Lab 10R

36-290 – Statistical Research Methodology

Week 10 Thursday - Fall 2021

Data

Below we read in the EMLINE_MASS dataset, in which the strengths of 10 emission lines are recorded for each of 21,046 galaxies, along with the galaxy masses.

```
rm(list=ls())
file.path = "https://raw.githubusercontent.com/pefreeman/36-290/master/EXAMPLE_DATASETS/EMLINE_MASS/emission_line_m
ass.Rdata"
load(url(file.path))
rm(file.path)
x = predictors$H_ALPHA
x.tmp = log10(x[x>0])
y = responses[x>0,1]
x = x.tmp
df = data.frame(x,y)
```

Today we are simply playing around with kernel density estimation and kernel regression, so all we are going to keep is the values for the strongest emission line, the so-called "H α " line at 656 nanometers (which we will call x), and the masses (which we will call y). We also filter the data so as to keep only positive emission line strengths, so that we can implement a logarithmic transformation for x.

Questions

Question 1

Do some EDA. First, use ggplot2 to create histograms of both x and y, and then use it to make a scatter plot of x and y. Don't worry about downsampling the amount of data; rather, change the transparency of the points by setting the alpha parameter to, e.g., 0.1.

```
library(ggplot2)
library(tidyverse)

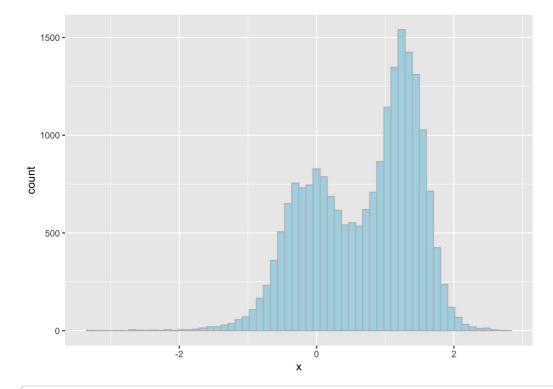
## — Attaching packages — tidyverse 1.3.1 —

## / tibble 3.1.4 / dplyr 1.0.7
## / tidyr 1.1.3 / stringr 1.4.0
## / readr 2.0.1 / forcats 0.5.1

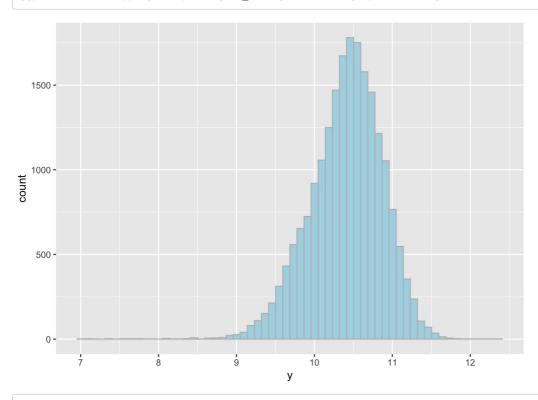
## / purrr 0.3.4

## — Conflicts — tidyverse_conflicts() —
## x dplyr::filter() masks stats::filter()
## x dplyr::lag() masks stats::lag()
```

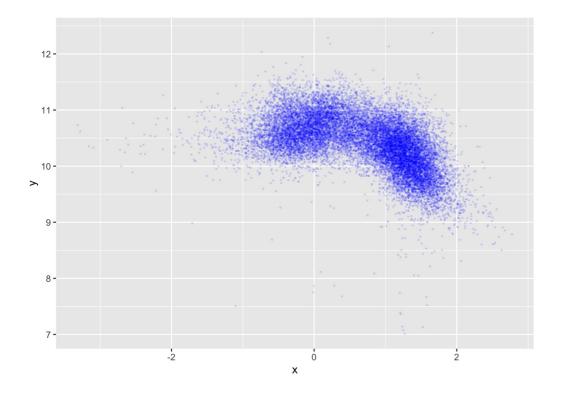
```
ggplot(data=df,mapping=aes(x)) + geom_histogram(color="grey",fill="lightblue",bins=60)
```



ggplot(data=df,mapping=aes(y)) + geom_histogram(color="grey",fill="lightblue",bins=60)



ggplot(data=df,mapping=aes(x,y)) + geom_point(color="blue",size=.3, alpha=.1)



