Computer Lab 4

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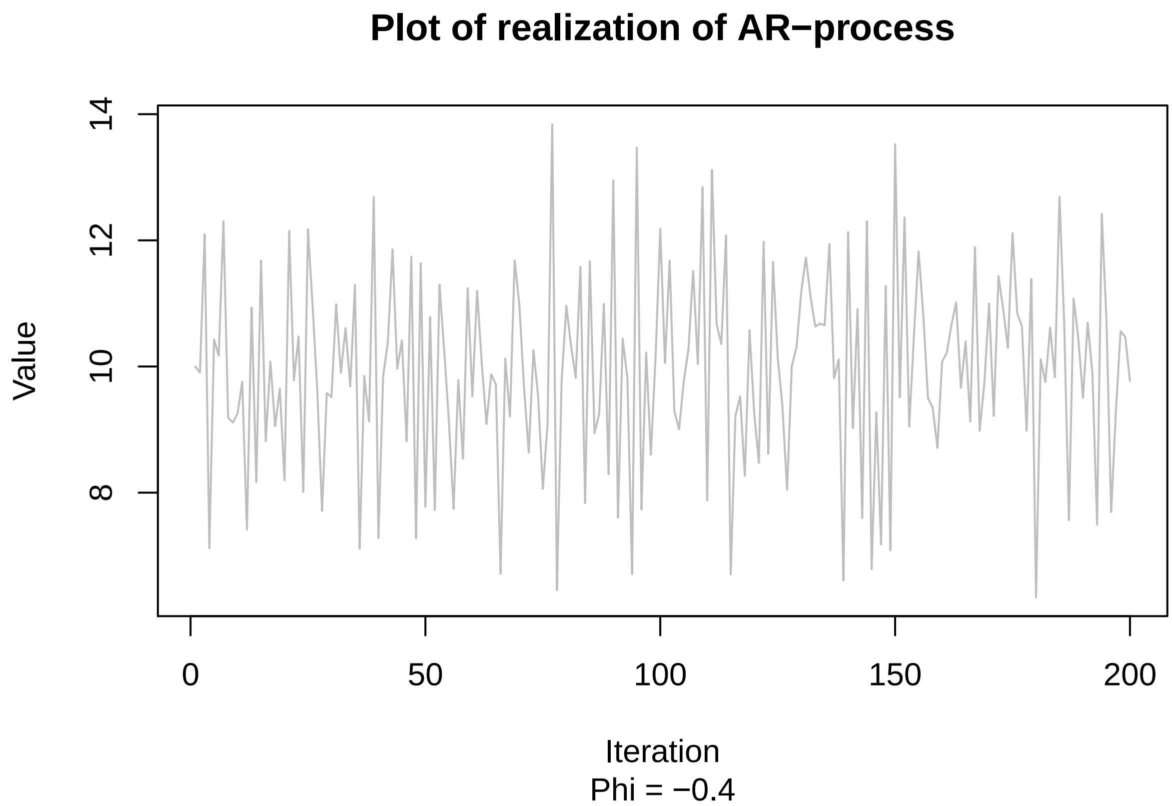
Assignment 1

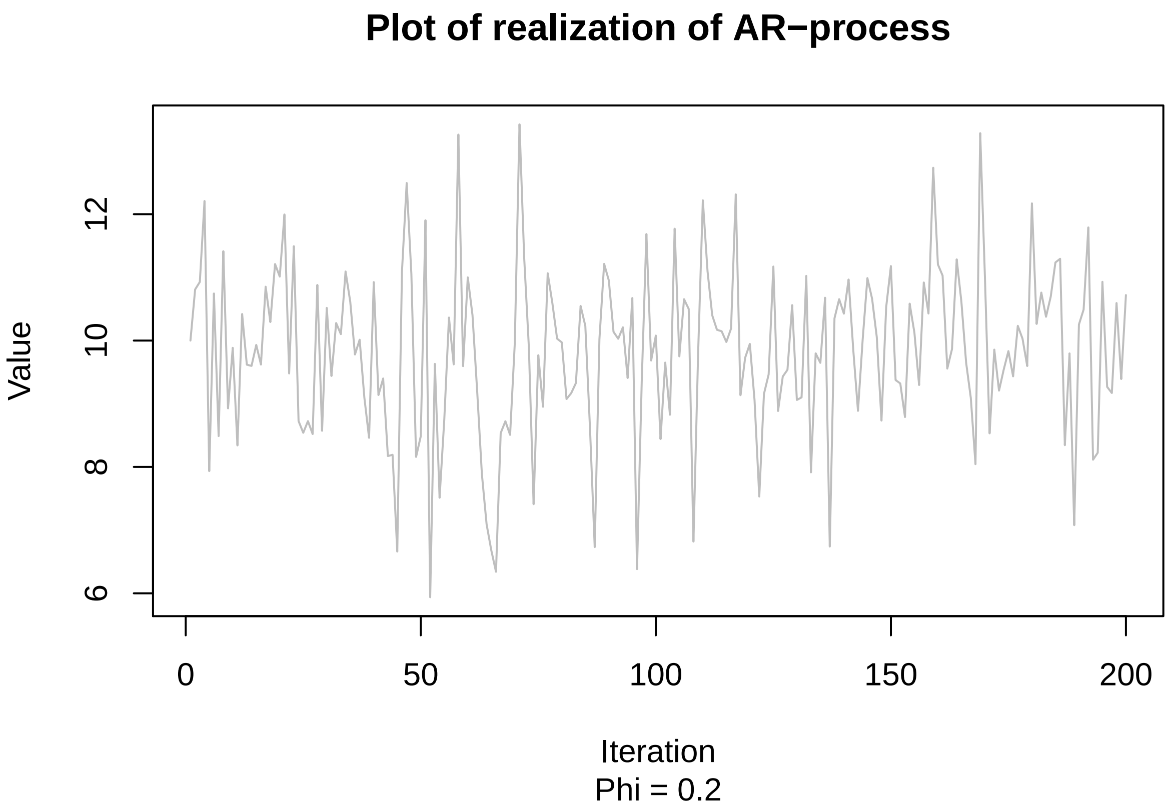
a)

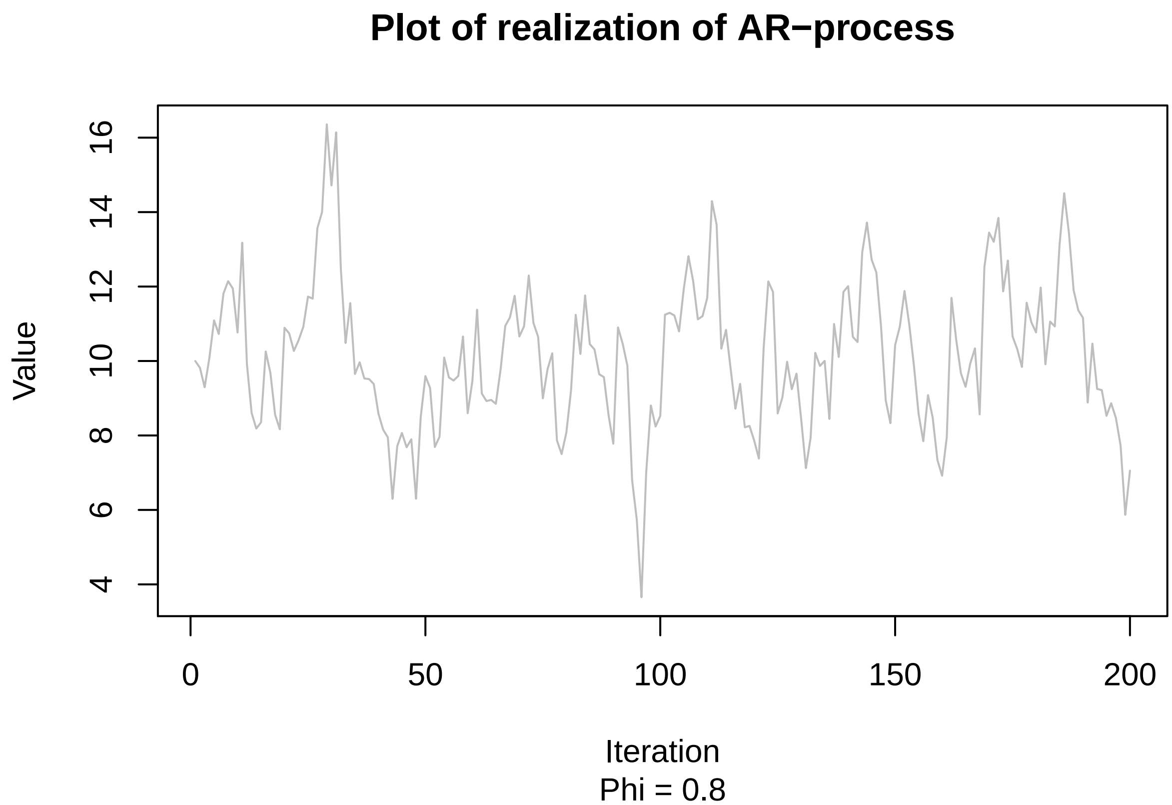
```
#install.packages("rstan")
mu=10
sigma_sq=2
T=200
x_init=mu
phi_vector=seq(-0.9,0.9,0.1)
results_matrix=matrix(0,200,length(phi_vector))
results_matrix[1,]=x_init
counter=1
set.seed(12345)

AR_process_function=function(mu, sigma_sq, T, phi) {
  x_init=mu
  result=rep(0,T)
  result[1]=x_init
  for (i in 2:T) {
    epsilon=rnorm(1,0,sqrt(sigma_sq))
    result[i]=mu+phi*(result[i-1]-mu)+epsilon
  }
  return(result)
}

results_matrix=matrix(0,T,length(phi_vector))
counter=1
for (phi in phi_vector) {
  results_matrix[,counter]=AR_process_function(mu,sigma_sq,T,phi)
  counter=counter+1
}
iter=seq(1,200,1)
counter=1
for (i in 1:length(phi_vector)) {
  if (counter %% 6 == 0) {
    plot(iter, results_matrix[,i], main="Plot of realization of AR-process",
         sub=paste("Phi =", phi_vector[i]),
         xlab="Iteration", ylab="Value", type="l", col="grey")
  }
  counter=counter+1
}
```


Plot of realization of AR−process
Value
14
12
10
8
0
50
100
150
200
Iteration
Phi = −0.4


Plot of realization of AR−process
Value
12
10
8
6
0
50
100
150
200
Iteration
Phi = 0.2



With ϕ-values below zero the process will oscillate faster but with ϕ-values above zero the process will be more correlated. The correlation between the different iterations increases as the phi-value becomes larger. This causes the oscillation to slow down and the process to move more slowly. This is also visualized in the above plots; when $\phi = -0.4$ the process oscillates much and moves back and forth, when $\phi = 0.2$ the process still oscillates quite much but less than before and finally when $\phi = 0.8$ we can see that the process moves more slowly and oscillates less - the correlation between the shorter lags are larger.

b)

```r
library(rstan)
```

```
## Warning: package 'rstan' was built under R version 3.6.3
```

```
## Loading required package: StanHeaders
```

```
## Warning: package 'StanHeaders' was built under R version 3.6.2
```

```
## Loading required package: ggplot2
```

```
## Warning: package 'ggplot2' was built under R version 3.6.3
```

```
## rstan (Version 2.19.3, GitRev: 2e1f913d3ca3)
```

```
## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)
```

```
## For improved execution time, we recommend calling
## Sys.setenv(LOCAL_CPPFLAGS = '-march=corei7 -mtune=corei7')
```

```
## although this causes Stan to throw an error on a few processors.
```

```
x=rep(0,T)
y=rep(0,T)
set.seed(12345)
x=AR_process_function(mu, sigma_sq, T, 0.3)
set.seed(12345)
y=AR_process_function(mu, sigma_sq, T, 0.95)

StanModel= '
data {
  int<lower=0> N;
  vector[N] y;
}

parameters {
  real mu;
  real phi;
  real<lower=0> sigma;
}
model {
  for (n in 2:N)
    y[n] ~ normal(mu + phi * (y[n-1]-mu), sigma);
}
'

data_x=list(N=T, y=x)
data_y=list(N=T, y=y)
fit_x=stan(model_code=StanModel, data=data_x)
fit_y=stan(model_code=StanModel, data=data_y)
```

```
## Warning: There were 27 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quant
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
library(rstan)
postDraws_x <- extract(fit_x)
postDraws_y <- extract(fit_y)
print(fit_x)
```

