MECENG 276 SDSE ASSIGNMENT 1

Exercise 1

First I had to define the functions

```
x = sp.symbols('x')

# Define functions using SymPy's Piecewise for conditional logic
def f1(x):
    return sp.Piecewise((0, x < -sp.pi/2), (0, x > sp.pi/2), (1/2 * sp.cos(x), True))

def f2(x):
    return 1/(1 + sp.exp(-x))

def f3(x):
    return sp.Piecewise((0, x < 0), (0, x > 1), ((x**3)/3, True))

def f4(x):
    return sp.Piecewise((0, x <= 0), (1/x, True))

def f5(x):
    return sp.Piecewise((0, x < 0), (2**(-x), True))
```

To test if the functions are valid pdf I have to test the axioms

```
# Test for Axiom 1: Check if f(x) >= 0 for all x in the range
def testAxiom1(f):
    # The reason this can be done with linspace is because the functions are continuous
    for val in np.linspace(-10, 10, 100):
        if f(val).evalf() < 0:
            return False
    return True

# Test for Axiom 2: Check if the integral of f(x) from -inf to inf equals 1
def testAxiom2(f, is_natural_numbers=False, include_zero=False):
    if is_natural_numbers:
        start = 0 if include_zero else 1
        sum_result = sp.Sum(f(x), (x, start, sp.oo))
        return sp.simplify(sum_result) == 1
    else:
        integral = sp.integrate(f(x), (x, -sp.oo, sp.oo))
        return sp.simplify(integral) == 1
```

All that was left to do was to run the code and check which of the functions are valid pdf

```
f1 ax1: True  ax2:  True
f2 ax1: True  ax2:  False
f3 ax1: True  ax2:  False
f4 ax1: True  ax2:  False
f5 ax1: True  ax2:  False
```

Exercise 2

First I had to define the function

```python
t = sp.symbols('t')

def f (t):
    # t measured in years
    return sp.Piecewise((0, t < 0), (0.1*sp.exp(-0.1*t), True))
```

The next step was to evaluate all the probabilities using simply

```python
# Mean lifetime
# We know that mean = E(t) = integral(t*f(t)) from 0 to inf
mean = sp.integrate(t*f(t), (t, 0, sp.oo))
print("Mean lifetime: ", round(mean), " years")

# Standard deviation
# Variance = E[ (Y - E[Y])^2] = E[Y^2] - E[Y]^2
# Standard deviation = sqrt(variance)
variance = sp.integrate((t**2)*f(t), (t, 0, sp.oo)) - mean**2
std_dev = sp.sqrt(variance)
print("Standard deviation: ", round(std_dev), " years")

# Probability of a system failing within  10 years
prob = sp.integrate(f(t), (t, 10, sp.oo))
print("Probability of failing within 10 years: ", round(1-prob, 3)*100, "%")

# The median lifetime of the system
# Median is the value of t such that P(t <= T) = 0.5
# We can solve this by integrating f(t) from 0 to T and setting it equal to 0.5
T = sp.symbols('T')
median = sp.solve((sp.integrate(f(t), (t, 0, T)) - 0.5), T)
print("Median lifetime: ", round(median[0], 2), " years")

# The probability that at least one out of 5 systems will fail within 3 years
# We can use the complement of the probability that all 5 systems will not fail within 3 years
p_system_not_fail = 1 - sp.integrate(f(t), (t, 0, 3))
p_one_system_fail = 1 - p_system_not_fail**5
print("Probability of at least one system failing within 3 years: ", round(prob, 3)*100, "%")

# The probability that all 5 systems will fail within 15 years
p_system_fail = sp.integrate(f(t), (t, 0, 15))
p_all_systems_fail = p_system_fail**5
print("Probability of all 5 systems failing within 15 years: ", round(p_all_systems_fail, 3)*100, "%")
```

The results:

