlation function has to be found by means of the soft-meson approximation which implies computation of the hard-scattering part of $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,q)$ in the limit $q\to 0$ [17]. It is worth to emphasize that soft-meson approximation is necessary to analyze tetraquark-meson-meson vertices: Strong couplings at vertices composed of two tetraquarks and a meson can be calculated by employing full version of the LCSR method [27].

It is evident, that soft approximation should by applied also to the phenomenological side of the sum rule. In the limit $q\to 0$ for the amplitude $\Pi^{\mathrm{Phys}}(p^2)$, we get

$$\Pi^{\mathrm{Phys}}(p^2)=g_1\frac{f_1m_1fm}{(p^2-\widetilde{m}^2)^2}\widetilde{m}^2+\cdots, \qquad (24)$$

where $\widetilde{m}^2=(m^2+m_1^2)/2$. The function $\Pi^{\mathrm{Phys}}(p^2)$ depends on one variable $p^2=p'^2$ and has the double pole at $p^2=\widetilde{m}^2$. The Borel transformation of $\Pi^{\mathrm{Phys}}(p^2)$ is given by the formula

$$\Pi^{\mathrm{Phys}}(M^2)=g_1f_1m_1fm\widetilde{m}^2\frac{e^{-\widetilde{m}^2/M^2}}{M^2}+\cdots. \qquad (25)$$

Besides ground-state contribution, the amplitude $\Pi^{\mathrm{Phys}}(M^2)$ in the soft limit contains unsuppressed terms which survive even after Borel transformation. These contaminations should be removed from $\Pi^{\mathrm{Phys}}(M^2)$ by applying the operator [15, 16]

$$\mathcal{P}(M^2,\widetilde{m}^2)=\left(1-M^2\frac{d}{dM^2}\right)M^2e^{\widetilde{m}^2/M^2}. \qquad (26)$$

After this operation, remaining undesired terms in $\Pi^{\mathrm{Phys}}(M^2)$ can be subtracted by usual manner in the context of quark-hadron duality assumption. It is clear, that we have to apply the operator $\mathcal{P}(M^2,\widetilde{m}^2)$ to QCD side of the sum rule as well. Then, the sum rule for the strong coupling g_1 reads

$$g_1=\frac{1}{f_1m_1fm\widetilde{m}^2}\mathcal{P}(M^2,\widetilde{m}^2)\Pi^{\mathrm{OPE}}(M^2,s_0), \qquad (27)$$

where $\Pi^{\mathrm{OPE}}(M^2,s_0)$ is Borel transformed and subtracted invariant amplitude $\Pi^{\mathrm{OPE}}(p^2)$ corresponding to the structure $g_{\mu\nu}$ in $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,q)$.

Prescriptions to calculate the correlation function $\Pi_{\mu\nu}^{\mathrm{OPE}}(p,q)$ in the soft approximation were presented in Ref. [17], and in many other publications, for this reason we do not concentrate here on details. Computations of $\Pi^{\mathrm{OPE}}(M^2,s_0)$ performed in accordance with this scheme lead to the expression

$$\Pi^{\mathrm{OPE}}(M^2,s_0)=\frac{f_\pi\mu_\pi}{48\pi^2}\int_{\mathcal{M}^2}^{s_0}\frac{ds(m_c^2-s)}{s^2}$$
$$\times\left(m_c^4+m_c^2s+6m_cm_ss-2s^2\right)e^{-s/M^2}$$
$$+\Pi_{\mathrm{NP}}(M^2), \qquad (28)$$

where the first term is the perturbative contribution to $\Pi^{\mathrm{OPE}}(M^2,s_0)$. The nonperturbative component $\Pi_{\mathrm{NP}}(M^2)$ of the correlation function is calculated with dimension-9 accuracy, and has the following form

