Project 1

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Problem 1

Part A

For Part A, the following statistics were calculated:

- Mean: 0.050198
- Variance: 0.010322
- Skewness: 0.120626
- Kurtosis: 0.230070

Part B

Part B compared a Normal Distribution with a T-Distribution. The observed kurtosis was higher than 0, suggesting heavier tails than a normal. Based on this, the T-Distribution would be preferable.

Part C

Both distributions were fitted using MLE. The more negative AIC and BIC values for the Normal Distribution indicated a better fit, relative to the initial kurtosis result. Therefore, a Normal Distribution was chosen as the better fit.

Problem 2

Part A: Pairwise Covariance Matrix

A pairwise approach was used to compute covariances between columns, ignoring rows lacking data for each pair. The matrix obtained was:

1.47048437	1.45421424	0.87726904	1.90322645	1.44436105
1.45421424	1.25207795	0.53954816	1.62191837	1.23787697
0.87726904	0.53954816	1.272425	1.17195897	1.091912
1.90322645	1.62191837	1.17195897	1.81446921	1.58972858
1.44436105	1.23787697	1.091912	1.58972858	1.39618646

Part B: Positive Semi-Definiteness Check

The eigenvalues were:

Eigenvalue

Eigenvalue
6.78670573
0.83443367
-0.31024286
0.02797828
-0.13323183

Two negative eigenvalues were found, indicating the matrix is not positive semi-definite.

Part C: Nearest PSD Matrix

Two methods were used: Rebenato and Jackel, and Higham's method.

Rebenato and Jackel

This approach sets all negative eigenvalues to zero after eigen-decomposition and reconstructs the matrix:

1.61513295	1.44196041	0.89714421	1.78042572	1.43379434
1.44196041	1.34696791	0.58508635	1.55455193	1.21140918
0.89714421	0.58508635	1.29891578	1.11595578	1.07669234
1.78042572	1.55455193	1.11595578	1.98316488	1.62137332
1.43379434	1.21140918	1.07669234	1.62137332	1.40493616

Higham's Method

1. With matrix A, set $Y_0 = A^{**}$, $\Delta S_0 = 0$.
2. $R_k = Y_{k-1} - \Delta S_{k-1}$.
3. Clamp negative eigenvalues of R_k to zero.
4. Update ΔS.
5. Repeat until the difference $||Y_k - Y_{k-1}||$ is below a tolerance or the maximum number of iterations is reached.

1.61513295	1.44196041	0.89714421	1.78042572	1.43379434
1.44196041	1.34696791	0.58508635	1.55455193	1.21140918
0.89714421	0.58508635	1.29891578	1.11595578	1.07669234
1.78042572	1.55455193	1.11595578	1.98316488	1.62137332
1.43379434	1.21140918	1.07669234	1.62137332	1.40493616

Part D: Overlapping Data Covariance

Excluding rows with missing data for any column produced the following covariance matrix (using same cov matrix methodology from part A):

0.41860366	0.39405407	0.42445735	0.41638241	0.43428682
0.39405407	0.39678563	0.40934344	0.3984012	0.42263077
0.42445735	0.40934344	0.4413601	0.42844141	0.44895733
0.41638241	0.3984012	0.42844141	0.43727358	0.44016735
0.43428682	0.42263077	0.44895733	0.44016735	0.46627249

Part E: Comparison of the Matrices from (C) and (D)

The matrices in (C) use maximum available pairs, which can create inconsistencies leading to negative eigenvalues. The overlapping-data matrix in (D) is PSD by construction but may discard many rows. The off-diagonal values in the near-PSD matrices can differ significantly from the overlapping-data approach because of sample size differences.

Problem 3

Part A: Fitting a Multivariate Normal

After loading the dataset, the sample mean vector and covariance matrix were computed for X_1 and X_2. The mean is approximately [0.0460, 0.0999], and the covariance matrix is:

0.0101622	0.00492354
0.00492354	0.02028441

Part B: Distribution of X_2 Given X_1 = 0.6

Method 1 (Conditional Formula) Using the standard bivariate normal conditional formula gave:

- Conditional Mean ~ 0.368325
- Conditional Variance ~ 0.017899

Method 2 (Linear Regression) A simple linear regression of X_2 on X_1 yielded:

- Conditional Mean ~ 0.368325
- Residual MSE ~ 0.017917

Both methods align well.

