COS-R403. Special Research Methods. *Forecasting I: Introduction*

Hands-on excercises

Day 2 of intensive 5-day course

University of Helsinki, Finland

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Source: https://github.com/christina-bohk-ewald/2020-COS-R403-forecasting-I-introducti on

Table of content:

0. Please run script of day 1

In particular, sections 1 through 3.

1. Some preparations in R

1.1 Open a new script for day 2 in R and save it to a folder of your choice.

1.2 Create a filepath to a folder where you would like to save your outcome. For example,

```
the.plot.path <- c("C:/plots")
```

1.3 You can then set the working directory to this outcome path

```
setwd(the.plot.path)
```

2. Analyze forecasted population structure by age and sex: population pyramids

On day 1 we have focused on total population size worldwide. On day 2 we want to look beneath the surface and analyze the global population structure by age and sex using various visualization techniques and quantitative metrics. We begin with the global population (age) pyramid for the calendar years 1950, 2019, 2050, and 2100. What has changed over time?

2.1 Global population (age) pyramid 1950

To construct a population (age) pyramid for the calendar year 1950, we first extract global population counts of that year from our data objects:

```
wom1950 <- wom_unwpp2019_1950_2020[,as.character(1950),1]
men1950 <- men_unwpp2019_1950_2020[,as.character(1950),1]
pop1950 <- (wom_unwpp2019_1950_2020[,as.character(1950),1] +
men_unwpp2019_1950_2020[,as.character(1950),1])

sum(wom1950/sum(pop1950))+sum(men1950/sum(pop1950))
```

```
## [1] 1
```

```
max(wom1950/sum(pop1950))*100
```

```
## [1] 1.521093
```

```
max(men1950/sum(pop1950))*100
```

