Contents

[Labs 2](#_Toc29387980)

[Lab 1 2](#_Toc29387981)

[Assignment 1 – Logistic regression and KKNN-method 2](#_Toc29387982)

[Assignment 2 – Probabilistic model and Bayesian model (max.likelihood computations) 4](#_Toc29387983)

[Assignment 4 – Linear regression, Ridge, LASSO, stepAIC 5](#_Toc29387984)

[Lab 2 8](#_Toc29387985)

[Assignment 1 - LDA and logistic regression. Draw decision boundary 8](#_Toc29387986)

[Assignment 2 – Split method, Tree modeling (gini and deviance), Naïve Bayes, loss matrix 9](#_Toc29387987)

[Assignment 4 – Principal components, PCA analysis, ICA analysis, Trace plots 12](#_Toc29387988)

[Lab 3 14](#_Toc29387989)

[Assignment 1 – Kernel methods, Smoothing coefficients 14](#_Toc29387990)

[Assignment 3 – Neural networks, weight initialization 16](#_Toc29387991)

[R 17](#_Toc29387992)

[Examples 17](#_Toc29387993)

[Linear regression 17](#_Toc29387994)

[Linear regression – confidence intervals 18](#_Toc29387995)

[Ridge regression 18](#_Toc29387996)

[Lasso regression 18](#_Toc29387997)

[Stepwise selection (stepAIC, specify which way under *direction*) 19](#_Toc29387998)

[Holdout method (dividation into train, valid, test) 19](#_Toc29387999)

[Cross-validation – K folds and different predictor sets 19](#_Toc29388000)

[Logistic regression 19](#_Toc29388001)

[Linear Discriminative Analysis (LDA) and Quadratic Discriminative Analysis (QDA) 19](#_Toc29388002)

[Naïve bayes 20](#_Toc29388003)

[Decision trees 20](#_Toc29388004)

[Least Absolute Deviation (LAD) regression 21](#_Toc29388005)

[Bootstrap 21](#_Toc29388006)

[Principle Component Analysis (PCA) 22](#_Toc29388007)

[Probabilistic PCA 23](#_Toc29388008)

[Independent Component Analysis (ICA) 23](#_Toc29388009)

[Distance between geographical points of interest 23](#_Toc29388010)

[Neural networks 23](#_Toc29388011)

# Labs

## Lab 1

### Assignment 1 – Logistic regression and KKNN-method

**#1: Read data and divide into test and train sets**

Dataframe=read.csv2("spambase.csv")

n=dim(Dataframe)[1]

set.seed(12345)

id=sample(1:n, floor(n\*0.5))

train=Dataframe[id,]

test=Dataframe[-id,]

**#2: Use logistic regression (functions glm(), predict()) to classify the training and test data by the classification Y(hat) = 1 if p(Y=1 | X) > 0.5, otherwise Y(hat)=0 and report the confusion matrices (use table()) and the misclassification rates for training and test data. Analyse the obtained results.**

#Create model for prediction

spammodel = glm(Spam~., family='binomial', data=train)

summary(spammodel)

#Predict values and create confusion matrix for traindata

predicted\_values\_train = predict(spammodel, newdata=train, type='response')

confusion\_matrix\_train = table(train$Spam, predicted\_values\_train>0.5)

print(confusion\_matrix\_train)

#Predict values and create confusion matrix for testdata

predicted\_values\_test = predict(spammodel, newdata=test, type='response')

confusion\_matrix\_test = table(test$Spam, predicted\_values\_test>0.5)

print(confusion\_matrix\_test)

#Create function for misclassification rate

missclass=function(conf\_matrix, fit\_matrix){

n=length(fit\_matrix[,1])

return(1-sum(diag(conf\_matrix))/n)

}

#Calculate missclassification rate for train and test data

missclass\_train = missclass(confusion\_matrix\_train, train)

print(missclass\_train)

missclass\_test = missclass(confusion\_matrix\_test, test)

print(missclass\_test)

#Conclusion: It is reasonable that the model performs better on the train data compared to the test data. However, the fact that the miscalculations are similar indicates that the model performs similarly on two different data sets which is generally how you want your model to behave. By analyzing the confusion matrices, it is notable that the model is wrong more times proportionally when trying to classify an email that is spam than an email that is not spam.

**#3: Use logistic regression to classify the test data by the classification principle: Same as above but with threshold 0.8. Compare the results. What effect did the new rule have.**

#Create confusion matrix where classification is based on threshold 0.8

confusion\_matrix\_train2 = table(train$Spam, predicted\_values\_train>0.8)

confusion\_matrix\_test2 = table(test$Spam, predicted\_values\_test>0.8)

print(confusion\_matrix\_train2)

print(confusion\_matrix\_test2)

#Calculate missclassification rate for train and test data with threshold 0.8

missclass\_train2 = missclass(confusion\_matrix\_train2, test)

print(missclass\_train2)

missclass\_test2 = missclass(confusion\_matrix\_test2, train)

print(missclass\_test2)

#Conclusion: The misclassification rates have similar results. Showing us that model is well fitted, since the model acts similar between trained and tested data. Although this classificiation principle gives us a higher misclassification rate, it lowered the risk of a non-spam being classified as spam substantially. Therefore we prefer this principle over the previous.

**#4: Use standard classifier kknn() with K=30 from package kknn, report the misclassification rates for the training and test data and compare the results with step 2.**

#Fetch package kkm

#install.packages("kknn")

library("kknn")

#Classify according to kknn with k=30 for test and train data sets

kknn\_30\_train = kknn(formula = Spam~., train, train, k=30)

kknn\_30\_test = kknn(formula = Spam~., train, test, k=30)

confusion\_matrix\_kknn30\_train = table(train$Spam, kknn\_30\_train$fitted.values>0.5)

missclass\_kknn30\_train = missclass(confusion\_matrix\_kknn30\_train, train)

confusion\_matrix\_kknn30\_test = table(test$Spam, kknn\_30\_test$fitted.values>0.5)

missclass\_kknn30\_test = missclass(confusion\_matrix\_kknn30\_test, test)

print(confusion\_matrix\_kknn30\_train)

print(missclass\_kknn30\_train)

print(confusion\_matrix\_kknn30\_test)

print(missclass\_kknn30\_test)

#Conclusion: The misclassification values between predictions of the different sets differ alot. This shows us that our model is not well fitted. The misclassification is lower for the trained data, since the model is fitted after these values. Compared to the results from using logistic regression to classify the data, the results from the KKNN model were significally worse on the test data. This implies that KKNN classification with K=30 is worse than logistic regression in this case.