Question 2

Create a density estimate for x using the <code>density()</code> function and the default bandwidth. Print the default bandwidth. Then overlay the density estimate on top of a density histogram. One creates a density histogram by adding an extra argument to <code>geom_histogram()</code>: <code>aes(y=..density..)</code>. One can then overlay the density estimate using an additional call to <code>geom_line()</code>, to which you pass a data frame with the x output of <code>density()</code> in one column and the y output of <code>density()</code> in the other.

```
d.estimatel=density(x, bw="nrd0", adjust =1)
d.estimatel
```

```
##
## Call:
##
   density.default(x = x, bw = "nrd0", adjust = 1)
##
##
  Data: x (20722 obs.);
                            Bandwidth 'bw' = 0.09539
##
##
          :-3.6016
                            :0.0000024
##
                    Min.
##
   1st Qu.:-1.9358
                    1st Qu.:0.0014043
   Median :-0.2701
##
                     Median :0.0221424
##
   Mean :-0.2701
                     Mean
                            :0.1499336
##
   3rd Qu.: 1.3957
                      3rd Qu.:0.2919921
                            :0.6761376
##
   Max.
          : 3.0615
                     Max.
```

```
"Default bandwith `bw`=0.09539"
```

```
## [1] "Default bandwith `bw`=0.09539"
```

```
df.estimate1 = data.frame(d.estimate1$x,d.estimate1$y)
df.estimate1
```

```
##
       d.estimate1.x d.estimate1.y
## 1
        -3.601622381 3.083997e-06
## 2
        -3.588583037 4.620915e-06
## 3
        -3.575543693 6.793147e-06
## 4
        -3.562504349 9.879722e-06
## 5
        -3.549465005 1.419070e-05
        -3.536425661 1.997696e-05
## 6
        -3.523386317 2.757447e-05
## 7
## 8
        -3.510346973 3.733631e-05
## 9
        -3.497307629 4.961376e-05
## 10
        -3.484268285 6.473235e-05
##
  11
        -3.471228941 8.296379e-05
## 12
        -3.458189597 1.044952e-04
        -3.445150253 1.293976e-04
## 13
## 14
        -3.432110909 1.578895e-04
```

```
## 15
        -3.419071565 1.893794e-04
## 16
        -3.406032221 2.232530e-04
## 17
        -3.392992877
                      2.588226e-04
##
  18
        -3.379953533
                      2.952656e-04
##
  19
        -3.366914189
                      3.316683e-04
## 20
        -3.353874845
                      3.670812e-04
## 21
        -3.340835501 4.005798e-04
## 22
        -3.327796157
                      4.313274e-04
## 23
        -3.314756813
                     4.586081e-04
## 24
        -3.301717469
                      4.814267e-04
##
  25
        -3.288678125
                      5.000455e-04
## 26
        -3.275638781
                      5.145124e-04
## 27
        -3.262599437
                      5.250493e-04
## 28
        -3.249560093 5.320004e-04
## 29
        -3.236520749 5.357734e-04
## 30
        -3.223481405 5.367814e-04
## 31
        -3.210442061 5.353891e-04
##
  32
        -3.197402717
                      5.318723e-04
## 33
        -3.184363373
                      5.263152e-04
## 34
        -3.171324029
                      5.186625e-04
## 35
        -3.158284685
                      5.090215e-04
## 36
        -3.145245341
                      4.973119e-04
## 37
        -3.132205997
                      4.834928e-04
##
  38
        -3.119166653
                      4.676096e-04
## 39
        -3.106127309
                      4.498315e-04
## 40
        -3.093087965
                      4.304730e-04
## 41
        -3.080048621 4.099972e-04
## 42
        -3.067009277
                      3.890008e-04
## 43
        -3.053969933 3.682626e-04
## 44
        -3.040930589