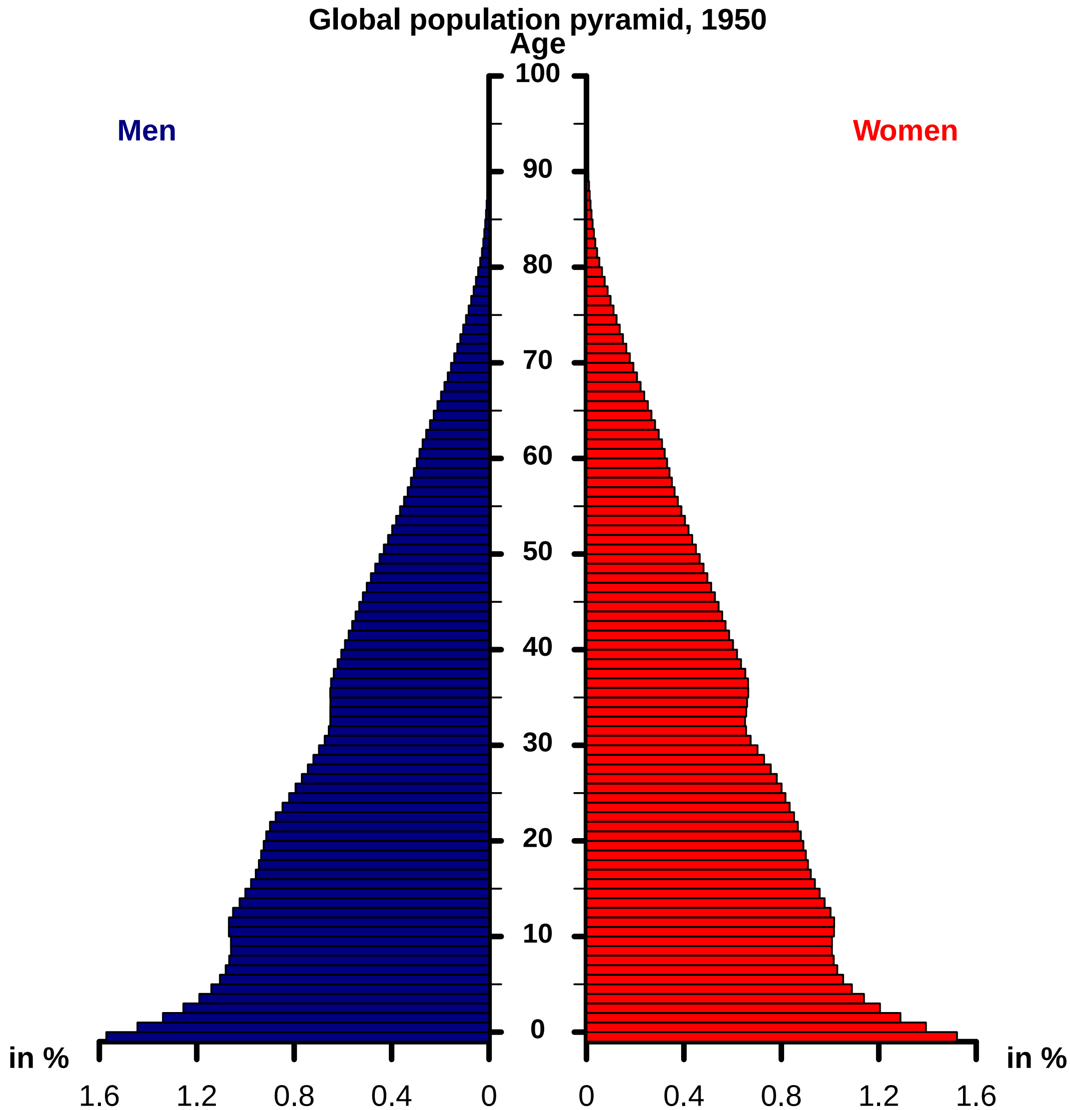
```
## [1] 1.571264
```

And we then display relative counts (in %) in a population age pyramid:

```
setwd(the.plot.path)

dev.off()

pdf(file="global-world-pop-pyramid-percent-1950.pdf", width=10, height=10, family="Times",
pointsize=20, onefile=TRUE)
```

```
require(wesanderson)
pal <- c("navy",wes_palette("Darjeeling1"))
```

```r
par(fig = c(0,1,0,1), las=1, mai=c(0.4,0.4,0.4,0.4))

    plot(x=-100,y=-100,xlim=c(-1.9-0.2,0.2+1.9),ylim=c(-2,105),xlab="",ylab="",
main="",axes=FALSE)

    axis(side=1,at=seq(-1.8,-0.2,0.4),labels=rev(seq(0.0,1.6,0.4)),lwd=3,pos=0)
    axis(side=1,at=seq(0.2,1.8,0.4),labels=seq(0.0,1.6,0.4),lwd=3,pos=0)

    segments(x0=-0.2,x1=-0.2,y0=0,y1=101,lwd=3)
    segments(x0=0.2,x1=0.2,y0=0,y1=101,lwd=3)
    segments(x0=-0.2,x1=-0.15,y0=seq(1,101,5),y1=seq(1,101,5),lwd=1)
    segments(x0=0.15,x1=0.2,y0=seq(1,101,5),y1=seq(1,101,5),lwd=1)
    segments(x0=-0.2,x1=-0.15,y0=seq(1,101,10),y1=seq(1,101,10),lwd=3)
    segments(x0=0.15,x1=0.2,y0=seq(1,101,10),y1=seq(1,101,10),lwd=3)

    text(x=rep(0,10),y=seq(-1.5,99.5,10),seq(0,100,10),font=2,pos=3,cex=0.9)
    for(age in 1:101){
        current.percent.men <- -0.2 - men1950[age]/sum(pop1950)*100
        current.percent.wom <- 0.2 + wom1950[age]/sum(pop1950)*100

        polygon(x=c( c(-0.2 , current.percent.men), c( current.percent.men , -0.2 ) ),
y=c(rep(age-1,2),rep(age,2)),col=pal[1])

        polygon(x=c( c(0.2 , current.percent.wom), c( current.percent.wom , 0.2 ) ),
y=c(rep(age-1,2),rep(age,2)),col=pal[2])
    }

    text(x=-1.85,y=-2,"in %",pos=2,font=2)
    text(x=1.85,y=-2,"in %",pos=4,font=2)

    text(x=-1.8,y=95,"Men",col=pal[1],pos=4,font=2)
    text(x=1.8,y=95,"Women",col=pal[2],pos=2,font=2)

    text(x=0,y=101.5,"Age",col="black",pos=3,font=2)
    text(x=0,y=104,"Global population pyramid, 1950",col="black",pos=3,font=2)
```