**#5: Repeat step 4 for K=1. Classify according to kknn with k=1 for test and train data sets. What does the decrease of K lead to and why?**  
  
kknn\_1\_train = kknn(formula = Spam~., train, train, k=1)

kknn\_1\_test = kknn(formula = Spam~., train, test, k=1)

confusion\_matrix\_kknn1\_train = table(train$Spam, kknn\_1\_train$fitted.values>0.5)

missclass\_kknn1\_train = missclass(confusion\_matrix\_kknn1\_train, train)

confusion\_matrix\_kknn1\_test = table(test$Spam, kknn\_1\_test$fitted.values>0.5)

missclass\_kknn1\_test = missclass(confusion\_matrix\_kknn1\_test, test)

print(confusion\_matrix\_kknn1\_train)

print(missclass\_kknn1\_train)

print(confusion\_matrix\_kknn1\_test)

print(missclass\_kknn1\_test)

#Conclusion: The misclassification rate for the training confusion matrix is zero since it compares each data point to itself, predicting all correct. This explains the high misclassification rate for the confusion matrix made on the test data. Using K=1 is a very unreliable method because it does not imply a large statistical advantage.

### Assignment 2 – Probabilistic model and Bayesian model (max.likelihood computations)

**#1: Import data**

Dataframe=read.csv2("machines\_csv.csv")

**#2: Assume probability model p(x|theta) = theta\*e^(-theta\*x) for x = Length in which observations are independent and identically distributed. What is the distribution type of x. Write a function that computes the log-likelihood log p(x|theta) for a given theta and a given data vector x. Plot the curve showing the dependence of log-likelihood on theta where the entire data is used for fitting. What is the maximum likelihood value of theta according to plot?**

#Compute a function for calculating the maximum likelihood of a function

loglikelihood=function(theta, x){

n = length(x[,1])

return(n\*log(theta)-theta\*sum(x))

}

#Plot curve for different theta values

theta\_curve = curve(-loglikelihood(x, Dataframe), xlab="Theta", from=min(Dataframe), to=max(Dataframe))

#Find maximum likelihood value of theta

theta\_max = function(x){

n=length(x[,1])

return(n/sum(x))

}

#Find maxtheta

max\_theta = theta\_max(Dataframe)

print(max\_theta)

#Conclusion: We can see from the probabilistic model that the distribution is of type exponential. The maximum likelihood value of theta is: 42.29453 The optimal theta for is: 1.126217

**#3: Repeat step 2 but use only 6 first observations from the data, and put the two log-likelihood curves (from step 2 and 3) in the same plot. What can you say about reliability of the maximum likelihood solution in each case?**

#New vector with first 6 values

y = matrix(Dataframe[1:6,1], nrow=length(Dataframe[1:6,1]), ncol=1)

print(y)

#Plot new curve on top of each other

curve(-loglikelihood(x, Dataframe), xlab="Theta", from=0, to=20, add=FALSE, col="red", ylim=c(0,100))

curve(-loglikelihood(x, y), xlab="Theta", from=0, to=20, add=TRUE, col="blue", ylim=c(0,100))

#Conclusion: The graph is increasing at a much slower pace when only using the first six values compared with to the graph when we use all data. The model with more data is more reliable since there is a more certain min-value from the graph whereas the one with only six values, the theta value could be anything from the minimum value and forward.

**#4: Assume now a Bayesian model with p(x|theta)=theta\*e^(-theta\*x) and a prior p(theta)=lambda\*e^(-lambda\*x), lambda=10. Write a function computing l(theta)=log(p(x|theta)\*p(theta)). What kind of measure is actually computed by this function? Plot the curve showing the dependence of l(theta) on theta computed using the entire data and overlay it with a plot from step 2. Find an optimal theta and compare your result with the previous findings.**

#Compute a function for calculating the likelihood of the bayesian function

bayesian\_likelihood=function(theta, lambda, x){

n = length(x[,1])

return(n\*log(theta)-theta\*sum(x)-lambda\*theta)

}

#Find maximum likelihood value of theta

bayesian\_theta\_max = function(lambda, x){

n=length(x[,1])

return(n/(sum(x)+lambda))

}

#Find maxtheta

bayesian\_max\_theta = bayesian\_theta\_max(10, Dataframe)

print(bayesian\_max\_theta)

#Plot new curve on top of each other

curve(-bayesian\_likelihood(x, 10, Dataframe), ylab="-Loglikelihood", xlab="Theta", from=0, to=10, add=FALSE, col="red", ylim=c(20,300))

curve(-loglikelihood(x, Dataframe), ylab="-Loglikelihood", xlab="Theta", from=0, to=10, add=TRUE, col="blue", ylim=c(20,300))

#Conclusion: When using an bayesian model we have a prior that gives the model information beforehand which helps fitting the model. The optimal theta is now 0.91 which is close to the datasets meanvalue, which makes sense that this model gives a better predicted value.

**#5: Use theta value found in step 2 and generate 50 new observations from p(x|theta)=theta\*e^(-theta\*x) (use standard number generators). Create the histograms of the original and the new data and make conclusions.**

#Generate 50 new observation using theta value from step 2

set.seed(12345)

newdata = rexp(50, rate = max\_theta)

print(newdata)

#Plot new data and old data in histogram

olddata = Dataframe$Length

print(olddata)

hist(newdata)

hist(olddata)

#Conclusion: The histogram shows us that the distribution is fairly similar between the actual and predicted data. This concludes model was accurately fitted to the correct distribution model.

### Assignment 4 – Linear regression, Ridge, LASSO, stepAIC

**#1: Read data and plot Moisture vs Protein**

Dataframe=read.csv2("tecator\_csv.csv")

n = length(Dataframe[,1])

print(Dataframe)

moisture = Dataframe$Moisture

protein = Dataframe$Protein

fat = Dataframe$Fat

plot(moisture, protein, type="p", ylab="Protein", xlab="Moisture", col="red")

#Conclusion: Looks like a linear relation so a linear regression model is appropriate

**#2: Consider model Mi in which Moisture is normally distributed, and the expected Moisture is a polynomial function of Protein including the polynomial terms up to power of i (i.e. M1 is a linear model, M2 is a quadratic model and so on). Report a probabilistic model that describes Mi. Why is it appropriate to use MSE criterion when fitting this model to a training data?**

#Conclusion: A probabilistic model describing M(i) is: M(i) = w0 + w1 ∗ X + w2 ∗ X2 + ... + wi ∗ Xi (3) The MSE criterion is a suitable method since it punishes outliers to a larger extent. This creates a better fitted model compared to when you punish the absolute value. This reduces the risk of an overfitted model.