                      3.486713e-04
##
  45
        -3.027891245
                      3.309100e-04
## 46
        -3.014851901
                      3.155768e-04
## 47
                      3.031404e-04
        -3.001812557
## 48
        -2.988773213
                      2.939272e-04
## 49
        -2.975733869
                      2.881285e-04
## 50
        -2.962694525
                     2.858258e-04
## 51
        -2.949655181
                      2.870330e-04
## 52
        -2.936615837
                      2.917496e-04
## 53
        -2.923576493
                      3.004397e-04
## 54
        -2.910537149 3.130008e-04
## 55
        -2.897497805 3.297324e-04
## 56
        -2.884458461 3.511439e-04
## 57
        -2.871419117
                      3.778501e-04
##
  58
        -2.858379773
                      4.105090e-04
## 59
        -2.845340429
                      4.497399e-04
## 60
        -2.832301085
                     4.960293e-04
## 61
        -2.819261741
                      5.496318e-04
## 62
        -2.806222397
                      6.105810e-04
## 63
        -2.793183053
                      6.788637e-04
## 64
                      7.525018e-04
        -2.780143709
## 65
        -2.767104365
                      8.299119e-04
## 66
        -2.754065021
                      9.091573e-04
## 67
        -2.741025677
                      9.880541e-04
## 68
        -2.727986333 1.064300e-03
## 69
        -2.714946989 1.135620e-03
## 70
        -2.701907645 1.199909e-03
##
  71
        -2.688868301 1.255380e-03
##
  72
        -2.675828957
                      1.300123e-03
## 73
        -2.662789613
                      1.333012e-03
## 74
        -2.649750269
                      1.354924e-03
## 75
        -2.636710925
                     1.366505e-03
## 76
        -2.623671581
                      1.368907e-03
## 77
        -2.610632237
                      1.363704e-03
##
  78
        -2.597592893
                      1.352774e-03
## 79
        -2.584553549
                      1.338173e-03
## 80
        -2.571514205
                     1.322006e-03
## 81
        -2.558474861 1.306286e-03
## 82
        -2.545435517 1.293143e-03
## 83
        -2.532396173 1.284078e-03
##
  84
        -2.519356829
                      1.279770e-03
## 85
        -2.506317485
                      1.280514e-03
## 86
        -2.493278141
                      1.286126e-03
## 87
        -2.480238797
                      1.295999e-03
## 88
        -2.467199453
                     1.309175e-03
## 89
        -2.454160109
                     1.324458e-03
## 90
        -2.441120765
                     1.340542e-03
  91
##
        -2.428081421
                      1.356145e-03
## 92
        -2.415042077
                      1.369844e-03
        -2.402002733 1.381008e-03
## 93
```

```
## 94
        -2.388963389 1.389399e-03
## 95
        -2.375924045 1.395158e-03
## 96
        -2.362884701 1.398769e-03
##
  97
        -2.349845357
                     1.400981e-03
## 98
        -2.336806013
                      1.402697e-03
## 99
        -2.323766669
                      1.404854e-03
## 100
       -2.310727325
                     1.408301e-03
## 101
       -2.297687981
                     1.413751e-03
## 102
       -2.284648637
                     1.421750e-03
## 103
       -2.271609293
                     1.432001e-03
##
  104
        -2.258569949
                      1.444140e-03
## 105
        -2.245530605
                      1.457614e-03
       -2.232491261 1.471774e-03
## 106
## 107
       -2.219451917 1.485981e-03
## 108
       -2.206412573 1.499699e-03