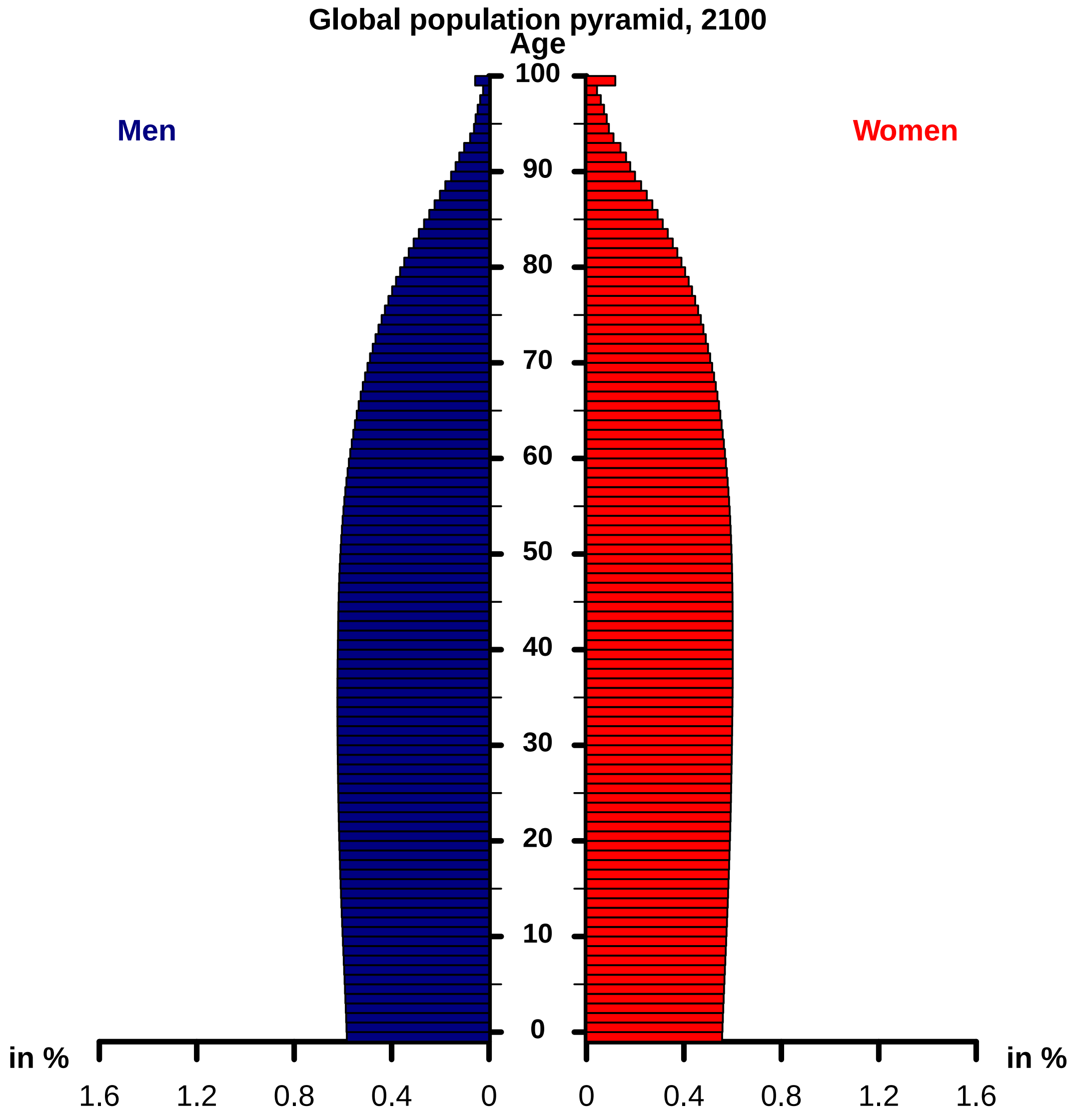

```
dev.off()
```

Brief data description. The population age pyramid depicts population counts of men on the left and women on the right by single years of age, 0 (bottom) through 100 (top), for the calendar year 1950. For the sake of comparability, this pyramid displays relative population counts, in %.

Something to think about. How would you describe this age and sex structure?

2.2 Global population (age) pyramids 2019, 2050, and 2100

Following the same procedure for 1950 above, you can display global population age pyramids for the calendar years 2019, 2050, and 2100. For illustative puposes you find the global population age pyramid for the calendar year 2100 below. Note that the scale of the horizontal axes is the same for 1950 and 2100:



Something to think about. Comparing the global population age pyramids of 1950, 2019, 2050, and 2100, how does the structure by age and sex is forecasted to change? Would you have an idea on how to visualize all these data in one plot? If yes, please try it.

3. Analyze forecasted population structure by sex: sex ratio

We use the sex ratio at birth to analyze the global population structure by sex.

3.1 Global sex ratio

To analyze the global sex ratio from 1950 to 2100, we first extract global population data for women and men from our data objects:

```
wom_age0 <- rbind(wom_unwpp2019_1950_2020[1,as.character(1950:2020),],
wom_unwpp2019_2020_2100[1,as.character(2021:2100),])
```

```r
men_age0 <- rbind(men_unwpp2019_1950_2020[1,as.character(1950:2020),],
men_unwpp2019_2020_2100[1,as.character(2021:2100),])

wom_ageTotal <- c(colSums(wom_unwpp2019_1950_2020[,as.character(1950:2020),1]),
colSums(wom_unwpp2019_2020_2100[,as.character(2021:2100),1]))

men_ageTotal <- c(colSums(men_unwpp2019_1950_2020[,as.character(1950:2020),1]),
colSums(men_unwpp2019_2020_2100[,as.character(2021:2100),1]))

min(men_age0/wom_age0)
```

```
## [1] 1.011898
```

```r
max(men_age0/wom_age0)
```

