Using Types in Composed Cryptographic Proofs

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Quick Primer: miTLS

- Formally verified implementation of TLS

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    - memory safety
    - functional correctness
    - cryptographic verification

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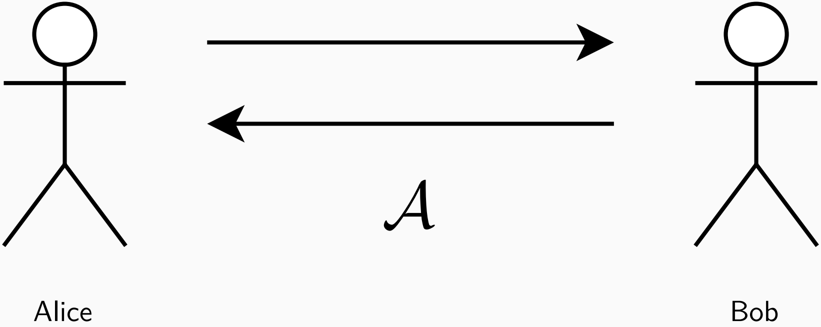
- Formally verified implementation of TLS
  - memory safety
  - functional correctness
  - cryptographic verification
- Not widely understood by cryptographers

What is Security?

Example: Authenticated Encryption

Alice and Bob expect confidentiality and authenticity and use some authenticated encryption scheme $\sigma = \{\mathsf{Enc}, \mathsf{Dec}\}$.



Real- vs Ideal Behaviour

We define Security in terms of Real- vs. Ideal behaviour of the authenticated encryption scheme σ.

Real Behaviour

Alice		**Bob**
m, k		k
$c \leftarrow\!\!\$\ \sigma.\mathrm{Enc}(k, m)$	$\xrightarrow{\quad c \quad}$	
		$m \leftarrow \sigma.\mathrm{Dec}(k, c)$

Ideal Behaviour

Ideal Behaviour

Alice		**Bob**
m, k		k
$c \leftarrow_{\$} \{0,1\}^{\|m\|}$		
$M[c] \leftarrow m$	$\xrightarrow{c}$	
		$m \leftarrow M[c]$

Security Game

Alice and Bob as Oracles

We turn Alice (encryption) and Bob (decryption) into oracles and bundle them into a package we call Authenticated Encryption (AE).

$$\mathcal{A} \xrightarrow{\mathsf{ENC}, \mathsf{DEC}} \boxed{\mathtt{AE}_\sigma^b}$$

Bit b determines if $\mathtt{AE}$ exhibits real $(b = 0)$ or ideal behaviour $(b = 1)$ of scheme σ.

Computational Indistinguishability

σ is AE-secure if for all efficient adversaries (i.e., algorithms) $\mathcal{A}$, we have that

$$\boxed{\mathtt{AE}_{\sigma}^{0}} \quad \overset{\mathcal{A}}{\approx} \quad \boxed{\mathtt{AE}_{\sigma}^{1}}$$

How to Prove this?

Building Blocks

Commonly, authenticated encryption schemes are built from two components:

- one providing confidentiality (encryption scheme ϵ)
- one providing authenticity (mac scheme μ)

$$
\begin{array}{l}
\underline{\sigma.\mathsf{Enc}(k, m)\qquad\qquad} \\
k_1 || k_2 \leftarrow k \\
c \leftarrow \epsilon.\mathsf{Enc}(k1, m) \\
tag \leftarrow \mu.\mathsf{MAC}(k2, c) \\
\textbf{return } (tag || c)
\end{array}
$$

Assumptions

One assumption for each component:

Assumption 1: Encryption scheme ϵ provides confidentiality

Assumption 2: MAC scheme μ provides authenticity

$$\forall \mathcal{A} : \texttt{Conf.}^0_\epsilon \overset{\mathcal{A}}{\approx} \texttt{Conf.}^1_\epsilon$$

$$\forall \mathcal{A} : \texttt{Auth.}^0_\mu \overset{\mathcal{A}}{\approx} \texttt{Auth.}^1_\mu$$

Reductions

- Goal: Show that $\forall \mathcal{A} : \mathtt{AE}^0_\sigma \overset{\mathcal{A}}{\approx} \mathtt{AE}^1_\sigma$.