$$\Pi_{\mathrm{NP}}(M^2)=\frac{\langle\overline{s}s\rangle f_\pi\mu_\pi m_c\left(2M^2+m_cm_s\right)}{12M^2}e^{-m_c^2/M^2}$$
$$+\langle\frac{\alpha_sG^2}{\pi}\rangle\frac{f_\pi\mu_\pi m_c}{144M^4}\int_0^1\frac{dx}{X^3}\left[m_c^2m_sX-2m_sXM^2\right.$$
$$\left.+m_c^3x+m_cxXM^2\right]e^{m_c^2/[M^2X]}+\frac{\langle\overline{s}g\sigma Gs\rangle f_\pi\mu_\pi}{72M^6}$$
$$\times\left(m_sM^4+m_sm_c^2M^2-3m_c^3M^2-m_sm_c^4\right)e^{-m_c^2/M^2}$$
$$+\langle\frac{\alpha_sG^2}{\pi}\rangle\langle\overline{s}s\rangle\frac{f_\pi\mu_\pi\pi^2m_c(3M^2-m_c^2)}{216M^8}$$
$$\times\left(2M^2+m_sm_c\right)e^{-m_c^2/M^2}+\langle\frac{\alpha_sG^2}{\pi}\rangle\langle\overline{s}g\sigma Gs\rangle$$
$$\times\frac{f_\pi\mu_\pi\pi^2m_c\left(m_c^5m_s+3M^2m_c^4-7m_c^3m_sM^2\right.}{1296M^{12}}$$
$$\left.-18m_c^2M^4+8m_cm_sM^4+18M^6\right)e^{-m_c^2/M^2}, \qquad (29)$$

where $X=x-1$.

Parameters	Values (in MeV units)
$m_1 = m[D_{s1}(2460)]$	2459.5 ± 0.6
$f_1 = f[D_{s1}(2460)]$	225 ± 25
$m_2 = m[D_{s0}^{*}(2317)]$	2317.8 ± 0.5
$f_2 = f[D_{s0}^{*}(2317)]$	202 ± 15
m_D	1869.65 ± 0.05
f_D	212.6 ± 0.7
m_{D_s}	1968.34 ± 0.07
f_{D_s}	249.9 ± 0.5
m_K	493.677 ± 0.0016
f_K	155.7 ± 0.3
m_π	139.57039 ± 0.00018
f_π	130.2 ± 1.2
m_ρ	775.26 ± 0.25
f_ρ	216 ± 3

TABLE I: Masses and decay constants of mesons used in numerical analyses.

Contributions to $\Pi_{\rm NP}(M^2)$ arise from the matrix element

$$\langle 0|\overline{d}i\gamma_5 u|\pi\rangle = f_\pi \mu_\pi, \tag{30}$$

where

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = -\frac{2\langle \overline{q}q\rangle}{f_\pi^2}. \tag{31}$$

The last equality in Eq. (31) is the relation between the mass m_π and decay constant f_π of the pion, masses of quarks, and quark condensate $\langle \overline{q}q\rangle$, which stems from the partial conservation of the axial-vector current.

In numerical computations of $\Pi^{\rm OPE}(M^2, s_0)$, we choose M^2 and s_0 within limits given by Eq. (13). Besides M^2 and s_0, Eq. (27) contains various vacuum condensates and spectroscopic parameters of the final-state mesons D_{s1} and π. The masses and decay constants of the mesons D_{s1} and π, as well as other parameters used in numerical analyses are borrowed from Ref. [28] and presented in Table I: For decay constants of the mesons D_{s0}^{*} and D_{s1}, we utilize the sum rule predictions from Refs. [23] and [29], respectively.