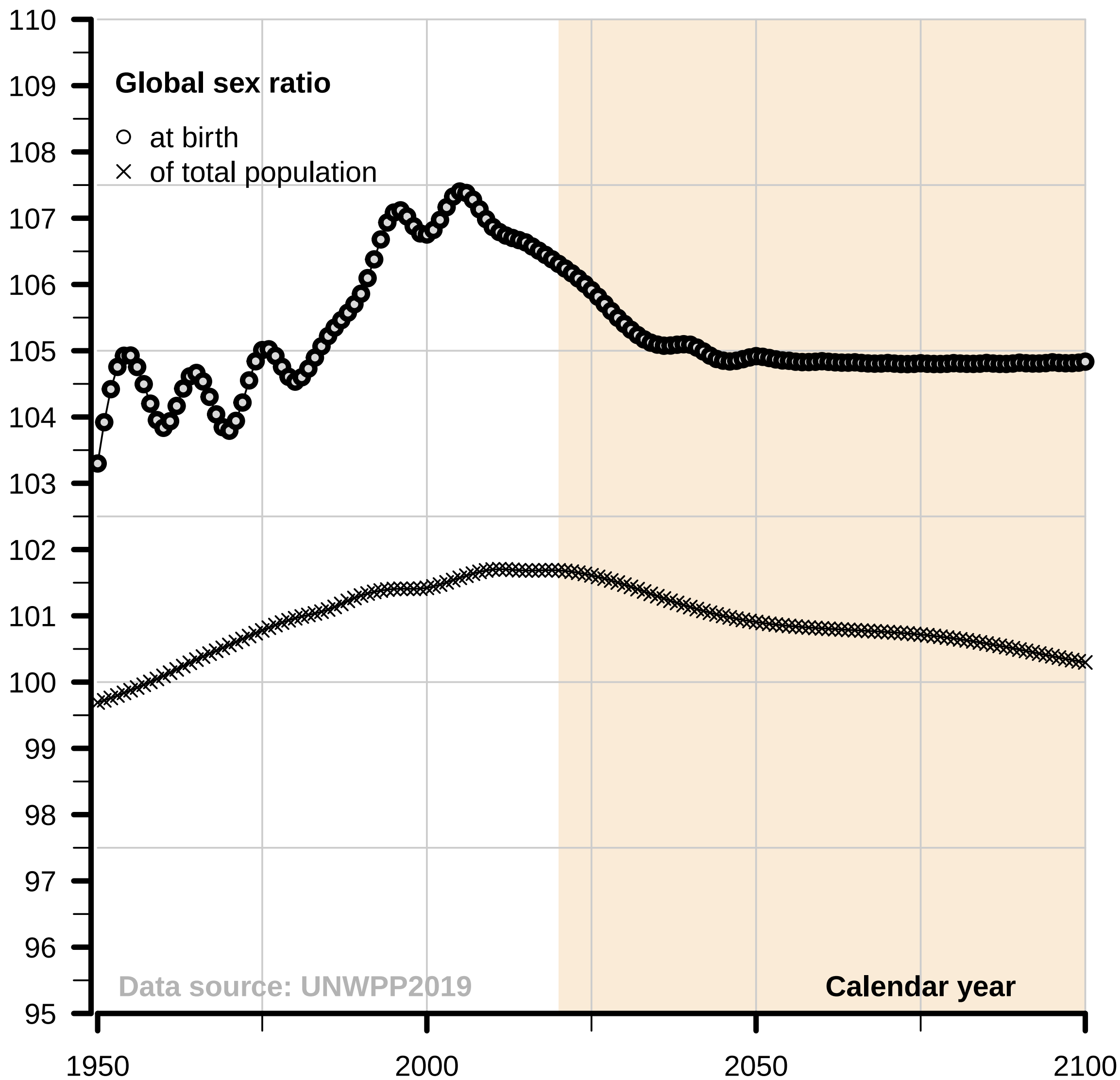
```
## [1] 1.102135
```

And we then calculate and display sex ratio at birth and sex ratio of the total population:

```r
setwd(the.plot.path)

dev.off()

pdf(file="global-world-pop-sex-ratio-1950-2100.pdf", width=10, height=10, family="Times",
pointsize=20, onefile=TRUE)
```

```r
require(wesanderson)
pal <- c("navy",wes_palette("Darjeeling1"))

par(fig = c(0,1,0,1), las=1, mai=c(0.4,0.4,0.4,0.4))

    plot(x=-100,y=-100,xlim=c(1949,2100),ylim=c(95,110),xlab="",ylab="",
main="",axes=FALSE)

    rect(xleft=2020, xright=2100, ybottom=95, ytop=110, col="antiquewhite", border=NA)
    segments(x0=seq(1975,2100,25),x1=seq(1975,2100,25),y0=95,y1=110,col=gray(0.8))
    segments(x0=1950,x1=2100,y0=seq(95,110,2.5),y1=seq(95,110,2.5),col=gray(0.8))

    axis(side=1,at=seq(1950,2100,25),labels=FALSE,lwd=1,pos=95)
    axis(side=1,at=seq(1950,2100,50),labels=TRUE,lwd=3,pos=95)
    axis(side=2,at=seq(95,110,0.5),labels=FALSE,lwd=1,pos=1949)
    axis(side=2,at=seq(95,110,1),labels=TRUE,lwd=3,pos=1949)

    text(x=1950,y=109,"Global sex ratio",pos=4,font=2)
    text(x=2075,y=95,"Calendar year",pos=3,font=2)
    text(x=1980,y=95,"Data source: UNWPP2019",pos=3,font=2,col=gray(0.7))

    ## World (UN release 1950:2019, UN forecast 2020:2100):
    lines(x=1950:2100, y=men_age0[,1]/wom_age0[,1]*100,lwd=1,col="black")
    points(x=1950:2100, y=men_age0[,1]/wom_age0[,1]*100,pch=21,lwd=3,
bg=gray(0.85),col="black")

    lines(x=1950:2100, y=men_ageTotal/wom_ageTotal*100,lwd=1,col="black")
    points(x=1950:2100, y=men_ageTotal/wom_ageTotal*100,pch=4,lwd=1,
bg=gray(0.85),col="black")
```

```
legend(x=1950,y=108.75,c("at birth", "of total population"),lty=FALSE,
pch=c(21,4), col=c("black","black"),bty="n")
```




```
dev.off()
```

Brief data description. The upper (circle) and lower (cross) lines represent sex ratio at birth and sex ratio of the total population, respectively, for the calendar years 1950 through 2100.

Something to think about. What does a sex ratio below and above 100 actually mean? How would you describe and interpret the development of the global sex ratio over time? What world region(s) could drive the global sex ratio into what direction?

4. Analyze forecasted population structure by age: mean age and dependency ratio

We use the mean age and the dependency ratio (for children and old age) to analyze the global population structure by age.

4.1 Mean age

To analyze the mean age of the global population, we first store required data in arrays by metric, calendar time, and world region. Note that this array could store the results of many more age metrics.