Part C: Cholesky-Based Simulation

A large sample (X_1, X_2) was generated from N(μ, Σ). After filtering for X_1 close to 0.6:

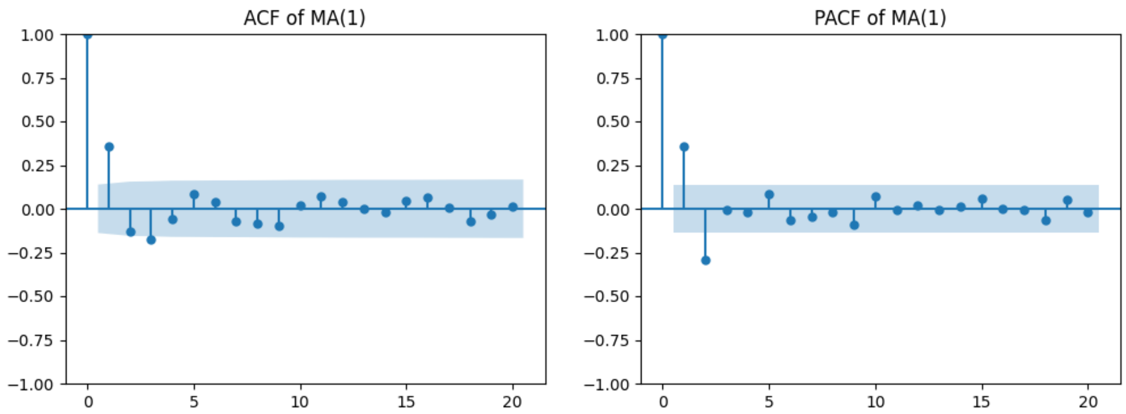
- Simulated Conditional Mean ~ 0.368784
- Simulated Conditional Variance ~ 0.014673

These are consistent with the analytical approach.

Problem 4

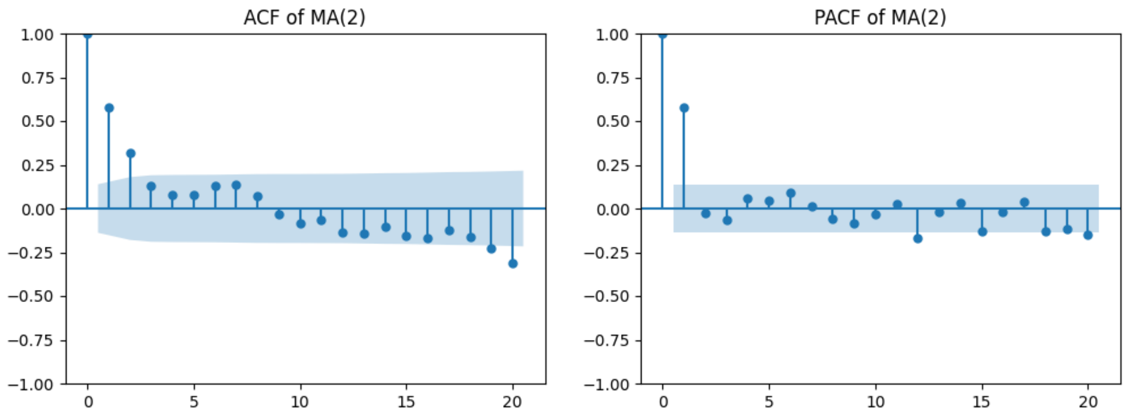
Part A: Simulating MA(1), MA(2), and MA(3)

