COS-R403. Special Research Methods. Forecasting I: Introduction

Lecturer: Christina Bohk-Ewald

Day 3 of intensive 5-day course

University of Helsinki, Finland
04.05.2020–08.05.2020

Third day's class:

- Recap main concepts of last lecture
- Some findings of previous lab session
- Introduction to mortality forecasting:
  - What kind of methods there are
  - The Lee-Carter method

Third day's class in the lab: Hands-on exercises in mortality forecasting with R

- Load mortality data from the Human Mortality Database
- Implement and use the Lee-Carter method
  - to fit and forecast US female and male mortality 50 years ahead
  - based on base period 1933-2017
- Compare LC mortality forecasts for US women and men, 2018-2067,
  - based on base periods of different length: 1933-2017 and 1988-2017
  - Explain how crucial model parameter values change.

  Present and discuss your findings.

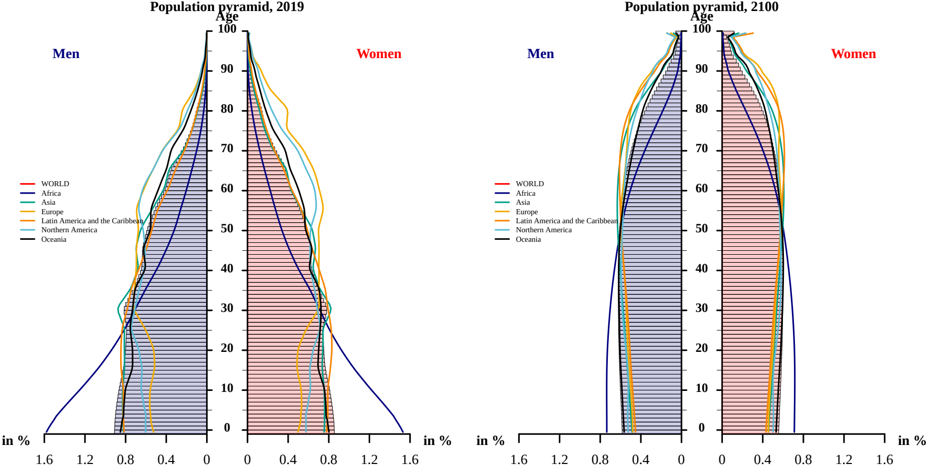
Please make sure to have installed R package `fds`. And to have at hand your username and password for the Human Mortality Database.

Recap main concepts of last lecture

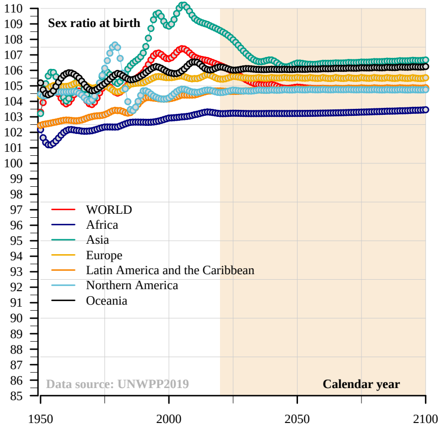
- What is the structure of a population (age) pyramid?
- What measures quantify population structure by sex?
- What measures quantify population structure by age?
- What is the conceptual difference between retrospective and prospective measures of aging?
- How does the cohort-component method work?
- What is the structure of Leslie's projection matrix?
- ...

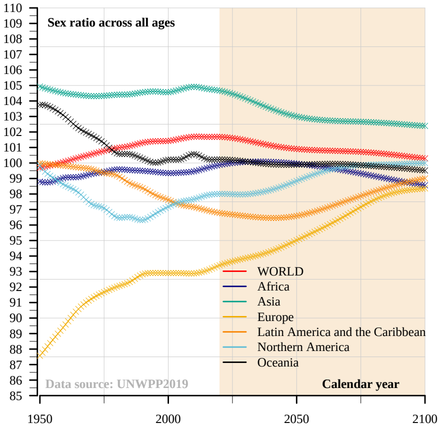
$\Rightarrow$ Questions?

Population (age) pyramid by world region

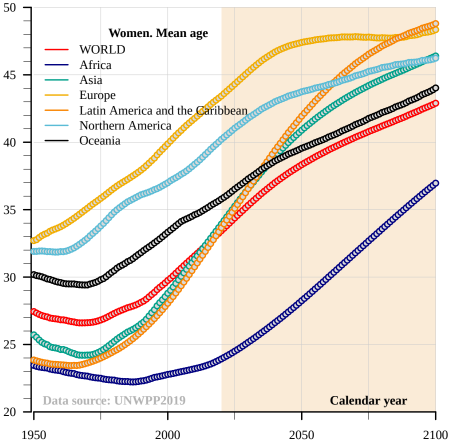


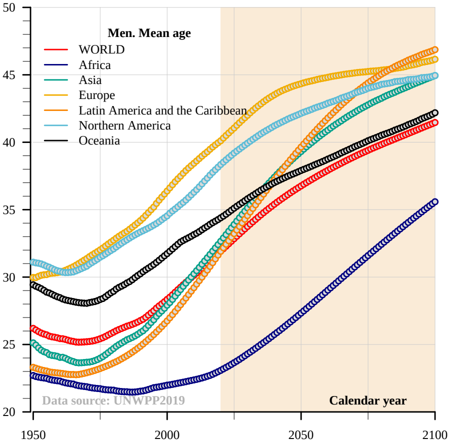
Sex ratio at birth and across all ages by world region