**#3: Divide the data into training and validation sets (50%/50%) and fit models Mi, i=1,...,6. For each model, record the training and the validation MSE and present a plot showing how training and validation MSE depend on i (write some R code to make this plot). Which model is best according to the plot? How do the MSE values change and why? Interpret this picture in terms of bias-variance tradeoff.**

colno\_protein = which(colnames(Dataframe)=="Protein")

colno\_moisture = which(colnames(Dataframe)=="Moisture")

moisture\_protein = Dataframe[colno\_protein:colno\_moisture]

set.seed(12345)

id=sample(1:n, floor(n\*0.5))

train=Dataframe[id,]

test=Dataframe[-id,]

moisture\_train = train$Moisture

moisture\_test = test$Moisture

#Create function for fitting linear regression models

fit\_moisture\_model = function(x) {

return(lm(formula = Moisture ~ poly(Protein, degree=x), data=train))

}

#Create predict function for training set

predict\_train = function(model){

return(predict(model, newdata = train))

}

#Create predict function for test set

predict\_test = function(model){

return(predict(model, newdata = test))

}

#Create function for calculating MSE, input parameters are vectors containing original data and predicted data

calcMSE = function(y, yhat){

return(sum((y-yhat)^2)/length(y))

}

#Create models and predictions and store MSE values in a vector for training and test data

vector\_train = c()

vector\_test = c()

for (i in 1:6){

fit = fit\_moisture\_model(i)

predicted\_train = predict\_train(fit)

predicted\_test = predict\_test(fit)

vector\_train[i] = calcMSE(moisture\_train, predicted\_train)

vector\_test[i] = calcMSE(moisture\_test, predicted\_test)

}

#Create numeric vector 1 through 6

models = c(1:6)

#Plot MSE for each model

plot(models, vector\_train, col="blue", xlab="Model", ylab="MSE")

par(new=TRUE)

plot(models, vector\_test, col="red", xlab="Model", ylab="MSE")

#Print MSE values

print(vector\_train)

print(vector\_test)

#Conclusion: Shown in the graphs we can see that for the tested values, M(3) has the lowest MSE and therefore being the model with the smallest error. In terms of bias, what we can see is that in the training model the predicted error, and MSE, is descending for a more complex model, because bias is defined by the ability for a model to fit data. This gives us an overfitted model, where we can see that the variance increases the more complex models because the MSE increases for the tested data and the MSE decreases for the training data. Variance is defined by the difference in predictions on different data sets.

#Use the entire data set in the following computations:

**#4: Fat response and Channel1-100 are predictors. Use stepAIC. How many variables were selected?**

#Fetch package stepAIC

#install.packages("MASS")

library("MASS")

#Perform stepAIC on fat

colno\_fat = which(colnames(Dataframe)=="Fat")

channel\_values = Dataframe[1:(colno\_fat-1)]

fit\_fat = lm(fat~., data = channel\_values)

step = stepAIC(fit\_fat, direction="both")

step$anova

summary(step)

#Conclusion: When we use StepAIC in total 63 variables were selected. These were chosen because not all variables are needed to predict a dependent variable.

**#5: Fit a Ridge regression model with same predictor and response variables. Plot model coefficient depend on the log of the penalty factor lambda and report how the coefficients change with lambda.**

#install.packages("glmnet")

library("glmnet")

covariates = scale(Dataframe[,2:(colno\_fat-1)])

response = scale(Dataframe[,colno\_fat])

ridge\_model = glmnet(as.matrix(covariates), response, alpha=0, family="gaussian")

plot(ridge\_model, xvar="lambda", label=TRUE)

#Coefficients goes towards 0 when lambda goes towards infinity

**#6: Repeat last step but with LASSO instead of Ridge. Differences?**

lasso\_model = glmnet(as.matrix(covariates), response, alpha=1, family="gaussian")

plot(lasso\_model, xvar="lambda", label=TRUE)

#Conclusion 5 and 6: The graphs below shows us that when we increase lambda fewer variables are selected to the models, since the coefficients goes towards zero. In the Lasso model, the penalty is the absolute value, and in the Ridge model the penalty is squared (penalizes large values more).

**#7: Choose the best model by cross validation for LASSO model. Report optimal lambda and how many variables that chosen by the model and make conclusions. Present also a plot showing the dependence of the CV score and comment how the CV score changes with lambda.**

lasso\_model\_optimal = cv.glmnet(as.matrix(covariates), response, alpha=1, family="gaussian", lambda=seq(0,1,0.001))

lasso\_model\_optimal$lambda.min

plot(lasso\_model\_optimal)

coef(lasso\_model\_optimal, s="lambda.min")

print(lasso\_model\_optimal$lambda.min)

#Conclusion: When the lambda increases in value, the MSE seems to strictly increase. From this model, we can make the conclusion that the optimal lambda = 0. This means, that all variables should be included for a better predicted model. Compared to the results from stepAIC, all variables where chosen since lambda = 0 instead of 63 that were chosen.

## Lab 2

### Assignment 1 - LDA and logistic regression. Draw decision boundary

**#1: Read data and plot carapace length versus rear width (obs coloured by sex). Do you think that this data is easy to classify by LDA? Motivate answer.**

RNGversion('3.5.1')

Dataframe=read.csv("australian-crabs.csv")

n = length(Dataframe[,1])

CL = Dataframe$CL

RW = Dataframe$RW

plot(CL, RW, main="Plot of carapace length versus rear width depending on sex", sub="Red = Female, Blue = Male",

col=c("red", "blue")[Dataframe$sex], xlab="CL", ylab="RW")

#Conclusion:

#Create function for misclassification rate

missclass=function(conf\_matrix, fit\_matrix){

n=length(fit\_matrix[,1])

return(1-sum(diag(conf\_matrix))/n)

}

**#2: LDA analysis with target Sex, and features CL and RW and proportional prior by using lda() function in package MASS Make a Scatter plot of CL versus RW colored by the predicted Sex and compare it with the plot in step 1. Compute the misclassification error and comment on the quality of fit.**

library("MASS")

model = lda(sex ~ CL+RW, data=Dataframe)

predicted = predict(model, data=Dataframe)

confusion\_matrix = table(Dataframe$sex, predicted$class)

misclass = missclass(confusion\_matrix, Dataframe)

print(confusion\_matrix)

print(misclass)

plot(CL, RW, main="Plot predicted values of CL and RW depending on sex", sub="Red = Female, Blue = Male",

col=c("red", "blue")[predicted$class], xlab="CL", ylab="RW")