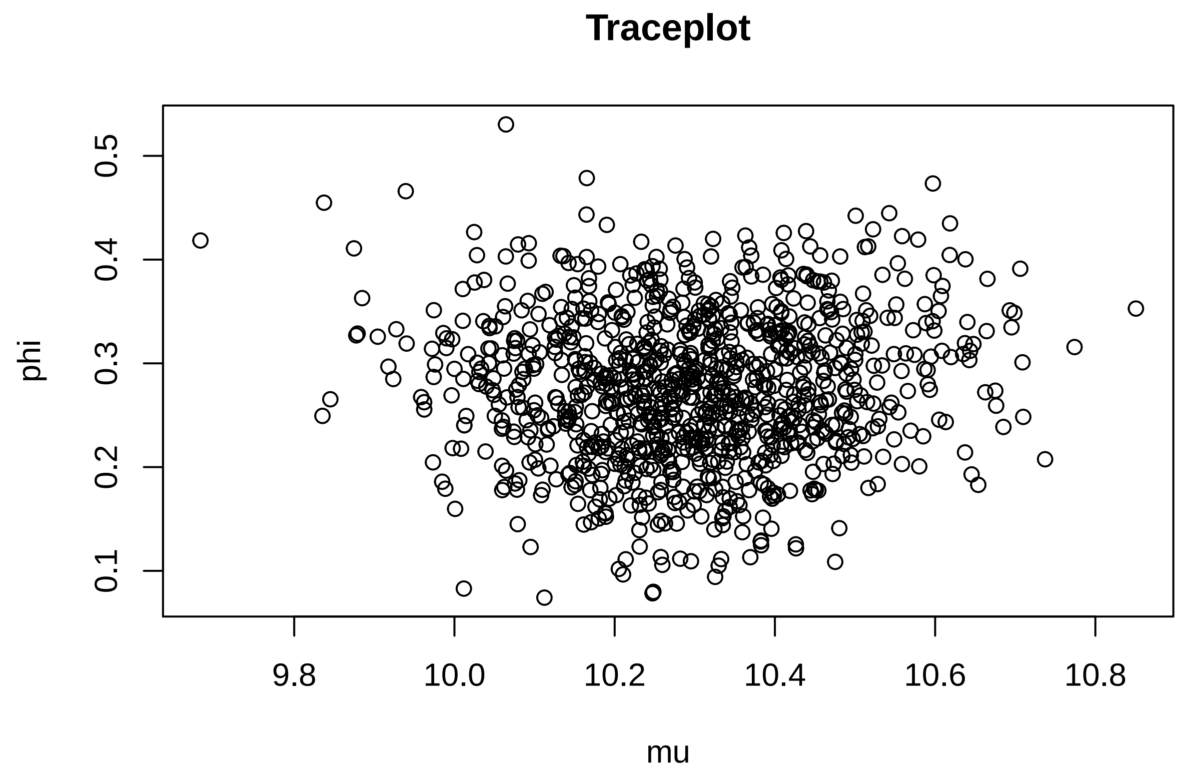
```
## Inference for Stan model: 59560b14e3970f232803cf9ed9d888cd.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##          mean se_mean   sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
## mu      10.30    0.00 0.15   10.00   10.19   10.29   10.40   10.60  3551    1
## phi      0.28    0.00 0.07    0.14    0.23    0.28    0.33    0.42  3293    1
## sigma    1.52    0.00 0.08    1.38    1.47    1.52    1.57    1.69  3513    1
## lp__  -182.29    0.03 1.23 -185.47 -182.89 -181.99 -181.37 -180.85  1974    1
##
## Samples were drawn using NUTS(diag_e) at Thu May 21 15:46:41 2020.
```

```
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

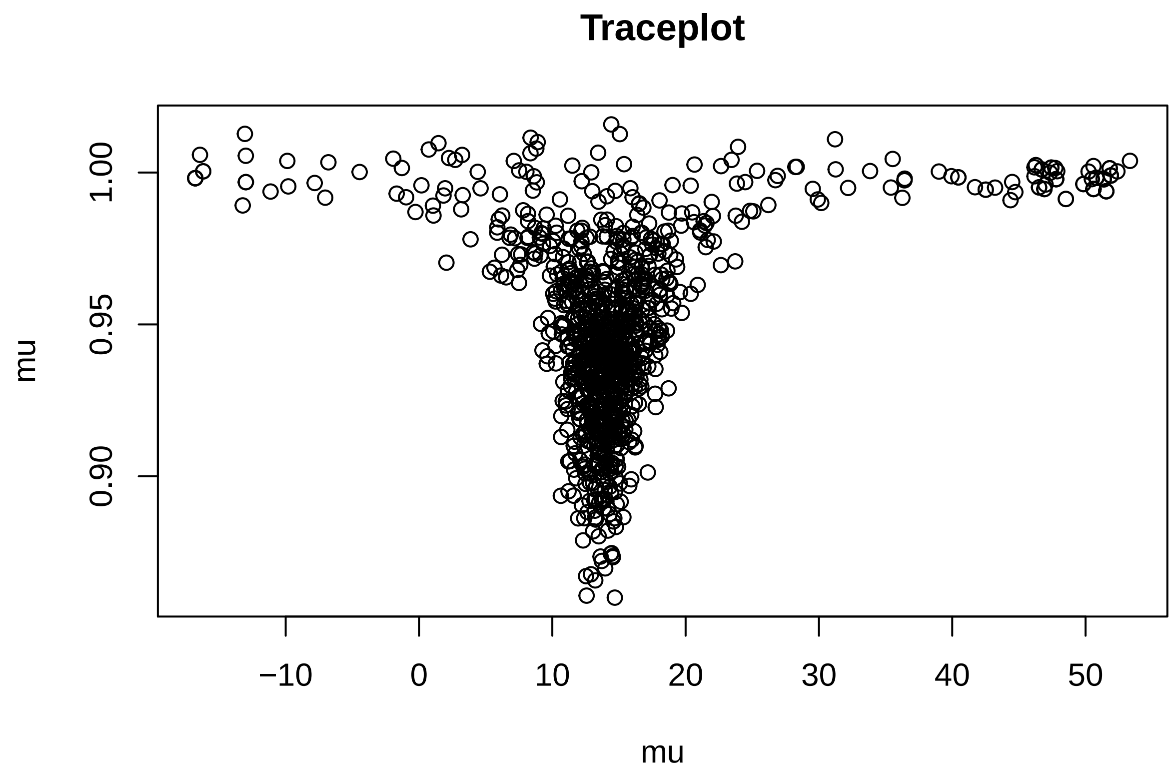
```r
print(fit_y)
```

```
## Inference for Stan model: 59560b14e3970f232803cf9ed9d888cd.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##           mean se_mean   sd    2.5%     25%     50%     75%   97.5% n_eff Rhat
## mu       14.70    0.56 6.73    4.10   12.63   14.07   15.74   32.60   147 1.01
## phi       0.95    0.00 0.03    0.89    0.93    0.95    0.97    1.00   672 1.01
## sigma     1.53    0.00 0.08    1.38    1.47    1.52    1.58    1.69  1138 1.00
## lp__   -182.44    0.05 1.40 -185.69 -183.27 -182.07 -181.35 -180.71   669 1.00
##
## Samples were drawn using NUTS(diag_e) at Thu May 21 15:46:44 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```r
mean_vector_x=fit_x@.MISC$summary$msd[,1]
cred_vector_x=fit_x@.MISC$summary$quan
eff_vector_x=fit_x@.MISC$summary$ess
mean_vector_y=fit_y@.MISC$summary$msd[,1]
cred_vector_y=fit_y@.MISC$summary$quan
eff_vector_y=fit_y@.MISC$summary$ess
# Do traceplots of the first chain
plot(postDraws_x$mu[1000:2000], postDraws_x$phi[1000:2000],ylab="phi", xlab="mu", main="Traceplot")
```




```
# Do traceplots of the first chain
plot(postDraws_y$mu[1000:2000],postDraws_y$phi[1000:2000],ylab="mu", xlab="mu",main="Traceplot")
```



The following information can be obtained from the output above for the two samplers respectively:

- μ_1: Mean = 10.2951189, 95 % cred. interval = [9.9957255, 10.6017443], No. of effective samples = 3551.1343101
- ϕ_1: Mean = 0.2784733, 95 % cred. interval = [0.1408476, 0.4172157], No. of effective samples = 3293.4961035
- σ_1: Mean = 1.523934, 95 % cred. interval = [1.3812962, 1.6854839], No. of effective samples = 3512.5548368
- μ_2: Mean = 14.6970165, 95 % cred. interval = [4.0950254, 32.6010794], No. of effective samples = 146.7804613
- ϕ_2: Mean = 0.9481794, 95 % cred. interval = [0.8916842, 1.0022372], No. of effective samples = 672.387384
- σ_2: Mean = 1.5266747, 95 % cred. interval = [1.3798615, 1.6889742], No. of effective samples = 1138.0110918

It is possible to estimate the true values of the parameters for the sample which used a ϕ=0.3 when obtaining the dataset used in the simulation. However, it is not as obvious to estimate the parameters' true values for the second sample, where ϕ=0.95 were used to obtain the dataset used in this particular simulation. The credible intervals for the parameters in the latter simulation are very wide and it is difficult to predict with certainty the true vale of the parameters. This might be due to the higher correlation between the lags caused by the higher value of ϕ.

The convergence of the samplers are different. For the first sample which used ϕ=0.3, the convergence is evident whilst for the second sample the posterior distribution is not obvious. This correlates with the fact that the credible intervals for the parameters on the second sample were very wide. What we can see from the posterior distribution obtained by the second sampler is that for lower values of phi the distribution centers around a value between 10 and 20. This is a behaviour similar to what is shown in the posterior for the first sampler, where ϕ was set to 0.3 initially, since this distribution was much tighter around the value of 10 for μ.

c)

```
campy=read.table("campy.dat", header=TRUE)
library(rstan)

StanModel_Pois = '
data {
  int<lower=0> T;
  int c[T];
}

parameters {
  real mu;
  real phi;
  real<lower=0> sigma;
  vector[T] x;
}

model {
  // Prior
  phi ~ uniform(-1,1);
  for (n in 2:T)
    x[n] ~ normal(mu + phi * (x[n-1]-mu), sigma);


  // Model/likelihood
  for (n in 1:T)
    c[n] ~ poisson(exp(x[n]));

}

generated quantities {
  vector[T] post_mean;
  post_mean = exp(x);
}
'

data=list(T=dim(campy)[1], c=campy$c)
fit_pois=stan(model_code=StanModel_Pois, data=data)
```

```
## Warning: There were 38 divergent transitions after warmup. Increasing adapt_delta above 0.8 may help
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quant
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
print(fit_pois)
```

```
## Inference for Stan model: 9aec8520e45696a8b7e216d186509f36.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##                   mean se_mean    sd    2.5%     25%     50%     75%   97.5%
```

```
## mu             2.41   0.01 0.21   2.11   2.31   2.40   2.49   2.77
## phi            0.82   0.00 0.06   0.69   0.78   0.82   0.86   0.95
## sigma          0.25   0.00 0.03   0.20   0.23   0.25   0.27   0.33
## x[1]           0.65   0.01 0.48  -0.40   0.35   0.68   0.98   1.52
## x[2]           0.98   0.01 0.35   0.23   0.76   0.99   1.22   1.62
## x[3]           1.23   0.01 0.28   0.64   1.04   1.23   1.43   1.75
## x[4]           1.37   0.00 0.25   0.87   1.20   1.38   1.55   1.86
## x[5]           1.72   0.00 0.24   1.23   1.56   1.72   1.88   2.17
## x[6]           2.00   0.00 0.22   1.56   1.85   2.00   2.15   2.44
## x[7]           2.16   0.00 0.22   1.73   2.02   2.16   2.31   2.58
## x[8]           2.07   0.00 0.21   1.65   1.93   2.07   2.21   2.47
## x[9]           1.97   0.00 0.22   1.54   1.82   1.97   2.12   2.40
## x[10]          2.04   0.00 0.21   1.61   1.89   2.04   2.19   2.46
## x[11]          2.17   0.00 0.21   1.75   2.02   2.17   2.31   2.57
## x[12]          2.11   0.00 0.21   1.67   1.97   2.11   2.26   2.52
## x[13]          1.99   0.00 0.21   1.56   1.85   2.00   2.14   2.40
## x[14]          1.98   0.00 0.22   1.54   1.83   1.99   2.13   2.40
## x[15]          2.06   0.00 0.21   1.64   1.92   2.07   2.21   2.46
## x[16]          2.06   0.00 0.21   1.63   1.92   2.06   2.20   2.46
## x[17]          2.20   0.00 0.20   1.80   2.06   2.20   2.34   2.60
## x[18]          2.11   0.00 0.20   1.71   1.97   2.11   2.25   2.51
## x[19]          2.05   0.00 0.21   1.64   1.91   2.05   2.19   2.45
## x[20]          2.03   0.00 0.21   1.61   1.90   2.04   2.17   2.45
## x[21]          2.21   0.00 0.20   1.81   2.08   2.21   2.36   2.60
## x[22]          2.35   0.00 0.19   1.96   2.22   2.35   2.48   2.71
## x[23]          2.37   0.00 0.20   1.97   2.24   2.37   2.50   2.77
## x[24]          2.30   0.00 0.20   1.92   2.17   2.31   2.43   2.68
## x[25]          2.33   0.00 0.20   1.93   2.20   2.33   2.46   2.70
## x[26]          2.23   0.00 0.20   1.83   2.10   2.24   2.37   2.61
## x[27]          2.25   0.00 0.20   1.85   2.11   2.25   2.39   2.63
## x[28]          2.31   0.00 0.19   1.92   2.18   2.31   2.44   2.68
## x[29]          2.06   0.00 0.21   1.64   1.92   2.06   2.20   2.45
## x[30]          2.03   0.00 0.21   1.60   1.89   2.03   2.17   2.44
## x[31]          2.20   0.00 0.20   1.79   2.06   2.20   2.34   2.59
## x[32]          2.37   0.00 0.20   1.97   2.23   2.37   2.51   2.75
## x[33]          2.30   0.00 0.19   1.90   2.17   2.30   2.43   2.67
## x[34]          2.39   0.00 0.19   2.02   2.26   2.39   2.52   2.75
## x[35]          2.56   0.00 0.19   2.18   2.43   2.56   2.69   2.92
## x[36]          2.50   0.00 0.18   2.14   2.38   2.50   2.63   2.85
## x[37]          2.41   0.00 0.19   2.05   2.29   2.41   2.53   2.77
## x[38]          2.27   0.00 0.20   1.85   2.15   2.28   2.40   2.66
## x[39]          2.12   0.00 0.21   1.72   1.99   2.13   2.27   2.51
## x[40]          2.09   0.00 0.21   1.67   1.95   2.09   2.24   2.49
## x[41]          1.98   0.00 0.22   1.55   1.84   1.99   2.13   2.40
## x[42]          1.96   0.00 0.22   1.52   1.82   1.97   2.11   2.38
## x[43]          1.87   0.00 0.23   1.42   1.71   1.87   2.02   2.29
## x[44]          1.79   0.00 0.23   1.31   1.64   1.80   1.95   2.23
## x[45]          1.87   0.00 0.22   1.41   1.72   1.88   2.02   2.28
## x[46]          1.97   0.00 0.22   1.53   1.82   1.97   2.12   2.40
## x[47]          2.16   0.00 0.21   1.73   2.02   2.17   2.30   2.56
## x[48]          2.17   0.00 0.20   1.76   2.03   2.18   2.31   2.56
## x[49]          2.24   0.00 0.20   1.84   2.11   2.24   2.38   2.62
## x[50]          2.28   0.00 0.20   1.89   2.15   2.28   2.42   2.67
## x[51]          2.25   0.00 0.21   1.84   2.11   2.25   2.39   2.65
```