Mean age by world region





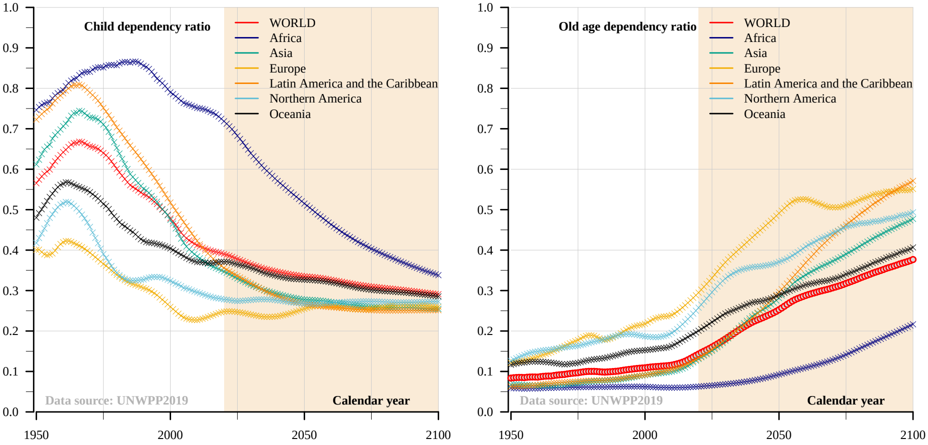
UNWPP2019 - key findings

7. The world's population is growing older, with persons over age 65 being the fastest-growing age group

By 2050, one in six people in the world will be over age 65 (16%), up from one in 11 in 2019 (9%). Regions where the share of the population aged 65 years or over is projected to double between 2019 and 2050 include Northern Africa and Western Asia, Central and Southern Asia, Eastern and South-Eastern Asia, and Latin America and the Caribbean. By 2050, one in four persons living in Europe and Northern America could be aged 65 or over. In 2018, for the first time in history, persons aged 65 or above outnumbered children under five years of age. The number of persons aged 80 years or over is projected to triple, from 143 million in 2019 to 426 million in 2050.

https://population.un.org/wpp/Publications/Files/WPP2019_10KeyFindings.pdf

Dependency ratio by world region



UNWPP2019 - key findings

8. Falling proportions of working-age people are putting pressure on social protection systems

The potential support ratio, which compares numbers of working-age people aged 25-64 to those over age 65, is falling around the world. In Japan, this ratio is 1.8, the lowest in the world. An additional 29 countries, mostly in Europe and the Caribbean, already have potential support ratios below three. By 2050, 48 countries, mostly in Europe, Northern America, and Eastern and South-Eastern Asia, are expected to have potential support ratios below two. These low values underscore the potential impact of population ageing on the labour market and economic performance as well as the fiscal pressures that many countries will face in the coming decades as they seek to build and maintain public systems of health care, pensions and social protection for older persons.

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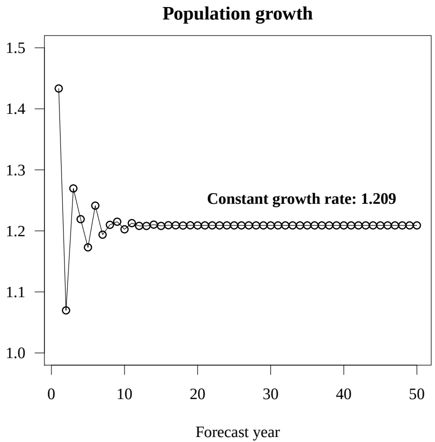
https://population.un.org/wpp/Publications/Files/WPP2019_10KeyFindings.pdf

Cohort-component method

Mini example for population with 3 age groups:

$$\begin{pmatrix} 3 \\ 0.8 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$



$\rightarrow$ Unrealistic assumption that mortality is time-invariant

$\Rightarrow$ How to forecast mortality that changes over age and time?

Mortality forecasting

Some empirical core questions:

- How long will people live in 10, 50, 100 years from now?
- How large will be mortality of 65-year-olds in 1-100 years from now?
- ...

Some urgent empirical questions of today:

- How many people will die from COVID-19?
- How large will be excess mortality from COVID-19?
- ...