1. **MA(1)**



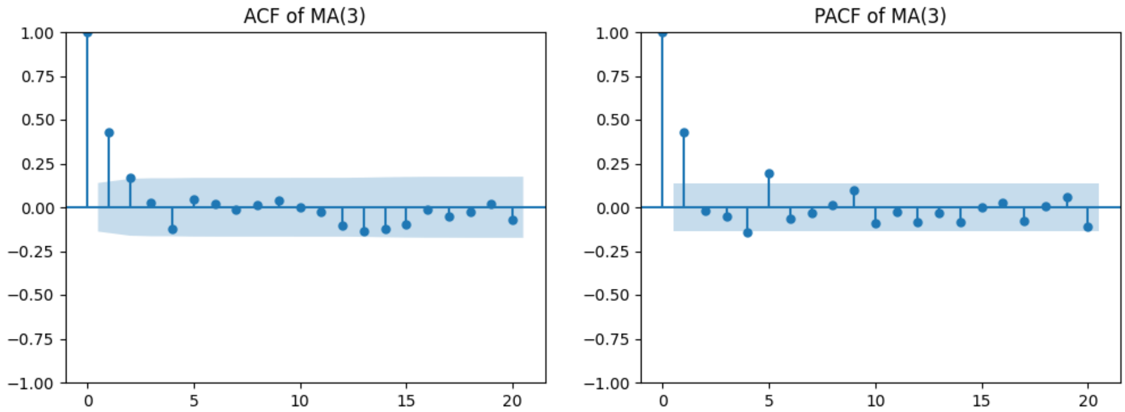
- **ACF**: Notice a significant spike at lag 1, then it drops near zero for higher lags.
- **PACF**: The first two lags are significant. Lag 17 is also significant.

2. **MA(2)**



- **ACF**: Typically shows two significant spikes (lags 1 and 2) before going near zero for larger lags.
- **PACF**: Although MA(1) doesn't exhibit gradual tapering, MA(2) PCF shows an even more aggressive drop after the first lag.

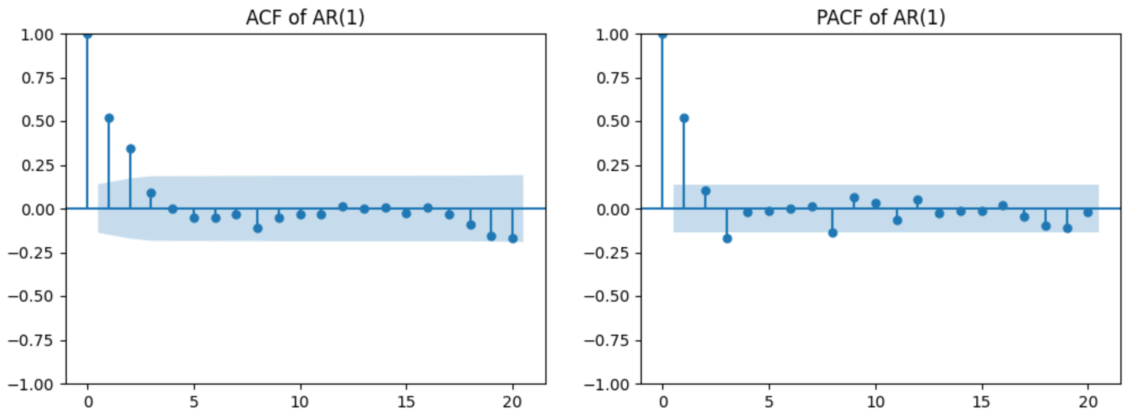
3. **MA(3)**



- **ACF**: Three significant autocorrelation spikes (lags 1, 2, and 3), dropping off afterward.
- **PACF**: Similar to MA(2) PCF, gradual tapering isn't seen.

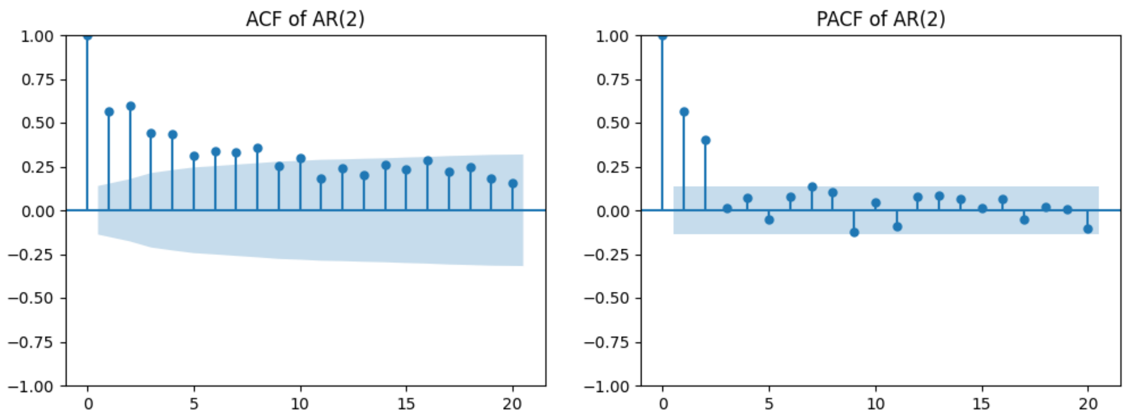
Part B: Simulating AR(1), AR(2), and AR(3)

