Should Banks Create Money?

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Abstract

The paper compares the welfare properties of the current fractional reserve banking (FB) system versus a narrow banking (NB) system where deposits are fully backed by reserves. The analysis shows that under sufficient competition FB is beneficial compared to NB because of higher interest payments on deposits. Since under FB deposits fund loans, banks have a higher income on their asset side which – if competition is sufficiently high – they pass on to deposit holders in the form of higher interest. This improves welfare because it compensates the deposit holders against inflation. A calibrated version of the model suggests however that the welfare gains of FB are relatively small, below 0.16% of yearly GDP in the US between 1984 and 2008

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1 Introduction

The appropriate "division of labor" between central banks and commercial banks in the money supply of an economy has been increasingly debated after the recent financial crisis. In Switzerland for example there was a public referendum on forcing banks to fully back their monetary liabilities with reserves, i.e. to impose a 100% reserve narrow banking (NB) system.[1] Also, many central banks think about introducing "Central Bank Digital Currencies" (CBDC), i.e. offering reserves not only to commercial banks but also to the general public.[2]

What is at stake if we consider moving from the current fractional reserve banking (FB) system, where commercial banks provide the majority of the money supply, towards another monetary system like 100% reserves, where money issued by central banks is more important? Usually this topic is phrased as an efficiency vs. stability tradeoff. FB is supposed to have efficiency benefits but at the cost of increasing financial instability compared to NB proposals. However, while the fragility of FB is relatively well understood – following the seminal paper by Diamond and Dybvig (1983) – the efficiency benefits of FB seem less clear. Brunnermeier and Niepelt (2019) for instance argue there are no specific efficiency gains from FB. Any FB allocation can be replicated with a NB or CBDC allocation, given appropriate transfers and open-market operations. This echoes the quantity theory: the quantity and the composition of money should be irrelevant for real allocations and welfare. On the other hand the fact that FB dominated monetary history suggests a socially useful function, e.g. increasing

[1] For the policy discussion in other countries, see `https://internationalmoneyreform.org/`. Pennacchi (2012) offers a good overview on NB proposals.

[2] This would introduce competition between commercial bank deposits and reserves from central banks. Niepelt (2020) and Bank for International Settlements (2018) give an overview on CBDC-proposals. A few central banks like Uruguay and Sweden have already implemented pilot CBDC projects.

beneficial long-term investment as in Diamond and Dybvig (1983). However, authors like Chari and Phelan (2014) show an inefficient FB system can persist over time due to a pecuniary externality over the price level. Does FB with its characteristic combination of financing investment and lending with monetary liabilities, even have a socially useful function compared to NB proposals like 100% reserves? If yes, what is it? And how important is it quantitatively?

To address these questions I build a banking model based on Lagos and Wright (2005) and more specifically on Berentsen et al. (2007). The model has the following main ingredients: A central bank issues potentially interest bearing reserves, which can circulate as money, controls inflation and the nominal interest rate. Commercial banks issue deposits and bonds, finance loans and hold reserves. Deposits are usable as a means of payment while bonds are illiquid. Under FB banks partially back deposits with reserves while under NB they fully do it. Banks have market power in the deposit market. Finally there are households who borrow from banks and use deposits or reserves to buy consumption goods. Holding money is costly, so the central bank sets a positive nominal interest rate above the Friedman rule. This "inflation tax" is the basic inefficiency in the model. To quantify the implications of the model I calibrate it with US-data from 1984–2008.

The main theoretical result is that FB is socially useful compared to a NB system if the banking sector is sufficiently competitive. FB then provides a means of payment with a higher return than NB. The mechanism is as follows: Under FB banks fund loans with deposits while under NB deposits are fully backed with reserves and loans are financed by bonds. Since loans have a higher return than reserves, banks have a higher income on their asset side under FB than under NB. With sufficient competition they pass on this higher income to their liability side in the form of higher interest payments on deposits. This is beneficial because it compensates the deposit holders against inflation or the money holding costs. The calibrated version of the model suggests however, that the welfare gains of FB due to higher interest payments are relatively small. They are around 0.1% (0.35%) of GDP at a nominal interest rate of 5% (10%) and below 0.16% of yearly GDP in the US between 1984 and 2008. If the central bank pays interest on reserves the welfare gains from FB are even smaller, and with a specific tax/transfer scheme they can be fully eliminated, confirming the equivalence argument of Brunnermeier and Niepelt (2019). To the extend that NB systems are able to resolve the inherent fragility of FB, this suggests that achieving financial stability with a NB system is not so costly.

The paper also contributes to the debate on the implications of introducing a CBDC. There are concerns that a CBDC lowers intermediation and bank lending and is thus detrimental for welfare, especially if the CBDC is interest bearing. Facing competition from CBDC, banks might need to increase their deposit rates and these higher funding costs could translate into higher loan rates and lower lending. This mechanism is for example present in Chiu et al. (2019) or in Keister and Sanches (2019). Chiu et al. (2019) show the transmission depends on competition. In particular, if competition is low the CBDC forces banks to rise the deposit rate and decreases bank profits but the loan rate is unaffected or even decreases because banks attract more deposits at higher deposit rates. This paper shows the transmission also depends on the role of other bank liabilities. If banks, or financial intermediaries more generally, use other liabilities than deposits to fund loans – in the model these are bonds – a higher interest on CBDC only increases the deposit rate but the loan rate is unaffected.[3] This formalizes the idea that the introduction of a CBDC might not affect loan rates

[3] In the model some households are savers and prefer to hold illiquid bonds to liquid deposits with lower return. Thus issuing bonds is always useful. This is in contrast to Chiu et al. (2019) where issuing bonds is also possible but bonds don't have an essential economic function.

too much because financial intermediaries can finance lending and investment with other, less liquid liabilities than deposits.

The rest of the paper is organized as follows: Section 2 shows the basic environment. Section 3 then presents the model under perfect competition and section 4 under imperfect competition. I calibrate the model in section 5 and finally conclude in section 6.

2 Environment

The environment is based on Lagos and Wright (2005) and in particular on Berentsen et al. (2007). Time is discrete and continues forever. Every period is divided into two sequential competitive markets called *first market* (FM) and *second market* (SM).[4] There is a perishable consumption good q in the FM and x in the SM.

There is a unit mass of infinitely lived households or agents. They discount future periods with β and are anonymous. At the beginning of every period households face a *preference shock* which divides them into two groups. With probability $s \in (0,1)$ an agent is a *seller* in the FM and produces with weakly convex disutility $c(q)$. With the inverse probability an agent is a *buyer* in the FM and consumes with strictly concave utility $u(q)$, satisfying the Inada-conditions. In the SM all agents consume x and produce h with quasi-linear utility $U(x)-h$. $U(x)$ is strictly concave in x and also satisfies the Inada-conditions.

The efficient quantities for consumption in the FM, q^* and q_s^*, and in the SM, x^*, equalize marginal utility of consumption and marginal disutility of production in both markets:

[4] Since both markets are competitive I follow Berentsen et al. (2007) and call these markets first and second market instead of the more common notations decentralized market (DM) and centralized market (CM) in the New Monetarist literature.

$$\frac{u'(q^*)}{c'(\frac{1-s}{s}q^*)} = 1, \quad q_s^* = \frac{1-s}{s}q^*, \quad U'(x^*) = 1. \tag{1}$$

Since households are anonymous buyers cannot issue debt to buy goods from sellers in the FM. Buyers thus find it useful to hold a means of payment. There are two means of payment in this economy: a public one coming from a *central bank* and a private one coming from *commercial banks.*

The central bank issues outside fiat money M or reserves. Reserves are potentially interest bearing with interest rate i_m and growth rate $\gamma = M/M_{-1}$.[5] Both households and banks can hold reserves.[6] The central bank manages the supply of reserves by lump-sum transfers τ to agents in the SM. Thus the central bank seignorage revenue in period t, $M - M_{-1}$, which is negative if the central bank reduces the money supply, is used for transfers τ and interest payments $i_m M_{-1}$. The nominal budget constraint of the central bank reads:

$$\tau + i_m M_{-1} = M - M_{-1} = (\gamma - 1)M_{-1} \tag{2}$$

An economy without interest on reserves resembles the monetary system before the financial crisis, where banks hold non-interest bearing reserves and households hold currency. On the other hand an economy with interest on reserves resembles a monetary system with an interest bearing CBDC, to which both, households and banks, have access.[7]

[5] In the whole paper I denote variables in a representative period t without subscript, the period before t with subscript -1 and the period after with $+1$.