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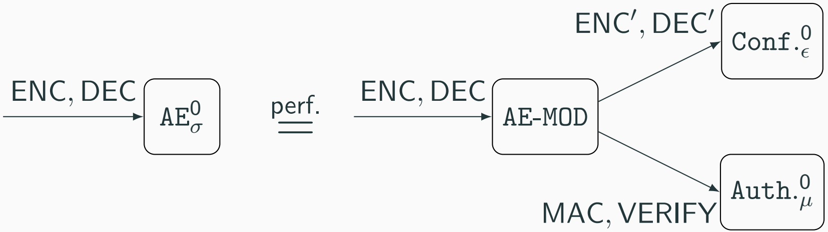
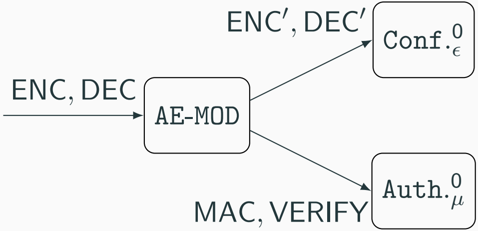
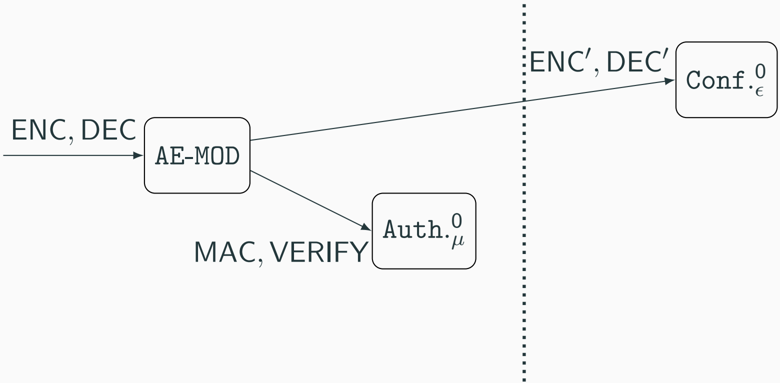


Proof

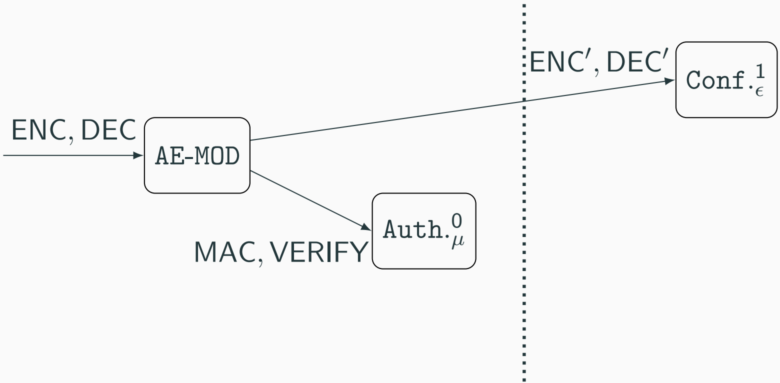


Proof

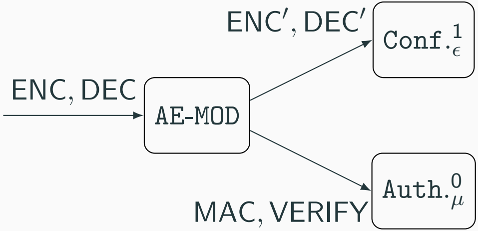


ENC, DEC
AE-MOD
ENC′, DEC′
$\mathtt{Conf.}^{0}_{\epsilon}$
MAC, VERIFY
$\mathtt{Auth.}^{0}_{\mu}$