## 109
       -2.193373229 1.512582e-03
## 110
       -2.180333885 1.524535e-03
##
  111
        -2.167294541
                      1.535734e-03
## 112
        -2.154255197
                      1.546813e-03
## 113
       -2.141215853
                     1.558615e-03
## 114
       -2.128176509
                     1.572063e-03
## 115
       -2.115137165
                     1.588124e-03
## 116
       -2.102097821
                     1.607727e-03
##
  117
        -2.089058477
                      1.631690e-03
        -2.076019133
## 118
                      1.660650e-03
## 119
       -2.062979789
                      1.695010e-03
## 120
       -2.049940445 1.734903e-03
## 121
       -2.036901101 1.780623e-03
## 122
       -2.023861757 1.831240e-03
## 123
        -2.010822413 1.885790e-03
##
  124
        -1.997783069
                      1.943347e-03
## 125
        -1.984743725
                      2.002936e-03
## 126
       -1.971704381 2.063626e-03
## 127
        -1.958665037
                      2.124645e-03
## 128
       -1.945625693
                     2.185481e-03
## 129
       -1.932586349 2.245983e-03
## 130
        -1.919547005
                      2.306441e-03
## 131
        -1.906507661
                      2.367990e-03
## 132
        -1.893468317
                      2.432201e-03
## 133
       -1.880428973 2.501091e-03
## 134
       -1.867389629 2.577026e-03
## 135
       -1.854350285 2.662603e-03
## 136
       -1.841310941 2.760497e-03
##
  137
        -1.828271597
                      2.873311e-03
## 138
        -1.815232253
                      3.003416e-03
## 139
        -1.802192909
                      3.152812e-03
## 140
       -1.789153565
                     3.323761e-03
## 141
       -1.776114221
                     3.518422e-03
## 142
       -1.763074877
                     3.734715e-03
## 143
        -1.750035533
                     3.971851e-03
## 144
                      4.228413e-03
        -1.736996189
## 145
        -1.723956845 4.502434e-03
## 146
       -1.710917501 4.791486e-03
## 147
       -1.697878157 5.092794e-03
## 148
       -1.684838813 5.403353e-03
## 149
       -1.671799469 5.720063e-03
##
  150
        -1.658760125
                     6.039840e-03
##
  151
        -1.645720781
                      6.359256e-03
## 152
        -1.632681437
                      6.675803e-03
## 153
        -1.619642093
                      6.987638e-03
## 154
        -1.606602749
                      7.293483e-03
## 155
        -1.593563405
                     7.592674e-03
## 156
        -1.580524061 7.885164e-03
##
  157
        -1.567484717
                      8.171503e-03
## 158
        -1.554445373
                      8.452780e-03
## 159
       -1.541406029 8.730527e-03
## 160
       -1.528366685 9.006701e-03
## 161
       -1.515327341 9.283820e-03
## 162
       -1.502287997
                      9.563937e-03
##
  163
        -1.489248653
                      9.848928e-03
##
        -1.476209309
  164
                      1.014048e-02
## 165
        -1.463169965
                      1.044007e-02
## 166
        -1.450130621 1.074906e-02
## 167
        -1.437091277
                     1.106873e-02
## 168
        -1.424051933 1.140045e-02
## 169
        -1.411012589
                     1.174599e-02
##
  170
        -1.397973245
                      1.210949e-02
## 171
       -1.384933901
                      1.249208e-02
## 172 -1.371894557 1.289709e-02
```

```
## 173
       -1.358855213 1.332831e-02
## 174
       -1.345815869 1.378988e-02
## 175
       -1.332776525 1.428604e-02
## 176
       -1.319737181 1.482087e-02