**#3: Repeat step 2 but use priors p(Male)=0.9 and p(Female)=0.1**

model2 = lda(sex ~ CL+RW, data=Dataframe, prior=c(1,9)/10)

predicted2 = predict(model2, data=Dataframe)

confusion\_matrix2 = table(Dataframe$sex, predicted2$class)

misclass2 = missclass(confusion\_matrix2, Dataframe)

print(confusion\_matrix2)

print(misclass2)

plot(CL, RW, main="Plot predicted values of CL and RW with priors p(Male)=0.9 and p(Female)=0.1"

, sub="Red = Female, Blue = Male", col=c("red", "blue")[predicted2$class], xlab="CL", ylab="RW")

**#4: Repeat step 2 but now with logistic regression (use function glm()). Compare with LDA results. Finally, report the equation of the decision boundary and draw the decision boundary in the plot of the classified data.**

model3 = glm(sex ~ CL+RW, data=Dataframe, family='binomial')

predicted3 = predict(model3, newdata=Dataframe, type='response')

sexvector = c()

for (i in predicted3) {

if (i>0.9) {

sexvector = c(sexvector, 'Male')

} else {

sexvector = c(sexvector, 'Female')

}

}

print(sexvector)

sexvector\_factor = as.factor(sexvector)

confusion\_matrix3 = table(Dataframe$sex, sexvector\_factor)

misclass3 = missclass(confusion\_matrix3, Dataframe)

print(confusion\_matrix3)

print(misclass3)

plot(CL, RW, main="Plot predicted values of CL and RW but with logistic regression",

col=c("red", "blue")[sexvector\_factor], xlab="CL", ylab="RW", xlim=c(0,50), ylim=c(0,20))

boundaryline = function(length, coefficientvector, prior) {

return(-coefficientvector[1]/coefficientvector[3]-(coefficientvector[2]/coefficientvector[3])\*length+log(prior/(1-prior))/coefficientvector[3])

}

par(new=TRUE)

curve(boundaryline(x, model3$coefficients, 0.9), xlab="CL", ylab="RW", col="green", from=0, to=50, xlim=c(0,50), ylim=c(0,20),

sub="Red = Female, Blue = Male, Green = Boundaryline")

### Assignment 2 – Split method, Tree modeling (gini and deviance), Naïve Bayes, loss matrix

**#1: Read data and divide into train, validation and test sets as 50/25/25.**

library("tree")

RNGversion('3.5.1')

data=read.csv2("creditscoring.csv")

n=dim(data)[1]

set.seed(12345)

id=sample(1:n, floor(n\*0.5))

train=data[id,]

id1=setdiff(1:n, id)

set.seed(12345)

id2=sample(id1, floor(n\*0.25))

valid=data[id2,]

id3=setdiff(id1,id2)

test=data[id3,]

#Create function for misclassification rate

misclass=function(conf\_matrix, fit\_matrix){

n=length(fit\_matrix[,1])

return(1-sum(diag(conf\_matrix))/n)

}

**#2: Fit a decision tree to train data using the measures of impurity gini and deviance. Report misclass rates and choose optimal measure moving forward.**

fit\_deviance=tree(good\_bad~., data=train, split="deviance")

predicted\_deviance=predict(fit\_deviance, newdata=test, type="class")

confusionmatrix\_deviance=table(test$good\_bad, predicted\_deviance)

misclass\_deviance=misclass(confusionmatrix\_deviance, test)

print(confusionmatrix\_deviance)

print(misclass\_deviance)

fit\_gini=tree(good\_bad~., data=train, split="gini")

predicted\_gini=predict(fit\_gini, newdata=test, type="class")

confusionmatrix\_gini=table(test$good\_bad, predicted\_gini)

misclass\_gini=misclass(confusionmatrix\_gini, test)

print(confusionmatrix\_gini)

print(misclass\_gini)

#Deviance has best misclass score

**#3: Use training and valid data to choose optimal tree depth. Present graphs of the dependence of deviances for training and validation data on the number of leaves. Report optimal tree, report it's depth and variables used by tree. Estimate misclassification rate for the test data.**

fit\_optimaltree=tree(good\_bad~., data=train, split="deviance")

summary(fit\_optimaltree)

trainScore=rep(0,15)

testScore=rep(0,15)

for(i in 2:15){

prunedTree=prune.tree(fit\_optimaltree, best=i)

pred=predict(prunedTree, newdata=valid, type="tree")

#Divide by two since double of data points

trainScore[i]=deviance(prunedTree)/2

testScore[i]=deviance(pred)

}

plot(2:15, trainScore[2:15], type="b", col="red", ylim=c(200,500))

points(2:15, testScore[2:15], type="b", col="blue")

min\_deviance=min(testScore[2:15])

print(min\_deviance)

optimal\_leaves=which(testScore[1:15] == min\_deviance)

print(optimal\_leaves)

#Optimal no of leaves is 4

finalTree=prune.tree(fit\_optimaltree, best=4)

summary(finalTree)

plot(finalTree)

text(finalTree, pretty=0)

#Final tree contains variables savings, duration and history. Since 3 vars => Depth of tree is 3.

predicted\_test=predict(finalTree, newdata=test, type="class")

confusionmatrix\_test=table(test$good\_bad, predicted\_test)

misclass\_test=misclass(confusionmatrix\_test, test)

print(confusionmatrix\_test)

print(misclass\_test)

**#4: Use traning data to perform classification using Naives bayes and report the confusion matrices and misclassification rates for the traning and for the test data. Compare with results from previous steps.**

#Load libraries

library(MASS)

library(e1071)

fit\_naive=naiveBayes(good\_bad~., data=train)

#Create function for predicting and creating confusion matrice and printing misclassification rate

compute\_naive=function(model,data){

predictedNaive=predict(model, newdata=data, type="class")

confusionmatrixNaive=table(data$good\_bad,predictedNaive)

misclass = misclass(confusionmatrixNaive, data)

print(confusionmatrixNaive)

print(misclass)

return(predictedNaive)

}

predictedNaive\_train=compute\_naive(fit\_naive,train)

predictedNaive\_test=compute\_naive(fit\_naive, test)