[6] In reality central banks issue two different types of outside money, reserves and currency. While currency is always non-interest bearing, reserves might bear interest, especially since the recent financial crisis. In the US for instance, the FED started to pay interest on reserves in October 2008. Also, households don't have access to reserves in the current system while banks can hold both.

[7] In equilibrium only banks hold reserves in this model. One could easily introduce a

Reserves are the numeraire in the economy. Let $1/\phi$ be the price of x in terms of reserves in the SM and let p be the price of q in terms of reserves in the following SM. Also define inflation π as the ratio of prices between two consecutive SM, i.e. $\pi = \phi_{-1}/\phi$. Under stationarity the real value of money is constant, $\phi M = \phi_{-1}M_{-1}$, and the money growth rate equals the inflation rate, $\gamma = \pi$. So the central bank perfectly controls long-run inflation and the nominal interest rate in this model, which I define as $1 + i = \gamma/\beta$, the product of inflation γ and the real interest rate $1/\beta$.

Throughout the paper I assume holding money is costly. This implies the central bank sets a positive nominal interest rate above the Friedman rule where $\gamma = \beta$ and above the interest rate on reserves:

$$i > 0 \quad , \quad i > i_m \tag{3}$$

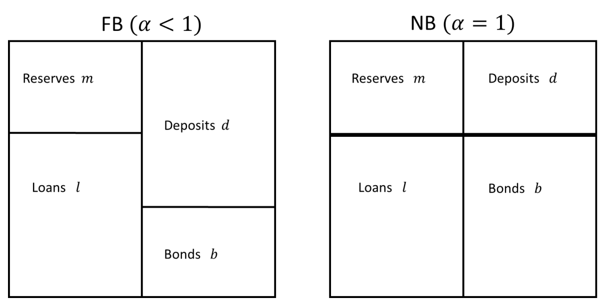
This assumption gives rise to the basic inefficiency of the environment in the form of an "inflation tax" on real FM activity. If holding money is costly, households hold too little and FM consumption and production is too low. Note that the preference shock aggravates this basic inefficiency. The risk to be a seller with costly idle money holdings in the FM makes acquiring money even less attractive ex-ante.

The second source of the money supply are profit-maximizing commercial banks. Banks can commit and monitor agents at no cost. This enables them to issue debt and to make loans. Banks acquire two types of assets: loans l at interest rate i_l and and reserves m. To finance these assets they issue deposits d at interest rate i_d and bonds at interest rate i_b. Deposits are the other means of

demand for reserves from households though, with an additional market in the FM where trades can only be settled with reserves. This would not affect the main results of the model.

payment in the economy besides reserves, while bonds are illiquid.[8] Deposits are subject to a reserve requirement. A bank issuing d deposits must back them at least with a fraction α of reserves, so $\alpha d \leq m$. Usually banks want to issue as many deposits as they can, so the reserve requirement binds. The reserve requirement then also determines the monetary system. If $\alpha < 1$ banks issue more deposits than reserves and we have a FB system and if $\alpha = 1$ banks fully back deposits with reserves and we have a NB system. The following figure shows the banking sector of the economy, operating either under FB (left) or under NB (right)

Figure 1: The banking sector under FB and NB



The NB system can be interpreted as a system where banks organize themselves in two legally or institutionally separated entities, e.g. with a "payment bank" and an "investment bank". The payment bank handles transactions and fully backs her monetary liabilities (deposits) with reserves. The investment bank makes investments and lends issuing non-monetary liabilities like bonds.[9] A FB system combines payments and investments in one institution. In particular, under FB banks can also fund loans with monetary liabilities like deposits.

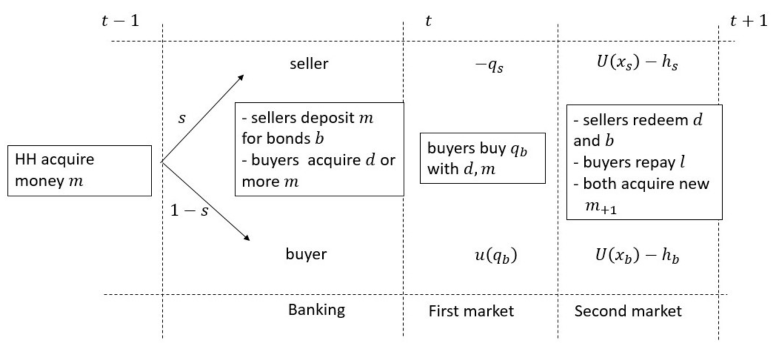
[8]Deposits and bonds might also be interpreted as two more or less liquid forms of deposits, like checking and time deposits.

[9]For such a bank to be "narrow" it must be clear that bond-holders don't have claims on reserves if e.g. loans or investments fail. The thick horizontal line in the NB system in figure 1 should illustrate this separation.

Financial contracts are nominal, formed after the preference shock in the FM, and fully redeemed in the following SM.[10] There will be two different market structures: In section 3 the banking sector is fully competitive. In section 4 however, there will be Cournot competition with N banks in the deposit market while the loan and the bond market are still perfectly competitive.

The following figure summarizes the structure and the sequence of events in this economy:

Figure 2: Sequence of events



In the SM in $t-1$ households acquire reserves m. As we will see, all agents acquire the same money holdings due to quasi-linear preferences. After the preference shock, households interact with banks. Since sellers don't need liquidity in the FM, they hold the assets with the highest return. In equilibrium this will be bonds. Buyers on the other hand want liquidity to buy goods in the FM. They borrow from banks to hold either reserves or deposits, depending on the return of these two kinds of money. In the FM buyers use their money to buy goods from the sellers. Finally, in the following SM sellers redeem deposits and bonds and buyers repay their loans. Both types then acquire new identical

[10]With linear utility in the second market there is no gain from spreading the redemption of debt or the repayment of loans over multiple periods. Thus assuming this kind of contracts is not constraining in this environment.

reserve holdings m_{+1}. As we will see banks have two roles in this environment. They create liquidity by issuing deposits and they provide insurance against the preference shock by issuing bonds.

3 The Model

3.1 Banks

Bank acquire reserves m and loans l by issuing deposits d and bonds b. They maximize the nominal value of her assets minus the value of liabilities, subject to the reserve constraint and a balance sheet constraint:

$$\max_{l,m,d,b} = m(1+i_m) + l(1+i_l) - d(1+i_d) - b(1+i_b) \tag{4}$$

$$s.t. \quad \alpha d \leq m \quad , \quad d+b = l+m$$

Using the balance sheet constraint with $m = d+b-l$ the problem rewrites as:

$$\max_{l,d,b} = l(i_l - i_m) - d(i_d - i_m) - b(i_b - i_m)$$

$$s.t. \quad l \leq (1-\alpha)d + b$$

If $i_l, i_d, i_b > i_m$ the bank wants to lend as much as possible and to hold as little reserves as possible. Thus the reserve constraint binds

$$l = (1-\alpha)d + b \tag{5}$$

and we can rewrite the objective function as:

$$\max_{d,b} \quad d\left((\alpha i_m + (1-\alpha)i_l - i_d\right) + b\left(i_l - i_b\right).$$

Any equilibrium with positive demand for deposits and bonds thus involves

$$i_d = \alpha i_m + (1-\alpha)i_l \tag{6}$$

$$i_b = i_l \tag{7}$$

and banks making zero profits. From (6) the deposit rate is a weighted average of the return on reserves paying and the return on loans. With a binding reserve requirement α is the ratio of reserves to deposits, and $1-\alpha$ is the ratio of loans financed by deposits to deposits. In equilibrium the loan rate will exceed reserve rate, so the deposit rate lies between the loan rate and the reserve rate. Under NB the deposit rate just equals the reserve rate, since deposits are only backed by reserves. From (7) the bond rate equals the loan rate since banks also finance loans with bonds. Consistent with the assumption of a binding reserve constraint, we thus get the following relations between the interest rates from the banking problem:

$$i_m \leq i_d < i_b = i \tag{8}$$

By issuing bonds banks can lend more because they don't need to hold reserves when issuing bonds,[11] but bonds are more expensive. This is the basic tradeoff between deposits and bonds for the bank.