Mortality forecasting

Some methodological core questions:

- How to best capture and forecast mortality dynamics, for example by age and calendar year (and birth cohort) with ongoing time?
- To what extent should we include factors that affect mortality dynamics? What are reliable sources of information?
- What are the characteristics of a *good* statistical forecast model?
  $\rightarrow$ Course in fall : *Scientific modeling and model assessment*

Mortality forecasting approaches

1. Extrapolation methods

- Model trends in mortality over age and time (and birth cohort)
- Are objective & data-driven
- Assume that basic trends in mortality were regular and would continue in years ahead

Mortality forecasting approaches

2. Explanation methods

- Take into account mortality that is e.g. attributable to health-related behavior (such as tobacco smoking) and/or causes of death
- Consider explanatory mechanisms / risk factors of mortality
- Are prone to model misspecification (due to high complexity)

Mortality forecasting approaches

3. Expert-based methods

- Use expert opinion to e.g. interpolate mortality between start and target value
- Are subjective & opinion-driven
- Might be biased (as experts tend to be overly confident)

Mortality forecasting approaches

1. Extrapolation methods
   - Model trends in mortality over age and time (and birth cohort)
   - Are objective & data-driven but assume basic trends in mortality to be regular and to continue in years ahead
2. Explanation methods
   - Take into account mortality that is e.g. attributable to health-related behavior (such as tobacco smoking) and/or causes of death
   - Consider explanatory mechanisms / risk factors of mortality but are prone to model misspecification (due to high complexity)
3. Expert-based methods
   - Use expert opinion to e.g. interpolate mortality between start and target value
   - Are subjective & opinion-driven and might be biased (as experts tend to be overly confident)
4. Mixture of methods above

$\longrightarrow$ Overview in e.g. Booth (2006) and Booth and Tickle (2008)

Mortality forecasting approaches

What approach would you prefer and why?

Would you have thought of another way to forecast mortality?

The Lee-Carter method

Modeling and Forecasting U.S. Mortality

RONALD D. LEE and LAWRENCE R. CARTER*

Time series methods are used to make long-run forecasts, with confidence intervals, of age-specific mortality in the United States from 1990 to 2065. First, the logs of the age-specific death rates are modeled as a linear function of an unobserved period-specific intensity index, with parameters depending on age. This model is fit to the matrix of U.S. death rates, 1933 to 1987, using the singular value decomposition (SVD) method; it accounts for almost all the variance over time in age-specific death rates as a group. Whereas e_0 has risen at a decreasing rate over the century and has decreasing variability, $\mathbf{k}(t)$ declines at a roughly constant rate and has roughly constant variability, facilitating forecasting. $\mathbf{k}(t)$, which indexes the intensity of mortality, is next modeled as a time series (specifically, a random walk with drift) and forecast. The method performs very well on within-sample forecasts, and the forecasts are insensitive to reductions in the length of the base period from 90 to 30 years; some instability appears for base periods of 10 or 20 years, however. Forecasts of age-specific rates are derived from the forecasts of $\mathbf{k}$, and other life table variables are derived and presented. These imply an increase of 10.5 years in life expectancy to 86.05 in 2065 (sexes combined), with a confidence band of plus 3.9 or minus 5.6 years, including uncertainty concerning the estimated trend. Whereas 46% now survive to age 80, by 2065 46% will survive to age 90. Of the gains forecast for person-years lived over the life cycle from now until 2065, 74% will occur at age 65 and over. These life expectancy forecasts are substantially lower than direct time series forecasts of e_0, and have far narrower confidence bands; however, they are substantially higher than the forecasts of the Social Security Administration's Office of the Actuary.

KEY WORDS: Demography; Forecast; Life expectancy; Mortality; Population; Projection.

From 1900 to 1988, life expectancy in the United States rose from 47 to 75 years. If it were to continue to rise at this same linear rate, life expectancy would reach 100 years in 2065, about seventy five years from now. The increase would

Next we fit the demographic model to U.S. data and evaluate its historical performance. Using standard time series methods, we then forecast the index of mortality and generate associated life table values at five-year intervals. Because we

Google Scholar Modeling and forecasting US mortality

Artikel Ungefähr 101.000 Ergebnisse (**0,10** Sek.)

Beliebige Zeit
Seit 2019
Seit 2018
Seit 2015
Zeitraum wählen...

Modeling and forecasting US mortality
RD Lee, LR Carter - Journal of the American statistical association, 1992 - Taylor & Francis
Time series methods are used to make long-run forecasts, with confidence intervals, of age-specific mortality in the United States from 1990 to 2065. First, the logs of the age-specific death rates are modeled as a linear function of an unobserved period-specific intensity ...
Zitiert von: 2888 Ähnliche Artikel Alle 17 Versionen

1. Golden standard to forecast mortality.
2. Published in 1992 and widely used since then.
3. Extrapolation method. Simple and robust.
4. Many extensions since 1992.

Use Lee-Carter model to fit and forecast US female mortality

We will look at the broad idea first
before we will also briefly look at methodological and technical details.

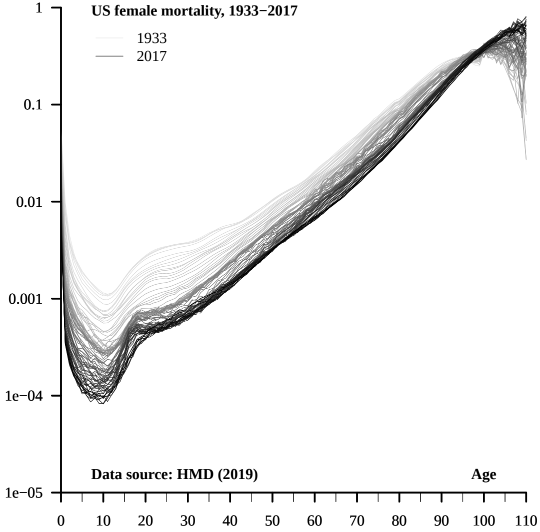
Example of US female mortality.

