Multivariable Functions

Christian Thieme

12/5/2020

```r
library(reticulate)
use_python("C:/Users/chris/anaconda3/", required = TRUE)
```

1. Find the equation of the regression line for the given points: (5.6, 8.8), (6.3, 12.4), (7, 14.8), (7.7, 18.2), (8.4, 20.8). Round any final values to the nearest hundredth, if necessary

First, we'll pull the x and y variables into two seperate vectors:

```r
x <- c(5.6,6.3,7,7.7,8.4)
y <- c(8.8,12.4,14.8,18.2,20.8)
```

Now, to get the regression line, we can use the `lm` function, passing in our dependent variable y and our independent variable x:

```r
lm(y~x)
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Coefficients:
## (Intercept)            x
##     -14.800        4.257
```

The output of the linear regression model tells us that the equation for the regression line is:

$y = 4.257x - 14.800$

2. Find all local maxima, local minima, and saddle points for the function: $f(x,y) = 24x - 6xy^2 - 8y^3$

These types of problems are often easier to solve with Python's `sympy`, so we'll use that below:

```python
import sympy as sym

x,y = sym.symbols("x y")
f = 24*x - 6*x*y**2 - 8*y**3
gradient = sym.derive_by_array(f, (x,y))
hessian = sym.Matrix(sym.derive_by_array(gradient, (x, y)))
#hessian
stationary_points = sym.solve(gradient, (x,y))

for p in stationary_points:
    value = f.subs({x: p[0], y: p[1]})
    hess = hessian.subs({x: p[0], y: p[1]})
    eigenvals = hess.eigenvals()
    if all(ev > 0 for ev in eigenvals):
        print("Local minimum at {} with value {}".format(p, value))
    elif all(ev < 0 for ev in eigenvals):
        print("Local maximum at {} with value {}".format(p, value))
    elif any(ev > 0 for ev in eigenvals) and any(ev < 0 for ev in eigenvals):
        print("Critical point at ({},{},{}) which are also saddle points".format(p[0],p
```

```
## Critical point at (-4,2,-64) which are also saddle points
## Critical point at (4,-2,64) which are also saddle points
```

Inspiration for code: https://stackoverflow.com/questions/50081980/finding-local-maxima-and-minima-of-user-defined-functions

3. A grocery store sells two brands of a product, the "house" brand and a "name" brand. The manager estimates that if she sells the "house" brand for x dollars and the "name" brand for y dollars, she will be able to sell $81 - 21x + 17y$ units of the "house" brand and $40 + 11x - 23y$ units of the "name" brand.

Step 1.: Find the revenue function R(x,y):

$R(x,y) = x(81 - 21x + 17y) + y(40 + 11x - 23y)$
$R(x,y) = 81x - 21x^2 + 17xy + 40y + 11xy + 23y^2$

Simplified, the revenue function is: $R(x,y) = 81x + 40y + 28xy - 21x^2 - 23y^2$

Step 2: What is the revenue if she sells the "house" brand for $2.30 and the "name" brand for $4.10?

To solve this, we just need to solve where R(2.30, 4.10):

```r
x <- 2.30
y <- 4.10

revenue <- 81*x + 40 * y + 28 * x * y -21*x**2 -23*y **2

revenue
```

```
## [1] 116.62
```

4. A company has a plant in Los Angeles and a plant in Denver. The firm is committed to produce a total of 96 units of a product each week. The total weekly cost is given by $C(x,y) = \frac{1}{6}x^2 + \frac{1}{6}y^2 + 7x + 25y + 700$ where x is the number of units produced in Los Angeles and y is the number of units produced in Denver. How many units should be produced in each plant to minimize the total weekly cost?

This problem is similar to the last, except this time we'll be looking for the derivative of the cost curve. In order to get the derivative, we'll need to solve the cost funtion. To do that we can take advantage of a key piece of information - we know that $x + y$ have committed to produce 96 units of product each week, so:

$x + y = 96$ which is we can use to solve for x: $x = 96 - y$, which we can then substitute into our cost function:

$C(96-y,y) = \frac{1}{6}(96-y)^2 + \frac{1}{6}y^2 + 7(96-y) + 25y + 700$

After several steps of simplification we get the following cost curve:

$C(y) = \frac{1}{3}y^2 - 14y + 2908$

Now, taking the derivative, we get:

$C'(y) = \frac{2}{3}y - 14$

To find the minimum of the derivative, we'll need to set it equal to 0 and solve:

$C'(y) = \frac{2}{3}y - 14 = 0$

Solving for y we get $y = 21$

Plugging this back into our original function $x + y = 96$ we get x is 75.

SO, we should produce **75 units in Los Angeles and 21 units in Denver**.

5. Evaluate the double integral on the given region and write your answer in exact form without decimals:

$\iint (e^{8x+3y})dA; R: 2 \le x \le 4 \quad and \quad 2 \le y \le 4$

```python
x, y = sym.symbols("x y")
f = sym.exp(8*x + 3*y)
res = sym.integrate(f, (y, 4, 2), (x, 4, 2))

res
```

```
## (1 - exp(6))*exp(22)/24 - (1 - exp(6))*exp(38)/24
```

I love `SYMPY`.