**#5: Use optimal tree and Naives Bayes to classify the test data by using principle: classified as 1 if 'good' bigger than 0.05, 0.1, 0.15, ..., 0.9, 0.95. Compute the TPR and FPR for two models and plot corresponsing ROC curves.**

#Writing function for classifying data

class=function(data, class1, class2, prior){

vector=c()

for(i in data) {

if(i>prior){

vector=c(vector,class1)

} else {

vector=c(vector,class2)

}

}

return(vector)

}

x\_vector=seq(0.05,0.95,0.05)

tpr\_tree=c()

fpr\_tree=c()

tpr\_naive=c()

fpr\_naive=c()

treeVector=c()

treeConfusion = c()

naiveConfusion = c()

treeClass = c()

naiveClass = c()

#Reusing optimal tree found in task 3 but returntype is response instead

set.seed(12345)

predictTree=data.frame(predict(finalTree, newdata=test, type="vector"))

predictNaive=data.frame(predict(fit\_naive, newdata=test, type="raw"))

for(prior in x\_vector){

treeClass = class(predictTree$good, 'good', 'bad', prior)

treeConfusion=table(test$good\_bad, treeClass)

if(ncol(treeConfusion)==1){

if(colnames(treeConfusion)=="good"){

treeConfusion=cbind(c(0,0), treeConfusion)

} else {

treeConfusion=cbind(treeConfusion,c(0,0))

}

}

totGood=sum(treeConfusion[2,])

totBad=sum(treeConfusion[1,])

tpr\_tree=c(tpr\_tree, treeConfusion[2,2]/totGood)

fpr\_tree=c(fpr\_tree, treeConfusion[1,2]/totBad)

print(fpr\_tree)

naiveClass=class(predictNaive$good, 'good', 'bad', prior)

naiveConfusion=table(test$good\_bad, naiveClass)

if(ncol(naiveConfusion)==1){

if(colnames(naiveConfusion)=="good"){

naiveConfusion=cbind(c(0,0), naiveConfusion)

} else {

naiveConfusion=cbind(naiveConfusion,c(0,0))

}

}

totGood=sum(naiveConfusion[2,])

totBad=sum(naiveConfusion[1,])

tpr\_naive=c(tpr\_naive, naiveConfusion[2,2]/totGood)

fpr\_naive=c(fpr\_naive, naiveConfusion[1,2]/totBad)

}

#Plot the ROC curves

plot(fpr\_naive, tpr\_naive, main="ROC curve", sub="Red = Naive Bayes, Blue = Tree", type="l", col="red", xlim=c(0,1),

ylim=c(0,1), xlab="FPR", ylab="TPR")

points(fpr\_tree, tpr\_tree, type="l", col="blue")

#Naive has greatest AOC => should choose Naive

**#6: Repeat Naive Bayes with loss matrix punishing with factor 10 if predicting good when bad and 1 if predicting bad when good.**

naiveModel=naiveBayes(good\_bad~., data=train)

train\_loss=predict(naiveModel, newdata=train, type="raw")

test\_loss=predict(naiveModel, newdata=test, type="raw")

confusion\_trainLoss=table(train$good\_bad, ifelse(train\_loss[,2]/train\_loss[,1]>10, "good", "bad"))

misclass\_trainLoss=misclass(confusion\_trainLoss, train)

print(confusion\_trainLoss)

print(misclass\_trainLoss)

confusion\_testLoss=table(test$good\_bad, ifelse(test\_loss[,2]/test\_loss[,1]>10, "good", "bad"))

misclass\_testLoss=misclass(confusion\_testLoss, test)

print(confusion\_testLoss)

print(misclass\_testLoss)

#The misclassification rates have changed since a higher punishment is given when predicting good creditscore when in fact it was bad (reasonable since bank loses money then). It is less worse to predict bad creditscore but turns out to be good (just a loss of customer). Due to this more errors occur mainly because fewer people are classified to have good creditscores.

### Assignment 4 – Principal components, PCA analysis, ICA analysis, Trace plots

#Read data

RNGversion('3.5.1')

data=read.csv2("NIRspectra.csv")

data$Viscosity=c()

n=dim(data)[1]

**#1: Conduct standard PCA using the feature space and provide a plot explaining how much variation is explained by each feature. Provide plot that show the scores of PC1 vs PC2. Are there unusual diesel fuels according to this plot.**

pcaAnalysis=prcomp(data)

lambda=pcaAnalysis$sdev^2

#Eigenvalues

print(lambda)

#Proportion of variation

propVar= lambda/sum(lambda)\*100

screeplot(pcaAnalysis)

print(propVar)

noOfVars=1

sumOfVariation=propVar[noOfVars]

while(sumOfVariation<99){

noOfVars=noOfVars+1

sumOfVariation=sumOfVariation+propVar[noOfVars]

}

#Print number of variables used

print(noOfVars)

#Print PC1 and PC2 in plot

plot(pcaAnalysis$x[,1],pcaAnalysis$x[,2], ylim=c(-10,10), type="p", col="blue", main="PC1 vs PC2", xlab="PC1", ylab="PC2")

#We can see from the graph that the data is very accurately described by PC1.

**#2: Make trace plots of the loadings of the components selected in step 1. Is there any principle component that is explained by mainly a few original features?**

U=pcaAnalysis$rotation

plot(U[,1], main="Traceplot, PC1", xlab="index", ylab="PC1", type="b")

plot(U[,2], main="Traceplot, PC2", xlab="index", ylab="PC2", type="b")

#We can see from graph that PC2 is not described by so many original features since it is close to zero for many of

#the features. The last 30 or so variables have an effect on PC2.

**#3: Perform independent Component Analysis (ICA) with no of components selected in step1 (set seed 12345). Check the documentation of R for fastICA method and do following:**

**# Compute W'=K\*W and present columns of W' in form of the trace plots. Compare with trace plots in step 2 and make**

**# conclusions. What kind of measure is represented by the matrix W'.**

**# Make a plot of the scores of the first two latent features and compare it with the score plot from step 1.**

#Install package fastICa

#install.packages("fastICA")

library("fastICA")

set.seed(12345)

icaModel = fastICA(data, n.comp=2, verbose=TRUE)

W=icaModel$W

K=icaModel$K

W\_est=K%\*%W

plot(W\_est[,1], main="Traceplot, ICA1", xlab="index", ylab="ICA1", type="b", col="red")

plot(W\_est[,2], main="Traceplot, ICA2", xlab="index", ylab="ICA2", type="b", col="red")

#Compared to the plots in step 2 the ICA1 follows in roughly the same pattern as PCA2 and ICA2 the same as PCA1.

plot(icaModel$S[,1], icaModel$S[,2], main="ICA1 vs ICA2", xlab="ICA1", ylab="ICA2", type="p", col="blue")

#We can see from the plot that the dat is pretty well described by ICA2 whereas ICA1 is not that significant in describing the data (since it is close to 0 most of the cases). Some outliers are however described by ICA1.

