Markov Chains and Random Walks - HW9

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Q1 - Smith is in jail and has 1 dollar; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets 'A' dollars, he wins 'A' dollars with probability .4 and loses 'A' dollars with probability .6. Find the probability that he wins 8 dollars before losing all of his money if:

- **A) He bets 1 dollar each time (timid strategy).**

```
set.seed(123)

win <- 0
trials <- 100000


for (i in seq(1, trials)) {
  #we'll simulate the game 100,000 times

  smiths_money <- 1 #how much money smith has. He starts with $1


  while (smiths_money > 0 & smiths_money < 8) {
    #while his money is greater than 0 and less than 8, keep playing the game

    outcome <-  sample(
        c(1, -1), #two outcomes, 1 or -1
        size = 1, #taking a single sample per turn
        replace = TRUE,
        prob = c(.4, .6) #.4 is the probability he wins
      )
    #using our probabilities, we'll take a single sample. 1 is a win,
    #-1 is a loss

    smiths_money = smiths_money + outcome # current $ Smith has

    if (smiths_money == 8) {  #if he wins, we increment win
      win <- win + 1
    }

  }

}

win_probability <- win/trials
```

Smith's probability of winning with the 'timid strategy' is: 0.02019.

- **B) He bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy).**

```
set.seed(123)

win <- 0
trials <- 100000
win_criteria <- 8


for (i in seq(1, trials)) {
  #we'll simulate the game 100,000 times

smiths_money <- 1 #how much money smith has. He starts with $1


while (smiths_money > 0 & smiths_money < win_criteria) {
  #while his money is greater than 0 and less than 8, keep playing the game

  max_bet <- (win_criteria - smiths_money) #bets as much as possible but not
  #more than necessary

  if (smiths_money < max_bet) {
      bet <- smiths_money #if money is less than the max bet, then he bets
      #all his money
    } else  {
      bet <- max_bet #otherwise he only bets what he needs to
    }

  outcome <-  sample(
      c(bet, -bet), #two outcomes using the bet
      size = 1, #taking a single sample per turn
      replace = TRUE,
      prob = c(.4, .6) #.4 is the probability he wins
    )
  #using our probabilities, we'll take a single sample.

  smiths_money = smiths_money + outcome # current $ Smith has

  }

  if (smiths_money == 8) {  #if he wins, we increment win
    win <- win + 1
    }

  }


win_probability_2 <- win/trials
```

Smith's probability of winning with the 'bold strategy' is: 0.06378.

- **C) Which strategy gives Smith the better chance of getting out of jail?**

Smith should use the 'bold strategy' because he's 3x more likely to get out of jail – even though his chances are incredibly slim anyway.