```
## x[52]                1.95     0.00  0.22     1.52     1.80     1.95     2.09     2.35
## x[53]                1.82     0.00  0.22     1.37     1.67     1.82     1.97     2.26
## x[54]                1.67     0.00  0.23     1.20     1.52     1.67     1.83     2.12
## x[55]                1.69     0.00  0.23     1.22     1.54     1.70     1.85     2.13
## x[56]                1.81     0.00  0.23     1.34     1.66     1.82     1.98     2.27
## x[57]                1.76     0.00  0.23     1.29     1.61     1.76     1.92     2.21
## x[58]                2.00     0.00  0.21     1.59     1.87     2.01     2.15     2.40
## x[59]                2.28     0.00  0.20     1.89     2.14     2.28     2.42     2.67
## x[60]                2.39     0.00  0.20     2.01     2.26     2.39     2.52     2.76
## x[61]                2.45     0.00  0.19     2.08     2.32     2.45     2.59     2.82
## x[62]                2.34     0.00  0.20     1.95     2.21     2.34     2.48     2.72
## x[63]                2.13     0.00  0.21     1.73     2.00     2.14     2.27     2.53
## x[64]                2.04     0.00  0.21     1.63     1.90     2.04     2.19     2.44
## x[65]                2.02     0.00  0.21     1.59     1.88     2.02     2.17     2.42
## x[66]                1.94     0.00  0.22     1.51     1.80     1.95     2.09     2.37
## x[67]                1.86     0.00  0.22     1.39     1.71     1.86     2.01     2.29
## x[68]                1.72     0.00  0.23     1.24     1.57     1.73     1.87     2.14
## x[69]                1.76     0.00  0.23     1.29     1.61     1.77     1.91     2.19
## x[70]                1.85     0.00  0.22     1.40     1.71     1.86     2.01     2.28
## x[71]                2.05     0.00  0.21     1.64     1.91     2.06     2.19     2.46
## x[72]                2.08     0.00  0.21     1.67     1.95     2.09     2.22     2.48
## x[73]                2.19     0.00  0.20     1.78     2.06     2.20     2.33     2.57
## x[74]                2.39     0.00  0.19     2.01     2.26     2.39     2.52     2.75
## x[75]                2.43     0.00  0.20     2.05     2.30     2.43     2.56     2.82
## x[76]                2.36     0.00  0.20     1.97     2.24     2.36     2.50     2.75
## x[77]                2.29     0.00  0.19     1.90     2.17     2.29     2.43     2.67
## x[78]                2.07     0.00  0.21     1.65     1.93     2.07     2.22     2.47
## x[79]                2.00     0.00  0.22     1.56     1.85     2.00     2.15     2.42
## x[80]                1.88     0.00  0.22     1.42     1.73     1.88     2.04     2.29
## x[81]                1.95     0.00  0.22     1.50     1.81     1.96     2.10     2.36
## x[82]                2.03     0.00  0.21     1.59     1.88     2.03     2.17     2.43
## x[83]                2.21     0.00  0.20     1.82     2.08     2.21     2.35     2.60
## x[84]                2.42     0.00  0.20     2.03     2.29     2.43     2.55     2.80
## x[85]                2.43     0.00  0.19     2.05     2.31     2.43     2.56     2.80
## x[86]                2.50     0.00  0.19     2.13     2.38     2.50     2.63     2.85
## x[87]                2.67     0.00  0.18     2.31     2.55     2.68     2.79     3.01
## x[88]                2.84     0.00  0.17     2.49     2.72     2.84     2.95     3.17
## x[89]                2.66     0.00  0.18     2.28     2.54     2.66     2.78     3.01
## x[90]                2.30     0.00  0.20     1.89     2.17     2.31     2.44     2.66
## x[91]                2.35     0.00  0.19     1.95     2.23     2.35     2.48     2.72
## x[92]                2.45     0.00  0.19     2.08     2.33     2.45     2.57     2.81
## x[93]                2.53     0.00  0.19     2.15     2.40     2.53     2.65     2.88
## x[94]                2.36     0.00  0.19     1.96     2.23     2.36     2.49     2.72
## x[95]                2.55     0.00  0.18     2.19     2.43     2.56     2.68     2.90
## x[96]                2.52     0.00  0.18     2.14     2.40     2.52     2.64     2.87
## x[97]                2.62     0.00  0.18     2.25     2.51     2.63     2.75     2.97
## x[98]                2.81     0.00  0.17     2.46     2.70     2.81     2.92     3.14
## x[99]                3.15     0.00  0.16     2.84     3.05     3.15     3.26     3.44
## x[100]               3.80     0.00  0.13     3.54     3.71     3.80     3.89     4.05
## x[101]               3.73     0.00  0.13     3.47     3.64     3.73     3.81     3.97
## x[102]               3.33     0.00  0.14     3.05     3.23     3.32     3.42     3.60
## x[103]               2.97     0.00  0.16     2.63     2.87     2.98     3.08     3.27
## x[104]               2.92     0.00  0.16     2.60     2.81     2.92     3.03     3.25
## x[105]               2.73     0.00  0.18     2.38     2.62     2.74     2.85     3.06
```

```
## x[106]            2.59    0.00  0.18    2.21    2.47    2.60    2.72    2.94
## x[107]            2.81    0.00  0.17    2.47    2.70    2.81    2.93    3.14
## x[108]            2.88    0.00  0.17    2.53    2.77    2.88    2.99    3.21
## x[109]            2.80    0.00  0.17    2.45    2.69    2.81    2.91    3.13
## x[110]            2.79    0.00  0.17    2.44    2.68    2.80    2.91    3.12
## x[111]            3.01    0.00  0.16    2.69    2.90    3.01    3.12    3.33
## x[112]            2.97    0.00  0.16    2.64    2.87    2.98    3.08    3.27
## x[113]            3.25    0.00  0.15    2.95    3.15    3.25    3.35    3.54
## x[114]            3.02    0.00  0.16    2.71    2.92    3.02    3.13    3.33
## x[115]            2.96    0.00  0.16    2.65    2.86    2.97    3.07    3.27
## x[116]            2.82    0.00  0.17    2.48    2.71    2.82    2.93    3.14
## x[117]            2.60    0.00  0.19    2.23    2.48    2.60    2.73    2.96
## x[118]            2.69    0.00  0.18    2.34    2.57    2.69    2.82    3.02
## x[119]            2.61    0.00  0.18    2.24    2.49    2.62    2.74    2.95
## x[120]            2.69    0.00  0.18    2.35    2.57    2.70    2.82    3.03
## x[121]            2.79    0.00  0.17    2.46    2.68    2.80    2.92    3.13
## x[122]            2.73    0.00  0.18    2.37    2.61    2.74    2.86    3.07
## x[123]            2.50    0.00  0.19    2.10    2.37    2.50    2.63    2.86
## x[124]            2.52    0.00  0.19    2.12    2.40    2.53    2.65    2.87
## x[125]            2.91    0.00  0.17    2.58    2.80    2.90    3.02    3.24
## x[126]            2.80    0.00  0.17    2.46    2.69    2.81    2.92    3.11
## x[127]            2.71    0.00  0.17    2.36    2.59    2.71    2.82    3.04
## x[128]            2.79    0.00  0.18    2.44    2.67    2.80    2.91    3.15
## x[129]            2.55    0.00  0.18    2.18    2.43    2.55    2.67    2.90
## x[130]            2.46    0.00  0.19    2.09    2.34    2.47    2.59    2.82
## x[131]            2.37    0.00  0.19    1.97    2.24    2.37    2.50    2.74
## x[132]            2.35    0.00  0.19    1.96    2.22    2.35    2.48    2.72
## x[133]            2.15    0.00  0.21    1.71    2.01    2.15    2.29    2.54
## x[134]            2.22    0.00  0.21    1.81    2.09    2.23    2.37    2.61
## x[135]            2.49    0.00  0.19    2.12    2.36    2.49    2.62    2.84
## x[136]            2.69    0.00  0.17    2.34    2.57    2.70    2.81    3.03
## x[137]            2.76    0.00  0.17    2.41    2.65    2.76    2.87    3.08
## x[138]            2.85    0.00  0.17    2.51    2.73    2.85    2.97    3.19
## x[139]            2.69    0.00  0.18    2.32    2.56    2.69    2.81    3.02
## x[140]            2.45    0.00  0.22    2.01    2.31    2.45    2.60    2.85
## post_mean[1]      2.14    0.02  1.00    0.67    1.42    1.96    2.67    4.56
## post_mean[2]      2.83    0.02  0.97    1.26    2.15    2.70    3.40    5.04
## post_mean[3]      3.55    0.02  1.00    1.89    2.83    3.42    4.16    5.78
## post_mean[4]      4.08    0.02  1.04    2.38    3.31    3.97    4.70    6.40
## post_mean[5]      5.72    0.02  1.37    3.43    4.76    5.57    6.53    8.75
## post_mean[6]      7.59    0.03  1.71    4.76    6.35    7.42    8.57   11.49
## post_mean[7]      8.89    0.03  1.92    5.66    7.51    8.70   10.05   13.15
## post_mean[8]      8.07    0.03  1.69    5.21    6.86    7.92    9.13   11.79
## post_mean[9]      7.34    0.03  1.64    4.67    6.18    7.18    8.30   11.06
## post_mean[10]     7.86    0.03  1.68    5.01    6.65    7.72    8.91   11.67
## post_mean[11]     8.91    0.03  1.88    5.76    7.57    8.76   10.06   13.11
## post_mean[12]     8.44    0.03  1.80    5.34    7.17    8.28    9.54   12.48
## post_mean[13]     7.52    0.03  1.62    4.77    6.38    7.36    8.49   11.07
## post_mean[14]     7.43    0.03  1.62    4.67    6.27    7.30    8.44   11.02
## post_mean[15]     8.05    0.03  1.71    5.14    6.82    7.93    9.08   11.76
## post_mean[16]     7.99    0.03  1.68    5.12    6.79    7.82    9.03   11.68
## post_mean[17]     9.22    0.03  1.89    6.04    7.86    9.05   10.41   13.40
## post_mean[18]     8.43    0.03  1.72    5.53    7.19    8.26    9.50   12.25
## post_mean[19]     7.93    0.03  1.68    5.13    6.73    7.74    8.97   11.64
```

```
## post_mean[20]     7.80    0.03  1.64    5.00    6.68    7.66    8.77   11.55
## post_mean[21]     9.35    0.03  1.90    6.08    7.99    9.15   10.54   13.53
## post_mean[22]    10.63    0.03  2.05    7.08    9.18   10.47   11.94   15.05
## post_mean[23]    10.89    0.03  2.15    7.17    9.38   10.66   12.21   15.90
## post_mean[24]    10.18    0.03  2.01    6.84    8.72   10.05   11.39   14.59
## post_mean[25]    10.47    0.03  2.07    6.90    9.02   10.28   11.75   14.95
## post_mean[26]     9.49    0.03  1.86    6.25    8.20    9.35   10.66   13.54
## post_mean[27]     9.65    0.03  1.93    6.34    8.26    9.53   10.86   13.88
## post_mean[28]    10.25    0.03  2.01    6.79    8.82   10.06   11.47   14.58
## post_mean[29]     7.99    0.03  1.68    5.13    6.79    7.81    9.04   11.61
## post_mean[30]     7.76    0.03  1.65    4.94    6.60    7.60    8.76   11.42
## post_mean[31]     9.18    0.03  1.89    6.00    7.83    9.01   10.34   13.30
## post_mean[32]    10.90    0.04  2.21    7.20    9.30   10.72   12.31   15.58
## post_mean[33]    10.12    0.03  1.96    6.68    8.72    9.99   11.35   14.41
## post_mean[34]    11.10    0.03  2.11    7.52    9.59   10.90   12.38   15.65
## post_mean[35]    13.14    0.04  2.48    8.89   11.38   12.96   14.71   18.52
## post_mean[36]    12.41    0.03  2.25    8.51   10.78   12.22   13.83   17.33
## post_mean[37]    11.33    0.03  2.11    7.76    9.88   11.16   12.60   15.98
## post_mean[38]     9.91    0.03  2.00    6.38    8.56    9.74   11.07   14.31
## post_mean[39]     8.55    0.03  1.75    5.59    7.33    8.38    9.64   12.29
## post_mean[40]     8.27    0.03  1.74    5.33    7.06    8.12    9.35   12.01
## post_mean[41]     7.44    0.03  1.61    4.72    6.32    7.29    8.39   11.03
## post_mean[42]     7.30    0.03  1.59    4.59    6.16    7.18    8.26   10.80
## post_mean[43]     6.63    0.03  1.51    4.15    5.54    6.46    7.56    9.91
## post_mean[44]     6.17    0.03  1.42    3.71    5.16    6.05    7.00    9.26
## post_mean[45]     6.63    0.02  1.47    4.11    5.59    6.52    7.54    9.78
## post_mean[46]     7.33    0.03  1.61    4.61    6.17    7.20    8.35   10.99
## post_mean[47]     8.86    0.03  1.85    5.61    7.57    8.74    9.97   12.99
## post_mean[48]     8.93    0.03  1.81    5.80    7.64    8.81   10.04   12.89
## post_mean[49]     9.58    0.03  1.95    6.27    8.21    9.43   10.75   13.76
## post_mean[50]    10.01    0.03  2.01    6.59    8.58    9.82   11.20   14.38
## post_mean[51]     9.67    0.03  1.99    6.31    8.23    9.51   10.87   14.12
## post_mean[52]     7.18    0.03  1.55    4.57    6.07    7.06    8.09   10.52
## post_mean[53]     6.31    0.03  1.42    3.94    5.30    6.18    7.14    9.54
## post_mean[54]     5.45    0.02  1.28    3.33    4.55    5.31    6.24    8.34
## post_mean[55]     5.58    0.02  1.29    3.39    4.68    5.45    6.39    8.44
## post_mean[56]     6.30    0.02  1.48    3.81    5.27    6.18    7.21    9.63
## post_mean[57]     5.98    0.03  1.38    3.63    5.01    5.84    6.79    9.09
## post_mean[58]     7.59    0.03  1.60    4.89    6.47    7.46    8.56   11.08
## post_mean[59]     9.97    0.03  2.04    6.61    8.49    9.79   11.22   14.50
## post_mean[60]    11.15    0.03  2.20    7.44    9.61   10.93   12.49   15.83
## post_mean[61]    11.82    0.04  2.29    7.96   10.20   11.63   13.29   16.85
## post_mean[62]    10.62    0.03  2.10    7.06    9.13   10.41   11.92   15.24
## post_mean[63]     8.62    0.03  1.78    5.62    7.36    8.48    9.71   12.52
## post_mean[64]     7.89    0.03  1.69    5.11    6.69    7.71    8.98   11.48
## post_mean[65]     7.69    0.03  1.65    4.91    6.52    7.53    8.75   11.23
## post_mean[66]     7.15    0.03  1.56    4.51    6.05    7.02    8.11   10.66
## post_mean[67]     6.56    0.03  1.48    4.03    5.52    6.42    7.48    9.87
## post_mean[68]     5.71    0.02  1.30    3.45    4.80    5.61    6.51    8.49
## post_mean[69]     5.96    0.02  1.36    3.62    5.00    5.86    6.77    8.97
## post_mean[70]     6.54    0.03  1.45    4.07    5.52    6.42    7.45    9.74
## post_mean[71]     7.98    0.03  1.68    5.17    6.77    7.83    8.97   11.72
## post_mean[72]     8.19    0.03  1.70    5.30    7.00    8.06    9.20   11.91
## post_mean[73]     9.13    0.03  1.81    5.94    7.84    9.01   10.24   13.05
```