For the coupling g_1, we get

$$g_1 = 0.36^{+0.08}_{-0.09}\ {\rm GeV}^{-1} \tag{32}$$

The partial width of the decay $Z_{\rm V} \to D_{s1}\pi$ can be found by means of the formula

$$\Gamma_1\left[Z_{\rm V} \to D_{s1}\pi\right] = \frac{g_1^2 m_1^2 \lambda}{24\pi}\left(3 + \frac{2\lambda^2}{m_1^2}\right), \tag{33}$$

where $\lambda \equiv \lambda\left(m, m_1, m_\pi\right)$ and

$$\begin{aligned}\lambda\left(a,b,c\right) \ =\ & \frac{1}{2a}\left[a^4 + b^4 + c^4\right. \\ & \left. -2\left(a^2b^2 + a^2c^2 + b^2c^2\right)\right]^{1/2}.\end{aligned} \tag{34}$$

By employing this expression, it is not difficult to obtain

$$\Gamma_1\left[Z_{\rm V} \to D_{s1}\pi\right] = 25.6^{+11.0}_{-7.5}\ {\rm MeV}. \tag{35}$$

The strong coupling and partial width of the second process $Z_{\rm V} \to D_{s0}^{*}\rho$ can be found by the same manner. Here, one starts from the correlation function

$$\widetilde{\Pi}_\nu(p,q) = i\int d^4x e^{ipx}\langle \rho(q)|\mathcal{T}\{J^{D_0}(x)J_\nu^{\dagger}(0)\}|0\rangle, \tag{36}$$

where the interpolating current of the scalar meson D_{s0}^{*} is denoted by $J^{D_0}(x)$ and determined by the expression

$$J^{D_0}(x) = \overline{s}_l(x)c_l(x). \tag{37}$$

In our studies, we use the matrix element $\langle 0|J^{D_0}|D_{s0}^{*}\rangle = f_2 m_2$, with m_2 and f_2 being the mass and decay constant of the D_{s0}^{*}. The vertex $Z_{\rm V}D_{s0}^{*}\rho$ is modeled in the form

$$\begin{aligned}&\langle D_{s0}^{*}\left(p\right)\rho(q)|Z_{\rm V}(p')\rangle = g_2\left[\left(q\cdot p'\right)\left(\epsilon^{*}\cdot\epsilon'\right)\right. \\ &\left. -\left(q\cdot\epsilon'\right)\left(p'\cdot\epsilon^{*}\right)\right],\end{aligned} \tag{38}$$

where ϵ_μ is the polarization vector of the ρ meson. Then, the phenomenological and QCD sides of the sum rule are given by expressions

$$\begin{aligned}\widetilde{\Pi}_\nu^{\rm Phys}(p,q) &= g_2\frac{m_2 f_2 m f}{\left(p^2 - m_2^2\right)\left(p'^2 - m^2\right)} \\ &\times\left(\frac{m_2^2 - m^2}{2}\epsilon_\nu^{*} + p'\cdot\epsilon^{*}q_\nu\right),\end{aligned} \tag{39}$$

and

$$\begin{aligned}\widetilde{\Pi}_\nu^{\rm OPE}(p,q) &= -i\int d^4x e^{ipx}\varepsilon\widetilde{\varepsilon}\left[\gamma_5\widetilde{S}_c^{lc}(x)\widetilde{S}_s^{ml}(-x)\right. \\ &\left.\times\gamma_\nu\gamma_5\right]_{\alpha\beta}\langle\rho(q)|\overline{u}_\alpha^{b}(0)d_\beta^{n}(0)|0\rangle,\end{aligned} \tag{40}$$

respectively. We perform calculations using the structures $\sim \epsilon_\nu^{*}$ in $\widetilde{\Pi}_\nu^{\rm Phys}$ and $\widetilde{\Pi}_\nu^{\rm OPE}$. The relevant amplitude $\widetilde{\Pi}^{\rm OPE}(M^2, s_0)$ receives contributions from the matrix elements of the ρ meson