Two-step procedure:

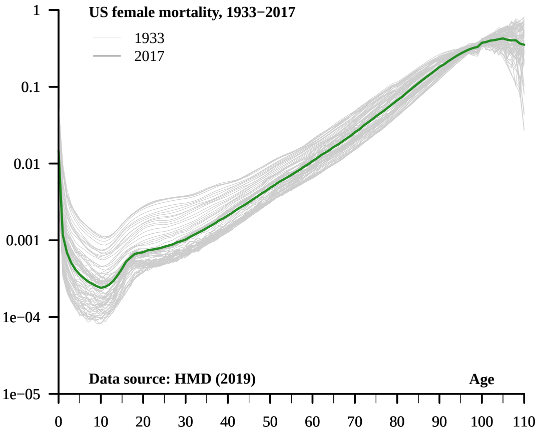
1. **Fit** Lee-Carter **model to** US female **mortality** by age and over time **in base period**.

2. **Forecast** US female **mortality** over time **in upcoming years**.

1. Fit Lee-Carter model to US female mortality, 1933-2017

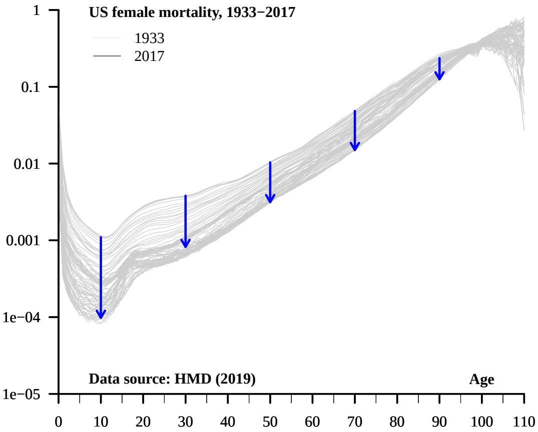


...with only few model parameters



$$\log m_{x,t} = \alpha_x + \beta_x \kappa_t$$

...with only with few model parameters



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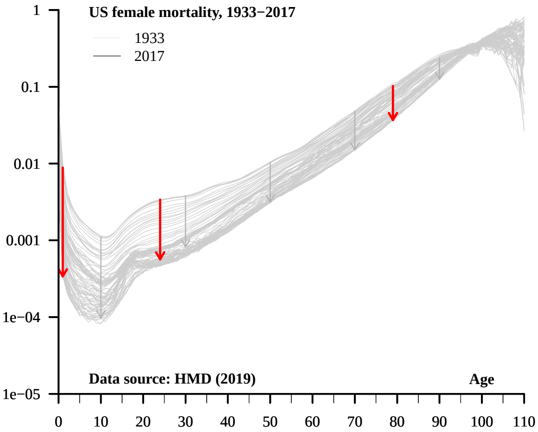
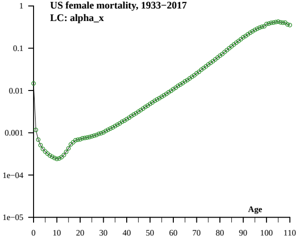
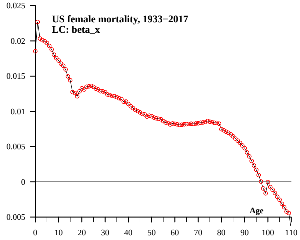


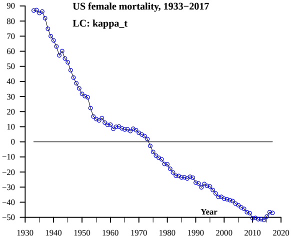
$$\log m_{x,t} = \alpha_x + \beta_x \kappa_t$$

Lee-Carter model fitted to US female mortality in base period 1933-2017

$$\log m_{x,t} = \alpha_x + \beta_x \kappa_t$$





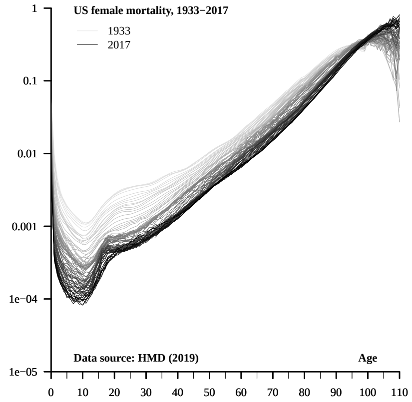


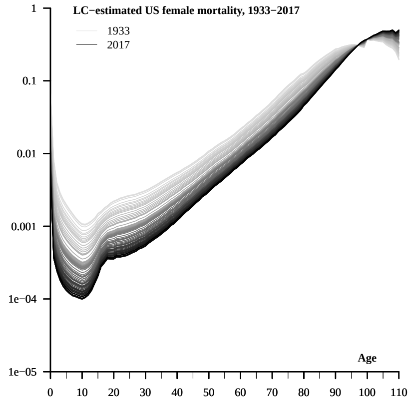
- α_x is the general shape of mortality across age x