## Lab 3

### Assignment 1 – Kernel methods, Smoothing coefficients

RNGversion('3.5.1')

## Assignment 1:

## Implement a kernel method to predict the hourly temperatures for a date and place in Sweden.

## To do so, you are provided with the files stations.csv and temps50k.csv. These

## files contain information about weather stations and temperature measurements in the stations

## at different days and times. The data have been kindly provided by the Swedish Meteorological

## and Hydrological Institute (SMHI).

## You are asked to provide a temperature forecast for a date and place in Sweden. The

## forecast should consist of the predicted temperatures from 4 am to 24 pm in an interval of 2

## hours. Use a kernel that is the sum of three Gaussian kernels:

## The first to account for the distance from a station to the point of interest.

## The second to account for the distance between the day a temperature measurement

## was made and the day of interest.

## The third to account for the distance between the hour of the day a temperature measurement

## was made and the hour of interest.

## Choose an appropriate smoothing coefficient or width for each of the three kernels above.

## Answer to the following questions:

## Show that your choice for the kernels' width is sensible, i.e. that it gives more weight

## to closer points. Discuss why your of definition of closeness is reasonable.

## Instead of combining the three kernels into one by summing them up, multiply them.

## Compare the results obtained in both cases and elaborate on why they may differ.

## Note that the file temps50k.csv may contain temperature measurements that are posterior

## to the day and hour of your forecast. You must filter such measurements out, i.e. they cannot

## be used to compute the forecast. Feel free to use the template below to solve the assignment.

set.seed(1234567890)

#install.packages("geosphere")

library(geosphere)

stations <- read.csv("stations.csv")

temps <- read.csv("temps50k.csv")

#A join operation on "station\_number"

st <- merge(stations,temps,by="station\_number")

n = dim(st)[1]

#Kernel weighting factors

h\_distance <- 100000

h\_date <- 20

h\_time <- 2

#Latitude of interest

a <- 59.4059

#Longitude of interest

b <- 18.0256

#Coordinates for Danderyd

#Create a vector of the point of interest

placeOI = c(a, b)

dateOI <- as.Date("1995-07-29") # The date to predict (up to the students), my birth date

timesOI = c("04:00:00", "06:00:00", "08:00:00", "10:00:00", "12:00:00", "14:00:00", "16:00:00", "18:00:00", "20:00:00",

"22:00:00", "24:00:00")

plotDist = function(dist, h){

u = dist/h

plot(dist, exp(-u^2), type="l", main="Plot of kernel wights for distances", xlab="Distance")

}

dist = seq(0, 100000, 1)

plotDist(dist, h\_distance)

plotDate = function(date, h){

u = date/h

plot(date, exp(-u^2), type="l", main="Plot of kernel wights for dates", xlab="Days")

}

date = seq(-182,182,1)

plotDate(date, h\_date)

plotTime = function(time, h){

u = time/h

plot(time, exp(-u^2), type="l", main="Plot of kernel wights for time", xlab="Hours")

}

time = seq(-12,12,1)

plotTime(time, h\_time)

#Remove posterior data

filter\_posterior = function(date, time, data){

return(data[which(as.numeric(difftime(strptime(paste(date, time, sep=" "), format="%Y-%m-%d %H:%M:%S"),

strptime(paste(data$date, data$time, sep=" "),format="%Y-%m-%d %H:%M:%S")))>0), ])

}

#A gaussian function for the difference in distance

gaussian\_dist = function(place, data, h) {

lat = data$latitude

long = data$longitude

points = data.frame(lat,long)

u = distHaversine(points, place)/h

return (exp(-u^2))

}

xy = gaussian\_dist(placeOI, st, h\_distance)

#A gaussian function for difference in days

gaussian\_day = function(date, data, h){

compare\_date = as.Date(data$date)

diff = as.numeric(date-compare\_date)

for (i in 1:length(diff)) {

if (diff[i] > 365) {

diff[i] = diff[i] %% 365

if(diff[i]>182){

diff[i]=365-diff[i]

}

}

}

u = diff/h

return (exp(-u^2))

}

#A gaussian function for difference in hours

gaussian\_hour = function(hour, data, h){

compare\_hour = strptime(data$time, format="%H:%M:%S")

compare\_hour = as.numeric(format(compare\_hour, format="%H"))

hour = strptime(hour, format="%H:%M:%S")

hour = as.numeric(format(hour, format="%H"))

diff = abs(hour-compare\_hour)

for (i in 1:length(diff)){

if(diff[i]>12){

diff[i] = 24-diff[i]

}

}

u=diff/h

return(exp(-u^2))

}

#Defining values that will be used in loop below

kernel\_sum = c()

kernel\_mult = c()

#Looping through time array and data points in nested loop to calculate the 11 kernel values

for (time in timesOI) {

filtered\_data = filter\_posterior(dateOI, time, st)

kernel\_dist = gaussian\_dist(placeOI, filtered\_data, h\_distance)

kernel\_day = gaussian\_day(dateOI, filtered\_data, h\_date)

kernel\_time = gaussian\_hour(time, filtered\_data, h\_time)

sum\_kernel = kernel\_dist+kernel\_day+kernel\_time

temp\_sum = sum(sum\_kernel \* filtered\_data$air\_temperature)/sum(sum\_kernel)

mult\_kernel = kernel\_dist\*kernel\_day\*kernel\_time

temp\_mult = sum(mult\_kernel \* filtered\_data$air\_temperature)/sum(mult\_kernel)

kernel\_sum = c(kernel\_sum, temp\_sum)

kernel\_mult = c(kernel\_mult, temp\_mult)

}

plot(kernel\_sum, type="o", main ="Temperature estimate through sum of factors", xlab="Time",

ylab="Est. temperature")

axis(1, at=1:length(timesOI), labels=timesOI)

plot(kernel\_mult, type="o", main="Temperature estimate through product of factors", xlab="Time",

ylab="Est. temperature")

axis(1, at=1:length(timesOI), labels=(timesOI))

