MECENG 276 - SDSE HW 2

Problem 1

Stationary points

Code:

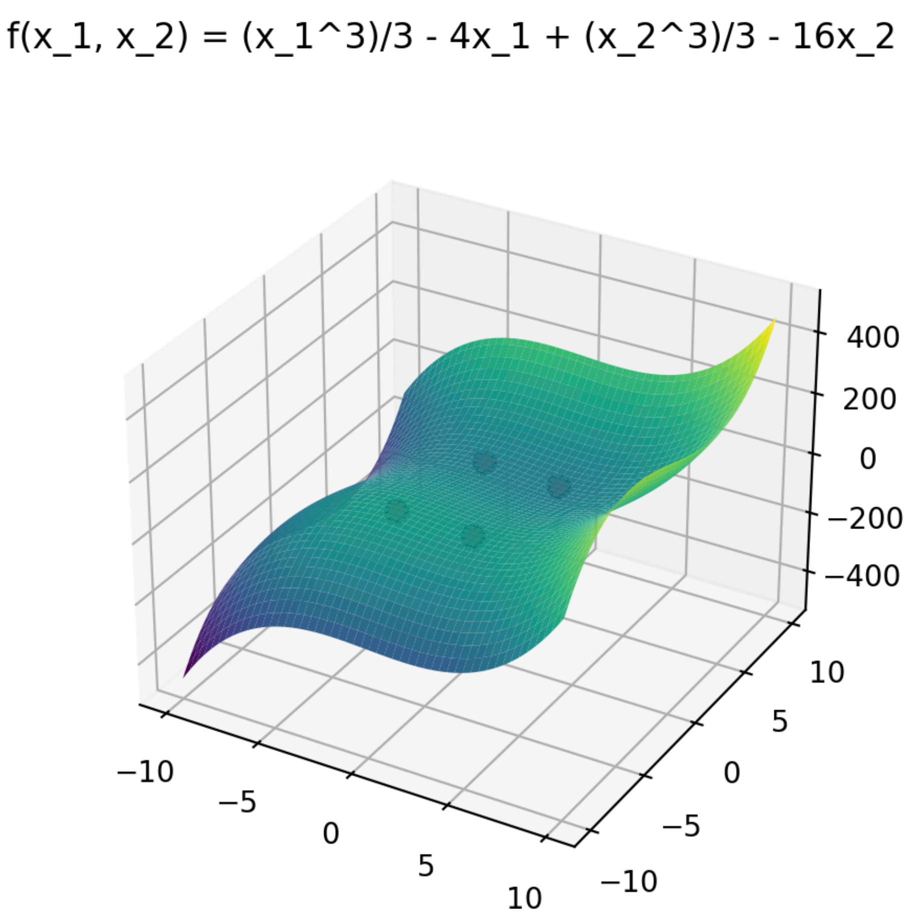
```
def f(x, y):
    return ((x**3)/3) - 4*x + ((y**3)/3) - 16*y

def stationary_points(f):
    x, y = sp.symbols('x y')
    f_x = sp.diff(((x**3)/3) - 4*x + ((y**3)/3) - 16*y, x)
    f_y = sp.diff(((x**3)/3) - 4*x + ((y**3)/3) - 16*y, y)
    stationary_points = sp.solve([f_x, f_y], (x, y))
    return stationary_points
```

Result:

```
[(-2, -4), (-2, 4), (2, -4), (2, 4)]
```

Plot with stationary points

```
def plot_surface(f, points):
    x1_vals = np.linspace(-10, 10, 100)
    x2_vals = np.linspace(-10, 10, 100)
    X_1, X_2 = np.meshgrid(x1_vals, x2_vals)
    Y = f(X_1, X_2)
    fig = plt.figure()
    fig.suptitle('f(x_1, x_2) = (x_1^3)/3 - 4x_1 + (x_2^3)/3 - 16x_2')
    ax = fig.add_subplot(111, projection='3d')
    ax.plot_surface(X_1, X_2, Y)
    for point in points:
        ax.scatter(point[0], point[1], f(point[0], point[1]), color='r')
    plt.show()
```



Local maxima: (-2, -4)

Local Minima: (2, 4)

Saddle points: (2, -4), (-2, 4)

Does this problem have a solution?

The problem does not have a finite solution, as the function would continue to decrease with arbitrary large x_1 and x_2 values. In other words, there are no global minimum.

Problem 2

To use the lookup table we have to find the Z value of the upper and lower bound.

Z = (X-expected value)/standard deviation

We get the Z values: -1.7 and 1.7.

This corresponds to the probabilities: 0.955 and 0.045.

Which gives us the probability of being within the limits of 91%

Problem 3

The mean of the total thickness can be calculated by adding each layers expected value multiplied by the amount of layers of that value.

Mean_T = 10*0.2 + 15*0.1 + 30*0.05 = 2 + 2 + 1.5 = 5mm

The standard deviation of the total thickness can be calculated by squaring the standard deviations, adding them and then the root.

SD_T =(10*4^2 + 15*3^2 + 30*0.5^2)^0.5 = (302.5)^0.5 = 17.4 micro m

Problem 4

Binomial Distribution

It is not appropriate to model the problem with a binomial distribution as the probability of success of each embryo placed is not independent of each other. We can see this if we calculate the expected variance for a binomial distribution and compare it to the given variance in the problem.

The expected value is 0.75 = n * p. This gives us a percentage of 0.25.

The variance can then be calculated as follows: n*p*(1-p) = 3 * 0.25 * 0.75 = 0.5625. This number does not match the variance provided in the problem (0.5).

Chebyshev and Upper Bound Calculation

$$P(L \geq 2) = P(L - \mu \geq 2 - 0.75 = 1.25)$$

$$\rightarrow P(|x - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

$$\frac{k\sigma}{\sigma} = \frac{1.25}{\sqrt{1/2}} = \sqrt{2} \cdot 1.25 = 1.77 = k$$

$$P(|L - \mu| \geq 1.25) \leq \frac{1}{1.77^2} \approx \underline{\underline{31.8\%}}$$

P(L>=2) Given Binomial Distribution

$$L \sim Bin(n = 3,\ s = 0.25) \rightarrow P(L \geq 2)?$$

$$P(L \geq 2) = P(L = 2) + P(L = 3)$$

$$= \binom{3}{2} p^2 (1-p)^1 + \binom{3}{3} \cdot p^3 \cdot (1-p)^0 \approx 0.16$$

$$\underline{\underline{16\%}}$$