As already explained in the environment the binding reserve requirement also

[11] From the binding reserve requirement (5) the marginal increase in the lending capacity by issuing deposits is only $1-\alpha$ while by issuing bonds it is 1.

determines whether banks operate under a FB system with $\alpha \in (0,1)$ or a NB system with $\alpha = 1$.

3.2 Buyers

Let $V_b(m)$ denote the value of entering period t as a buyer with m reserves, and let $V(m_{b+1})$ denote the expected value of a buyer entering period $t+1$ with m_{b+1} reserves. The problem of a buyer reads:

$$V_b(m) = \max_{q_b,x_b,h_b,m_{b+1},l_b,d_b,m'_b,b_b} \quad u(q_b) - h_b + U(x_b) + \beta V(m_{b+1})$$

$$s.t. \quad m + l_b \leq b_b + m'_b + d_b \tag{9}$$

$$pq_b \leq m'_b(1+i_m) + d_b(1+i_d) \tag{10}$$

$$h_b + \phi\left(m'_b(1+i_m) + d_b(1+i_d) - pq_b\right) + \phi\tau + b_b\phi(1+i_b) \tag{11}$$

$$= x_b + \phi m_{b+1} + l_b\phi(1+i_l)$$

When buyers enter the period they first acquire financial assets and borrow from banks. They use their reserves m and borrow l_b to acquire bonds b_b, reserves m'_b and deposits d_b as shown in (9). In the FM buyers use their money to buy goods from sellers. They either use reserves or deposits, see (10).[12] When buyers enter the SM they get resources from working h_b, from monetary wealth left over after the FM, $m'_b(1+i_m)+d_b(1+i_d)-pq_b$, from real transfers of the central bank, $\phi\tau$ and from their bond holdings $b_b\phi(1+i_b)$. They use these resources to acquire consumption goods x_b, new money holdings ϕm_{b+1} and repay loans $l_b\phi(1+i_l)$ as shown in (11).

[12]Note that p is the price of q in terms of reserves in the *next* SM. This is why the interest rates i_m and i_d show up in the budget constraint. One might think of $m'_b(1+i_m)+d_b(1+i_d)$ as the "monetary wealth" of a buyer.

In the following δ denotes the Lagrange multiplier for (9) and λ the Lagrange multiplier for (10). Substituting (11) into the objective function, the problem yields the following first-order conditions, assuming interior solutions for q_b, x_b, m_{b+1} and l_b:

$$q_b : \qquad u'(q_b) = p\,(\phi + \lambda) \tag{12}$$

$$x_b : \qquad U'(x_b) = 1 \tag{13}$$

$$m_{b+1} : \qquad \beta V'(m_{b+1}) = \phi \tag{14}$$

$$l_b : \qquad \delta = \phi(1 + i_l) \tag{15}$$

$$d_b : \qquad (1 + i_d)(\phi + \lambda) \leq \delta \tag{16}$$

$$m_b' : \qquad (1 + i_m)(\phi + \lambda) \leq \delta \tag{17}$$

$$b_b : \qquad \phi(1 + i_b) \leq \delta \tag{18}$$

Clearly buyers want to hold either reserves or deposits. Otherwise they cannot consume in the FM. From the bank problem (8) we know that the deposit rate is weakly above the interest on reserves. Thus the marginal benefit of deposits, the left-hand side of (16), is weakly higher than the marginal benefit of reserves, the left-hand side of (17). Therefore buyers weakly prefer holding deposits, and I assume this also holds if they are indifferent. So (16) binds and (17) is slack. Combining (16) and (15) the multiplier of the liquidity constraint reads:

$$\lambda = \phi \left(\frac{1 + i_l}{1 + i_d} - 1 \right)$$

The expression says that as long as there is a spread between the loan rate and the deposit rate, buyers use all their deposits in the FM, i.e. they are liquidity constrained. From the bank problem we know the loan rate is above the deposit rate so this holds. We also know the bond rate equals the loan rate. Using this in (15) and (18), we see buyers are indifferent between holding bonds or not. I assume they do not hold bonds. Finally from (13) buyers consume the efficient quantity in the SM, and from (14) the expected marginal value of new reserve holdings equals marginal costs. Summarizing, the solution to the buyer problem reads:

$$u'(q_b) = p\phi\frac{1+i}{1+i_d} \tag{19}$$

$$x_b = x^* \tag{20}$$

$$\beta V'(m_{b+1}) = \phi \tag{21}$$

$$d_b = \frac{pq_b}{1+i_d} \tag{22}$$

$$l_b = d_b - m \tag{23}$$

$$m_b' = 0 \quad , \quad b_b = 0 \tag{24}$$

This also yields the envelope condition to the buyer problem:

$$V_b'(m) = \delta = \phi(1+i_l) \tag{25}$$

3.3 Sellers

The problem for sellers is analogue to the buyers' problem. Only that they don't face a liquidity constraint in the FM because they don't want to consume:

$$
\begin{aligned}
V_s(m) = \max_{q_s, x_s, h_s, m_{s+1}, l_s, d_s, m_s', b_s} \quad & -c(q_s) - h_s + U(x_s) + \beta V(m_{s+1}) \\
s.t. \quad & m + l_s \leq b_s + m_s' + d_s \\
& h_s + \phi\left(m_s'(1+i_m) + d_s(1+i_d) + pq_s\right) + \phi\tau + b_s\phi(1+i_b) \\
& = x_s + \phi m_{s+1} + l_s\phi(1+i_l)
\end{aligned}
$$

Again I replace h_s in the objective function with the budget constraint.[13] Assuming interior solutions for q_s, x_s and m_{s+1} we get the following first-order conditions:

$$
\begin{aligned}
& q_s: && p\phi = c'(q_s) \\
& x_s: && U'(x_s) = 1 && (26) \\
& m_{s+1}: && \beta V'(m_{s+1}) = \phi && (27) \\
& l_s: && \delta \leq \phi(1+i) \\
& d_s: && (1+i_d)\phi \leq \delta \\
& m_s': && (1+i_m)\phi \leq \delta \\
& b_s: && \phi(1+i_b) \leq \delta && (28)
\end{aligned}
$$

Since sellers don't need liquidity in the FM, their choice of financial assets is only driven by return considerations. From the banking problem and (8) we know the bond rate is higher than the deposit rate and the reserve rate. Therefore sellers only want to hold bonds and (28) holds with equality. Because

[13] This assumes $h_s > 0$ which requires a scaling condition on $U(.)$. As we will see sellers also consume the efficient quantity in the SM $x_s = x^*$. So $h_s > 0$ holds if x^* is sufficiently big.

the bond rate equals the loan rate, sellers are indifferent between borrowing and not borrowing. I assume they don't borrow. From (26) also sellers consume the efficient amount in the SM, and the first-order condition for new reserve holdings, (27), is identical to the condition for buyers, (14). In particular the condition does not depend on the agents type or their wealth in the SM. So buyers and sellers choose the same reserve holdings in the SM: $m_{b+1} = m_{s+1} = m_{+1}$. This is an implication of quasi-linear utility introduced by Lagos and Wright (2005). We can summarize the solution to the seller problem as:

$$c'(q_s) = p\phi \tag{29}$$

$$x_s = x^* \tag{30}$$

$$\beta V'(m_{+1}) = \phi \tag{31}$$

$$d_s = l_s = m'_s = 0 \tag{32}$$

$$b_s = m \tag{33}$$

Sellers produce in the FM to equalize marginal disutility and utility of production, they consume the efficient quantity x^* in the SM and they acquire the same money holdings as buyers in the SM. They only hold bonds because these have the highest return and they don't borrow. The envelope condition for sellers is thus:

$$V'_s(m) = \delta = \phi(1 + i_b) \tag{34}$$

Iterating (34) and the envelope condition of buyers (25) one period forward, the optimality condition for optimal reserve holdings in the SM rewrites as:

$$\phi = \beta V'(m_{+1}) = \beta s V_s'(m_{+1}) + \beta(1-s)V_b'(m_{+1}) \tag{35}$$
$$= \beta s \phi_{+1}(1 + i_{b+1}) + \beta(1-s)\phi_{+1}(1 + i_{l+1})$$

This says that the marginal expected value of reserves equals the marginal value of reserves for buyers plus the marginal value for sellers times the respective probabilities. For sellers the marginal value of reserves is the real bond rate because they deposit reserves to acquire bonds. For buyers the marginal value of reserves is the real loan rate because if they bring more reserves they can borrow less and they save on the loan interest payments.