$$\begin{aligned}\langle 0|\overline{d}\gamma_\mu u|\rho\rangle \ &=\ f_\rho m_\rho\epsilon_\mu, \\ \langle 0|\overline{d}g\widetilde{G}_{\mu\nu}\gamma_\nu\gamma_5 u|\rho\rangle \ &=\ f_\rho m_\rho^3\zeta_4\epsilon_\mu,\end{aligned} \tag{41}$$

where m_ρ and f_ρ are the mass and decay constant of the ρ meson, and $\widetilde{G}_{\mu\nu}$ is the dual gluon field tensor $\widetilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}/2$. The second equality in Eq. (41) is the matrix element of the twist-4 operator [30]. The parameter $\zeta_{4\rho}$ was evaluated in the context of QCD sum rule approach at the renormalization scale $\mu = 1$ GeV in Ref. [31] and is equal to $\zeta_{4\rho} = 0.07 \pm 0.03$.

The correlation function $\widetilde{\Pi}^{\rm OPE}(M^2, s_0)$ has the form

$$\begin{aligned}\widetilde{\Pi}^{\rm OPE}(M^2,s_0) &= \frac{f_\rho m_\rho}{16\pi^2}\int_{\mathcal{M}^2}^{s_0}\frac{ds(s - m_c^2)}{s} \\ &\times\left(m_c^2 + 2m_c m_s - s\right)e^{-s/M^2} + \frac{f_\rho m_\rho^3\zeta_{4\rho}}{32\pi^2} \\ &\times\int_{\mathcal{M}^2}^{s_0}\frac{ds(m_c^4 - s^2)}{s^2}e^{-s/M^2} + \widetilde{\Pi}_{\rm NP}(M^2),\end{aligned} \tag{42}$$

where $\widetilde{\Pi}_{\rm NP}(M^2)$ is nonperturbative component of $\widetilde{\Pi}^{\rm OPE}$: We refrain from providing its explicit expression here.

Omitting further details, we write down results for the strong coupling g_2 and partial width of the process $Z_{\rm V} \to D_{s0}^{*}\rho$

$$g_2 = (7.0 \pm 1.1) \cdot 10^{-1}\ \mathrm{GeV}^{-1},$$
$$\Gamma_2 \left[Z_{\rm V} \to D_{s0}^{*}\rho\right] = 8.7^{+3.3}_{-2.3}\ \mathrm{MeV}. \qquad (43)$$

Returning to other S-wave decays listed above, we assume that couplings corresponding to vertices $Z_{\rm V}DK_1(1270)$ etc., are the same order of g_1 and g_2. Then widths of these decays are suppressed, because the factor $m_{*}^2\lambda\left(3+2\lambda^2/m_{*}^2\right)$ (m_* is a mass of a final meson) is smaller for two final-state mesons of approximately equal mass than in the case of light and heavy mesons. We also take into account that the full width $\Gamma_{\rm full}$ of the tetraquark $Z_{\rm V}$ is formed mainly due to P-wave processes $Z_{\rm V} \to DK$ and $Z_{\rm V} \to D_s\pi$, and for this reason consider only two dominant S-wave decays.

B. Decays $Z_{\rm V} \to DK$ and $Z_{\rm V} \to D_s\pi$

In this subsection, we consider the P-wave decays $Z_{\rm V} \to DK$ and $Z_{\rm V} \to D_s\pi$ of the tetraquark $Z_{\rm V}$. Treatments of these processes do not differ from analysis carried out above, differences being mainly in meson-tetraquark vertices and matrix elements of final-state mesons employed in calculations.

Let us concentrate on the decay $Z_{\rm V} \to DK$. The correlation function to find a sum rule for the strong coupling G_1 of vertex $Z_{\rm V}DK$ is given by the formula

$$\Pi_{\nu}(p,q) = i\int d^4x e^{ipx}\langle K(q)|\mathcal{T}\{J^D(x)J_{\nu}^{\dagger}(0)\}|0\rangle, \quad (44)$$

where $J^D(x)$ is the interpolating current

$$J^D(x) = \overline{d}_l(x)i\gamma_5 c_l(x), \qquad (45)$$

for the pseudoscalar meson D.