```
## post_mean[74]     11.07    0.03  2.14    7.43    9.58   10.88   12.38   15.69
## post_mean[75]     11.56    0.04  2.28    7.76    9.93   11.40   12.93   16.76
## post_mean[76]     10.83    0.03  2.14    7.19    9.36   10.59   12.14   15.62
## post_mean[77]     10.10    0.03  1.96    6.72    8.74    9.92   11.33   14.43
## post_mean[78]      8.10    0.03  1.70    5.20    6.90    7.95    9.18   11.80
## post_mean[79]      7.56    0.03  1.68    4.75    6.36    7.42    8.57   11.29
## post_mean[80]      6.71    0.02  1.50    4.14    5.66    6.58    7.66    9.92
## post_mean[81]      7.20    0.03  1.55    4.46    6.12    7.08    8.16   10.61
## post_mean[82]      7.75    0.03  1.67    4.92    6.58    7.60    8.76   11.39
## post_mean[83]      9.31    0.03  1.86    6.15    8.00    9.13   10.44   13.45
## post_mean[84]     11.48    0.03  2.23    7.63    9.90   11.36   12.85   16.41
## post_mean[85]     11.62    0.03  2.24    7.76   10.05   11.39   12.97   16.50
## post_mean[86]     12.38    0.03  2.29    8.39   10.80   12.22   13.80   17.35
## post_mean[87]     14.70    0.04  2.61   10.06   12.80   14.55   16.35   20.32
## post_mean[88]     17.30    0.05  2.98   12.01   15.15   17.12   19.09   23.83
## post_mean[89]     14.50    0.04  2.66    9.82   12.62   14.31   16.19   20.30
## post_mean[90]     10.17    0.04  1.99    6.65    8.76   10.04   11.46   14.31
## post_mean[91]     10.68    0.03  2.05    7.03    9.27   10.49   11.96   15.17
## post_mean[92]     11.77    0.03  2.18    7.99   10.24   11.60   13.11   16.54
## post_mean[93]     12.73    0.04  2.40    8.56   11.06   12.52   14.21   17.87
## post_mean[94]     10.73    0.03  2.04    7.08    9.29   10.56   12.03   15.20
## post_mean[95]     13.07    0.03  2.41    8.92   11.38   12.89   14.54   18.16
## post_mean[96]     12.64    0.03  2.31    8.52   11.04   12.46   14.07   17.65
## post_mean[97]     14.02    0.04  2.54    9.53   12.26   13.81   15.65   19.47
## post_mean[98]     16.85    0.04  2.88   11.69   14.87   16.64   18.62   23.11
## post_mean[99]     23.61    0.06  3.66   17.09   21.05   23.35   26.02   31.21
## post_mean[100]    44.96    0.10  5.81   34.54   40.92   44.58   48.74   57.12
## post_mean[101]    41.88    0.08  5.28   32.23   38.22   41.62   45.17   52.97
## post_mean[102]    28.12    0.06  3.98   21.01   25.36   27.79   30.59   36.63
## post_mean[103]    19.75    0.05  3.19   13.92   17.57   19.60   21.74   26.39
## post_mean[104]    18.85    0.04  3.12   13.41   16.65   18.63   20.76   25.76
## post_mean[105]    15.62    0.04  2.72   10.84   13.68   15.50   17.37   21.40
## post_mean[106]    13.57    0.04  2.45    9.10   11.86   13.45   15.12   18.83
## post_mean[107]    16.90    0.04  2.88   11.83   14.91   16.69   18.67   23.08
## post_mean[108]    18.04    0.05  3.07   12.57   15.90   17.85   19.98   24.66
## post_mean[109]    16.73    0.04  2.87   11.54   14.79   16.57   18.42   22.92
## post_mean[110]    16.56    0.04  2.81   11.48   14.64   16.36   18.36   22.58
## post_mean[111]    20.51    0.04  3.36   14.67   18.22   20.29   22.53   28.01
## post_mean[112]    19.78    0.04  3.17   14.02   17.55   19.64   21.83   26.34
## post_mean[113]    25.99    0.05  3.90   19.10   23.25   25.68   28.40   34.30
## post_mean[114]    20.75    0.04  3.25   15.02   18.46   20.54   22.80   27.92
## post_mean[115]    19.59    0.04  3.07   14.22   17.44   19.42   21.48   26.24
## post_mean[116]    16.96    0.04  2.85   11.90   14.98   16.81   18.77   23.15
## post_mean[117]    13.72    0.04  2.56    9.28   11.90   13.53   15.31   19.26
## post_mean[118]    14.98    0.04  2.62   10.39   13.09   14.79   16.70   20.56
## post_mean[119]    13.86    0.04  2.51    9.35   12.11   13.73   15.43   19.13
## post_mean[120]    15.01    0.04  2.64   10.48   13.08   14.85   16.72   20.62
## post_mean[121]    16.61    0.04  2.91   11.66   14.52   16.40   18.50   22.79
## post_mean[122]    15.63    0.04  2.79   10.74   13.60   15.42   17.43   21.65
## post_mean[123]    12.34    0.04  2.33    8.19   10.69   12.19   13.82   17.40
## post_mean[124]    12.64    0.04  2.34    8.36   11.03   12.50   14.16   17.72
## post_mean[125]    18.54    0.05  3.15   13.18   16.37   18.24   20.50   25.60
## post_mean[126]    16.68    0.04  2.78   11.72   14.73   16.54   18.46   22.47
## post_mean[127]    15.19    0.04  2.62   10.58   13.38   15.07   16.77   20.98
```

```
## post_mean[128]   16.60    0.04  2.98   11.50   14.44   16.41   18.37   23.29
## post_mean[129]   12.98    0.04  2.37    8.82   11.36   12.80   14.45   18.15
## post_mean[130]   11.95    0.03  2.22    8.06   10.38   11.79   13.33   16.71
## post_mean[131]   10.93    0.03  2.10    7.19    9.44   10.75   12.19   15.49
## post_mean[132]   10.70    0.03  2.09    7.11    9.25   10.51   11.96   15.25
## post_mean[133]    8.74    0.03  1.83    5.53    7.46    8.62    9.86   12.66
## post_mean[134]    9.43    0.03  1.92    6.12    8.11    9.30   10.65   13.56
## post_mean[135]   12.27    0.04  2.31    8.34   10.63   12.11   13.77   17.16
## post_mean[136]   14.98    0.04  2.63   10.38   13.12   14.81   16.62   20.73
## post_mean[137]   15.99    0.04  2.72   11.16   14.09   15.81   17.65   21.77
## post_mean[138]   17.56    0.05  3.03   12.34   15.40   17.28   19.41   24.31
## post_mean[139]   14.90    0.04  2.67   10.22   12.99   14.74   16.63   20.50
## post_mean[140]   11.86    0.04  2.57    7.46   10.03   11.62   13.48   17.34
## lp__           2656.03    0.61 14.52 2626.79 2646.42 2656.37 2665.74 2684.17
##                n_eff Rhat
## mu               242 1.03
## phi              790 1.00
## sigma            665 1.00
## x[1]            1874 1.00
## x[2]            1830 1.00
## x[3]            2445 1.00
## x[4]            3137 1.00
## x[5]            3627 1.00
## x[6]            3506 1.00
## x[7]            3153 1.00
## x[8]            3934 1.00
## x[9]            3818 1.00
## x[10]           3549 1.00
## x[11]           3825 1.00
## x[12]           4106 1.00
## x[13]           3626 1.00
## x[14]           2703 1.00
## x[15]           3552 1.00
## x[16]           3434 1.00
## x[17]           3562 1.00
## x[18]           4126 1.00
## x[19]           3917 1.00
## x[20]           3530 1.00
## x[21]           4436 1.00
## x[22]           3475 1.00
## x[23]           4074 1.00
## x[24]           3865 1.00
## x[25]           3871 1.00
## x[26]           4570 1.00
## x[27]           4578 1.00
## x[28]           4106 1.00
## x[29]           3892 1.00
## x[30]           3472 1.00
## x[31]           3779 1.00
## x[32]           3630 1.00
## x[33]           3514 1.00
## x[34]           3954 1.00
## x[35]           4271 1.00
## x[36]           4350 1.00
```

```
## x[37]            4094 1.00
## x[38]            4340 1.00
## x[39]            3540 1.00
## x[40]            3560 1.00
## x[41]            3347 1.00
## x[42]            3598 1.00
## x[43]            3530 1.00
## x[44]            3111 1.00
## x[45]            3521 1.00
## x[46]            3678 1.00
## x[47]            3869 1.00
## x[48]            3972 1.00
## x[49]            3969 1.00
## x[50]            4261 1.00
## x[51]            3756 1.00
## x[52]            3769 1.00
## x[53]            3184 1.00
## x[54]            2821 1.00
## x[55]            2843 1.00
## x[56]            3544 1.00
## x[57]            2755 1.00
## x[58]            3936 1.00
## x[59]            4519 1.00
## x[60]            4481 1.00
## x[61]            3745 1.00
## x[62]            4311 1.00
## x[63]            3803 1.00
## x[64]            3514 1.00
## x[65]            3174 1.00
## x[66]            3209 1.00
## x[67]            3021 1.00
## x[68]            2945 1.00
## x[69]            2800 1.00
## x[70]            2760 1.00
## x[71]            2638 1.00
## x[72]            3378 1.00
## x[73]            3717 1.00
## x[74]            4190 1.00
## x[75]            3925 1.00
## x[76]            4342 1.00
## x[77]            4204 1.00
## x[78]            4049 1.00
## x[79]            4190 1.00
## x[80]            3619 1.00
## x[81]            3391 1.00
## x[82]            3763 1.00
## x[83]            3802 1.00
## x[84]            5287 1.00
## x[85]            4911 1.00
## x[86]            4887 1.00
## x[87]            4626 1.00
## x[88]            3785 1.00
## x[89]            3546 1.00
## x[90]            3040 1.00
```

```
## x[91]           3797 1.00
## x[92]           4102 1.00
## x[93]           4618 1.00
## x[94]           3854 1.00
## x[95]           5029 1.00
## x[96]           4701 1.00
## x[97]           4130 1.00
## x[98]           4136 1.00
## x[99]           4040 1.00
## x[100]          3471 1.00
## x[101]          4624 1.00
## x[102]          4904 1.00
## x[103]          4075 1.00
## x[104]          5140 1.00
## x[105]          4766 1.00
## x[106]          3426 1.00
## x[107]          4843 1.00
## x[108]          4568 1.00
## x[109]          4110 1.00
## x[110]          4067 1.00
## x[111]          5721 1.00
## x[112]          5120 1.00
## x[113]          5076 1.00
## x[114]          5845 1.00
## x[115]          5495 1.00
## x[116]          4165 1.00
## x[117]          4242 1.00
## x[118]          4625 1.00
## x[119]          4659 1.00
## x[120]          4690 1.00
## x[121]          4609 1.00
## x[122]          4552 1.00
## x[123]          4066 1.00
## x[124]          4002 1.00
## x[125]          3857 1.00
## x[126]          3964 1.00
## x[127]          3975 1.00
## x[128]          4586 1.00
## x[129]          4100 1.00
## x[130]          4169 1.00
## x[131]          4541 1.00
## x[132]          4688 1.00
## x[133]          3514 1.00
## x[134]          3447 1.00
## x[135]          3966 1.00
## x[136]          4688 1.00
## x[137]          5086 1.00
## x[138]          4201 1.00
## x[139]          5130 1.00
## x[140]          4965 1.00
## post_mean[1]    1913 1.00
## post_mean[2]    1847 1.00
## post_mean[3]    2499 1.00
## post_mean[4]    3087 1.00
```

```
## post_mean[5]    3643 1.00
## post_mean[6]    3609 1.00
## post_mean[7]    3233 1.00
## post_mean[8]    4012 1.00
## post_mean[9]    3842 1.00
## post_mean[10]   3564 1.00
## post_mean[11]   3729 1.00
## post_mean[12]   4027 1.00
## post_mean[13]   3615 1.00
## post_mean[14]   2766 1.00
## post_mean[15]   3550 1.00
## post_mean[16]   3512 1.00
## post_mean[17]   3543 1.00
## post_mean[18]   4053 1.00
## post_mean[19]   3817 1.00
## post_mean[20]   3538 1.00
## post_mean[21]   4484 1.00
## post_mean[22]   3581 1.00
## post_mean[23]   4020 1.00
## post_mean[24]   3873 1.00
## post_mean[25]   3928 1.00
## post_mean[26]   4753 1.00
## post_mean[27]   4594 1.00
## post_mean[28]   4161 1.00
## post_mean[29]   3964 1.00
## post_mean[30]   3494 1.00
## post_mean[31]   3871 1.00
## post_mean[32]   3703 1.00
## post_mean[33]   3627 1.00
## post_mean[34]   4005 1.00
## post_mean[35]   4440 1.00
## post_mean[36]   4410 1.00
## post_mean[37]   4157 1.00
## post_mean[38]   4228 1.00
## post_mean[39]   3706 1.00
## post_mean[40]   3604 1.00
## post_mean[41]   3412 1.00
## post_mean[42]   3648 1.00
## post_mean[43]   3433 1.00
## post_mean[44]   3162 1.00
## post_mean[45]   3662 1.00
## post_mean[46]   3905 1.00
## post_mean[47]   3985 1.00
## post_mean[48]   3924 1.00
## post_mean[49]   3911 1.00
## post_mean[50]   4279 1.00
## post_mean[51]   3552 1.00
## post_mean[52]   3795 1.00
## post_mean[53]   3161 1.00
## post_mean[54]   2885 1.00
## post_mean[55]   2937 1.00
## post_mean[56]   3640 1.00
## post_mean[57]   2884 1.00
## post_mean[58]   3982 1.00
```