Finally we derive the market clearing conditions. Market clearing for reserves in the SM reads:

$$(1-s)m_{b+1} + s m_{s+1} = m_{+1} = M \tag{36}$$

Since buyers and sellers both acquire the same reserve holdings and they together have mass one, individual reserve holdings equal aggregate reserve holdings.

Suppose the banking sector in total supplies D deposits, B bonds, holds L loans and R reserves. Market clearing for reserves, bonds, deposits and loans then reads:

$$M_{-1} = R + (1-s)m_b' + s m_s' = R \tag{37}$$
$$B = (1-s)b_b + s b_s = s M_{-1} \tag{38}$$
$$D = (1-s)d_b = \frac{M_{-1}}{\alpha} \tag{39}$$
$$L = (1-s)l_b = \left(\frac{1-\alpha}{\alpha} + s\right) M_{-1} \tag{40}$$

From (37), the total amount of reserves brought into the period by buyers and sellers, M_{-1}, must equal the total reserve holdings of banks R and the reserve holdings of buyers and sellers. Since the later don't hold reserves after interacting with the banks, banks hold all reserves and $R = M_{-1}$. In the bond market (38) banks supply B bonds, which must equal total demand from sellers because only sellers hold bonds. In the deposit market (39) banks issue D deposits which must equal demand from buyers because only buyers hold deposits. Since banks hold all the reserves from (37) banks issue M_{-1}/α deposits, due to the binding reserve requirement. The loan market (40) equalizes total loans by banks with total demand from buyers.

Ultimately the FM and the SM must clear as follows:

$$(1-s)q_b = sq_s \tag{41}$$

$$(1-s)x_b + sx_s = x^* = (1-s)h_b + sh_s \tag{42}$$

3.4 Equilibrium

To solve for equilibrium interest rates we combine the expression for optimal reserve holdings, (35), with the relations for interest rates from the bank problem, (6) and (7), and apply stationarity where $\gamma = \phi/\phi_{+1}$. This yields:

$$i_l = i_b = i \tag{43}$$

$$i_d = (1-\alpha)i + \alpha i_m \tag{44}$$

The equilibrium loan rate and the bond rate equal the nominal interest rate and are independent of reserve requirement. The equilibrium deposit rate is a weighted average of the nominal interest rate and the interest on reserves.

By assumption the interest on reserves is below the nominal interest rate, so the deposit rate is between the reserve rate and the nominal interest rate and decreases in the reserve requirement. Under NB the deposit rate equals the rate on reserves. Thus the deposit rate is higher under FB than under NB.

To get equilibrium consumption in the FM we combine optimal consumption (19) with optimal production (29) and use the equilibrium expressions for the interest rates and market clearing (41):

$$\frac{u'(q_b)}{c'(\frac{1-s}{s}q_b)} = \frac{1+i}{1+i_d} \tag{45}$$

The rest of the equilibrium allocation is then given by:

$$\begin{aligned}
q_s &= \frac{1-s}{s}q_b \\
\phi D &= (1-s)q_b\frac{c'(q_s)}{1+i_d} \\
\phi M_{-1} &= \alpha\phi D \\
x_b &= x_s = x^* \\
h_s &= x^* - \phi M_{-1}i - c'(q_s)q_s \\
h_b &= x^* + \frac{q_b c'(q_s)(1+i)}{1+i_d} - \phi M_{-1}i
\end{aligned} \tag{46}$$

We can focus on the allocation in the FM given by (45) for welfare analysis, since SM consumption is always efficient. From (45) FM-consumption decreases in the spread between the nominal interest rate and the deposit rate.[14] Without spread, the allocation achieves the efficient allocation in (1). Therefore the welfare comparison between FB and NB depends on which system is able to

[14]The left-hand side of (45) decreases in q_b, so a higher spread must imply a lower q_b.

provide a higher deposit rate. The following proposition summarizes the main welfare implications of the two banking systems:

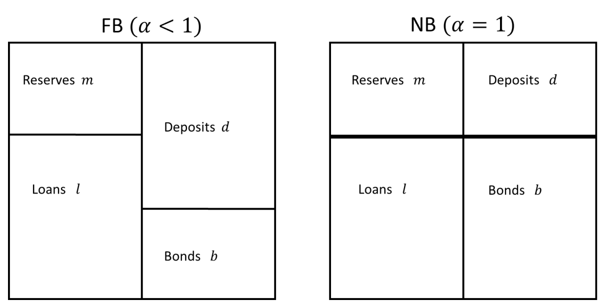
Proposition 1. *Suppose holding money is costly, banking is fully competitive and the banking system either operates under FB with $\alpha \in (0,1)$ or under NB with $\alpha = 1$. Then there is a stationary equilibrium in which:*

i) welfare is higher under FB than under NB

ii) FB and NB both provide perfect insurance against the preference shock

iii) higher interest on reserves increases welfare in both systems but relatively more under NB

Remember that the basic difference between a FB and a NB system is that under FB deposits also fund loans while under NB deposits only finance reserves and loans are fully financed by bonds. This is shown again in the following figure.



The return on loans is higher than the return on reserves in equilibrium. So banks have a higher income on their asset side under FB than under NB. With perfect competition they will pass on this higher income to the deposit holders in form of higher interest payments.[15] This is beneficial because it compensates the deposit holders against inflation or the money holding costs. Higher interest payments under FB thus reduce the welfare costs of inflation.

[15]The next section discusses this conclusion under imperfect competition.

Proposition 1 contradicts the quantity theory. If banks issue more deposits – induced by a lower reserve requirement – the real allocation improves. In contrast to the quantity theory the composition and the quantity of money matter here.[16] Can we restore neutrality with an appropriate tax/transfer system as Brunnermeier and Niepelt (2019) argue? Proposition 1 shows that simply increasing the reserve rate is not enough. Although from *iii*) the deposit rate and welfare increase more under NB in this case, the deposit rate and welfare remain higher under FB from *i*). We can restore neutrality only if we allow for system specific interest rates on reserves, say i_m under FB and $i'_m > i_m$ under NB. In particular, suppose the central bank under NB pays reserve rate equal to the deposit rate under FB, i.e. she pays $i'_m = i_d = (1-\alpha)i + \alpha i_m$. Then logically the allocations under FB and NB are identical. The central bank can finance these higher interest payments by lower transfers or higher taxes, in line with her budget constraint (2). In equilibrium this means buyers work more and sellers work less in the SM. But these changes do not matter for welfare since both groups have linear disutility of working in the SM.

Proposition 1 shows also that banks provide insurance against the preference or liquidity shock. Note that the equilibrium allocation (45) is independent from the preference shock s. The reason is that banks issue bonds paying an interest equal to the nominal interest rate. This means sellers, who acquire bonds in the banking period, are perfectly compensated for the money holding costs. In other words, the risk of becoming a seller with idle, costly money holdings is irrelevant if bonds paying the nominal interest rate are available. This also holds under NB. In fact, in appendix A.2 I show that the NB allocation is identical to an economy without preference shock and banks.

The NB allocation is also equivalent to the basic version of Berentsen et al.

[16] Note however that an increase in deposits induced by a one time change in the level of reserves is neutral.

(2007). In this model there are only reserves and banks reallocate these after the preference shock. Thus the proposition shows how the model of Berentsen et al. (2007) can be interpreted as a NB economy where banks issue fully backed deposits and bonds.[17]

There is a debate on whether it matters if banks are modelled as institutions who influence the money supply by issuing partially backed deposits "ex nihilo" or as institutions who only intermediate already existing funds like outside money (Jakab and Kumhof, 2015; Andolfatto, 2018). As explained above, these two modelling approaches exactly correspond to the difference between FB and NB in the model. Under FB banks create deposits "ex nihilo" and influence the money supply while under NB they are pure "intermediators" of reserves as in Berentsen et al. (2007). Proposition 1 shows this can make a difference and thus provides a counterexample to Andolfatto (2018) who finds no substantial effect in his model.