Then, the physical side of the sum rule has the form

$$\Pi_{\nu}^{\rm Phys}(p,q) = \frac{\langle 0|J^D|D\left(p\right)\rangle}{p^2-m_D^2}\langle D\left(p\right)K(q)|Z_{\rm V}(p')\rangle$$
$$\times\frac{\langle Z_{\rm V}(p')|J_{\nu}^{\dagger}|0\rangle}{p'^2-m^2}+\cdots . \qquad (46)$$

Using the matrix elements

$$\langle 0|J^D|D\rangle = \frac{f_D m_D^2}{m_c},\ \langle D\left(p\right)K(q)|Z_{\rm V}(p')\rangle = G_1 p\cdot\epsilon', \qquad (47)$$

for $\Pi_{\nu}^{\rm Phys}$, we find

$$\Pi_{\nu}^{\rm Phys}(p,q) = G_1\frac{f_D m_D^2 f m}{2m_c(p^2-m_D^2)(p'^2-m^2)}$$
$$\times\left[\left(-1+\frac{m_D^2-m_K^2}{m^2}\right)p_{\nu}+\left(1+\frac{m_D^2-m_K^2}{m^2}\right)q_{\nu}\right]$$
$$+\cdots . \qquad (48)$$

In expressions above, m_D and f_D are the mass and decay constant of the D meson, respectively.

In terms of quark propagators the same correlation function $\Pi_{\upsilon}(p,q)$ is determined by the expression

$$\Pi_{\nu}^{\rm OPE}(p,q) = \int d^4x e^{ipx}\varepsilon\widetilde{\varepsilon}\left[\gamma_5\widetilde{S}_c^{lc}(x)\gamma_5\widetilde{S}_d^{nl}(-x)\right.$$
$$\left.\times\gamma_5\gamma_{\nu}\right]_{\alpha\beta}\langle K(q)|\overline{u}_{\alpha}^{b}(0)s_{\beta}^{m}(0)|0\rangle. \qquad (49)$$

Operations necessary to derive the sum rule for the coupling G_1 have just been explained above, that is why we do not consider these questions. Let us note that the sum rule for G_1 has been obtained using the structures proportional p_{ν}. The local matrix element of K meson which contributes to $\Pi_{\nu}^{\rm OPE}(M^2,s_0)$ is

$$\langle 0|\overline{d}i\gamma_5 u|\pi\rangle = \frac{f_K m_K^2}{m_s}, \qquad (50)$$

where m_K and f_K are the mass and decay constant of the K meson.

The width of the process $Z_{\rm V} \to DK$ can be found by means of the formula

$$\Gamma_3\left[Z_{\rm V} \to DK\right] = \frac{G_1^2\lambda^3(m,m_D,m_K)}{24\pi m^2}. \qquad (51)$$

In sum rule computations of the coupling G_1 the Borel and continuum threshold parameters are chosed as in Eq. (13). The spectroscopic parameters of the mesons D and K are collected in Table I. Our predictions read

$$G_1 = 4.5^{+0.7}_{-0.5},$$
$$\Gamma_3\left[Z_{\rm V} \to DK\right] = 30.4^{+10.3}_{-6.8}\ \mathrm{MeV}. \qquad (52)$$

For the second P-wave decay $Z_{\rm V} \to D_s\pi$, we find

$$G_2 = 7.1^{+1.1}_{-0.9},$$
$$\Gamma_4\left[Z_{\rm V} \to D_s\pi\right] = 76.1^{+25.7}_{-17.1}\ \mathrm{MeV}. \qquad (53)$$