### Assignment 3 – Neural networks, weight initialization

## Assignment3:

## Train a neural network to learn the trigonometric sine function. To do so, sample 50 points

## uniformly at random in the interval [0,10]. Apply the sine function to each point. The resulting

## pairs are the data available to you. Use 25 of the 50 points for training and the rest for validation.

## The validation set is used for early stop of the gradient descent. That is, you should

## use the validation set to detect when to stop the gradient descent and so avoid overfitting.

## Stop the gradient descent when the partial derivatives of the error function are below a given

## threshold value. Check the argument threshold in the documentation.Consider threshold

## values i/1000 with i = 1,...,10. Initialize the weights of the neural network to random values in

## the interval [-1, 1]. Use a neural network with a single hidden layer of 10 units. Use the default values

## for the arguments not mentioned here. Choose the most appropriate value for

## the threshold. Motivate your choice. Provide the final neural network learned with the chosen

## threshold. Feel free to use the following template.

RNGversion('3.5.1')

#install.packages("neuralnet")

library(neuralnet)

set.seed(1234567890)

Var <- runif(50, 0, 10)

trva <- data.frame(Var, Sin=sin(Var))

train <- trva[1:25,] # Training

valid <- trva[26:50,] # Validation

n = dim(valid)[1]

# Random initialization of the weights in the interval [-1, 1]

winit <- runif(31, -1, 1)

trainScore = rep(0,10)

validScore = rep(0,10)

for(i in 1:10) {

nn\_temp <- neuralnet(Sin~Var, data=train, hidden=10, threshold=i/1000, startweights=winit)

nn = as.data.frame(nn\_temp$net.result)

pred=predict(nn\_temp, newdata=valid)

trainScore[i] = 1/n\*sum((nn[,1]-train$Sin)^2)

validScore[i] = 1/n\*sum((pred-valid$Sin)^2)

}

plot(1:10, trainScore[1:10], type="b", col="red", xlab="Threshold index", ylab="MSE")

points(1:10, validScore[1:10], type="b", col="blue")

min\_error=min(validScore[1:10])

print(min\_error)

optimal\_i=which(validScore[1:10] == min\_error)

print(optimal\_i)

optimal\_nn = neuralnet(Sin~Var, data=train, hidden=10, threshold=optimal\_i/1000, startweights=winit)

plot(optimal\_nn)

# Plot of the predictions (black dots) and the data (red dots)

plot(prediction(optimal\_nn)$rep1)

points(trva, col = "red")

# R

## Examples

### Linear regression

fit=lm(formula, data, subset, weights,…)

* data is the data frame containing the predictors and response values
* formula is expression for the model
* subset which observations to use (training data)?
* weights should weights be used?

fit is object of class lm containing various regression results.

• Useful functions (many are generic, used in many other models)

– Get details about the particular function by ”.”, for ex. predict.lm

summary(fit)

predict(fit, newdata, se.fit, interval)

coefficients(fit) # model coefficients

confint(fit, level=0.95) # CIs for model parameters

fitted(fit) # predicted values

residuals(fit) # residuals

**Example of ordinary least squares regression**

mydata=read.csv2("Bilexempel.csv")

fit1=lm(Price~Year, data=mydata)

summary(fit1)

fit2=lm(Price~Year+Mileage+Equipment, data=mydata)

summary(fit2)

### Linear regression – confidence intervals

fitted <- predict(fit1, interval = "confidence")

# plot the data and the fitted line

attach(mydata)

plot(Year, Price)

lines(Year, fitted[, "fit"])

# plot the confidence bands

lines(Year, fitted[, "lwr"], lty = "dotted", col="blue")

lines(Year, fitted[, "upr"], lty = "dotted", col="blue")

detach(mydata)

### Ridge regression

Use package glmnet with alpha = 0 (Ridge)

data=read.csv("machine.csv", header=F)

covariates=scale(data[,3:8])

response=scale(data[, 9])

model0=glmnet(as.matrix(covariates), response, alpha=0,family="gaussian")

plot(model0, xvar="lambda", label=TRUE)

**Choosing best model using cross-validation**

model=cv.glmnet(as.matrix(covariates), response, alpha=0,family="gaussian")

model$lambda.min

plot(model)

coef(model, s="lambda.min")

**How good is this model in prediction?**

ind=sample(209, floor(209\*0.5))

data1=scale(data[,3:9])

train=data1[ind,]

test=data1[-ind,]

covariates=train[,1:6]

response=train[, 7]

model=cv.glmnet(as.matrix(covariates), response, alpha=1,family="gaussian", lambda=seq(0,1,0.001))

y=test[,7]

ynew=predict(model, newx=as.matrix(test[, 1:6]), type="response")

#Coefficient of determination

sum((ynew-mean(y))^2)/sum((y-mean(y))^2)

sum((ynew-y)^2)

### Lasso regression

Use package glmnet with alpha = 1 (LASSO)

### Stepwise selection (stepAIC, specify which way under *direction*)

library(MASS)

fit <- lm(V9~.,data=data.frame(data1))

step <- stepAIC(fit, direction="both")

step$anova

summary(step)

### Holdout method (dividation into train, valid, test)

**How to partition into train/test**  
  
n=dim(data)[1]

set.seed(12345)

id=sample(1:n, floor(n\*0.7))

train=data[id,]

test=data[-id,]

**How to partition into train/valid/test**

n=dim(data)[1]

set.seed(12345)

id=sample(1:n, floor(n\*0.4))

train=data[id,]

id1=setdiff(1:n, id)

set.seed(12345)

id2=sample(id1, floor(n\*0.3))

valid=data[id2,]

id3=setdiff(id1,id2)

test=data[id3,]

### Cross-validation – K folds and different predictor sets

**Try models with different predictor sets**

data=read.csv("machine.csv", header=F)

library(cvTools)

fit1=lm(V9~V3+V4+V5+V6+V7+V8, data=data)

fit2=lm(V9~V3+V4+V5+V6+V7, data=data)

fit3=lm(V9~V3+V4+V5+V6, data=data)

f1=cvFit(fit1, y=data$V9, data=data,K=10, foldType=" consecutive")

f2=cvFit(fit2, y=data$V9, data=data,K=10, foldType=" consecutive")

f3=cvFit(fit3, y=data$V9, data=data,K=10, foldType=" consecutive")

res=cvSelect(f1,f2,f3)

plot(res)

### Logistic regression

Use glm() with family=”binomial”

* Predicted probabilities: predict(fit, newdata, type=”response”)

### Linear Discriminative Analysis (LDA) and Quadratic Discriminative Analysis (QDA)

Syntax in R: Library MASS

lda(formula, data, ..., subset, na.action)  
•Prior – classprobabiliies  
•Subset– indices, if training data should be used   
  
qda(formula, data, ..., subset, na.action)  
predict(..)