```
## post_mean[59]   4569 1.00
## post_mean[60]   4449 1.00
## post_mean[61]   3581 1.00
## post_mean[62]   4418 1.00
## post_mean[63]   3642 1.00
## post_mean[64]   3465 1.00
## post_mean[65]   3228 1.00
## post_mean[66]   3187 1.00
## post_mean[67]   3408 1.00
## post_mean[68]   3161 1.00
## post_mean[69]   2995 1.00
## post_mean[70]   2814 1.00
## post_mean[71]   2725 1.00
## post_mean[72]   3418 1.00
## post_mean[73]   3797 1.00
## post_mean[74]   4330 1.00
## post_mean[75]   3843 1.00
## post_mean[76]   4162 1.00
## post_mean[77]   4273 1.00
## post_mean[78]   4162 1.00
## post_mean[79]   4302 1.00
## post_mean[80]   3826 1.00
## post_mean[81]   3593 1.00
## post_mean[82]   3904 1.00
## post_mean[83]   3955 1.00
## post_mean[84]   5250 1.00
## post_mean[85]   4904 1.00
## post_mean[86]   4995 1.00
## post_mean[87]   4674 1.00
## post_mean[88]   3856 1.00
## post_mean[89]   3585 1.00
## post_mean[90]   3231 1.00
## post_mean[91]   3947 1.00
## post_mean[92]   4185 1.00
## post_mean[93]   4581 1.00
## post_mean[94]   3963 1.00
## post_mean[95]   5021 1.00
## post_mean[96]   4873 1.00
## post_mean[97]   4096 1.00
## post_mean[98]   4200 1.00
## post_mean[99]   4117 1.00
## post_mean[100]  3502 1.00
## post_mean[101]  4457 1.00
## post_mean[102]  4792 1.00
## post_mean[103]  4010 1.00
## post_mean[104]  5119 1.00
## post_mean[105]  4830 1.00
## post_mean[106]  3440 1.00
## post_mean[107]  4717 1.00
## post_mean[108]  4490 1.00
## post_mean[109]  4176 1.00
## post_mean[110]  4323 1.00
## post_mean[111]  5774 1.00
## post_mean[112]  5323 1.00
```

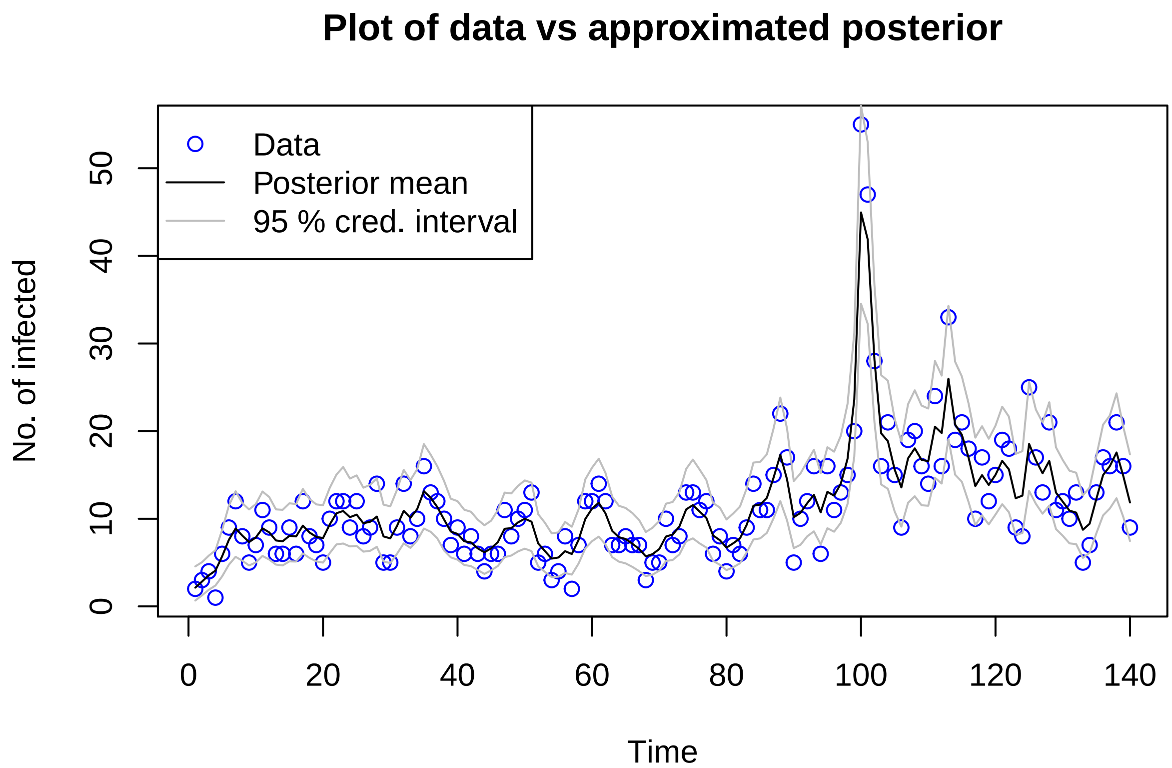
```
## post_mean[113]  5048 1.00
## post_mean[114]  6030 1.00
## post_mean[115]  5280 1.00
## post_mean[116]  4247 1.00
## post_mean[117]  4358 1.00
## post_mean[118]  4655 1.00
## post_mean[119]  4706 1.00
## post_mean[120]  4637 1.00
## post_mean[121]  4724 1.00
## post_mean[122]  4587 1.00
## post_mean[123]  4077 1.00
## post_mean[124]  4011 1.00
## post_mean[125]  3845 1.00
## post_mean[126]  3904 1.00
## post_mean[127]  4000 1.00
## post_mean[128]  4623 1.00
## post_mean[129]  4098 1.00
## post_mean[130]  4293 1.00
## post_mean[131]  4469 1.00
## post_mean[132]  4780 1.00
## post_mean[133]  3738 1.00
## post_mean[134]  3740 1.00
## post_mean[135]  3886 1.00
## post_mean[136]  4888 1.00
## post_mean[137]  5141 1.00
## post_mean[138]  4029 1.00
## post_mean[139]  5398 1.00
## post_mean[140]  4975 1.00
## lp__              564 1.00
##
## Samples were drawn using NUTS(diag_e) at Thu May 21 15:47:37 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```r
pois_mean_list=fit_pois@.MISC$summary$msd
post_mean=pois_mean_list[grep("post_mean", rownames(pois_mean_list)),]

plot(campy$c, col="blue", ylab="No. of infected", xlab="Time")
points(post_mean[,1], col="black", type="l")

quantiles=fit_pois@.MISC$summary$quan
quantiles_post_mean=quantiles[grep("post_mean", rownames(quantiles)),]
cred_interval_post_mean=matrix(0,dim(quantiles_post_mean)[1], 2)
cred_interval_post_mean[,1]=quantiles_post_mean[,1]
cred_interval_post_mean[,2]=quantiles_post_mean[,ncol(quantiles_post_mean)]

lines(cred_interval_post_mean[,1], col="gray", lty=1)
lines(cred_interval_post_mean[,2], col="gray", lty=1)
title(main="Plot of data vs approximated posterior")
legend("topleft", box.lty= 1, pch=c(1,NaN,NaN),
       legend=c("Data", "Posterior mean", "95 % cred. interval"),
       col=c("blue", "black", "gray"), lwd=c(NaN,1,1), lty=c(NaN, 1, 1))
```



As seen in the plot above, the posterior mean follows the data accurately. Almost all of the datapoints are inside the credible intervals which aren't that wide which indicates that the approximated posterior resembles the reality shown by the data well.

```
StanModel_Pois_Prior = '
data {
  int<lower=0> T;
  int c[T];
}

parameters {
  real mu;
  real phi;
  real<lower=0> sigma;
  vector[T] x;
}

model {
  // Prior
  phi ~ uniform(-1,1);
  sigma ~ scaled_inv_chi_square(140, 0.15);
  for (n in 2:T)
    x[n] ~ normal(mu + phi * (x[n-1]-mu), sigma);


  // Model/likelihood
  for (n in 1:T)
```

```
    c[n] ~ poisson(exp(x[n]));

}

generated quantities {
  vector[T] post_mean;
  post_mean = exp(x);
}
'
fit_pois_prior=stan(model_code=StanModel_Pois_Prior, data=data)
```

```
## Warning: There were 1224 divergent transitions after warmup. Increasing adapt_delta above 0.8 may hel
## http://mc-stan.org/misc/warnings.html#divergent-transitions-after-warmup

## Warning: Examine the pairs() plot to diagnose sampling problems

## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#bulk-ess

## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quant
## Running the chains for more iterations may help. See
## http://mc-stan.org/misc/warnings.html#tail-ess
```

```
print(fit_pois_prior)
```

```
## Inference for Stan model: f2ffac418cea06e34582d92ad0fc4b77.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##                     mean se_mean   sd  2.5%   25%   50%   75% 97.5%
## mu                  3.59    0.08 1.66  2.41  2.68  2.94  3.69  9.06
## phi                 0.99    0.00 0.01  0.96  0.98  0.99  1.00  1.00
## sigma               0.04    0.00 0.01  0.03  0.03  0.04  0.04  0.05
## x[1]                1.72    0.00 0.15  1.38  1.62  1.73  1.82  1.99
## x[2]                1.74    0.00 0.14  1.44  1.65  1.75  1.84  2.00
## x[3]                1.76    0.00 0.13  1.49  1.68  1.77  1.85  2.01
## x[4]                1.79    0.00 0.12  1.54  1.71  1.80  1.88  2.02
## x[5]                1.83    0.00 0.11  1.59  1.75  1.83  1.90  2.04
## x[6]                1.86    0.00 0.11  1.64  1.79  1.86  1.94  2.07
## x[7]                1.89    0.00 0.10  1.68  1.83  1.90  1.96  2.09
## x[8]                1.91    0.00 0.10  1.71  1.85  1.92  1.98  2.11
## x[9]                1.93    0.00 0.10  1.74  1.87  1.94  2.00  2.12
## x[10]               1.96    0.00 0.09  1.77  1.90  1.96  2.02  2.14
## x[11]               1.98    0.00 0.09  1.80  1.92  1.98  2.04  2.16
## x[12]               2.00    0.00 0.09  1.83  1.94  2.00  2.06  2.17
## x[13]               2.01    0.00 0.09  1.84  1.95  2.01  2.07  2.18
## x[14]               2.03    0.00 0.09  1.87  1.97  2.03  2.09  2.20
## x[15]               2.05    0.00 0.09  1.88  1.99  2.05  2.11  2.21
## x[16]               2.07    0.00 0.09  1.90  2.01  2.07  2.12  2.23
## x[17]               2.08    0.00 0.08  1.92  2.03  2.08  2.14  2.25
## x[18]               2.10    0.00 0.08  1.93  2.05  2.10  2.15  2.26
## x[19]               2.11    0.00 0.08  1.95  2.06  2.11  2.17  2.27
## x[20]               2.13    0.00 0.08  1.97  2.07  2.13  2.18  2.29
## x[21]               2.15    0.00 0.08  1.99  2.09  2.15  2.20  2.31
## x[22]               2.17    0.00 0.08  2.00  2.11  2.17  2.22  2.33
```

```
## x[23]              2.18    0.00 0.08    2.01    2.13    2.18    2.23    2.34
## x[24]              2.19    0.00 0.08    2.02    2.13    2.18    2.24    2.34
## x[25]              2.19    0.00 0.08    2.03    2.14    2.19    2.25    2.35
## x[26]              2.20    0.00 0.08    2.04    2.14    2.20    2.25    2.36
## x[27]              2.20    0.00 0.08    2.04    2.15    2.20    2.26    2.36
## x[28]              2.21    0.00 0.08    2.05    2.15    2.21    2.26    2.36
## x[29]              2.20    0.00 0.08    2.04    2.15    2.21    2.26    2.35
## x[30]              2.21    0.00 0.08    2.05    2.15    2.21    2.26    2.36
## x[31]              2.22    0.00 0.08    2.06    2.16    2.22    2.27    2.37
## x[32]              2.23    0.00 0.08    2.07    2.17    2.23    2.28    2.38
## x[33]              2.23    0.00 0.08    2.07    2.18    2.23    2.28    2.38
## x[34]              2.23    0.00 0.08    2.08    2.18    2.23    2.29    2.39
## x[35]              2.24    0.00 0.08    2.08    2.18    2.24    2.29    2.40
## x[36]              2.23    0.00 0.08    2.08    2.18    2.23    2.28    2.38
## x[37]              2.22    0.00 0.08    2.06    2.16    2.22    2.27    2.38
## x[38]              2.20    0.00 0.08    2.04    2.14    2.20    2.25    2.37
## x[39]              2.18    0.00 0.08    2.02    2.13    2.18    2.23    2.34
## x[40]              2.17    0.00 0.08    2.01    2.11    2.17    2.22    2.32
## x[41]              2.15    0.00 0.08    1.99    2.09    2.15    2.20    2.31
## x[42]              2.14    0.00 0.08    1.97    2.08    2.14    2.19    2.30
## x[43]              2.13    0.00 0.08    1.96    2.07    2.13    2.18    2.29
## x[44]              2.12    0.00 0.08    1.95    2.06    2.12    2.17    2.28
## x[45]              2.11    0.00 0.08    1.94    2.06    2.12    2.17    2.27
## x[46]              2.12    0.00 0.08    1.95    2.06    2.12    2.17    2.27
## x[47]              2.12    0.00 0.08    1.95    2.07    2.12    2.17    2.27
## x[48]              2.12    0.00 0.08    1.95    2.07    2.12    2.17    2.28
## x[49]              2.12    0.00 0.08    1.95    2.06    2.12    2.17    2.28
## x[50]              2.12    0.00 0.08    1.95    2.06    2.12    2.18    2.28
## x[51]              2.11    0.00 0.08    1.95    2.06    2.11    2.17    2.27
## x[52]              2.10    0.00 0.08    1.93    2.04    2.10    2.15    2.26
## x[53]              2.09    0.00 0.09    1.92    2.03    2.09    2.15    2.26
## x[54]              2.08    0.00 0.08    1.91    2.02    2.08    2.14    2.25
## x[55]              2.08    0.00 0.08    1.91    2.03    2.08    2.14    2.24
## x[56]              2.09    0.00 0.08    1.92    2.04    2.09    2.15    2.25
## x[57]              2.10    0.00 0.08    1.93    2.04    2.10    2.16    2.26
## x[58]              2.11    0.00 0.08    1.95    2.06    2.12    2.17    2.27
## x[59]              2.13    0.00 0.08    1.97    2.08    2.13    2.19    2.29
## x[60]              2.14    0.00 0.08    1.98    2.09    2.14    2.20    2.31
## x[61]              2.15    0.00 0.08    1.99    2.09    2.15    2.21    2.31
## x[62]              2.15    0.00 0.08    1.99    2.09    2.15    2.20    2.30
## x[63]              2.14    0.00 0.08    1.97    2.09    2.14    2.19    2.30
## x[64]              2.13    0.00 0.08    1.97    2.08    2.14    2.19    2.29
## x[65]              2.13    0.00 0.08    1.97    2.08    2.14    2.19    2.29
## x[66]              2.13    0.00 0.08    1.96    2.07    2.13    2.18    2.28
## x[67]              2.13    0.00 0.08    1.96    2.07    2.13    2.18    2.28
## x[68]              2.13    0.00 0.08    1.96    2.07    2.13    2.19    2.28
## x[69]              2.14    0.00 0.08    1.97    2.08    2.14    2.19    2.29
## x[70]              2.15    0.00 0.08    1.99    2.10    2.15    2.21    2.31
## x[71]              2.17    0.00 0.08    2.01    2.12    2.17    2.23    2.33
## x[72]              2.19    0.00 0.08    2.03    2.14    2.19    2.24    2.35
## x[73]              2.21    0.00 0.08    2.05    2.16    2.21    2.27    2.37
## x[74]              2.23    0.00 0.08    2.07    2.18    2.23    2.29    2.39
## x[75]              2.25    0.00 0.08    2.09    2.20    2.25    2.30    2.41
## x[76]              2.26    0.00 0.08    2.11    2.21    2.26    2.32    2.42
```