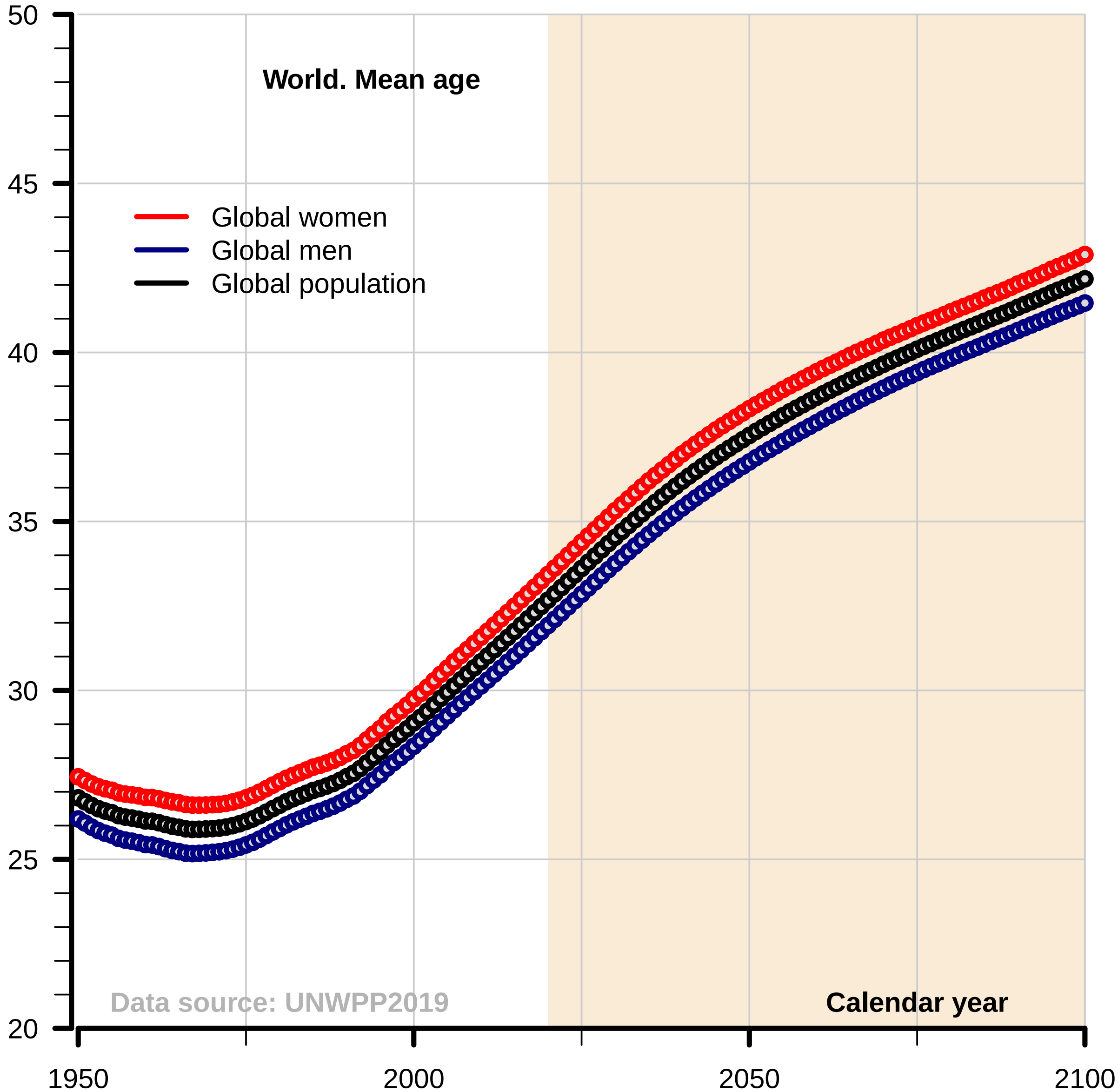
```
world_regions <- c("WORLD","Africa","Asia","Europe","Latin America and the Caribbean",
"Northern America","Oceania")

row.names <- "mean_age"
column.names <- as.character(c(1950:2100))
matrix.names <- world_regions
mean_age_wom_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

mean_age_men_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

mean_age_pop_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

for(wr in 1:7){
    current.wr <- world_regions[wr]
    wom.obs <- apply(X=wom_unwpp2019_1950_2020[,,wr],2,
function(X){sum(c(0:100)*X)/sum(X)})

    men.obs <- apply(X=men_unwpp2019_1950_2020[,,wr],2,
function(X){sum(c(0:100)*X)/sum(X)})

    pop.obs <- apply(X=(wom_unwpp2019_1950_2020[,,wr]+men_unwpp2019_1950_2020[,,wr]),2,
function(X){sum(c(0:100)*X)/sum(X)})

    wom.forecast <- apply(X=wom_unwpp2019_2020_2100[,as.character(2021:2100),wr],2,
function(X){sum(c(0:100)*X)/sum(X)})

    men.forecast <- apply(X=men_unwpp2019_2020_2100[,as.character(2021:2100),wr],2,
function(X){sum(c(0:100)*X)/sum(X)})

    pop.forecast <- apply(X=(wom_unwpp2019_2020_2100[,as.character(2021:2100),wr] +
men_unwpp2019_2020_2100[,as.character(2021:2100),wr]),2,
function(X){sum(c(0:100)*X)/sum(X)})

    mean_age_wom_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- wom.obs
    mean_age_men_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- men.obs
    mean_age_pop_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- pop.obs

    mean_age_wom_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- wom.forecast
    mean_age_men_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- men.forecast
    mean_age_pop_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- pop.forecast
}
```

Please explain: *apply()*.