### Naïve bayes

Use package e1071

library(MASS)

library(e1071)

n=dim(housing)[1]

ind=rep(1:n, housing[,5])

housing1=housing[ind,-5]

fit=naiveBayes(Sat~., data=housing1)

fitYfit=predict(fit, newdata=housing1)

table(Yfit,housing1$Sat)

### Decision trees

Use package *tree*, alternative *rpart*

tree(formula, data, weights, control, split = c("deviance", "gini"), ...)

print(), summary(), plot(), text()

**Example**

library(tree)

n=dim(biopsy)[1]

fit=tree(class~., data=biopsy)

plot(fit)

text(fit, pretty=0)

fit

summary(fit)

**Misclassification results**

Yfit=predict(fit, newdata=biopsy, type="class")

table(biopsy$class,Yfit)

**Selecting optimal tree by penalizing**

* Use Cv.Tree()

set.seed(12345)

ind=sample(1:n, floor(0.5\*n))

train=biopsy[ind,

]valid=biopsy[-ind,]

fit=tree(class~.,data=train)

set.seed(12345)

cv.res=cv.tree(fit)

plot(cv.res$size, cv.res$dev, type="b", col="red")

plot(log(cv.res$k), cv.res$dev, type="b", col="red")

**Selecting optimal tree by train/validation**

fit=tree(class~., data=train)

trainScore=rep(0,9)

testScore=rep(0,9)

for(i in 2:9) {

prunedTree=prune.tree(fit,best=i)

pred=predict(prunedTree, newdata=valid, type="tree")

trainScore[i]=deviance(prunedTree)

testScore[i]=deviance(pred)

}

plot(2:9, trainScore[2:9], type="b", col="red", ylim=c(0,250))

points(2:9, testScore[2:9], type="b", col="blue")

**Use final tree decided by methods above**

finalTree=prune.tree(fit, best=5)

Yfit=predict(finalTree, newdata=valid, type="class")

table(valid$class,Yfit)

### Least Absolute Deviation (LAD) regression

Use package L1pack

### Bootstrap

Use package boot

Functions:

* Boot()
* Boot.ci() – 1 parameter
* Envelope() – many parameters

Example: boot(data, statistic, R, sim=”ordinary”, ran.gen = function(d, p) d, mle=NULL, …)

Random random generation for parametric bootstrap:

* Rnorm()
* Runif()
* …

**Nonparametric bootstrap:**

* Write a function statistic that depends on dataframe and index and returns the estimator

library(boot)

data2=data[order(data$Area),]#reorderingdata accordingto Area

# computingbootstrapsamples

f=function(data, ind){

data1=data[ind,]# extractbootstrapsample

res=lm(Price~Area, data=data1) #fit linearmodel

#predictvaluesfor all Area valuesfrom the original data

priceP=predict(res,newdata=data2)

return(priceP)

}

res=boot(data2, f, R=1000) #make bootstrap

**Parametric bootstrap**

mle=lm(Price~Area, data=data2)

rng=function(data, mle ) {

data1=data.frame(Price=data$Price, Area=data$Area)

n=length(data$Price)

#generatenew Price

data1$Price=rnorm(n,predict(mle, newdata=data1),sd(mle$residuals))

return(data1)

}

f1=function(data1){

res=lm(Price~Area, data=data1) #fit linearmodel

#predictvaluesfor all Area valuesfrom the original data

priceP=predict(res,newdata=data2)

return(priceP)

}

res=boot(data2, statistic=f1, R=1000, mle =mle,ran.gen=rng , sim="parametric")

**Bootstrap confidence bands for linear model**

e=envelope(res) #computeconfidencebands

fit=lm(Price~Area, data=data2)

priceP=predict(fit)

plot(Area, Price, pch=21, bg="orange")

points(data2$Area,priceP,type="l") #plotfittedline

#plotcofidencebands

points(data2$Area,e$point[2,], type="l", col="blue")

points(data2$Area,e$point[1,], type="l", col="blue")

**Bootstrap prediction bands for linear model**

mle=lm(Price~Area, data=data2)

f1=function(data1){

res=lm(Price~Area, data=data1) #fit linearmodel

#predictvaluesfor all Area valuesfrom the original data

priceP=predict(res,newdata=data2)

n=length(data2$Price)

predictedP=rnorm(n,priceP, sd(mle$residuals))

return(predictedP)

}

res=boot(data2, statistic=f1, R=10000, mle=mle, ran.gen=rng, sim="parametric")

### Principle Component Analysis (PCA)

* Formulas: Prcomp(), biplot(), screeplot()
* Use package pls for making Principle component regression

mydata=read.csv2("tecator.csv")

data1=mydata

data1$Fat=c()

res=prcomp(data1)

lambda=res$sdev^2

#eigenvalues

lambda

#proportion of variation

sprintf("%2.3f",lambda/sum(lambda)\*100)

screeplot(res)

**Principal component loadings (U)**

U=res$rotation

head(U)

**Data in (PC1, PC2) – scores (Z)**

plot(res$x[,1], res$x[,2], ylim=c(-5,15))

**Trace plots**

U= res$rotation

plot(U[,1], main="Traceplot, PC1")

plot(U[,2],main="Traceplot, PC2")

### Probabilistic PCA

* Use pcaMethods from Bioconductor

### Independent Component Analysis (ICA)

R package: fastICA

S <- cbind(sin((1:1000)/20), rep((((1:200)-100)/100), 5))

A < - matrix(c(0.291, 0.6557, -0.5439, 0.5572), 2, 2)

X < - S %\*% A #mixing signals

a < - fastICA(X, 2) #now separate them

### Distance between geographical points of interest

Use package geosphere

Use function distHaversine(x, y) which returns the distance between the two points whereas x and y are 2 dimensional vectors/data frames (i.e. pairs of coordinates of latitude and longitude)

### Neural networks

Use package neuralnet

Use function neuralnet(formula, data, hidden, threshold, startweights)

* Formula – X~Y
* Data – the data used
* Hidden – the number of nodes in each hidden layers (can be vector of c(5, 3) which corresponds to first hidden layer with 5 nodes and second hidden layer with 3 nodes)
* Threshold – the threshold of the partial derivatives of the error function as stopping criteria
* Startweights – a vector containing starting values of the weights (needs to be initialized), if NULL random initialization