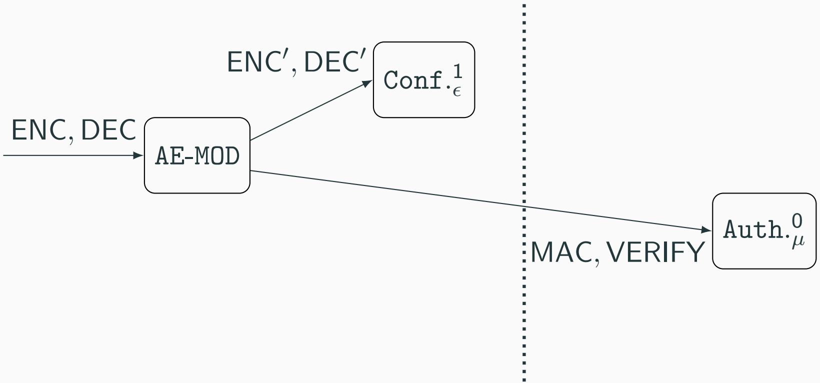
Proof


ENC, DEC
AE-MOD
$\mathrm{ENC}', \mathrm{DEC}'$
$\texttt{Conf.}_{\epsilon}^{1}$
MAC, VERIFY
$\texttt{Auth.}_{\mu}^{0}$

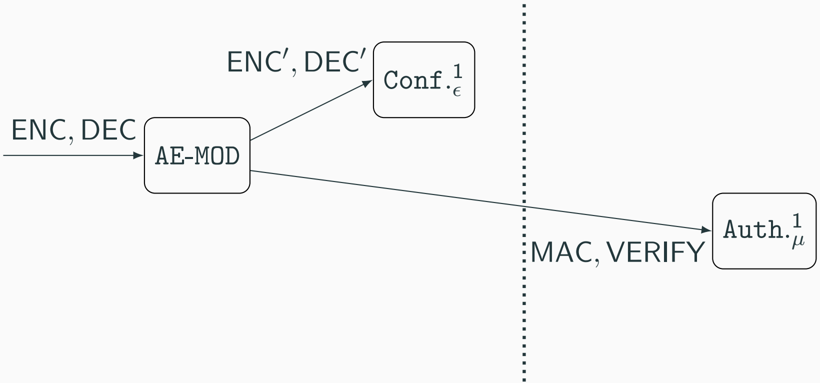
Proof



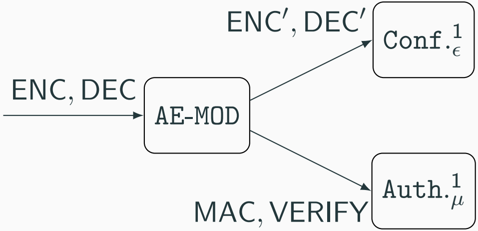
Proof



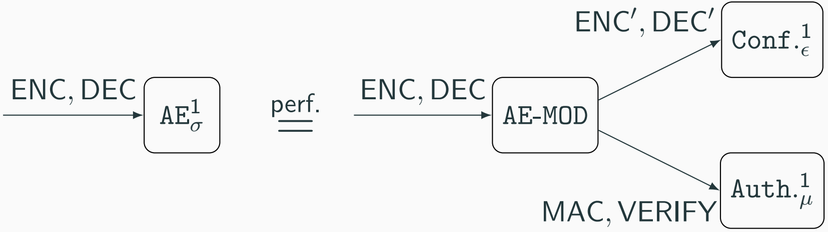
Proof



Proof



Proof con'd


ENC, DEC
$\mathtt{AE}^1_\sigma$
perf.
ENC, DEC
AE-MOD
ENC′, DEC′
$\mathtt{Conf.}^1_\epsilon$
MAC, VERIFY
$\mathtt{Auth.}^1_\mu$

Proof cont'd

$$\mathtt{AE}_\sigma^0 \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \to \frac{\mathtt{Conf.}_\mu^0}{\mathtt{Auth.}_\epsilon^0}$$

$$\wr\wr$$

$$\mathtt{AE}_\sigma^1 \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \to \frac{\mathtt{Conf.}_\mu^1}{\mathtt{Auth.}_\epsilon^1}$$

Proof cont'd

$$\mathtt{AE}_\sigma^0 \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \to \frac{\mathtt{Conf.}_\mu^0}{\mathtt{Auth.}_\epsilon^0}$$

$\wr\wr$ by assumptions

$$\mathtt{AE}_\sigma^1 \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \to \frac{\mathtt{Conf.}_\mu^1}{\mathtt{Auth.}_\epsilon^1}$$

Proof cont'd

$$\mathtt{AE}_\sigma^0 \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \rightarrow \frac{\mathtt{Conf.}_\mu^0}{\mathtt{Auth.}_\epsilon^0}$$

$\wr\wr$ by assumptions

$$\mathtt{AE}_\sigma^1 \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \rightarrow \frac{\mathtt{Conf.}_\mu^1}{\mathtt{Auth.}_\epsilon^1}$$

How to prove $\overset{\text{perf.}}{=}$?