As explained above, a monetary system with interest on reserves might be interpreted as an interest bearing CBDC system. Proposition 1 shows this would be beneficial. The mechanism again works over the deposit rate. From (44) the deposit rate increases with the reserve rate, because banks have a higher income from the reserves backing their deposits in both systems. The deposit rate and welfare increase more under NB because an increase in the reserve rate increases the deposit rate one to one under NB, while under FB the marginal increase is only α.

It is interesting to relate this result to the disintermediation arguments accompanying the introduction of interest-bearing CBDC. For instance, Chiu et al. (2019) or Keister and Sanches (2019) argue that an interest bearing CBDC

[17] In footnote 9 of Berentsen et al. (2007) the authors also make the interpretation of their model as a NB economy.

might crowd out investment and lower intermediation. In these models, higher interest on reserves also increases the deposit rate. But there higher deposit rates also translate into higher loan rates and thus funding costs for borrowers, which reduces lending and intermediation and seems detrimental for welfare. The essential difference between the two types of models seems to be that in Chiu et al. (2019) financing lending over bonds – although it is possible – is not used in equilibrium. Here however, financing lending over bonds serves a useful economic function because sellers can thus save at a higher interest rate. The loan rate and the bond rate and then pinned down by the nominal interest rate, independent from changes in the reserve rate.[18]

4 Imperfect Competition

The quantitative analysis below will demonstrate that a perfect competition model cannot explain the observed spread between the nominal interest rate and the deposit rate. In this section I thus introduce imperfect competition. In particular, following Chiu et al. (2019), I assume Cournot competition with N banks in the deposit market, while the bond and loan market are still perfectly competitive. This means an individual bank j anticipates the effects of her own deposit supply on the deposit rate over market clearing, taking the deposit issuance of other banks $i \neq j$ as given, but ignores similar effects on the loan and the bond rate. From the perspective of bank j the total real supply of deposits is given as

$$\phi D = \phi \left(\sum_{i \neq j}^{N} d_i + d_j \right) \tag{47}$$

[18] In Keister and Sanches (2019) banks cannot issue other liabilities than deposits. Another point is that a decrease in real lending or disintermediation must not mean a decrease in welfare. In fact, a replication of a basic perfect competition version of the model by Chiu et al. (2019) indicates that a decrease in intermediation induced by higher interest on reserves is *beneficial* because there was overinvestment initially.

where d_j is the own supply of deposits and $\sum_{i \neq j}^{N} d_i$ is the supply of all other banks. The bank anticipates total real demand for deposits from buyers from (19), (22) and (29) as:

$$\phi D = \begin{cases} (1-s)\frac{q_b u'(q_b)}{1+i} & \text{if} \quad i_d \geq i_m \\ 0 & \text{if} \quad i_d < i_m \end{cases} \tag{48}$$

The real demand for deposits is only positive for deposit rates weakly higher than the reserve rate. If $i_d < i_m$, buyers switch from holding deposits to holding reserves and the demand for deposits is zero.

Equalizing (47) and (48) yields a connection between the deposits issued by bank j and the deposit rate, which I denote as $i_d(\phi D)$, where ϕD is given by (47). The interesting case is when the real demand for deposits (48) increases with the deposit rate, i.e. when $\partial \phi D / \partial i_d > 0$ and $\partial i_d / \partial \phi D > 0$. This means that if the bank issues more deposits, buyers must be compensated with a higher return for holding these additional deposits. Only in this case the bank faces a tradeoff when issuing deposits.[19] From (45) buyers consume more at higher deposit rates. Whether they finance this higher consumption by holding more, less or the same level of real deposits depends on preferences however. From (48) the real demand for deposits increases in the deposit rate only if the coefficient of relative risk aversion is below 1:

$$-\frac{q_b u''(q_b)}{u'(q_b)} < 1. \tag{49}$$

In the following I assume this condition holds.[20]

[19]In the opposite case, i.e. if a bank issues more deposits and buyers are willing to hold these deposits at a lower deposit rate, the bank would always issue as many deposits as she could, such that i_d is as low as possible, i.e. $i_d = i_m$.

[20]Using CRRA-utility, the calibration yields $\sigma = 0.16$. So the data supports this assumption.

The problem of representative bank j is very similar to the perfect competition model. In particular, the steps up until the binding reserve constraint are identical. Then the banking problem reads:

$$\max_{d_j, b_j} \quad d_j \left((\alpha i_m + (1-\alpha)i - i_d(\phi D)) + b_j \left(i - i_b \right) \right)$$

The only difference to before is that the bank now takes into account that the deposit rate depends on how many deposits she issues. The first order condition for deposits becomes:

$$(1-\alpha)i + \alpha i_m - i_d = \phi d_j \frac{\partial i_d(\phi D)}{\partial \phi D} \tag{50}$$

Compared to the perfect competition model the additional term on the right-hand side introduces a spread between the deposit rate under perfect competition and under imperfect competition. I denote the deposit rate under perfect competition as $i_d^{PC} = (1-\alpha)i + \alpha i_m$ in the following.

In a symmetric equilibrium all banks issue the same amount of deposits, i.e. $d_j = d_i = d = D/N$. (50) then becomes the familiar equality between a type of Lerner index and the inverse interest-elasticity of real deposits multiplied by the number of banks N (Freixas and Rochet, 2008, p. 78–80).

$$\frac{i_d^{PC} - i_d}{i_d} = \frac{1}{\varepsilon_D(i_d)} \frac{1}{N} \quad \text{where} \quad \varepsilon_D(i_d) = \frac{i_d}{\phi D} \frac{\partial \phi D}{\partial i_d} \tag{51}$$

(51) says that the spread between the deposit rate under perfect and under imperfect competition depends on the interest-elasticity of deposit demand and on the degree of competition. The spread goes to zero if the banking sector

becomes perfectly competitive, i.e. if $N \to \infty$, or if real deposit demand is perfectly elastic, i.e. if $\varepsilon_D \to \infty$. Denoting the solution of (51) with i_d^*, the equilibrium deposit rate under imperfect competition solves:

$$i_d = \max\{i_d^*, i_m\} \leq i_d^{PC} \tag{52}$$

If competition or the elasticity are very low, banks might like to set the deposit rate below the interest on reserves, i.e. they set $i_d^* < i_m$. But then buyers prefer to hold reserves and banks oose their cheap funding over deposits. Therefore banks set the deposit rate to the lowest possible value such that buyers are indifferent. The deposit rate is thus bounded below at i_m. Note that under NB the deposit rate under perfect competition is just the reserve rate. So in this system the imperfect competition rate is always below the reserve rate and banks set $i_d = i_m$. The following proposition summarizes the welfare implications of the two banking systems under imperfect competition:

Proposition 2. *Suppose holding money is costly, there is Cournot competition with N banks in the deposit market and the banking system either operates under FB with $\alpha \in (0, 1)$ or under NB with $\alpha = 1$. Then there is a stationary equilibrium in which:*

i) welfare under FB is higher than under NB if banking is sufficiently competitive, i.e. if $N > \bar{N}$

where

$$\bar{N} = \frac{i_m}{(1-\alpha)(i - i_m)\varepsilon_D(i_m)} \tag{53}$$

Proposition 2 shows that with imperfect competition FB does not always dominate NB in terms of welfare. Although banks still generate higher income on the asset side under FB they only pass on this higher income to deposit holders if competition is sufficiently high. $N > \bar{N}$ is more likely, if the elasticity is high and it does not make sense for banks to substantially reduce the deposit rate, if the nominal interest rate is high and banks get a higher income on their assets independent of competition, or if the reserve requirement is low. Note that as the economy becomes a NB economy the threshold $\bar{N}$ goes to infinity and $N > \bar{N}$ can never hold. In this case, as we already saw, banks will always set the deposit rate to the reserve rate, independent of competition.[21]

5 Quantitative Assessment

The previous two sections argued that FB can be useful because of higher interest payments on deposits. In this section I quantify the importance of these benefits. I thus calibrate the imperfect competition model to the US economy using yearly data on interest rates, monetary aggregates and nominal GDP. I interpret the model as representing yearly steady states given by a particular nominal interest rate.