Decay channels of the tetraquark $Z_{\rm V}$ considered in this section allow us to evaluate its full width

$$\Gamma_{\rm full} = 141^{+30}_{-20}\ \mathrm{MeV}. \qquad (54)$$

It is clear that $Z_{\rm V}$ can be classified as an exotic meson of wide width. Its main decay modes are processes $Z_{\rm V} \to D_s\pi$ and $Z_{\rm V} \to DK$ with branching ratios $\mathcal{BR}(Z_{\rm V} \to D_s\pi) \approx 0.54$ and $\mathcal{BR}(Z_{\rm V} \to DK) \approx 0.22$, respectively. Contribution of the decay $Z_{\rm V} \to D_{s1}\pi$ is also considerable with estimate $\mathcal{BR}(Z_{\rm V} \to D_s\pi) \approx 0.18$.

IV. CONCLUSIONS AND FINAL NOTES

In this article, we have studied the doubly charged vector tetraquark $Z_{\rm V}^{++} = [uc][\overline{s}\overline{d}]$ and calculated its mass m and width $\Gamma_{\rm full}$. The parameters of $Z_{\rm V}^{++}$ have been evaluated using the QCD two-point and light-cone sum rules. It is worth noting that doubly charged tetraquarks $Z_{\overline{c}s} = [sd][\overline{uc}]$ with quantum numbers $J^{\mathrm{P}} = 0^+$, 0^- and 1^+ were investigated in our paper [11].

Interest to these yet hypothetical particles is twofold: They are built of four quarks of different flavors, and moreover bear two units of electric charge. Recently the LHCb collaboration observed structures $X_{0(1)}(2900)$ which may be interpreted as exotic mesons containing four different quarks [4, 5]. The structures $X_{0(1)}$ were seen as resonance-like peaks in the mass distribution of D^-K^+ mesons. The master process to discover $X_{0(1)}$ was the weak decay of the B meson $B^+ \to D^+D^-K^+$.

It is remarkable that the process $B^+ \to D^+D^-K^+$ and data collected during its exploration can be employed to observe another tetraquarks, namely states with quark content $[uc][\overline{s}\overline{d}]$. In fact, analysis of the invariant mass distribution of mesons D^+K^+ may lead to information about doubly charged scalar and vector tetraquarks $[uc][\overline{s}\overline{d}]$. Decays to final mesons D^+K^+ are among favored channels of the scalar Z_{S}^{++} and vector Z_{V}^{++} particles: Relevant branching ratios are equal to $\mathcal{BR}(Z_{\mathrm{S}} \to DK) \approx 0.86$ and $\mathcal{BR}(Z_{\mathrm{V}} \to DK) \approx 0.22$, respectively. The decay of the B meson $B^+ \to D^-D_s^+\pi^+$ is also useful to see states Z_{S}^{++} and Z_{V}^{++}, because processes $Z_{\mathrm{S}} \to D_s\pi$ and $Z_{\mathrm{V}} \to D_s\pi$ have considerable branching ratios $\mathcal{BR}(Z_{\mathrm{S}} \to D_s\pi) \approx 0.12$ and $\mathcal{BR}(Z_{\mathrm{V}} \to D_s\pi) \approx 0.54$. Results obtained for spectroscopic parameters and decay channels of Z_{V}^{++}, as well as information on the tetraquarks $Z_{\overline{c}s}$ may be useful for such experimental investigations. Existence of tetraquarks with content $[ud][\overline{c}\overline{d}]$ may be confirmed in decay $B^+ \to D^-D_s^+\pi^+$ as well: These neutral states may be fixed in the invariant mass distribution of $D^-\pi^+$ mesons.

It is evident, that three-meson weak decays of B meson are sources of valuable information on four-quark exotic states. Data collected by various collaborations in running experiments may be utilized for these purposes. New decays of B meson can open wide prospects to study numerous four-quark states. Indeed, final-state mesons in such processes can be combined to form different pairs and their invariant mass distributions can be explored to find resonance-type enhancements. In any case, additional experimental and theoretical studies are necessary to take advantage of the emerging opportunities.

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