```
## x[77]           2.27    0.00  0.08    2.12    2.22    2.27    2.33    2.43
## x[78]           2.28    0.00  0.08    2.12    2.22    2.28    2.33    2.44
## x[79]           2.29    0.00  0.08    2.14    2.23    2.29    2.34    2.45
## x[80]           2.30    0.00  0.08    2.14    2.25    2.30    2.36    2.46
## x[81]           2.33    0.00  0.08    2.17    2.27    2.33    2.38    2.48
## x[82]           2.36    0.00  0.08    2.20    2.30    2.35    2.41    2.51
## x[83]           2.39    0.00  0.08    2.23    2.34    2.39    2.44    2.54
## x[84]           2.43    0.00  0.08    2.27    2.38    2.43    2.48    2.57
## x[85]           2.46    0.00  0.08    2.31    2.41    2.46    2.51    2.60
## x[86]           2.50    0.00  0.08    2.34    2.45    2.50    2.55    2.65
## x[87]           2.53    0.00  0.08    2.39    2.48    2.53    2.58    2.68
## x[88]           2.56    0.00  0.07    2.42    2.51    2.56    2.61    2.71
## x[89]           2.58    0.00  0.07    2.43    2.53    2.58    2.63    2.73
## x[90]           2.59    0.00  0.07    2.45    2.55    2.60    2.64    2.74
## x[91]           2.62    0.00  0.07    2.49    2.57    2.62    2.67    2.77
## x[92]           2.65    0.00  0.07    2.51    2.60    2.65    2.70    2.79
## x[93]           2.68    0.00  0.07    2.55    2.64    2.68    2.73    2.82
## x[94]           2.71    0.00  0.07    2.58    2.67    2.72    2.76    2.84
## x[95]           2.76    0.00  0.07    2.62    2.71    2.76    2.81    2.89
## x[96]           2.80    0.00  0.07    2.67    2.76    2.81    2.85    2.93
## x[97]           2.86    0.00  0.07    2.72    2.81    2.86    2.90    3.00
## x[98]           2.92    0.00  0.07    2.78    2.87    2.92    2.97    3.06
## x[99]           2.98    0.00  0.08    2.84    2.93    2.98    3.03    3.14
## x[100]          3.05    0.00  0.08    2.90    2.99    3.05    3.11    3.23
## x[101]          3.07    0.00  0.08    2.92    3.01    3.06    3.12    3.24
## x[102]          3.05    0.00  0.08    2.91    2.99    3.04    3.10    3.20
## x[103]          3.01    0.00  0.07    2.88    2.96    3.01    3.06    3.16
## x[104]          2.99    0.00  0.07    2.86    2.94    2.99    3.04    3.12
## x[105]          2.96    0.00  0.07    2.84    2.92    2.96    3.01    3.10
## x[106]          2.94    0.00  0.07    2.82    2.90    2.94    2.99    3.08
## x[107]          2.94    0.00  0.06    2.81    2.90    2.94    2.98    3.07
## x[108]          2.93    0.00  0.07    2.80    2.89    2.93    2.98    3.06
## x[109]          2.93    0.00  0.06    2.80    2.88    2.93    2.97    3.06
## x[110]          2.92    0.00  0.07    2.80    2.88    2.92    2.97    3.06
## x[111]          2.93    0.00  0.07    2.80    2.88    2.92    2.97    3.06
## x[112]          2.92    0.00  0.07    2.79    2.88    2.92    2.97    3.06
## x[113]          2.92    0.00  0.07    2.79    2.88    2.92    2.97    3.06
## x[114]          2.90    0.00  0.07    2.77    2.86    2.90    2.95    3.03
## x[115]          2.88    0.00  0.07    2.75    2.83    2.88    2.92    3.01
## x[116]          2.85    0.00  0.07    2.72    2.81    2.85    2.90    2.98
## x[117]          2.82    0.00  0.07    2.69    2.78    2.82    2.87    2.95
## x[118]          2.81    0.00  0.07    2.66    2.76    2.81    2.85    2.94
## x[119]          2.79    0.00  0.07    2.65    2.74    2.79    2.83    2.93
## x[120]          2.77    0.00  0.07    2.63    2.73    2.77    2.82    2.91
## x[121]          2.76    0.00  0.07    2.62    2.72    2.76    2.81    2.90
## x[122]          2.75    0.00  0.07    2.61    2.70    2.75    2.80    2.88
## x[123]          2.73    0.00  0.07    2.59    2.68    2.73    2.78    2.87
## x[124]          2.72    0.00  0.07    2.57    2.67    2.72    2.77    2.86
## x[125]          2.72    0.00  0.07    2.58    2.68    2.72    2.77    2.86
## x[126]          2.71    0.00  0.07    2.57    2.66    2.71    2.76    2.85
## x[127]          2.70    0.00  0.07    2.55    2.65    2.70    2.74    2.84
## x[128]          2.68    0.00  0.07    2.53    2.63    2.68    2.73    2.82
## x[129]          2.66    0.00  0.08    2.51    2.61    2.66    2.71    2.80
## x[130]          2.64    0.00  0.08    2.49    2.59    2.64    2.69    2.79
```

```
## x[131]           2.63    0.00  0.08    2.47    2.57    2.63    2.68    2.78
## x[132]           2.62    0.00  0.08    2.46    2.56    2.62    2.67    2.76
## x[133]           2.61    0.00  0.08    2.45    2.55    2.61    2.66    2.76
## x[134]           2.61    0.00  0.08    2.46    2.56    2.61    2.66    2.77
## x[135]           2.63    0.00  0.08    2.46    2.57    2.63    2.68    2.79
## x[136]           2.64    0.00  0.08    2.48    2.59    2.64    2.70    2.81
## x[137]           2.65    0.00  0.09    2.48    2.59    2.65    2.71    2.82
## x[138]           2.66    0.00  0.09    2.48    2.60    2.66    2.72    2.83
## x[139]           2.66    0.00  0.09    2.47    2.59    2.66    2.73    2.84
## x[140]           2.65    0.00  0.10    2.45    2.59    2.66    2.72    2.84
## post_mean[1]     5.63    0.02  0.84    3.99    5.05    5.62    6.18    7.31
## post_mean[2]     5.74    0.02  0.80    4.23    5.19    5.73    6.27    7.38
## post_mean[3]     5.89    0.02  0.77    4.44    5.36    5.87    6.39    7.47
## post_mean[4]     6.04    0.02  0.73    4.66    5.54    6.03    6.53    7.52
## post_mean[5]     6.25    0.02  0.71    4.90    5.77    6.23    6.71    7.70
## post_mean[6]     6.47    0.02  0.70    5.16    6.00    6.45    6.93    7.91
## post_mean[7]     6.67    0.02  0.69    5.36    6.21    6.66    7.13    8.05
## post_mean[8]     6.81    0.02  0.67    5.55    6.35    6.79    7.24    8.21
## post_mean[9]     6.95    0.02  0.67    5.70    6.50    6.93    7.40    8.33
## post_mean[10]    7.11    0.02  0.67    5.89    6.66    7.08    7.55    8.50
## post_mean[11]    7.27    0.02  0.67    6.07    6.80    7.24    7.71    8.68
## post_mean[12]    7.40    0.02  0.66    6.22    6.92    7.38    7.84    8.77
## post_mean[13]    7.51    0.02  0.66    6.33    7.04    7.48    7.94    8.84
## post_mean[14]    7.64    0.02  0.66    6.47    7.17    7.61    8.08    8.99
## post_mean[15]    7.78    0.02  0.67    6.52    7.33    7.76    8.21    9.15
## post_mean[16]    7.92    0.02  0.68    6.68    7.43    7.90    8.36    9.33
## post_mean[17]    8.07    0.02  0.68    6.80    7.60    8.03    8.51    9.50
## post_mean[18]    8.18    0.02  0.68    6.92    7.74    8.14    8.62    9.54
## post_mean[19]    8.29    0.02  0.67    7.05    7.83    8.27    8.72    9.68
## post_mean[20]    8.42    0.02  0.69    7.15    7.94    8.39    8.85    9.90
## post_mean[21]    8.59    0.02  0.70    7.32    8.11    8.56    9.04   10.07
## post_mean[22]    8.75    0.02  0.72    7.38    8.27    8.72    9.21   10.24
## post_mean[23]    8.86    0.02  0.73    7.46    8.37    8.84    9.33   10.38
## post_mean[24]    8.93    0.02  0.73    7.55    8.44    8.89    9.40   10.42
## post_mean[25]    9.00    0.02  0.73    7.63    8.49    8.97    9.46   10.53
## post_mean[26]    9.02    0.02  0.74    7.67    8.53    9.01    9.49   10.56
## post_mean[27]    9.07    0.02  0.73    7.69    8.56    9.04    9.54   10.58
## post_mean[28]    9.11    0.02  0.72    7.76    8.63    9.09    9.59   10.59
## post_mean[29]    9.08    0.02  0.71    7.69    8.60    9.07    9.55   10.52
## post_mean[30]    9.11    0.02  0.72    7.76    8.62    9.10    9.59   10.59
## post_mean[31]    9.20    0.02  0.72    7.87    8.70    9.18    9.66   10.73
## post_mean[32]    9.29    0.02  0.73    7.92    8.78    9.27    9.78   10.80
## post_mean[33]    9.32    0.02  0.73    7.93    8.82    9.29    9.81   10.80
## post_mean[34]    9.36    0.02  0.75    7.98    8.85    9.31    9.85   10.96
## post_mean[35]    9.39    0.02  0.76    8.00    8.86    9.37    9.87   10.98
## post_mean[36]    9.33    0.02  0.75    7.98    8.81    9.30    9.81   10.86
## post_mean[37]    9.21    0.02  0.74    7.87    8.69    9.18    9.69   10.77
## post_mean[38]    9.05    0.02  0.74    7.73    8.53    9.02    9.53   10.64
## post_mean[39]    8.89    0.02  0.73    7.55    8.38    8.85    9.34   10.41
## post_mean[40]    8.76    0.02  0.71    7.44    8.27    8.72    9.22   10.21
## post_mean[41]    8.61    0.02  0.70    7.30    8.12    8.60    9.05   10.04
## post_mean[42]    8.51    0.02  0.70    7.20    8.02    8.50    8.97    9.95
## post_mean[43]    8.41    0.02  0.69    7.10    7.94    8.39    8.87    9.86
## post_mean[44]    8.34    0.02  0.69    7.05    7.86    8.32    8.79    9.74
```

```
## post_mean[45]     8.31    0.02 0.69    6.98    7.85    8.31    8.76    9.67
## post_mean[46]     8.32    0.02 0.69    7.01    7.86    8.33    8.76    9.71
## post_mean[47]     8.35    0.02 0.69    7.04    7.90    8.35    8.79    9.71
## post_mean[48]     8.35    0.02 0.69    7.04    7.89    8.34    8.79    9.74
## post_mean[49]     8.35    0.02 0.69    7.06    7.88    8.34    8.80    9.75
## post_mean[50]     8.34    0.02 0.70    7.03    7.87    8.33    8.81    9.76
## post_mean[51]     8.30    0.02 0.69    7.00    7.82    8.28    8.77    9.67
## post_mean[52]     8.18    0.02 0.68    6.92    7.70    8.16    8.63    9.55
## post_mean[53]     8.10    0.02 0.69    6.81    7.62    8.06    8.55    9.55
## post_mean[54]     8.05    0.02 0.68    6.78    7.57    8.03    8.51    9.46
## post_mean[55]     8.05    0.02 0.67    6.72    7.60    8.02    8.49    9.41
## post_mean[56]     8.11    0.02 0.68    6.84    7.66    8.08    8.55    9.51
## post_mean[57]     8.17    0.02 0.69    6.87    7.70    8.16    8.63    9.63
## post_mean[58]     8.32    0.02 0.69    7.01    7.84    8.29    8.76    9.71
## post_mean[59]     8.46    0.02 0.70    7.15    7.98    8.43    8.92    9.92
## post_mean[60]     8.56    0.02 0.70    7.26    8.08    8.53    9.03   10.03
## post_mean[61]     8.60    0.02 0.71    7.32    8.10    8.58    9.09   10.04
## post_mean[62]     8.59    0.02 0.70    7.31    8.11    8.57    9.05   10.02
## post_mean[63]     8.53    0.02 0.70    7.20    8.05    8.51    8.98    9.94
## post_mean[64]     8.48    0.02 0.69    7.14    8.02    8.48    8.95    9.87
## post_mean[65]     8.46    0.02 0.70    7.15    7.98    8.46    8.91    9.90
## post_mean[66]     8.43    0.02 0.69    7.12    7.96    8.42    8.89    9.82
## post_mean[67]     8.42    0.02 0.69    7.11    7.94    8.41    8.88    9.80
## post_mean[68]     8.43    0.02 0.70    7.07    7.96    8.43    8.89    9.82
## post_mean[69]     8.51    0.02 0.70    7.15    8.04    8.50    8.98    9.92
## post_mean[70]     8.64    0.02 0.71    7.29    8.16    8.62    9.09   10.10
## post_mean[71]     8.81    0.02 0.72    7.43    8.32    8.78    9.27   10.26
## post_mean[72]     8.97    0.02 0.73    7.60    8.47    8.94    9.42   10.50
## post_mean[73]     9.15    0.02 0.74    7.76    8.64    9.12    9.64   10.72
## post_mean[74]     9.37    0.02 0.76    7.94    8.86    9.34    9.84   10.91
## post_mean[75]     9.53    0.02 0.77    8.08    8.99    9.49   10.01   11.11
## post_mean[76]     9.64    0.02 0.77    8.22    9.10    9.60   10.13   11.25
## post_mean[77]     9.74    0.02 0.78    8.32    9.19    9.69   10.24   11.39
## post_mean[78]     9.79    0.02 0.79    8.35    9.23    9.77   10.30   11.45
## post_mean[79]     9.91    0.02 0.81    8.47    9.34    9.88   10.42   11.62
## post_mean[80]    10.04    0.02 0.81    8.52    9.50   10.02   10.58   11.71
## post_mean[81]    10.28    0.02 0.82    8.76    9.72   10.25   10.80   11.93
## post_mean[82]    10.57    0.02 0.84    9.01   10.00   10.53   11.13   12.32
## post_mean[83]    10.95    0.02 0.85    9.34   10.36   10.92   11.51   12.68
## post_mean[84]    11.35    0.02 0.87    9.71   10.77   11.34   11.92   13.08
## post_mean[85]    11.75    0.02 0.90   10.04   11.15   11.75   12.31   13.51
## post_mean[86]    12.17    0.02 0.93   10.43   11.54   12.16   12.76   14.11
## post_mean[87]    12.62    0.03 0.95   10.89   11.96   12.58   13.24   14.57
## post_mean[88]    13.02    0.02 0.97   11.24   12.36   12.98   13.66   15.04
## post_mean[89]    13.26    0.02 0.98   11.41   12.58   13.22   13.91   15.35
## post_mean[90]    13.43    0.02 0.98   11.63   12.75   13.42   14.06   15.44
## post_mean[91]    13.77    0.03 1.00   12.00   13.05   13.75   14.42   15.89
## post_mean[92]    14.18    0.03 1.00   12.33   13.50   14.15   14.84   16.29
## post_mean[93]    14.67    0.03 1.03   12.76   13.98   14.65   15.33   16.76
## post_mean[94]    15.13    0.02 1.04   13.19   14.42   15.12   15.79   17.17
## post_mean[95]    15.83    0.02 1.08   13.70   15.10   15.81   16.53   17.94
## post_mean[96]    16.55    0.02 1.12   14.43   15.80   16.53   17.28   18.81
## post_mean[97]    17.47    0.03 1.22   15.12   16.64   17.43   18.24   20.00
## post_mean[98]    18.58    0.04 1.34   16.12   17.66   18.53   19.41   21.38
```