- β_x is the change of mortality at age x
- $\beta_x > 0$: mortality decline, $\beta_x < 0$: mortality increase

- κ_t is an index of the level of mortality over time t
- direction and slope indicate strong mortality decline

...and one more thing

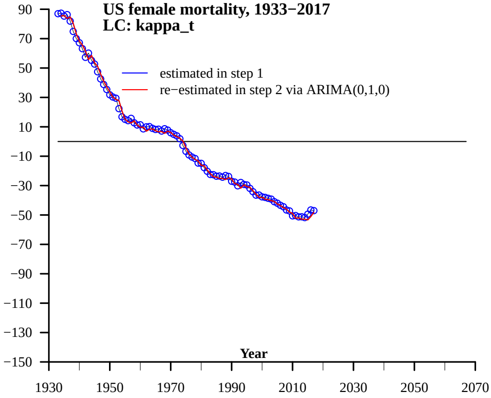




to account for residuals between LC-estimated and observed mortality:

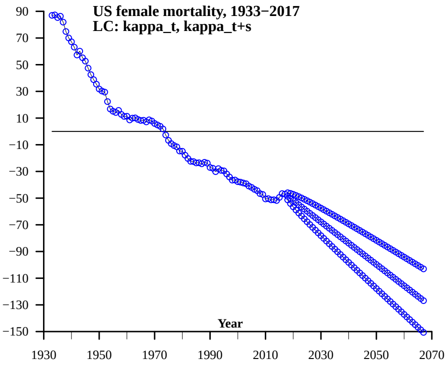
$$\log m_{x,t} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$$

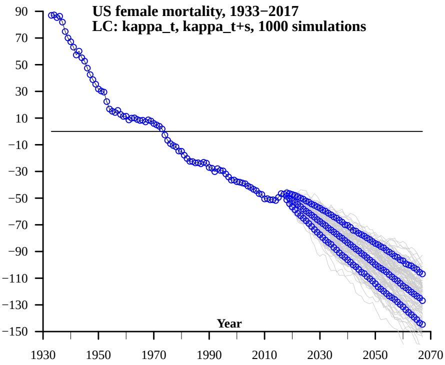
2. Use fitted LC-model to forecast US female mortality s years ahead via time index k_t



- κ_t is an index of the level of mortality over time t
- **Fit** estimated κ_t in base period using a time series model, e.g. ARIMA(0,1,0)
  - Random walk with drift: $\kappa_t = \kappa_{t-1} + \delta + \epsilon_t$
    - with δ being a drift term
    - and ϵ_t being an error term

2. Use fitted LC-model to forecast US female mortality s years ahead via time index k_t

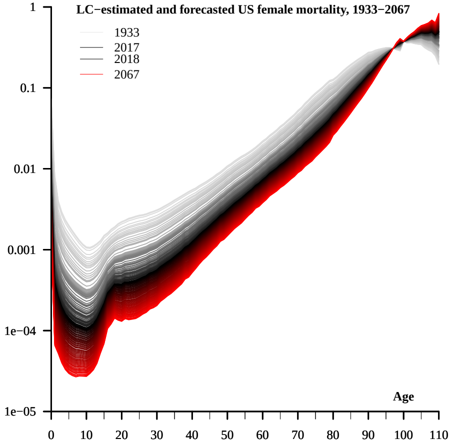


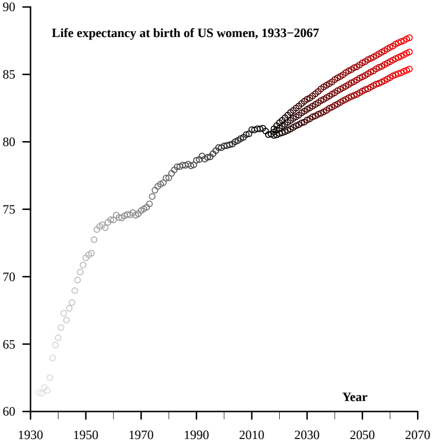


ARIMA(0,1,0), random walk with drift: $\kappa_t = \kappa_{t-1} + \delta + \epsilon_t$

80% **prediction intervals** based on statistical theory (left) & simulation (right)