We then plot the mean age of global women, global men, and global women and men:

```
setwd(the.plot.path)

dev.off()

pdf(file="global-world-pop-mean-age-1950-2100.pdf", width=10, height=10, family="Times",
pointsize=20, onefile=TRUE)
```

```
require(wesanderson)
pal <- c("navy",wes_palette("Darjeeling1"))

par(fig = c(0,1,0,1), las=1, mai=c(0.4,0.4,0.0,0))

    plot(x=-100,y=-100,xlim=c(1949,2100),ylim=c(20,50),xlab="",ylab="",
main="",axes=FALSE)

    rect(xleft=2020, xright=2100, ybottom=20, ytop=50, col="antiquewhite", border=NA)
    segments(x0=seq(1975,2100,25),x1=seq(1975,2100,25),y0=20,y1=50,col=gray(0.8))
    segments(x0=1950,x1=2100,y0=seq(20,50,5),y1=seq(20,50,5),col=gray(0.8))

    axis(side=1,at=seq(1950,2100,25),labels=FALSE,lwd=1,pos=20)
    axis(side=1,at=seq(1950,2100,50),labels=TRUE,lwd=3,pos=20)
    axis(side=2,at=seq(20,50,1),labels=FALSE,lwd=1,pos=1949)
    axis(side=2,at=seq(20,50,5),labels=TRUE,lwd=3,pos=1949)

    text(x=1950+25,y=48,"World. Mean age",pos=4,font=2)
    text(x=2075,y=20,"Calendar year",pos=3,font=2)
    text(x=1980,y=20,"Data source: UNWPP2019",pos=3,font=2,col=gray(0.7))

    ## Global women (UN release 1950:2100):
    lines(x=1950:2100, y=mean_age_wom_unwpp2019_1950_2100[1,,1], lwd=1,col=pal[2])
    points(x=1950:2100, y=mean_age_wom_unwpp2019_1950_2100[1,,1], pch=21,lwd=3,
bg=gray(0.85),col=pal[2])

    ## Global men (UN release 1950:2100):
    lines(x=1950:2100, y=mean_age_men_unwpp2019_1950_2100[1,,1], lwd=1,col=pal[1])
    points(x=1950:2100, y=mean_age_men_unwpp2019_1950_2100[1,,1], pch=21,lwd=3,
bg=gray(0.85),col=pal[1])

    ## Global population (UN release 1950:2100):
    lines(x=1950:2100, y=mean_age_pop_unwpp2019_1950_2100[1,,1], lwd=1,col="black")
    points(x=1950:2100, y=mean_age_pop_unwpp2019_1950_2100[1,,1], pch=21,lwd=3,
bg=gray(0.85),col="black")

    legend(x=1955,y=45,c("Global women","Global men","Global population"),
col=c(pal[2],pal[1],"black"),lty=1,bty="n",lwd=3)
```




```r
dev.off()
```

Brief data description. The red, blue, and black dotted lines represent the mean age of global women, global men, and global women and men, respectively, from 1950 to 2100.

Something to think about. What does a mean age of 25 (compared to e.g. 40) years old actually mean? How would you describe and interpret the development of the global mean age over time? What world region(s) could drive the global mean age into what direction?

Something more to think about. What do you think: how do mortality and fertility (day 1) play into the mean age of the global population?

4.2 Old age dependency ratio and child dependency ratio

To analyze the old age dependency ratio and child dependency ratio, we first store the required data in arrays by metric, calendar time, and world region.

```r
world_regions <- c("WORLD","Africa","Asia","Europe","Latin America and the Caribbean",
"Northern America","Oceania")

row.names <- "mean_age"
column.names <- as.character(c(1950:2100))
matrix.names <- world_regions
oadr_wom_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

oadr_men_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

oadr_pop_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

cdr_wom_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

cdr_men_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

cdr_pop_unwpp2019_1950_2100 <- array(NA,dim=c(1,151,7),
dimnames = list(row.names,column.names,matrix.names))

for(wr in 1:7){
    current.wr <- world_regions[wr]

    ## Old age dependency ratio:
    wom.obs <- apply(X=wom_unwpp2019_1950_2020[,,wr],2,
function(X){sum(X[66:101])/sum(X[16:65])})
    men.obs <- apply(X=men_unwpp2019_1950_2020[,,wr],2,
function(X){sum(X[66:101])/sum(X[16:65])})

    pop.obs <- apply(X=(wom_unwpp2019_1950_2020[,,wr]+men_unwpp2019_1950_2020[,,wr]),2,
function(X){sum(X[66:101])/sum(X[16:65])})

    wom.forecast <- apply(X=wom_unwpp2019_2020_2100[,as.character(2021:2100),wr],2,
function(X){sum(X[66:101])/sum(X[16:65])})

    men.forecast <- apply(X=men_unwpp2019_2020_2100[,as.character(2021:2100),wr],2,
function(X){sum(X[66:101])/sum(X[16:65])})

    pop.forecast <- apply(X=(wom_unwpp2019_2020_2100[,as.character(2021:2100),wr] +
men_unwpp2019_2020_2100[,as.character(2021:2100),wr]),2,
function(X){sum(X[66:101])/sum(X[16:65])})

    oadr_wom_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- wom.obs
    oadr_men_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- men.obs
    oadr_pop_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- pop.obs

    oadr_wom_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- wom.forecast
    oadr_men_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- men.forecast
    oadr_pop_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- pop.forecast
```

```r
    ## Child dependency ratio:
    wom.obs <- apply(X=wom_unwpp2019_1950_2020[,,wr],2,
function(X){sum(X[1:15])/sum(X[16:65])})

    men.obs <- apply(X=men_unwpp2019_1950_2020[,,wr],2,
function(X){sum(X[1:15])/sum(X[16:65])})

    pop.obs <- apply(X=(wom_unwpp2019_1950_2020[,,wr] +
men_unwpp2019_1950_2020[,,wr]),2,function(X){sum(X[1:15])/sum(X[16:65])})

    wom.forecast <- apply(X=wom_unwpp2019_2020_2100[,as.character(2021:2100),wr],2,
function(X){sum(X[1:15])/sum(X[16:65])})

    men.forecast <- apply(X=men_unwpp2019_2020_2100[,as.character(2021:2100),wr],2,
function(X){sum(X[1:15])/sum(X[16:65])})

    pop.forecast <- apply(X=(wom_unwpp2019_2020_2100[,as.character(2021:2100),wr] +
men_unwpp2019_2020_2100[,as.character(2021:2100),wr]),2,
function(X){sum(X[1:15])/sum(X[16:65])})

    cdr_wom_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- wom.obs
    cdr_men_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- men.obs
    cdr_pop_unwpp2019_1950_2100[,as.character(1950:2020),wr] <- pop.obs

    cdr_wom_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- wom.forecast
    cdr_men_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- men.forecast
    cdr_pop_unwpp2019_1950_2100[,as.character(2021:2100),wr] <- pop.forecast
}
```