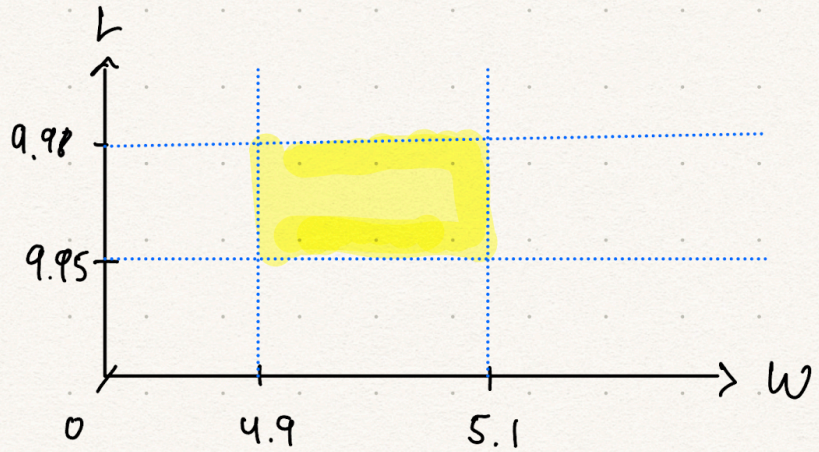
(I was a bit unsure about the standard deviation, it seems too big, but I dont see where my calculation is wrong)

```
Mean lifetime:  10  years
Standard deviation:  10  years
Probability of failing within 10 years:  63.2 %
Median lifetime:  6.93  years
Probability of at least one system failing within 3 years:  36.8 %
Probability of all 5 systems failing within 15 years:  28.3 %
```

Exercise 3

Case 1 : $L < 9.98$

$$\Omega \in \{(L, W) : L \in [9.95, 9.98), W \in [4.9, 5.1]\}$$

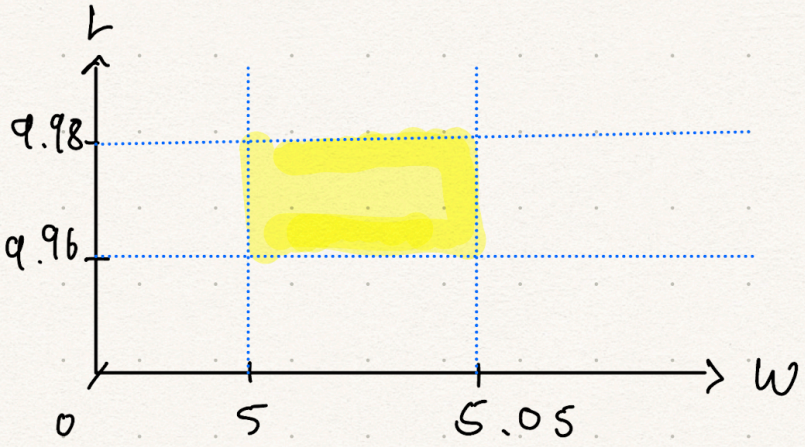


Probability of event is the length of the interval in comparison to the original interval

$$\frac{9.98 - 9.95}{10.5 - 9.95} = \underline{0.3}$$

Case 2 : $L \in [9.96, 9.98]$ and $W \in [5, 5.05]$

$$\Omega \in \{(L, W) : L \in [9.96, 9.98], W \in [5, 5.05]\}$$



The p. is $p\text{-}L \cdot p\text{-}W$

$$p\text{-}L = \frac{9.98 - 9.96}{10.05 - 9.95} = 0.2$$

$$p\text{-}W = \frac{0.05}{0.2} = 0.25$$

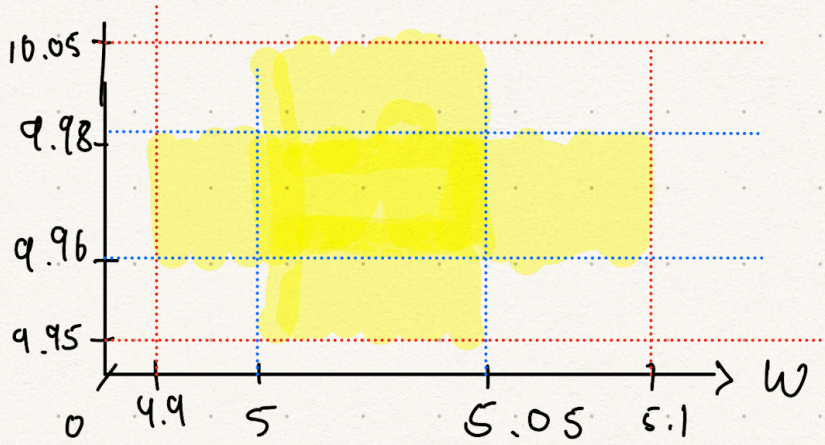
$$p = 0.2 \cdot 0.25 = 0.05 = 5\%$$

Case 3: $L \in [9.96, 9.98]$ or $W \in [5, 5.05]$

$\Omega_1 \in \{(L, W) : L \in [9.96, 9.98], W \in [4.9, 5.1]\}$

$\Omega_2 \in \{(L, W) : L \in [9.95, 10.05], W \in [5, 5.05]\}$

$\Omega = \{\Omega_1, \Omega_2\}$



The p. is $p_L + p_W - p_L \cap p_W$

$p = 0.25 + 0.2 - 0.05 = 0.4 = 40\%$