```
## post_mean[99]    19.84    0.05  1.51    17.10    18.80    19.75    20.78    23.08
## post_mean[100]   21.23    0.07  1.80    18.12    19.97    21.06    22.34    25.20
## post_mean[101]   21.58    0.07  1.82    18.57    20.29    21.42    22.73    25.47
## post_mean[102]   21.09    0.06  1.65    18.27    19.92    20.95    22.09    24.62
## post_mean[103]   20.41    0.05  1.47    17.80    19.37    20.31    21.35    23.50
## post_mean[104]   19.92    0.04  1.37    17.47    18.95    19.84    20.83    22.74
## post_mean[105]   19.41    0.03  1.29    17.04    18.54    19.31    20.25    22.14
## post_mean[106]   19.03    0.03  1.27    16.71    18.17    18.96    19.86    21.77
## post_mean[107]   18.93    0.03  1.23    16.65    18.10    18.89    19.70    21.50
## post_mean[108]   18.82    0.03  1.25    16.48    17.97    18.77    19.61    21.36
## post_mean[109]   18.68    0.03  1.22    16.42    17.86    18.63    19.42    21.24
## post_mean[110]   18.62    0.03  1.23    16.37    17.79    18.56    19.40    21.27
## post_mean[111]   18.70    0.03  1.25    16.37    17.83    18.63    19.50    21.31
## post_mean[112]   18.62    0.03  1.25    16.30    17.77    18.56    19.43    21.24
## post_mean[113]   18.66    0.03  1.26    16.34    17.79    18.61    19.48    21.33
## post_mean[114]   18.24    0.02  1.21    15.98    17.40    18.21    19.03    20.71
## post_mean[115]   17.82    0.02  1.20    15.56    17.00    17.76    18.57    20.28
## post_mean[116]   17.34    0.02  1.16    15.17    16.56    17.28    18.11    19.78
## post_mean[117]   16.85    0.02  1.14    14.67    16.06    16.84    17.60    19.10
## post_mean[118]   16.57    0.02  1.14    14.35    15.79    16.56    17.32    18.89
## post_mean[119]   16.27    0.03  1.13    14.13    15.51    16.24    16.99    18.68
## post_mean[120]   16.07    0.03  1.13    13.94    15.30    16.03    16.81    18.36
## post_mean[121]   15.88    0.03  1.13    13.76    15.12    15.84    16.61    18.20
## post_mean[122]   15.65    0.03  1.11    13.55    14.88    15.63    16.38    17.88
## post_mean[123]   15.38    0.02  1.09    13.36    14.62    15.35    16.10    17.55
## post_mean[124]   15.23    0.02  1.09    13.13    14.50    15.19    15.93    17.40
## post_mean[125]   15.26    0.02  1.08    13.25    14.52    15.21    15.98    17.48
## post_mean[126]   15.06    0.02  1.08    13.05    14.33    15.02    15.73    17.35
## post_mean[127]   14.84    0.02  1.07    12.82    14.12    14.81    15.54    17.04
## post_mean[128]   14.65    0.03  1.08    12.61    13.91    14.64    15.34    16.85
## post_mean[129]   14.33    0.03  1.08    12.30    13.59    14.31    15.03    16.51
## post_mean[130]   14.07    0.03  1.08    12.03    13.31    14.06    14.78    16.29
## post_mean[131]   13.86    0.03  1.07    11.87    13.11    13.84    14.55    16.08
## post_mean[132]   13.73    0.03  1.07    11.70    13.00    13.71    14.43    15.84
## post_mean[133]   13.60    0.03  1.07    11.59    12.87    13.55    14.29    15.80
## post_mean[134]   13.68    0.03  1.09    11.66    12.94    13.63    14.37    15.97
## post_mean[135]   13.87    0.03  1.14    11.70    13.10    13.82    14.60    16.26
## post_mean[136]   14.10    0.03  1.17    11.94    13.30    14.04    14.88    16.54
## post_mean[137]   14.25    0.03  1.23    11.99    13.38    14.21    15.06    16.73
## post_mean[138]   14.37    0.03  1.29    11.99    13.47    14.33    15.22    16.91
## post_mean[139]   14.34    0.03  1.35    11.86    13.38    14.30    15.26    17.03
## post_mean[140]   14.28    0.03  1.43    11.64    13.26    14.24    15.23    17.14
## lp__           3026.26    1.27 20.93 2982.04 3012.72 3027.14 3041.09 3064.61
##                n_eff Rhat
## mu               389 1.01
## phi             1280 1.00
## sigma            320 1.01
## x[1]            1442 1.00
## x[2]            1333 1.00
## x[3]            1360 1.00
## x[4]            1405 1.00
## x[5]            1499 1.00
## x[6]            1508 1.00
## x[7]            1471 1.00
```

```
## x[8]              1500 1.00
## x[9]              1628 1.00
## x[10]             1543 1.00
## x[11]             1493 1.00
## x[12]             1435 1.00
## x[13]             1492 1.00
## x[14]             1615 1.00
## x[15]             1775 1.00
## x[16]             1757 1.00
## x[17]             1801 1.00
## x[18]             1806 1.00
## x[19]             1816 1.00
## x[20]             1819 1.00
## x[21]             1729 1.00
## x[22]             1718 1.00
## x[23]             1645 1.00
## x[24]             1629 1.00
## x[25]             1579 1.00
## x[26]             1465 1.00
## x[27]             1509 1.00
## x[28]             1584 1.00
## x[29]             1573 1.00
## x[30]             1706 1.00
## x[31]             1736 1.00
## x[32]             1794 1.00
## x[33]             1697 1.00
## x[34]             1709 1.00
## x[35]             1677 1.00
## x[36]             1710 1.00
## x[37]             1824 1.00
## x[38]             1920 1.00
## x[39]             1911 1.00
## x[40]             1881 1.00
## x[41]             1924 1.00
## x[42]             1830 1.00
## x[43]             1753 1.00
## x[44]             1554 1.00
## x[45]             1482 1.00
## x[46]             1611 1.00
## x[47]             1730 1.00
## x[48]             1787 1.00
## x[49]             1798 1.00
## x[50]             1782 1.00
## x[51]             1773 1.00
## x[52]             1455 1.00
## x[53]             1261 1.00
## x[54]             1217 1.00
## x[55]             1130 1.00
## x[56]             1086 1.00
## x[57]             1254 1.00
## x[58]             1323 1.00
## x[59]             1397 1.00
## x[60]             1538 1.00
## x[61]             1568 1.00
```

```
## x[62]           1626 1.00
## x[63]           1722 1.00
## x[64]           1620 1.00
## x[65]           1597 1.00
## x[66]           1484 1.00
## x[67]           1378 1.00
## x[68]           1227 1.00
## x[69]           1213 1.00
## x[70]           1280 1.00
## x[71]           1376 1.00
## x[72]           1350 1.00
## x[73]           1372 1.00
## x[74]           1497 1.00
## x[75]           1592 1.00
## x[76]           1637 1.00
## x[77]           1390 1.00
## x[78]           1333 1.00
## x[79]           1264 1.00
## x[80]           1226 1.00
## x[81]           1215 1.00
## x[82]           1261 1.00
## x[83]           1320 1.00
## x[84]           1594 1.00
## x[85]           1582 1.00
## x[86]           1592 1.00
## x[87]           1382 1.00
## x[88]           1586 1.00
## x[89]           1792 1.00
## x[90]           1605 1.00
## x[91]           1622 1.00
## x[92]           1585 1.00
## x[93]           1656 1.00
## x[94]           1724 1.00
## x[95]           2057 1.00
## x[96]           2028 1.00
## x[97]           1782 1.00
## x[98]           1265 1.00
## x[99]            937 1.00
## x[100]           651 1.00
## x[101]           627 1.01
## x[102]           774 1.00
## x[103]          1071 1.00
## x[104]          1194 1.00
## x[105]          1543 1.00
## x[106]          1754 1.00
## x[107]          2205 1.00
## x[108]          2034 1.00
## x[109]          1959 1.00
## x[110]          2250 1.00
## x[111]          2079 1.00
## x[112]          2278 1.00
## x[113]          2166 1.00
## x[114]          2368 1.00
## x[115]          2484 1.00
```

```
## x[116]          2502 1.00
## x[117]          2358 1.00
## x[118]          2099 1.00
## x[119]          1967 1.00
## x[120]          1796 1.00
## x[121]          1834 1.00
## x[122]          1844 1.00
## x[123]          1977 1.00
## x[124]          1950 1.00
## x[125]          2038 1.00
## x[126]          1994 1.00
## x[127]          1907 1.00
## x[128]          1862 1.00
## x[129]          1783 1.00
## x[130]          1594 1.00
## x[131]          1531 1.00
## x[132]          1644 1.00
## x[133]          1544 1.00
## x[134]          1370 1.00
## x[135]          1309 1.00
## x[136]          1581 1.00
## x[137]          1698 1.00
## x[138]          1766 1.00
## x[139]          1853 1.00
## x[140]          1855 1.00
## post_mean[1]    1408 1.00
## post_mean[2]    1311 1.00
## post_mean[3]    1324 1.00
## post_mean[4]    1381 1.00
## post_mean[5]    1494 1.00
## post_mean[6]    1527 1.00
## post_mean[7]    1495 1.00
## post_mean[8]    1510 1.00
## post_mean[9]    1656 1.00
## post_mean[10]   1586 1.00
## post_mean[11]   1547 1.00
## post_mean[12]   1486 1.00
## post_mean[13]   1530 1.00
## post_mean[14]   1619 1.00
## post_mean[15]   1784 1.00
## post_mean[16]   1742 1.00
## post_mean[17]   1794 1.00
## post_mean[18]   1811 1.00
## post_mean[19]   1818 1.00
## post_mean[20]   1819 1.00
## post_mean[21]   1729 1.00
## post_mean[22]   1725 1.00
## post_mean[23]   1672 1.00
## post_mean[24]   1664 1.00
## post_mean[25]   1602 1.00
## post_mean[26]   1508 1.00
## post_mean[27]   1540 1.00
## post_mean[28]   1603 1.00
## post_mean[29]   1608 1.00
```

```
## post_mean[30]   1730 1.00
## post_mean[31]   1771 1.00
## post_mean[32]   1818 1.00
## post_mean[33]   1728 1.00
## post_mean[34]   1742 1.00
## post_mean[35]   1712 1.00
## post_mean[36]   1744 1.00
## post_mean[37]   1850 1.00
## post_mean[38]   1930 1.00
## post_mean[39]   1914 1.00
## post_mean[40]   1886 1.00
## post_mean[41]   1944 1.00
## post_mean[42]   1837 1.00
## post_mean[43]   1763 1.00
## post_mean[44]   1608 1.00
## post_mean[45]   1521 1.00
## post_mean[46]   1656 1.00
## post_mean[47]   1805 1.00
## post_mean[48]   1847 1.00
## post_mean[49]   1833 1.00
## post_mean[50]   1809 1.00
## post_mean[51]   1784 1.00
## post_mean[52]   1458 1.00
## post_mean[53]   1253 1.00
## post_mean[54]   1240 1.00
## post_mean[55]   1170 1.00
## post_mean[56]   1126 1.00
## post_mean[57]   1276 1.00
## post_mean[58]   1332 1.00
## post_mean[59]   1391 1.00
## post_mean[60]   1533 1.00
## post_mean[61]   1573 1.00
## post_mean[62]   1628 1.00
## post_mean[63]   1733 1.00
## post_mean[64]   1642 1.00
## post_mean[65]   1632 1.00
## post_mean[66]   1509 1.00
## post_mean[67]   1398 1.00
## post_mean[68]   1244 1.00
## post_mean[69]   1225 1.00
## post_mean[70]   1298 1.00
## post_mean[71]   1394 1.00
## post_mean[72]   1366 1.00
## post_mean[73]   1381 1.00
## post_mean[74]   1515 1.00
## post_mean[75]   1613 1.00
## post_mean[76]   1651 1.00
## post_mean[77]   1414 1.00
## post_mean[78]   1367 1.00
## post_mean[79]   1275 1.00
## post_mean[80]   1245 1.00
## post_mean[81]   1225 1.00
## post_mean[82]   1259 1.00
## post_mean[83]   1322 1.00
```