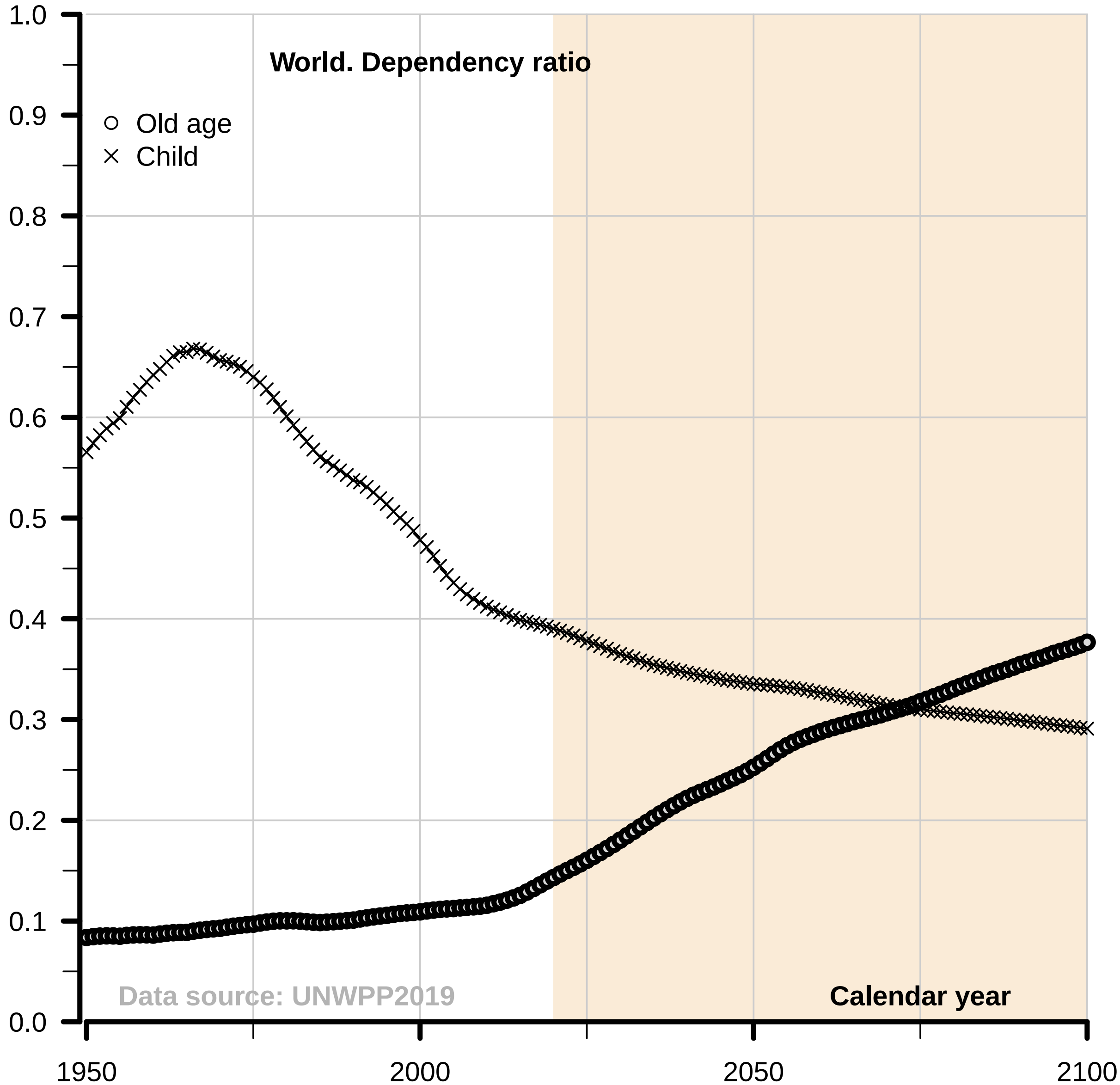
Something to think about. How would an array look like that contains old age dependency ratio and child dependency ratio by calendar year, world region, and gender? What is more convenient for you: multiple smaller arrays or fewer larger arrays?

That is it. We have all the data we need to plot the old age dependency ratio and child dependency ratio:

```r
setwd(the.plot.path)

dev.off()

pdf(file="global-world-pop-oadr-and-cdr-1950-2100.pdf", width=10, height=10,
family="Times", pointsize=20, onefile=TRUE)
```

```r
require(wesanderson)
pal <- c("navy",wes_palette("Darjeeling1"))

par(fig = c(0,1,0,1), las=1, mai=c(0.4,0.4,0.0,0))

    plot(x=-100,y=-100,xlim=c(1949,2100),ylim=c(0,1),xlab="",ylab="",
main="",axes=FALSE)

    rect(xleft=2020, xright=2100, ybottom=0, ytop=1, col="antiquewhite", border=NA)
    segments(x0=seq(1975,2100,25),x1=seq(1975,2100,25),y0=0,y1=1,col=gray(0.8))
    segments(x0=1950,x1=2100,y0=seq(0,1,0.2),y1=seq(0,1,0.2),col=gray(0.8))
```

```
    axis(side=1,at=seq(1950,2100,25),labels=FALSE,lwd=1,pos=0)
    axis(side=1,at=seq(1950,2100,50),labels=TRUE,lwd=3,pos=0)
    axis(side=2,at=seq(0,1,0.05),labels=FALSE,lwd=1,pos=1949)
    axis(side=2,at=seq(0,1,0.1),labels=TRUE,lwd=3,pos=1949)

    text(x=1950+25,y=0.95,"World. Dependency ratio",pos=4,font=2)
    text(x=2075,y=0,"Calendar year",pos=3,font=2)
    text(x=1980,y=0,"Data source: UNWPP2019",pos=3,font=2,col=gray(0.7))

    ## Global pop (UN release and forecast 1950:2100):
    lines(x=1950:2100,y=oadr_pop_unwpp2019_1950_2100[1,,1],lwd=1,col="black")
    points(x=1950:2100,y=oadr_pop_unwpp2019_1950_2100[1,,1],pch=21,lwd=3,
bg=gray(0.85),col="black")

    lines(x=1950:2100,y=cdr_pop_unwpp2019_1950_2100[1,,1],lwd=1,col="black")
    points(x=1950:2100,y=cdr_pop_unwpp2019_1950_2100[1,,1],pch=4,lwd=1,
bg=gray(0.85),col="black")

    legend(x=1950,y=0.925,c("Old age", "Child"),lty=FALSE, pch=c(21,4),
col=c("black","black"),bty="n")
```




```r
dev.off()
```