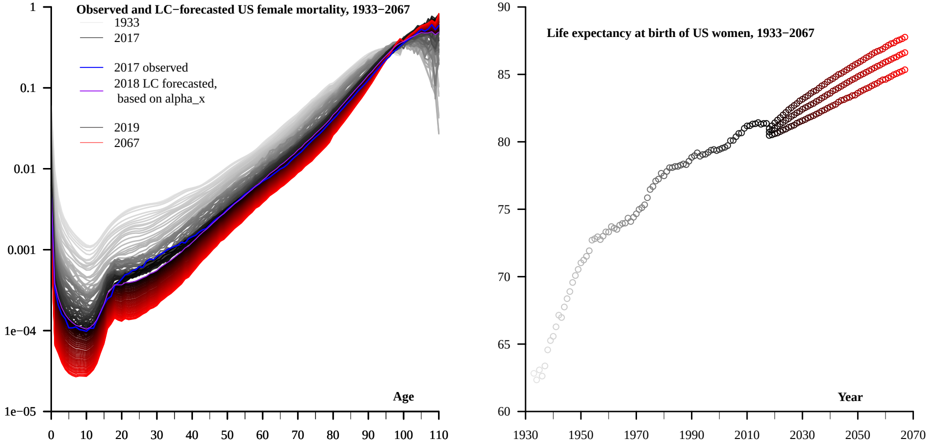
Forecasting US female mortality 50 years ahead using base period 1933-2017





κ_t point estimates are based on median of 1000 simulated trajectories

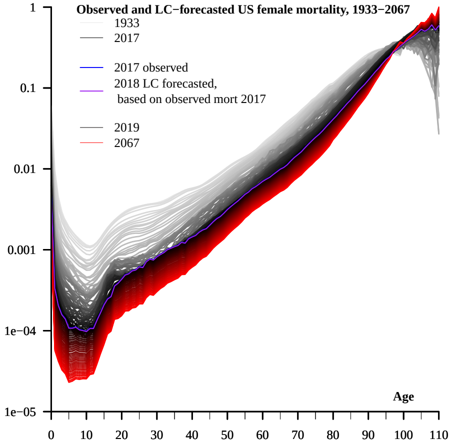
Forecasting US female mortality 50 years ahead using base period 1933-2017, **jump-off-bias**

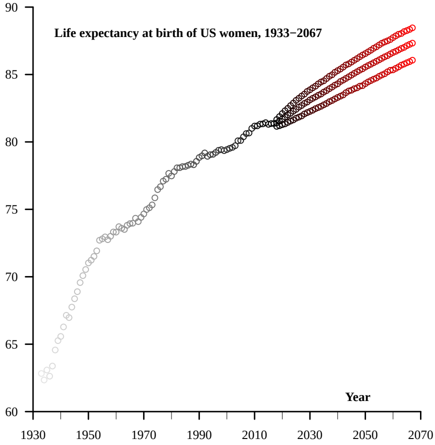


κ_t point estimates are based on median of 1000 simulated trajectories

Forecasting US female mortality 50 years ahead using base period 1933-2017, **corrected for jump-off-bias:**

$$\log m_{x,t} = m_{x,2017} + \beta_x \kappa_t^* + \epsilon_{x,t}$$





κ_t point estimates are based on median of 1000 simulated trajectories

Think about it.

What does the Lee-Carter model do?

What are the main steps?

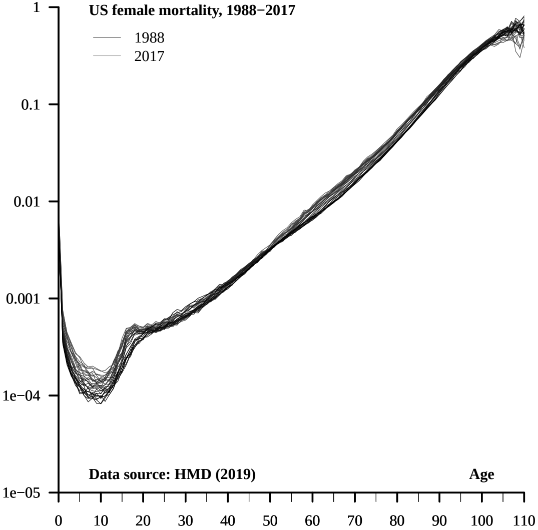
Could we forecast US female mortality differently
although we always use the Lee-Carter model?

What are high-impact settings & parameters of the LC model?

What impacts US female mortality forecast with LC model?

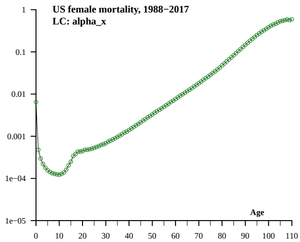
- Observed levels and trends in base period (α_x, β_x, and κ_t)
- Fitting procedure (e.g. singular value decomposition, maximum likelihood)
- Forecast time index κ_t
  - Time series model
  - Prediction intervals (based on e.g. simulation or statistical theory)
- Implementation (e.g. different R-packages)

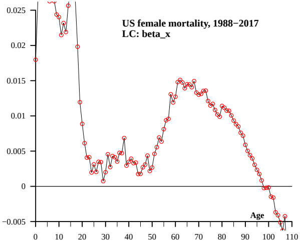
Impact base period: fit LC model to US female mortality, 1988-2017