- Inline all the code?
- Use a proof assistant?

- Functional programming language, inspired by F#, ML, OCaml and others
- Support for dependent types, refinement types and monadic effects
- Developed by Microsoft Research and Inria, as well as the F* community
- Available on Github[1]!

[1] https://github.com/FStarLang/FStar

Code Equivalence cont'd

Implement oracles in $\texttt{AE}^b$ as interface. Any code verified against that interface is code-equivalent to AE^b.

```
val enc: m:message → ST ciphertext
  (ensures
    if b=0 then
      c = sigma.enc(k,m)
    else
      c = rand (length m)
      ∧ M[c] = m
  )
```

Code Equivalence cont'd

Implement oracles in $\mathtt{AE}^b$ as interface. Any code verified against that interface is code-equivalent to AE^b.

```
val enc: m:message → ST ciphertext
  (ensures (λ s c s' ->
    if b=0 then
      c = sigma.enc(k,m)
      ∧ s == s'
    else
      c = rand (length m)
      M[c] = m
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    if b=0 then
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    else
      c = rand (length m)
      ∧ sel s' M == upd (sel s M) c m
  ))
```

Code Equivalence cont'd

Implement oracles in $\texttt{AE}^b$ as interface. Any code verified against that interface is code-equivalent to AE^b.

```
val enc: n:nonce → m:message → ST ciphertext
  (ensures (λ s c s' →
    if b=0 then
      c = sigma.enc(k,n,m)
      ∧ s == s'
    else
      c = rand (length m)
      ∧ sel s' M == upd (sel s M) (n,c) m
  ))
```

Code Equivalence cont'd

Implement oracles in $\mathtt{AE}^b$ as interface. Any code verified against that interface is code-equivalent to AE^b.

```
val enc: n:nonce → m:message → ST ciphertext
  (requires (λ s → ∀ c . fresh M (n,c) s))
  (ensures (λ s c s' →
    if b=0 then
      c = sigma.enc(k,n,m)
      s == s'
    else
      c = rand (length m)
      ∧ sel s' M == upd (sel s M) (n,c) p
  ))
```

Code Equivalence cont'd

Implement oracles in `AE`^{b} as interface. Any code verified against that interface is code-equivalent to AE^b.

```
val enc: ap:ae_package → n:nonce → m:message
→ ST ciphertext
  (requires (λ s → ∀ c . fresh ap.M (n,c) s))
  (ensures (λ s c s' →
    if b=0 then
      c = sigma.enc(k,n,m)
      s == s'
    else
      c = rand (length m)
      ∧ sel s' ap.M == upd (sel s ap.M) (n,c) p
  ))
```

Proof cont'd

$$\mathtt{AE}_{\sigma}^{0} \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \rightarrow \frac{\mathtt{Conf.}_{\epsilon}^{0}}{\mathtt{Auth.}_{\mu}^{0}}$$

$$\wr\wr$$

$$\mathtt{AE}_{\sigma}^{1} \overset{\text{perf.}}{=} \mathtt{AE\text{-}MOD} \rightarrow \frac{\mathtt{Conf.}_{\epsilon}^{1}}{\mathtt{Auth.}_{\mu}^{1}}$$

Limitations

- We might be limited to a subset of protocols/security notions
- $\mathrm{F}^\star$ can't do probabilistic reasoning
- We have to change the modelling at some places to accomodate $\mathrm{F}^\star$
- Works best with real- vs ideal behaviour games

References

- Brzuska, Deligant-Lavaud, Kohbrok, Kohlweiss: State Separation for Code-Based Game-Playing Proofs
  Asiacrypt preprint.
- Bhargavan et al.: Implementing and Proving the TLS 1.3 Record Layer
  ePrint.
- A verified implementation of AEAD (slightly more complicated) here.

Future Work

- Prove the TLS 1.3 Key Schedule secure using our methodology
- Give a simple example of how to combine our methodology with $\mathrm{F}^{\star}$
- More protocols!