Brief data description. The upper (cross) and lower (circle) lines represent the child dependency ratio and old age dependency ratio, respectively, for the calendar years 1950 through 2100.

Something to think about. What does a dependency ratio of 0.3 actually mean? How would you describe and interpret the development of the global dependency ratio (child vs old age) over time?

Something more to think about. What do you think: how does the global dependency ratio reflect the developments of the global mean age (section 4.1)?

5. Now is the time to do this analysis again (sections 2 through 4) for your assigned world region. Feel free to adapt or change R code as you wish :-)

6. Project population with cohort-component method

We now switch from exploring existing forecasts to generating forecasts in the first place :-)

Looking closer at the cohort-component method, we project an exemplary population that has only three age groups. At the beginning, this population consists of three persons: one person in each age group. Fertility is zero, one, and two for age groups 1, 2, and 3, respectively. Survival probabilities are 0.8 from age group 1 to 2 and 0.5 from age group 2 to 3. Persons in age group 3 do not survive.

6.1 Population vector and Leslie matrix

In a first step we construct the population vector:

```
P <- matrix(rep(1,3),nr=3,nc=1)
P
```

```
##      [,1]
## [1,]    1
## [2,]    1
## [3,]    1
```

Please explain: *rep(1,3)*.

In a second step we construct the Leslie (projection) matrix:

```
fertility <- c(0,1,2)
mortality <- c(0.8,0.5)

L <- rbind(fertility,cbind(diag(mortality),matrix(0,nr=2,nc=1)))
rownames(L) <- 1:3
L
```

```
##   [,1] [,2] [,3]
## 1  0.0  1.0    2
## 2  0.8  0.0    0
## 3  0.0  0.5    0
```

Please explain: *diag()*.

Something to think about. How do we make sure in the Leslie matrix that no one survives in age group 3? Will there still be persons in age group 3?

And in a third step we combine the population vector P and the Leslie matrix L via matrix multiplication to project our exemplary population one year forward:

```
L%*%P
```

```
##   [,1]
## 1  3.0
## 2  0.8
## 3  0.5
```

Please explain: *%·%*.

Something to think about. Why is the order of L and P so critical when doing matrix multiplication? What is the total population size after one year? Did population size increase or decrease between t_0 and t_1?

6.2 Project population size

We now project our exemplary population 100 years ahead assuming that fertility and mortality will not change over time.

```
P_ahead <- matrix(0,nr=3,nc=101)
P_ahead[,1] <- P

for(year in 1:100){
    P_ahead[,year+1] <- L%*%P_ahead[,year]
}
P_ahead[,c(26,51,76,101)]
```

```
##            [,1]      [,2]      [,3]      [,4]
## [1,] 195.32303 22463.567 2583482.7 297120349
## [2,] 129.24867 14864.135 1709489.6 196604435
## [3,]  53.44671  6147.246  706980.4  81308180
```

Something to think about. How many people are projected to live after 50 and 100 years?

Please plot how the total population size is projected to develop within 100 years starting from the jump-off year t_0.

6.3 Constant growth rate

We now calculate the constant growth rate of our exemplary population:

```
P_ahead[,91]/P_ahead[,90]
```

```
## [1] 1.209008 1.209008 1.209008
```

```
P_ahead[,101]/P_ahead[,100]
```

```
## [1] 1.209008 1.209008 1.209008
```

```
eigen(L)$values
```

```
## [1]  1.2090077+0.0000000i -0.6045039+0.5443113i -0.6045039-0.5443113i
```

```
eigen(L)$values[1]
```

```
## [1] 1.209008+0i
```

Please explain: *eigen(L)*.

Something to think about. How large is the constant growth rate of our exemplary population? Does it indicate positive or negative population growth? What are the two alternative ways to approximate / derive the constant growth rate?

6.4 Constant age structure

We now calculate the constant age structure of our exemplary population:

```
P_ahead[,100]/sum(P_ahead[,100])
```

```
## [1] 0.5167014 0.3419012 0.1413974
```

```
eigen(L)$vectors
```

```
##               [,1]                  [,2]                  [,3]
## [1,] 0.8130530+0i -0.6547522+0.000000i -0.6547522+0.000000i
## [2,] 0.5379969+0i  0.4785255+0.430877i  0.4785255-0.430877i
```

```
## [3,] 0.2224952+0i -0.0413627-0.393633i -0.0413627+0.393633i
```

```r
eigen(L)$vectors[,1]/sum(eigen(L)$vectors[,1])
```

```
## [1] 0.5167014+0i 0.3419012+0i 0.1413974+0i
```

Something to think about. How large are the proportions of age groups 1, 2, and 3 in the long-run? Describe the procedure how you can derive the constant age structure from the Leslie (projection) matrix.

Please note that population size and structure can be forecasted in way more refined ways within the cohort-component method. Try to think of ways how to improve our simple example above in order to make it more realistic.