Following the welfare costs of inflation literature pioneered by Lucas (2000), I fit the parameters to match aggregate real money demand or inverse velocity, denoted with z.[22] z relates money balances held to nominal GDP. In the model total money balances held are total deposits D which also equals nominal output in the FM. Nominal output in the SM is given by x^*/ϕ so total nominal GDP is given by $D + x^*/\phi$. This gives the following real money demand expression:

$$z = \frac{D}{D + x^*/\phi} = \frac{1}{1 + \frac{x^*}{\phi D}} \tag{54}$$

[21]Note that $\bar{N}$ does not go to zero if $i_m \to 0$ as one might think from (53). Since the elasticity has i_m in the numerator this cancels with the i_m in the numerator of (53).

[22]In the New Monetarist literature see in particular Lagos and Wright (2005), Craig and Rocheteau (2008) and Berentsen et al. (2015). See also Lucas (2013) for similar arguments with a cash-in-advance model.

Real deposit holdings decrease in the nominal interest rate, so $\partial z/\partial i < 0$. Following the literature I parametrize $u(q) = q^{1-\sigma}/(1-\sigma)$, $c(q) = q$ and $U(x) = B\log(x)$. Furthermore I set the interest on reserves to zero, i.e. $i_m = 0$, since the FED didn't pay interest in the considered period. This implies the stationary allocation is given by:

$$x^* = B \tag{55}$$

$$q_b = \left(\frac{1+i_d}{1+i}\right)^{1/\sigma} \tag{56}$$

$$\phi D = (1-s)\frac{(1+i_d)^{\frac{1-\sigma}{\sigma}}}{(1+i)^{\frac{1}{\sigma}}} \tag{57}$$

$$i_d = \max\left\{0, \frac{1+i_d^{PC}}{1+\frac{\sigma}{1-\sigma}\frac{1}{N}} - 1\right\} \tag{58}$$

Note that $\partial\phi D/\partial i_d > 0$, i.e. real deposits increase in the deposit rate if $\sigma < 1$.

I calibrate the model using yearly, averaged US-data from 1984 until 2008.[23] The reason for this specific period is that from 1933 the Glass-Steagall Act prohibited commercial banks to pay interest on bank deposits under Regulation Q. So with this regulation the basic mechanism of the model was not operative. Only in the 1980s Regulation Q was gradually lifted. Particularly, in 1982 banks were allowed to issue interest-bearing money market deposit accounts (MMDA). Following Lucas and Nicolini (2015) I assume it took the banks two years to adjust to this new type of account and from 1984 on the mechanism was operative. I stop after 2008 because nominal interest rates were close to

[23] Lucas and Nicolini (2015) argue that money demand models often fail to match the behaviour of real money demand at high frequency "in the sense that temporary changes in the short term interest rates have relatively little impact on real money balances" (p. 58). As a robustness exercise I thus also used HP-filtered data with a smoothing parameter of 100 to calibrate the model. While – unsurprisingly – the fit of such a model is slightly better, the parameter estimates and thus the welfare results are practically identical to using yearly data.

zero after 2008, which violates the basic assumption of the model that holding money is costly. The following table summarizes the correspondence between model variables and data:

Table 1: Model variables and data

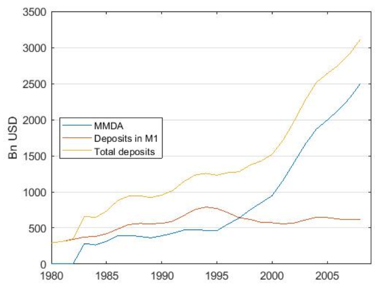
Model	**Data**
deposits D	M1 − currency + MMDA
reserves M	M0 − currency
nominal interest rate i	3-Month T-Bill rate
interest on deposits i_d	weighted average interest on deposits in M1 and interest on MMDA

Data sources: FRED, Lucas and Nicolini (2015).

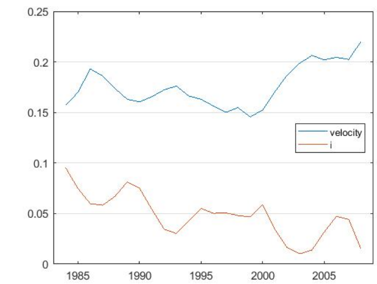
For deposits I use the deposit component of M1 combined with the MMDA data provided by Lucas and Nicolini (2015). I add MMDAs because they perform a similar economic function to traditional deposit accounts and money demand is not stable without this correction, as has been shown by Lucas and Nicolini (2015). I exclude currency because the model is silent on this and calibrations including currency yield very similar results. For reserves I use M0 minus currency. Thus $\alpha = M/D$ measures bank reserve holdings. For the nominal interest rate I use the 3-month T-bill rate. Finally I calculate the deposit rate based on data from Lucas and Nicolini (2015) on the average deposit rate of M1 deposits and MMDA accounts, weighted by the shares of the two aggregates.[24] The following figure plots a) the evolution of MMDA, deposits in M1 and total deposits since 1980, b) money demand and the nominal interest rate since 1984, c) the evolution of bank reserve holdings since 1980 and d) the nominal interest rate, the deposit rate from the data and the deposit rate implied by the model under perfect competition.

[24] For a few years there is no interest rate data for both series. I use a linear extrapolation for these years.

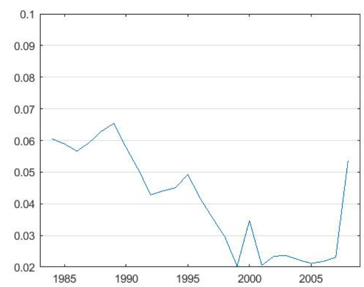
Figure 3: The Data



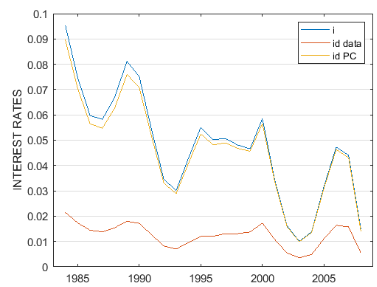
(a) Deposit components



(b) Money demand and i



(c) Reserve ratio α



(d) Interest rates

Subfigure a) shows how important it is to include MMDA in total deposits. While still zero in 1980 MMDA strongly increase to over 280 bn USD in 1983, accounting for over 40% of total deposits. After 1995 MMDA dominate the evolution of total deposits.[25] Subfigure c) shows that bank reserve holdings are close to zero over the whole period. The mean for 1984 to 2008 is just 4%. Subfigure b) shows that the negative relationship between money demand and the nominal interest rate implied by the theory looks plausible in the data. But as already pointed out above this hinges crucially on the inclusion of MMDA's in total deposits. Finally subfigure d) shows that the perfect competition model

[25]The strong increase of MMDA after the mid-90s is related to the introduction of so called "sweep"-deposit accounts in 1994. This was another response to a relaxation of Regulation Q. Sweep-accounts essentially allowed banks to automatically move funds from traditional deposit accounts to MMDA. See Lucas and Nicolini (2015) and Berentsen et al. (2015).

clearly fails in explaining the evolution of the deposit rate. The model predicts $i_d^{PC} = (1-\alpha)i$, which essentially implies the deposit rate should equal the nominal interest rate since the reserve ratio is close to zero. In the data however, the deposit rate is much lower. To account for this spread it is crucial to use the model with imperfect competition developed in section 4.