##
  177
        -1.306697837
                     1.539803e-02
       -1.293658493 1.602048e-02
## 178
## 179
       -1.280619149 1.669237e-02
## 180
       -1.267579805 1.741623e-02
## 181
       -1.254540461 1.818704e-02
## 182
       -1.241501117 1.900263e-02
##
  183
        -1.228461773
                     1.985992e-02
## 184
        -1.215422429
                     2.075523e-02
       -1.202383085 2.168461e-02
## 185
## 186
       -1.189343741 2.264425e-02
## 187
       -1.176304397 2.363095e-02
## 188
       -1.163265053 2.464246e-02
## 189
       -1.150225709 2.568000e-02
##
  190
        -1.137186365
                     2.674457e-02
## 191
        -1.124147021
                     2.783887e-02
## 192
        -1.111107677
                     2.896890e-02
## 193
        -1.098068333 3.014257e-02
## 194
       -1.085028989
                     3.136952e-02
## 195
       -1.071989645
                     3.266074e-02
##
  196
        -1.058950301
                     3.402819e-02
        -1.045910957
## 197
                     3.548424e-02
       -1.032871613 3.704115e-02
## 198
## 199
       -1.019832269 3.872481e-02
## 200
       -1.006792925 4.053519e-02
## 201
       -0.993753581 4.247813e-02
## 202
       -0.980714237 4.455902e-02
##
  203
        -0.967674893
                     4.678108e-02
## 204
        -0.954635549
                     4.914555e-02
## 205
       -0.941596205 5.165197e-02
## 206
       -0.928556861 5.429867e-02
## 207
       -0.915517517
                    5.708331e-02
## 208
       -0.902478173 6.000631e-02
## 209
        -0.889438829
                     6.307824e-02
## 210
        -0.876399485
                     6.628542e-02
## 211
       -0.863360141 6.963149e-02
       -0.850320797 7.312294e-02
## 212
## 213
       -0.837281453 7.676929e-02
## 214
       -0.824242109 8.058299e-02
## 215
       -0.811202765 8.457918e-02
##
  216
        -0.798163421
                     8.877522e-02
## 217
        -0.785124077
                     9.318996e-02
## 218
       -0.772084733
                     9.785727e-02
## 219
       -0.759045389 1.028049e-01
## 220
       -0.746006045 1.080311e-01
## 221
       -0.732966701 1.135484e-01
## 222
        -0.719927357
                     1.193646e-01
## 223
        -0.706888013
                     1.254822e-01
## 224
       -0.693848669 1.318976e-01
## 225
       -0.680809325 1.386006e-01
## 226
       -0.667769981 1.455746e-01
## 227
       -0.654730637 1.527971e-01
## 228
       -0.641691293 1.602560e-01
## 229
        -0.628651949 1.679015e-01
## 230
        -0.615612605
                     1.756896e-01
## 231
       -0.602573261 1.835852e-01
## 232
       -0.589533917 1.915548e-01
## 233
       -0.576494573 1.995675e-01
## 234
       -0.563455229 2.075958e-01
## 235
       -0.550415885
                     2.156160e-01
##
  236
        -0.537376541
                     2.236081e-01
## 237
        -0.524337197
                     2.315549e-01
## 238
       -0.511297853 2.394313e-01
## 239
       -0.498258509 2.472265e-01
## 240
       -0.485219165 2.549225e-01
       -0.472179821 2.624974e-01
## 241
##
  242
        -0.459140477
                     2.699245e-01
## 243
        -0.446101133
                     2.771722e-01
## 244
        -0.433061789
                     2.842044e-01
## 245
       -0.420022445 2.909813e-01
## 246
       -0.406983101 2.974605e-01
## 247
        -0.393943757 3.035880e-01
## 248
        -0.380904413 3.092898e-01
##
  249
        -0.367865069
                     3.145668e-01
## 250
       -0.354825725
                     3.193939e-01
## 251 -0.341786381 3.237556e-01
```