```
## post_mean[84]   1606 1.00
## post_mean[85]   1584 1.00
## post_mean[86]   1596 1.00
## post_mean[87]   1365 1.00
## post_mean[88]   1571 1.00
## post_mean[89]   1792 1.00
## post_mean[90]   1572 1.00
## post_mean[91]   1585 1.00
## post_mean[92]   1529 1.00
## post_mean[93]   1627 1.00
## post_mean[94]   1733 1.00
## post_mean[95]   2102 1.00
## post_mean[96]   2059 1.00
## post_mean[97]   1812 1.00
## post_mean[98]   1253 1.00
## post_mean[99]    930 1.00
## post_mean[100]   628 1.00
## post_mean[101]   612 1.01
## post_mean[102]   760 1.00
## post_mean[103]  1035 1.00
## post_mean[104]  1175 1.00
## post_mean[105]  1520 1.00
## post_mean[106]  1709 1.00
## post_mean[107]  2118 1.00
## post_mean[108]  1993 1.00
## post_mean[109]  1880 1.00
## post_mean[110]  2239 1.00
## post_mean[111]  2060 1.00
## post_mean[112]  2267 1.00
## post_mean[113]  2166 1.00
## post_mean[114]  2362 1.00
## post_mean[115]  2465 1.00
## post_mean[116]  2487 1.00
## post_mean[117]  2357 1.00
## post_mean[118]  2129 1.00
## post_mean[119]  1995 1.00
## post_mean[120]  1807 1.00
## post_mean[121]  1848 1.00
## post_mean[122]  1859 1.00
## post_mean[123]  1985 1.00
## post_mean[124]  1950 1.00
## post_mean[125]  2043 1.00
## post_mean[126]  2001 1.00
## post_mean[127]  1917 1.00
## post_mean[128]  1866 1.00
## post_mean[129]  1776 1.00
## post_mean[130]  1570 1.00
## post_mean[131]  1514 1.00
## post_mean[132]  1632 1.00
## post_mean[133]  1531 1.00
## post_mean[134]  1327 1.00
## post_mean[135]  1185 1.00
## post_mean[136]  1507 1.00
## post_mean[137]  1600 1.00
```

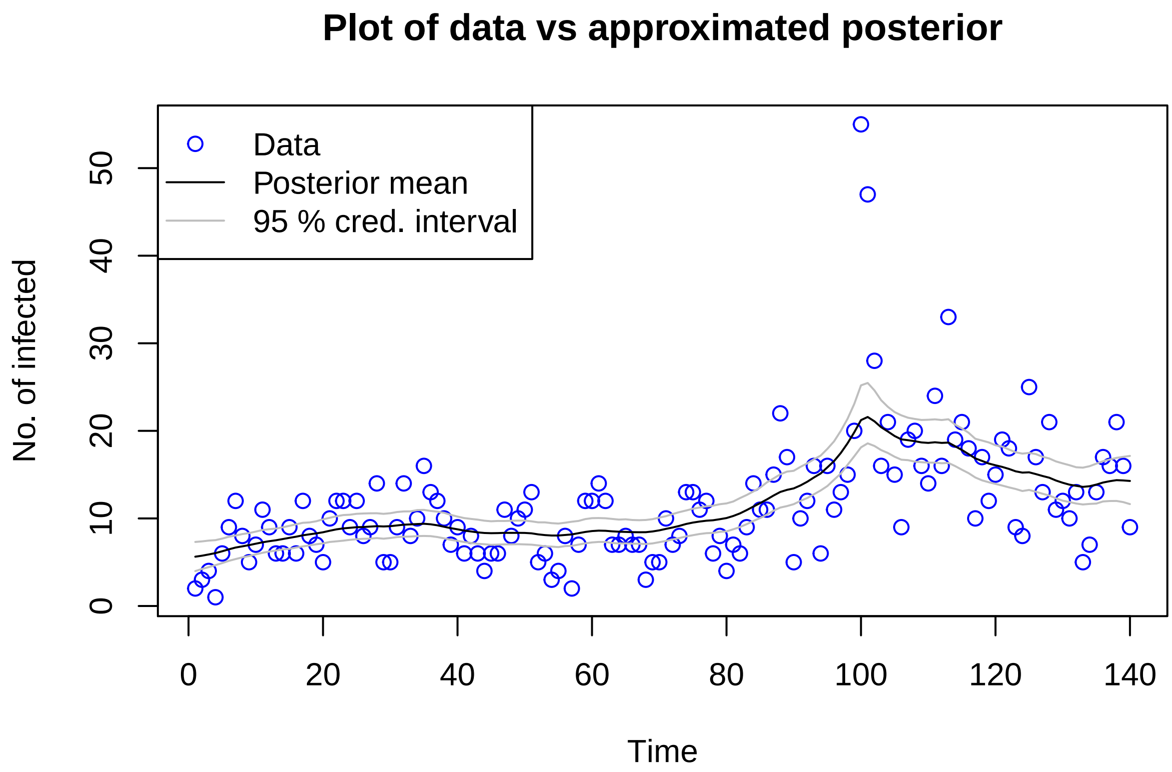
```
## post_mean[138]  1737 1.00
## post_mean[139]  1822 1.00
## post_mean[140]  1826 1.00
## lp__             270 1.02
##
## Samples were drawn using NUTS(diag_e) at Thu May 21 15:48:53 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
pois_mean_list_prior=fit_pois_prior@.MISC$summary$msd
post_mean_prior=pois_mean_list_prior[grep("post_mean", rownames(pois_mean_list)),]

plot(campy$c, col="blue", ylab="No. of infected", xlab="Time")
points(post_mean_prior[,1], col="black", type="l")

quantiles_prior=fit_pois_prior@.MISC$summary$quan
quantiles_post_mean_prior=quantiles_prior[grep("post_mean", rownames(quantiles)),]
cred_interval_post_mean_prior=matrix(0,dim(quantiles_post_mean)[1], 2)
cred_interval_post_mean_prior[,1]=quantiles_post_mean_prior[,1]
cred_interval_post_mean_prior[,2]=quantiles_post_mean_prior[,ncol(quantiles_post_mean)]

lines(cred_interval_post_mean_prior[,1], col="gray", lty=1)
lines(cred_interval_post_mean_prior[,2], col="gray", lty=1)
title(main="Plot of data vs approximated posterior")
legend("topleft", box.lty= 1, pch=c(1,NaN,NaN),
       legend=c("Data", "Posterior mean", "95 % cred. interval"),
       col=c("blue", "black", "gray"), lwd=c(NaN,1,1), lty=c(NaN, 1, 1))
```



Now when we have specified a small prior for sigma, it is noteable in the new plot that the posterior mean varies less and moves more smoothly. The consequence of this is that more datapoints lie outside of the credible interval which suggests that the approximated posterior does not resemble the reality described by the data as accurately as before. However, by defining a prior for sigma which indicates that the posterior mean does not vary as much, one can avoid overfitting when the model is applied to a new dataset.

Appendix

```
## Assignment 1:
## a) Write a function in R that simulate data from the AR(1)-process: xt=mu+phi(x(t-1)-mu) + epsilon(t
## epsilon(t)~N(0,sigma^2), for given values of mu, phi, and sigma^2. Start the process at x1=mu and th
## values for xt for t=2,3,...,T and return the vector x1:T containing all time points. Use mu=10, sigm
## T=200 and look at different realizations (simulation) of x1:T for values of phi between -1 and 1 (th
## interval of phi where the AR-process is stable). Include a plot of at least one realization in the r
## effect does the value of phi have on x1:t

#install.packages("rstan")
mu=10
sigma_sq=2
T=200
x_init=mu
phi_vector=seq(-0.9,0.9,0.1)
results_matrix=matrix(0,200,length(phi_vector))
results_matrix[1,]=x_init
counter=1
```

```
set.seed(12345)

AR_process_function=function(mu, sigma_sq, T, phi) {
  x_init=mu
  result=rep(0,T)
  result[1]=x_init
  for (i in 2:T) {
    epsilon=rnorm(1,0,sqrt(sigma_sq))
    result[i]=mu+phi*(result[i-1]-mu)+epsilon
  }
  return(result)
}

results_matrix=matrix(0,T,length(phi_vector))
counter=1
for (phi in phi_vector) {
  results_matrix[,counter]=AR_process_function(mu,sigma_sq,T,phi)
  counter=counter+1
}
iter=seq(1,200,1)
counter=1
for (i in 1:length(phi_vector)) {
  if (counter %% 6 == 0) {
    plot(iter, results_matrix[,i], main="Plot of realization of AR-process", sub=paste("Phi =", phi_vec
         xlab="Iteration", ylab="Value", type="l", col="grey")
  }
  counter=counter+1
}

## b) Use your function from a) to simulate two AR(1)-processes, x1:T with phi=0.3 and y1:T with phi=0.
## treat the values of mu, phi and sigma^2 as unknown and estimate them using MCMC. Implement Stan-code
## samples from the posterior of the three parameters, using suitable non-informative priors of your ch
## [Hint: Look at the time-series models examples in the Stan user's guide/reference manual, and note t
## parametizations used here.]
## i) Report the posterior mean, 95% credible intervals and the number of effective posterior samples f
## three inferred parameters for each of the simulated AR(1)-process. Are you able to estimate the true
## ii)  For each of the two data sets, evaluate the convergence of the samplers and plot the joint post
## mu and phi. Comments?

library(rstan)

x=rep(0,T)
y=rep(0,T)
set.seed(12345)
x=AR_process_function(mu, sigma_sq, T, 0.3)
set.seed(12345)
y=AR_process_function(mu, sigma_sq, T, 0.95)

StanModel= '
data {
  int<lower=0> N;
  vector[N] y;
}
```

```
parameters {
  real mu;
  real phi;
  real<lower=0> sigma;
}
model {
  for (n in 2:N)
    y[n] ~ normal(mu + phi * (y[n-1]-mu), sigma);
}
'

data_x=list(N=T, y=x)
data_y=list(N=T, y=y)
fit_x=stan(model_code=StanModel, data=data_x)
fit_y=stan(model_code=StanModel, data=data_y)
postDraws_x <- extract(fit_x)
postDraws_y <- extract(fit_y)
print(fit_x)
print(fit_y)

# Do traceplots of the first chain
plot(postDraws_x$mu[1000:2000], postDraws_x$phi[1000:2000],ylab="phi", xlab="mu", main="Traceplot")

# Do traceplots of the first chain
plot(postDraws_y$mu[1000:2000],postDraws_y$phi[1000:2000],ylab="mu", xlab="mu",main="Traceplot")

## c) The data campy.dat contain the number of cases of campylobacter infections in the north of the pr
## Quebec (Canada) in four week intervals from January 1990 to the end of October 2000. It has 13 obser
## year and 140 observations in total. Assume that the number of infections ct at each time point follo
## independent Poisson distribution when conditioned on a latend AR(1)-process xt, that is
## ct given xt ~ Poisson(exp(xt)), where xt is an AR(1)-process as in a). Implement and estimate the mo
## using suitable priors of your choice. Produce a plot that contains both the data and the posterior m
## 95 % credible intervals for the latent intensity theta_t=exp(xt) over time.
## [Hint: Should xt be seen as data or parameters]

campy=read.table("campy.dat", header=TRUE)
library(rstan)

StanModel_Pois = '
data {
  int<lower=0> T;
  int c[T];
}

parameters {
  real mu;
  real phi;
  real<lower=0> sigma;
  vector[T] x;
}

model {
  // Prior
```

```
  phi ~ uniform(-1,1);
  for (n in 2:T)
    x[n] ~ normal(mu + phi * (x[n-1]-mu), sigma);


  // Model/likelihood
  for (n in 1:T)
    c[n] ~ poisson(exp(x[n]));

}

generated quantities {
  vector[T] post_mean;
  post_mean = exp(x);
}
'

data=list(T=dim(campy)[1], c=campy$c)
fit_pois=stan(model_code=StanModel_Pois, data=data)
print(fit_pois)
pois_mean_list=fit_pois@.MISC$summary$msd
post_mean=pois_mean_list[grep("post_mean", rownames(pois_mean_list)),]

plot(campy$c, col="blue", ylab="No. of infected", xlab="Time")
points(post_mean[,1], col="black", type="l")

quantiles=fit_pois@.MISC$summary$quan
quantiles_post_mean=quantiles[grep("post_mean", rownames(quantiles)),]
cred_interval_post_mean=matrix(0,dim(quantiles_post_mean)[1], 2)
cred_interval_post_mean[,1]=quantiles_post_mean[,1]
cred_interval_post_mean[,2]=quantiles_post_mean[,ncol(quantiles_post_mean)]

lines(cred_interval_post_mean[,1], col="gray", lty=21)
lines(cred_interval_post_mean[,2], col="gray", lty=21)
title(main="Plot of data vs approximated posterior")
legend("topleft", box.lty= 1, pch=c(1,NaN,NaN), legend=c("Data", "Posterior mean", "95 % cred. interval
       col=c("blue", "black", "gray"), lwd=c(NaN,1,1), lty=c(NaN, 1, 21))

## d) Now, assume that we have a prior belief that the true underlying intensity theta_t varies more sm
## the data suggests. Change the prior for sigma_sq so that it becomes informative about that the AR(1)
## increments epsilon_t should be small. Re-estimate the model using Stan with the new prior and produc
## plot as in c). Has the posterior for theta_t changed?

StanModel_Pois_Prior = '
data {
  int<lower=0> T;
  int c[T];
}

parameters {
  real mu;
  real phi;
  real<lower=0> sigma;
```

```
  vector[T] x;
}

model {
  // Prior
  phi ~ uniform(-1,1);
  sigma ~ scaled_inv_chi_square(140, 0.15);
  for (n in 2:T)
    x[n] ~ normal(mu + phi * (x[n-1]-mu), sigma);


  // Model/likelihood
  for (n in 1:T)
    c[n] ~ poisson(exp(x[n]));

}

generated quantities {
  vector[T] post_mean;
  post_mean = exp(x);
}
'
fit_pois_prior=stan(model_code=StanModel_Pois_Prior, data=data)
print(fit_pois_prior)
pois_mean_list_prior=fit_pois_prior@.MISC$summary$msd
post_mean_prior=pois_mean_list_prior[grep("post_mean", rownames(pois_mean_list)),]

plot(campy$c, col="blue", ylab="No. of infected", xlab="Time")
points(post_mean_prior[,1], col="black", type="l")

quantiles_prior=fit_pois_prior@.MISC$summary$quan
quantiles_post_mean_prior=quantiles_prior[grep("post_mean", rownames(quantiles)),]
cred_interval_post_mean_prior=matrix(0,dim(quantiles_post_mean)[1], 2)
cred_interval_post_mean_prior[,1]=quantiles_post_mean_prior[,1]
cred_interval_post_mean_prior[,2]=quantiles_post_mean_prior[,ncol(quantiles_post_mean)]

lines(cred_interval_post_mean_prior[,1], col="gray", lty=21)
lines(cred_interval_post_mean_prior[,2], col="gray", lty=21)
title(main="Plot of data vs approximated posterior")
legend("topleft", box.lty= 1, pch=c(1,NaN,NaN), legend=c("Data", "Posterior mean", "95 % cred. interval
       col=c("blue", "black", "gray"), lwd=c(NaN,1,1), lty=c(NaN, 1, 21))
```