For the calibration I choose s, σ, B and N to minimize the sum of squared residuals between money demand in the data D/GDP and in the model, given by (54).[26]

$$\min_{s,\sigma,B,N} \quad \sum_{1984}^{2008} \left(\frac{D_t}{GDP_t} - \frac{1}{1 + \frac{B(1+i_{d_t})}{(1-s)q_{b_t}}} \right)^2 \tag{59}$$

As it turns out, calibrating B and s separately will just estimate the same constant $K = B/(1-s)$ in the denominator of (59) which can be satisfied for different combinations of B and s. Thus the choice of s does not affect the results and following Lagos and Wright (2005) we will set it to $s = 0.5$.[27] The remaining results are then $\sigma = 0.16$, $B = 1.82$ and $N = 4$. This is very close to similar calibrations.[28]

The following figure shows the fit of the model. It plots money demand in the data and the predicted money demand from the model against the nominal interest rate. The figure shows a satisfactory overall fit but the model has

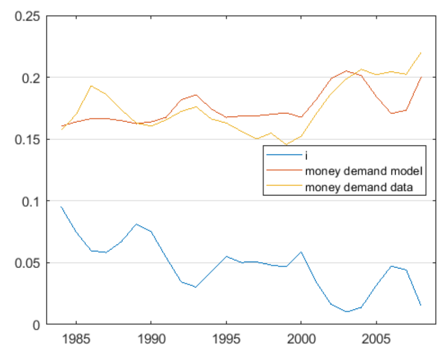
[26] One could also calibrate the model to match data moments, in particular the mean money demand, the mean deposit rate and the average elasticity of money demand to the nominal interest rate. This yields very similar numbers.

[27] B cannot be too small however, which implies s cannot be too small for a given K. Otherwise the condition for positive hours for sellers in the second market, $h_s > 0$ given by (46) cannot hold. Lagos and Wright (2005) arrive at the same conclusion.

[28] Lagos and Wright (2005) find $\sigma = 0.16, B = 1.97$ for a similar specification, Craig and Rocheteau (2008) find $\sigma = 0.14, B = 1.82$. The estimated parameters of Berentsen et al. (2015) are slightly higher, $\sigma = 0.31$ and $B = 2.2$. Chiu et al. (2019) find $\sigma = 0.48, B = 1.79$ and $N = 9$.

difficulties to match the data in particular subperiods. These are periods where the data badly satisfies the negative relationship between money demand and the nominal interest rate embedded in the model. For instance between the mid-1990s and 2002 money demand decreases but the nominal interest rate is constant or also slightly decreases in the data, which the model cannot account for.

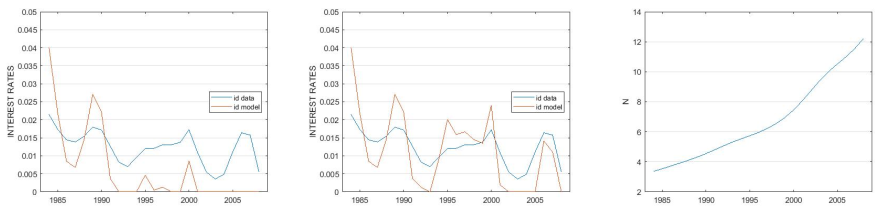
Figure 4: Fit of money demand



The following figure evaluates the fit of the model with respect to the deposit rate. It plots the deposit rate in the data against a) the model implied interest rate with $N = 4$ and b) with $N = 4$ between 1984–1991 and $N = 6$ between 1992–2008 and c) the implied yearly N_t to perfectly match trend i_d according to (58) using HP-filtered data with a smoothing parameter of 100.[29]

[29]With raw data N_t would be extremely volatile. I keep $\sigma = 0.16$ here although the parameter might be slightly different in a full calibration with filtered data.

Figure 5: Fit of the deposit rate



(a) (b) (c)

(a) $N = 4$, (b) $N = 4$ (1984–1991), $N = 6$ (1992–2008), (c) implied yearly N_t to match trend i_d (HP-filtered data)

The figures illustrate the difficulties of the imperfect competition model to match the deposit rate in the data with a constant N. The deposit rate is much less volatile than the model implied deposit rate, which closely follows the movements in the nominal interest rate. Also, assuming a constant N over the whole time period is probably not a good approximation of reality. As (b) shows the fit with two different N for 1984–1991 and 1992–2008 is much better, and the implied N to match the trend in i_d is increasing from (c). Both figures indicate increasing competition in the sample period which seems plausible given the deregulation of the financial sector in this period. For the welfare calculations I will thus use different values for N, taking $N = 4$ as a lower bound.

To quantify the welfare gains from FB I follow the welfare costs of inflation literature and calculate the fraction of steady state consumption or GDP agents would give up under FB to be at the NB welfare level without interest on reserves. The fraction Δ_{FB} is given by

$$W_{FB}(\Delta_{FB}) = (1-s)u(q_b \cdot \Delta_{FB}) - sq_s + U(x^* \cdot \Delta_{FB}) - x^* = W_{NB} \qquad (60)$$
$$\text{with} \quad W_{NB} = (1-s)u(q_b|_{\alpha=1}) - sq_s|_{\alpha=1} + U(x^*) - x^*$$

where $1 - \Delta_{FB}$ then measures the welfare gains of FB due to higher interest on deposits. The following table shows $1 - \Delta_{FB}$ for different levels of N and the nominal interest rate. Besides monopoly with $N = 1$ and perfect competition with $N = \infty$, the table also displays slightly higher competition levels, $N = 6$ and $N = 9$, the value used by Chiu et al. (2019).

Table 2: Welfare gains from FB

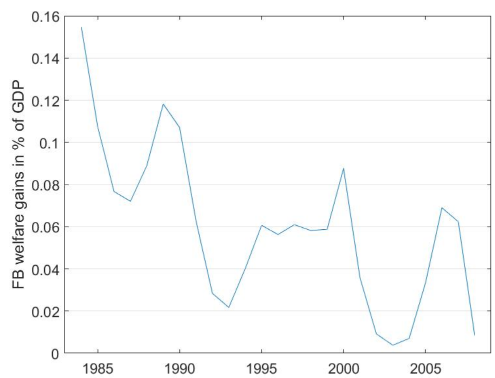
			$\mathbf{1 - \Delta_{FB}}$		
i	$N = 1$	$N = 4$	$N = 6$	$N = 9$	$N = \infty$
2%	0	0	0	0	0.03
5%	0	0	0.07	0.10	0.13
10%	0	0.30	0.37	0.40	0.43
14.4%	0	0.62	0.69	0.72	0.74

Notes: Numbers display % of consumption or GDP households give up going from FB to the NB welfare level. Both systems are assumed to operate with $i_m = 0$. The reserve requirement under FB is at the mean $\alpha = 0.04$.

The table shows welfare gains from FB are relatively small, especially for moderate inflation rates. They are around 0.1% of GDP at a nominal interest rate of 5% and around 0.35% of GDP at a nominal interest rate of 10%. At higher inflation rates, numbers are higher. At a nominal interest rate of 14.4% – which corresponds to 10% inflation and a real interest rate of 4% – the welfare gains of FB are around 0.7% of GDP. The effects of competition are highly non-linear. In a monopoly the bank always pays the same deposit rate as under NB, i.e. $i_d = i_m = 0$. So there are no benefits of FB. On the other hand, the difference in welfare between a fairly concentrated banking system with say $N = 4$ and a fully competitive one with $N = \infty$ are relatively small.

We can also apply the model to the US economy in the sample period, taking the deposit rate directly from the data. The numbers show how much agents value being in the FB system in place at that time versus a NB system without interest on reserves. As the figure shows, their valuation of the benefits of FB is quite low, below 0.16% of GDP at the beginning of the period and even lower thereafter. The welfare gains closely follow the evolution of the nominal interest rate in the sample period.

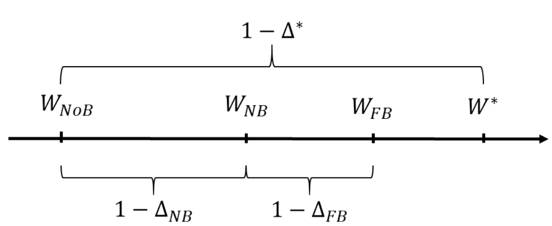
Figure 6: Historical welfare gains from FB in the US 1984–2008



Notes: The calculations directly use the deposit rate from the data. NB operates under zero interest on reserves.

To put the welfare gains from FB into a relative perspective I also compare them with two other reference points: welfare in an economy without banks given in appendix A.1, W_{NoB}, and first best welfare, W^*[30]. The following figure introduces some notation:

Figure 7: Welfare measures



[30]This occurs if the central bank runs the Friedman rule and sets $\gamma = \beta$.