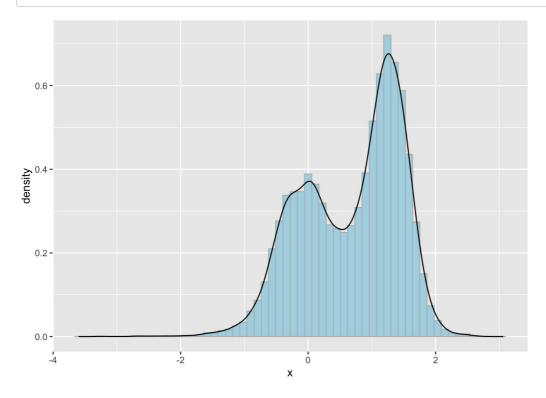
```
## 252
       -0.328747037 3.276469e-01
## 253
       -0.315707693 3.310743e-01
## 254
       -0.302668349 3.340554e-01
        -0.289629005
##
  255
                     3.366185e-01
##
  256
        -0.276589661
                      3.388019e-01
##
  257
        -0.263550317
                      3.406353e-01
## 258
       -0.250510973 3.421853e-01
##
  259
        -0.237471629
                     3.435327e-01
## 260
        -0.224432285
                     3.447381e-01
## 261
        -0.211392941 3.458609e-01
##
  262
        -0.198353597
                      3.469573e-01
## 263
        -0.185314253
                      3.480786e-01
       -0.172274909 3.492690e-01
## 264
## 265
       -0.159235565 3.505642e-01
## 266
       -0.146196221 3.519895e-01
## 267
       -0.133156877 3.535731e-01
##
  268
        -0.120117533 3.553024e-01
##
  269
        -0.107078189
                      3.571554e-01
## 270
        -0.094038845
                      3.591018e-01
## 271
        -0.080999501 3.611000e-01
## 272
        -0.067960157
                      3.630978e-01
## 273
        -0.054920813 3.650350e-01
## 274
        -0.041881469
                      3.668448e-01
##
  275
        -0.028842125
                      3.684569e-01
## 276
        -0.015802781
                      3.697987e-01
## 277
        -0.002763437
                      3.707506e-01
## 278
         0.010275907 3.712871e-01
## 279
         0.023315251 3.713592e-01
## 280
         0.036354595 3.709300e-01
##
         0.049393939 3.699759e-01
  281
##
  282
         0.062433283
                      3.684874e-01
## 283
         0.075472627
                      3.664687e-01
## 284
         0.088511971 3.639372e-01
## 285
         0.101551315 3.609224e-01
## 286
         0.114590659
                     3.574485e-01
## 287
         0.127630003 3.535538e-01
##
  288
         0.140669347
                      3.493321e-01
## 289
         0.153708691
                      3.448390e-01
## 290
         0.166748035 3.401299e-01
## 291
         0.179787379 3.352583e-01
## 292
         0.192826723 3.302754e-01
## 293
         0.205866067 3.252290e-01
## 294
         0.218905411 3.201636e-01
##
  295
         0.231944755
                      3.151207e-01
##
  296
         0.244984099
                      3.101463e-01
## 297
         0.258023443
                      3.052823e-01
## 298
         0.271062787
                     3.005580e-01
## 299
         0.284102131 2.960046e-01
## 300
         0.297141475 2.916507e-01
##
  301
         0.310180819
                      2.875224e-01
##
  302
         0.323220163
                      2.836423e-01
## 303
         0.336259507
                      2.800287e-01
## 304
         0.349298851 2.766949e-01
## 305
         0.362338195 2.736484e-01
## 306
         0.375377539 2.709212e-01
## 307
         0.388416883 2.684796e-01
##
  308
         0.401456227
                      2.663048e-01
##
  309
         0.414495571
                      2.643796e-01
## 310
         0.427534915
                      2.626848e-01
## 311
         0.440574259
                      2.612011e-01
## 312
         0.453613603
                     2.599105e-01
## 313
         0.466652947
                     2.587989e-01
## 314
         0.479692291 2.578572e-01
##
  315
         0.492731635
                      2.570855e-01
## 316
         0.505770979
                      2.565083e-01
## 317
         0.518810323 2.561219e-01
## 318
         0.531849667 2.559475e-01
## 319
         0.544889011 2.560108e-01
         0.557928355 2.563394e-01
## 320
##
  321
         0.570967699
                      2.569617e-01
##
  322
         0.584007043
                      2.579043e-01
## 323
         0.597046387
                      2.591907e-01
## 324
         0.610085731 2.608398e-01
## 325
         0.623125075
                     2.628839e-01
## 326
         0.636164419 2.653422e-01
## 327
         0.649203763
                      2.681844e-01
##
  328
         0.662243107
                      2.714066e-01
## 329
         0.675282451
                      2.750028e-01
## 330
         0.688321795 2.789662e-01
```

```
## 331
         0.701361139 2.832913e-01
## 332
         0.714400483 2.879746e-01
##
  333
         0.727439827 2.930161e-01
##
  334
         0.740479171 2.984200e-01
##
   335
         0.753518515
                       3.042274e-01
##
  336
         0.766557859