1. **AR(1)**



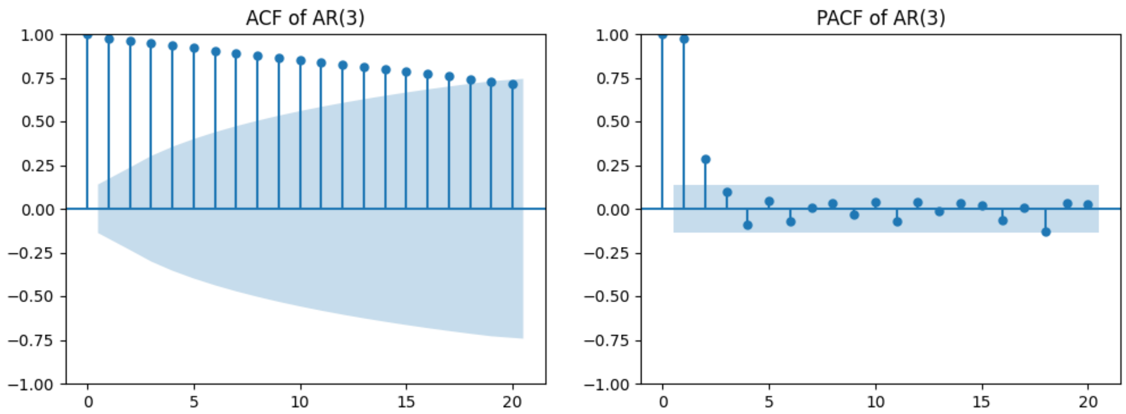
- **ACF**: It tails off exponentially, but it cuts off after lag 2.
- **PACF**: Shows a clear significant spike at lag 1, then becomes very small afterward.

2. **AR(2)**



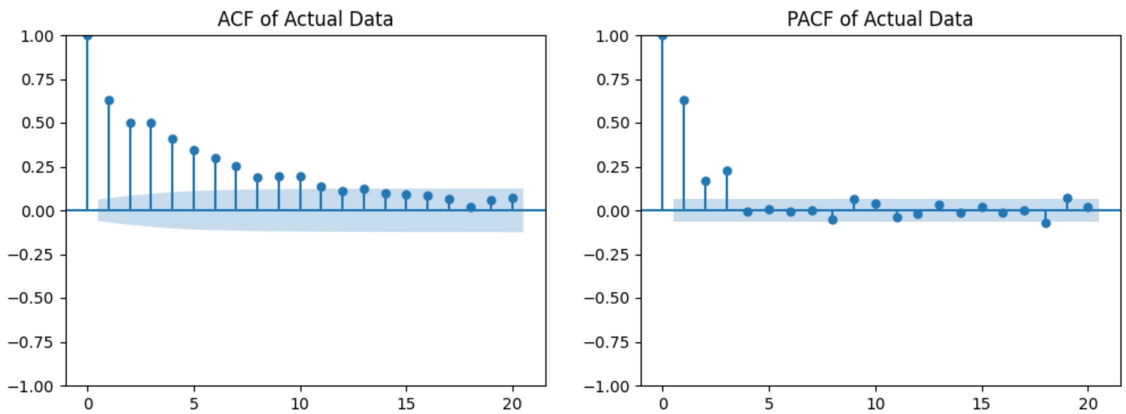
- **ACF**: It decays as expected for an AR model.
- **PACF**: Significant spikes at lags 1 and 2, then mostly zero for higher lags.

3. **AR(3)**



- **ACF**: It decays, but it decays at an even slower rate than seen in AR(20).
- **PACF**: Three significant partial autocorrelation spikes, followed by lower values afterward. This confirms AR(3) structure.

Part C: Examining `problem4.csv`



The ACF gradually declines, and the PACF has three prominent lags. This suggests an AR(3) model.