Analogue to (60), Δ_{NB} is the fraction of consumption households give up under NB to be equally well of as without a banking system. So Δ_{NB} solves $W_{NB}(\Delta_{NB}) = W_{NoB}$. Remember that banks have two roles in this economy: they provide insurance against preference shocks and they provide a means of payment with higher return if competition is sufficiently high. Without interest on reserves narrow banks only have the first function and fractional reserve banks might have both functions. In this sense $1 - \Delta_{NB}$ measures the value of a banking system providing liquidity insurance and $1 - \Delta_{FB}$ measures the value of a banking system providing interest bearing money. Finally, Δ^* is the fraction of consumption households would give up under the Friedman rule to be at the welfare level of an economy without banks. So Δ^* solves $W^*(\Delta^*) = W_{NoB}$ and $1 - \Delta^*$ measures the total welfare costs of inflation of an economy without banks, comparable to similar estimates like in Lagos and Wright (2005). The following table contrasts the welfare gains of FB due to higher deposit rates to this two other measures:

Table 3: Total welfare costs, welfare gains from liquidity insurance and welfare gains from FB

i	$\mathbf{1-\Delta^*}$	$\mathbf{1-\Delta_{NB}}$	$\mathbf{1-\Delta_{FB}}$
2%	0.09	0.07	0
5%	0.43	0.31	0
10%	1.14	0.79	0.30
14.4%	1.73	1.12	0.62

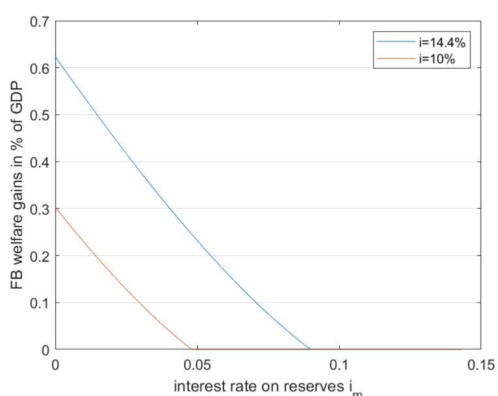
Notes: Numbers display % of consumption or GDP. Again $i_m = 0$ and FB operates with $\alpha = 0.04$ and $N = 4$.

Total welfare costs are in line with similar models. The maximal welfare costs of 1.73% for 10% inflation and a 4% real interest rate correspond to 1.62% in the model of Lagos and Wright (2005), table 1 column (2), who use a slightly higher value for B in their calibration.

Table 3 allows to compare the value of the two functions of banks in this economy. It shows that the welfare gains from liquidity insurance are much higher than the gains due to higher interest on deposits, especially for low and moderate inflation rates. For example at 10% nominal interest rate, liquidity insurance compensates households for nearly 70% of total welfare costs while higher interest payments on deposits make up for only 26% of total welfare costs.[31]

Finally I evaluate what happens to the welfare advantages of FB if the central bank pays interest on reserves. As mentioned above this might be interpreted as a monetary system with an interest bearing CBDC. From proposition 1, *iii*) we know that interest on reserves increases welfare in both systems but the increase is higher under NB. The reason is that under NB a marginal increase in the reserve rate increases the deposit rate one to one while under FB the marginal increase is only α. So paying interest on reserves further reduces the welfare gains from FB, i.e. it makes NB relatively more attractive. The following figure shows by how much:

Figure 8: Welfare gains of FB with interest on reserves



Notes: Numbers are in % of GDP. FB operates with $\alpha = 0.04$ and $N = 4$.

[31]The sum of the welfare gains from liquidity insurance and the welfare gains from FB do not exactly add up to total welfare costs.

The red line shows how the welfare gains of FB at a 10% nominal interest rate evolve if the central bank starts paying interest on reserves. The blue line does the same for a 14.4% nominal interest rate. The starting values at $i_m = 0$ correspond to the values for a 10% and 14.4% nominal interest rate in the last column of table 3. The figure shows how paying interest on reserves significantly decreases the welfare gains from FB. In the case of a 10% nominal interest rate, increasing the reserve rate up to 5% gradually reduces the welfare gains of FB from 0.3% of GDP to zero, and similarly for a nominal interest rate of 14.4%. Thus paying interest on reserves substantially reduces the welfare advantages of FB. This confirms the intuition of Friedman (1960) who advocated a NB system paying interest on reserves.

6 Conclusion

The paper aimed to come up with a theoretical and a quantitative answer to the question, what is at stake if we move from the current FB system, with its characteristic combination of financing investments or lending with monetary liabilities like deposits, to a NB system where investment and lending must be financed with non-monetary liabilities like bonds. The paper argues that, indeed the unique feature of FB provides a socially useful function, especially if the banking system is competitive and inflation is high. In this case FB provides a means of payment with higher return that compensates money holders against inflation. The calibration suggests however, that the quantitative importance of these benefits is relatively small and gets even smaller if the central bank pays interest on reserves. Thus to the extent that NB systems are able to resolve the instability issues associated to the unique financing structure of FB, the social costs of financial safety seem relatively small. Obviously, they should be compared to the social costs of financial instability due to FB.

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Appendix A

A.1 Economy with reserves and preference shock

In this economy there are no banks and the only source of money is the central bank issuing outside money or reserves. This is the basic model of Lagos and Wright (2005) with perfect competition and interest on outside money. Without banks the optimality condition for reserve holdings, (35), looks different. Since sellers cannot deposit their reserves in banks, their marginal value of reserves next period is just $\phi_{+1}(1+i_{m+1})$, the real value of reserves next period. Buyers on the other hand can consume a little bit more if they bring more reserves. Since they are liquidity constrained if holding money is costly, their FM consumption is $q_b = m(1+i_m)/p$. Thus the marginal value of bringing more reserves into next period for a buyer is $u'(q_{b+1})(1+i_{m+1})/p_{+1}$ and (35) becomes:

$$\phi = \beta(1-s)\frac{u'(q_{b+1})(1+i_{m+1})}{p_{+1}} + \beta s\phi_{+1}(1+i_{m+1}).$$

Optimal production still equals $c'(q_s) = p\phi$. Thus the stationarity equilibrium consumption in the FM without banks solves:

$$\frac{u'(q_b)}{c'(\frac{1-s}{s}q_b)} = \left(\frac{1+i}{1+i_m} - s\right)\frac{1}{1-s} \tag{61}$$

The right-hand side of (61) is strictly higher than the right-hand side of (45). So the allocation without banks is strictly worse than the allocation with banks. Also note that the right-hand side of (61) increases with s. So the higher the risk to become a seller with costly idle money holdings, the worse the allocation. Banks are especially valuable if the risk of becoming a seller is high.

A.2 Economy with reserves and no preference shock

In this economy there are no banks and there is a fixed measure s of sellers and $1 - s$ of buyers. Again money is only provided by the central bank in the form of reserves. This is a basic version of Rocheteau and Nosal (2017) with interest on outside money. In such an environment only buyers will acquire reserves if holding money is costly. Thus their optimal reserve holdings read analogue to (35):

$$\phi = \beta(1-s)\frac{u'(q_{b+1})(1+i_{m+1})}{p_{+1}}$$

Under stationarity this becomes:

$$\frac{u'(q_b)}{c'(\frac{1-s}{s}q_b)} = \frac{1+i}{1+i_m} \tag{62}$$

This is exactly the same allocation as in the perfect competition model under NB, i.e. in (45) with $\alpha = 1$.

Appendix B

B.1 proof of proposition 2

Proof. Apart from the determination of the deposit rate, (52), the equilibrium conditions under imperfect competition are the same as under perfect competition. In particular, the deposit rate and the bond rate still equal the nominal interest rate, $i_l = i_b = i$, and optimal FM consumption still solves (45). This implies total real deposits, ϕD are independent of N and $i_d^{PC} = \alpha i_m + (1-\alpha)i$. Threshold $\bar{N}$ then solves (51) at $i_d = i_m$. If N increases from $\bar{N}$ the right-hand side of (51) decreases. Thus the equation can only hold if the left-hand side decreases too which implies i_d must increase. Therefore $i_d > i_m$ if $N > \bar{N}$. The opposite logic holds for $N < \bar{N}$. ■