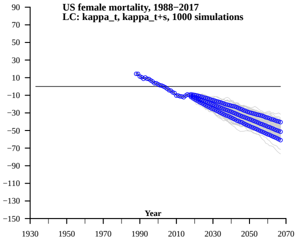


Lee-Carter model fitted to mortality in base period: 1988-2017 (focus on more recent trends)

$$\log m_{x,t} = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$$







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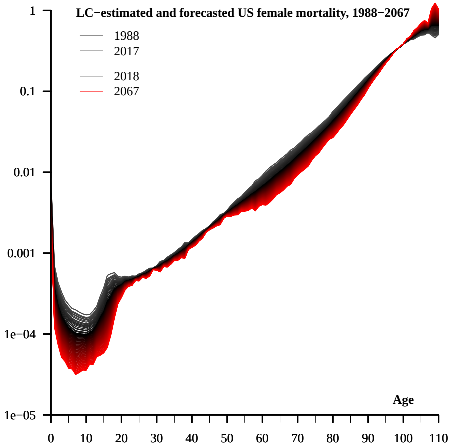
- κ_t is an index of the level of mortality over time t
- direction and slope indicate moderate mortality decline

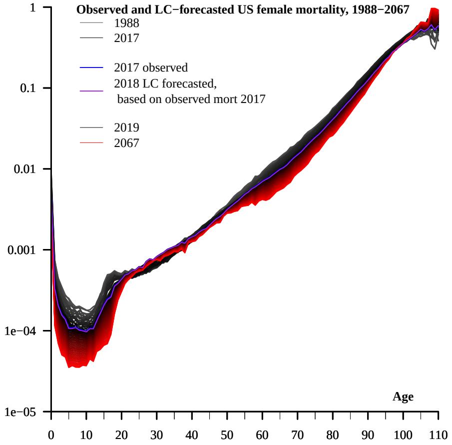
Forecasting US female mortality 50 years ahead using base period 1988-2017

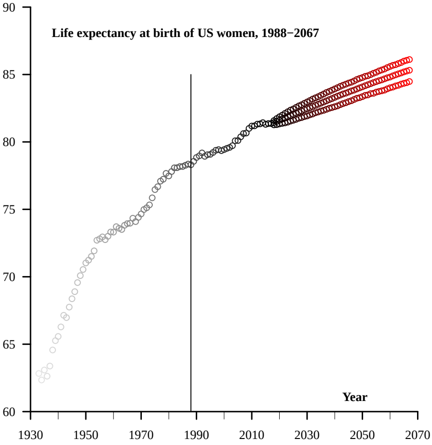


κ_t point estimates are based on median of 1000 simulated trajectories

Forecasting US female mortality 50 years ahead using base period 1988-2017, **corrected for jump-off-bias:**

$$\log m_{x,t} = m_{x,2017} + \beta_x \kappa^*_t + \epsilon_{x,t}$$





Think about it.

What are the benefits of the Lee-Carter model?

What trends does the Lee-Carter model capture?

What are the caveats concerning the Lee-Carter model?

What trends does it not capture?

Methodological and technical details on generating mortality forecasts with the LC method

1. Fit the Lee-Carter model to mortality by age and time in base period.
2. Forecast mortality by age s years ahead.

1. Fit LC model to mortality in base period in 8 steps

1. Put mortality rates $m_{x,t}$ in matrix by age (rows) and year (columns)
2. Calculate natural logarithm of mortality rates: $\ln m_{x,t}$
3. Calculate α_x as mean mortality over time for each age x
4. Calculate central (or normalized) log mortality rates $M_{x,t}$ as difference between $\ln m_{x,t}$ and α_x
5. Estimate β_x and κ_t applying singular value decomposition (SVD) to central log mortality rates ($M_{x,t}$)

1. Fit LC model to mortality in base period in 8 steps

5. Estimate β_x and κ_t applying singular value decomposition (SVD) to central log mortality rates ($M_{x,t}$)
   1. $svd(M_{x,t}) = UDV$; with $M[x,t]$, $U[t,t]$, $D[1,t]$, and $V[x,x]$
   2. $\beta_x = \frac{V[,1]}{\sum V[,1]}$
   3. $\kappa_t = D[1,1]U[,1]sumV[,1]$
   4. Check that $\sum \beta_x = 1$ and $\sum \kappa_t = 0$

1. Fit LC model to mortality in base period in 8 steps

6. Plot α_x, β_x, κ_t for plausibility checks
7. If desired, re-fit κ_t to e.g. total death counts, deaths counts by age, life expectancy with iterative process.
8. Fit mortality in base period putting parameter values into LC model function: $\log \hat{m}_{x,t} = \alpha_x + \beta_x \kappa_t$

2. Forecast mortality by age s years ahead in 3 steps

1. **Fit estimated κ_t in base period** using a time series model, e.g. ARIMA(0,1,0)
   - Lee and Carter suggest random walk with drift, ARIMA(0,1,0): $\kappa_{t+s} = \kappa_{t-1} + \delta + \epsilon_t$
     - δ is a drift term
     - ϵ_t is an error term