Part D

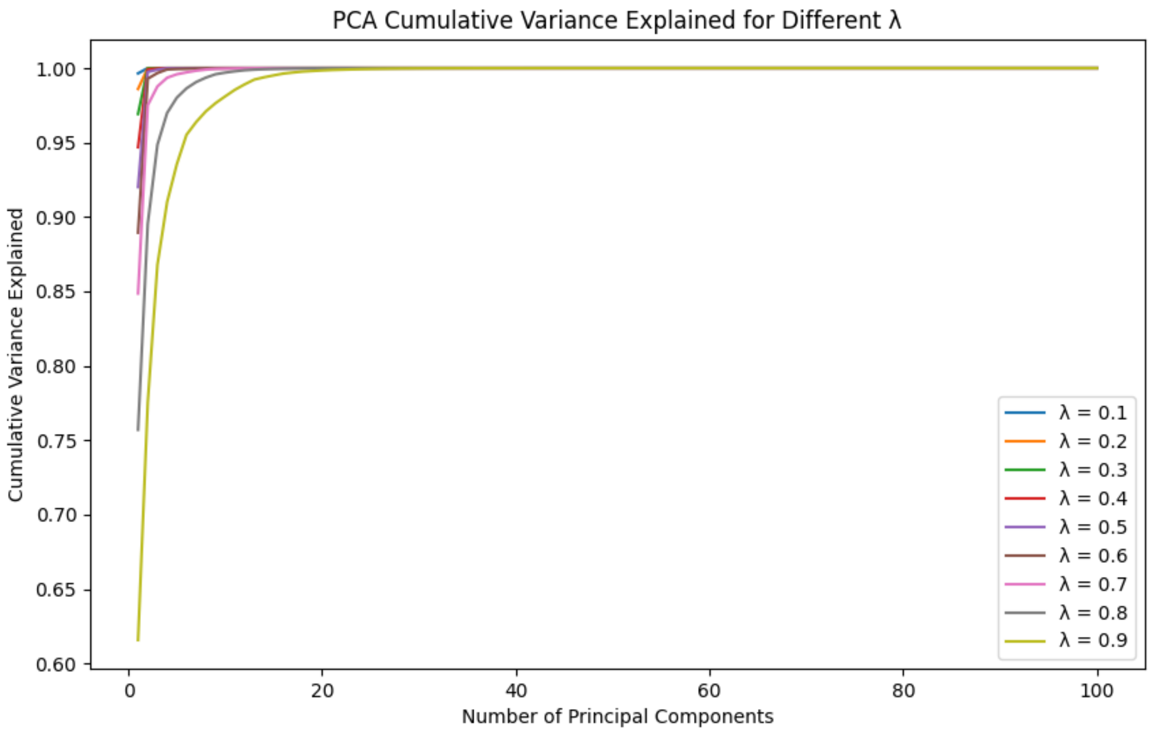
Model fit comparison using AIC and AICc:

```
(1, 0, 0) => (AIC=-1669.089, AICc=-1669.065)
(2, 0, 0) => (AIC=-1696.092, AICc=-1696.051)
(3, 0, 0) => (AIC=-1746.282, AICc=-1746.221)
(0, 0, 1) => (AIC=-1508.927, AICc=-1508.903)
(0, 0, 2) => (AIC=-1559.251, AICc=-1559.211)
(0, 0, 3) => (AIC=-1645.133, AICc=-1645.073)
```

The best model by AIC is AR(3).

Problem 5

Part B



Part C

Larger lambda values decay more slowly, so older observations remain significant and the covariance matrix is more stable. Smaller lambda values make recent data more influential, producing a sharper drop in the leading principal components.

Problem 6

Part A

A Cholesky Root simulation was performed with a near-psd matrix for stability. The matrix was generated using the near-psd method of Rebenato and Jackel.

Part C

The Frobenius norm relative to the original is 0.000006 for Cholesky-based and 0.000022 for PCA-based, indicating the Cholesky-based matrix is closer.

Part D

The Cholesky-based matrix preserves the entire covariance structure, leading to a gradual cumulative variance. The PCA-based method explains 66% by the first component, 84% by the seventh, and 100% by the 29th, reflecting fewer non-zero eigenvalues and a lower rank.

Part F: Tradeoffs

The Cholesky approach is quicker and retains the full covariance. PCA offers dimension reduction and noise filtering, which can be beneficial in high-dimensional settings or when dropping minor components is preferred.