2. Forecast mortality by age s years ahead in 3 steps

2. **Forecast** κ_t s **years ahead** with fitted time series model:
   $\kappa_{t+s} = \kappa_{t-1} + \delta + \epsilon_t$
   - Point and interval forecasts of κ_t can be based on simulation:
     - ★ Simulate N trajectories for κ_{t+s}
       using estimate of δ (of fitted ARIMA(0,1,0) model)
     - ★ Draw $\epsilon_t \sim \mathcal{N}(0, \sigma^2_{\epsilon_t})$
       for each trajectory and year t,
       with $\sigma^2_{\epsilon_t}$ being the estimated variance of the residuals
       of the fitted ARIMA(0,1,0) model
     - ★ Determine median and 80% prediction intervals for κ_{t+s}
       using quantiles (0.1, 0.5, and 0.9)
       of the distribution comprising the N trajectories

2. Forecast mortality by age s years ahead in 3 steps

3. **Forecast age-specific mortality s years ahead**
   inserting parameter values into LC model function:
   $\log m_{x,t+s} = \alpha_x + \beta_x \kappa_{t+s} + \epsilon_{x,t}$

That is it

Yes, it is a lot. Try to wrap your mind around it. Take your time. Think about it. Talk to others about it. Have a look into the paper.

Please make a note of upcoming questions

What you have learned today about demographic forecasting

- Describe different approaches to forecast mortality
- Explain the method of Lee and Carter
- Discuss pros and cons of the Lee-Carter method

Third day's class in the lab: Hands-on exercises in mortality forecasting with R

- Load mortality data from the Human Mortality Database
- Implement and use the Lee-Carter method
  - to fit and forecast US female and male mortality 50 years ahead
  - based on base period 1933-2017
- Compare LC mortality forecasts for US women and men, 2018-2067,
  - based on base periods of different length: 1933-2017 and 1988-2017
  - Explain how crucial model parameter values change.

  Present and discuss your findings.

Please make sure to have installed R package `fds`. And to have at hand your username and password for the Human Mortality Database.

Course learning materials

Course learning materials on GitHub:

https://github.com/christina-bohk-ewald/2020-course-COS-R403-forecasting-1-introduction

R programming

Some functions we will use:

- read.hmd()
- data[[2]][,t]
- colorRampPalette()
- svd()

→ Get information about what they are and how to use them

Human Mortality Database (HMD)

- `https://www.mortality.org/`
- High-quality data (e.g. death counts, population exposure) for mostly highly developed countries
- Overview: `https://www.mortality.org/Public/Overview.php`

$\longrightarrow$ Explore the Human Mortality Database

Recommended learning material for today's class

- **Lee, R. D., & Carter, L. R. (1992)**
  Modeling and forecasting U.S. mortality. Journal of the American Statistical Association, 87(419), 659-671.
- **Booth, H. (2006)**
  Demographic forecasting: 1980 to 2005 in review. International Journal of Forecasting, 22(3), 547-581.
- **Booth, H., & Tickle, L. (2008)**
  Mortality modelling and forecasting: A review of methods. Annals of Actuarial Science, 3(1-2), 3-43.

Recommended learning material for today's class

- **Bohk, C., & Rau, R. (2016)**
  Changing mortality patterns and their predictability: the case of the United States. In Dynamic Demographic Analysis (pp. 69-89). Springer, Cham.
- **Rau, R., Bohk-Ewald, C., Muszyńska, M., & Vaupel, J. (2017)**
  Visualizing Mortality Dynamics in the Lexis Diagram (Vol. 44). Springer.
- **Bohk-Ewald, C., Ebeling, M., & Rau, R. (2017)**
  Lifespan disparity as an additional indicator for evaluating mortality forecasts. Demography, 54(4), 1559-1577.
- **Bohk-Ewald, C., Li, P., & Myrskylä, M. (2018)**
  Forecast accuracy hardly improves with method complexity when completing cohort fertility. Proceedings of the National Academy of Sciences, 115(37), 9187-9192.

Recommended learning material for today's class

- **UNWPP2019**: `https://population.un.org/wpp/`
  Publications, Graphs, & Data files.
- **Raftery, A. E., Gerland, P., and Ševčíková, H. (2013)**
  Bayesian probabilistic projections of life expectancy for all countries. Demography, 50(3), 777–801.
- **Alho, J. and Spencer, B. (1997)**
  The practical specification of the expected error of population forecasts. Journal of Official Statistics, 13(3), 203–225.
- **Preston, S., Heuveline, P., and Guillot, M. (2000)**
  Demography: measuring and modeling population processes
  Blackwell Publishers Ltd.
- **Alho, J. and Spencer, B. (2006)**
  Statistical demography and forecasting
  Springer Science & Business Media.

Thank you for your attention!

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Third day's class in the lab: Hands-on exercises in mortality forecasting with R

- Load mortality data from the Human Mortality Database
- Implement and use the Lee-Carter method
  - to fit and forecast US female and male mortality 50 years ahead
  - based on base period 1933-2017
- Compare LC mortality forecasts for US women and men, 2018-2067,
  - based on base periods of different length: 1933-2017 and 1988-2017
  - Explain how crucial model parameter values change.

  Present and discuss your findings.

Please make sure to have installed R package `fds`. And to have at hand your username and password for the Human Mortality Database.