                       3.104399e-01
## 337
         0.779597203
                      3.170579e-01
##
  338
         0.792636547
                      3.240999e-01
## 339
         0.805675891
                      3.315846e-01
##
  340
         0.818715235
                      3.395293e-01
##
  341
         0.831754579
                       3.479487e-01
## 342
         0.844793923
                      3.568534e-01
## 343
         0.857833267 3.662492e-01
## 344
         0.870872611 3.761359e-01
## 345
         0.883911955 3.865617e-01
## 346
         0.896951299 3.974554e-01
##
  347
         0.909990643
                     4.087877e-01
##
  348
         0.923029987
                       4.205273e-01
## 349
         0.936069331
                      4.326374e-01
## 350
         0.949108675
                      4.450758e-01
##
  351
         0.962148019
                      4.577962e-01
## 352
         0.975187363
                      4.707486e-01
##
  353
         0.988226707
                      4.838807e-01
##
  354
         1.001266051
                      4.971394e-01
## 355
         1.014305395
                      5.104629e-01
## 356
         1.027344739
                      5.237794e-01
## 357
         1.040384083
                      5.370277e-01
## 358
         1.053423427
                     5.501434e-01
## 359
         1.066462771 5.630580e-01
##
  360
         1.079502115
                      5.756983e-01
##
  361
         1.092541459
                       5.879861e-01
##
  362
         1.105580803
                      5.998381e-01
## 363
         1.118620147
                       6.111665e-01
## 364
         1.131659491
                      6.218368e-01
## 365
         1.144698835
                       6.317285e-01
## 366
         1.157738179
                       6.407959e-01
##
  367
         1.170777523
                       6.489557e-01
##
  368
         1.183816867
                       6.561343e-01
## 369
         1.196856211
                      6.622702e-01
## 370
         1.209895555
                      6.673164e-01
## 371
         1.222934899
                      6.712423e-01
## 372
         1.235974243 6.740337e-01
## 373
         1.249013587
                      6.756930e-01
##
  374
         1.262052931
                      6.761376e-01
##
  375
         1.275092275
                       6.754722e-01
## 376
         1.288131619
                       6.737798e-01
## 377
         1.301170963
                      6.711077e-01
## 378
         1.314210307
                       6.675035e-01
## 379
         1.327249651
                      6.630123e-01
##
  380
         1.340288995
                       6.576741e-01
##
  381
         1.353328339
                       6.515225e-01
## 382
         1.366367683
                      6.445835e-01
## 383
         1.379407027
                      6.368760e-01
## 384
         1.392446371 6.283166e-01
## 385
         1.405485715 6.190106e-01
## 386
         1.418525059
                      6.089650e-01
##
  387
         1.431564403
                      5.981877e-01
##
  388
         1.444603747
                       5.866902e-01
##
  389
         1.457643091
                      5.744895e-01
##
  390
         1.470682435
                      5.616085e-01
## 391
         1.483721779
                       5.480772e-01
## 392
         1.496761123
                      5.339324e-01
##
  393
         1.509800467
                       5.192000e-01
##
  394
         1.522839811
                       5.039088e-01
## 395
         1.535879155
                      4.881691e-01
## 396
                      4.720405e-01
         1.548918499
## 397
         1.561957843 4.555849e-01
## 398
         1.574997187 4.388661e-01
## 399
         1.588036531 4.219488e-01
##
  400
         1.601075875
                      4.048984e-01
## 401
         1.614115219
                      3.877804e-01
## 402
         1.627154563
                      3.706601e-01
##
  403
         1.640193907
                      3.536115e-01
## 404
         1.653233251
                     3.367104e-01
## 405
         1.666272595
                      3.200129e-01
## 406
                      3.035759e-01
         1.679311939
##
  407
         1.692351283
                      2.874526e-01
## 408
         1.705390627
                      2.716921e-01
## 409
         1.718429971 2.563382e-01
```

```
## 410
         1.731469315 2.414293e-01
## 411
         1.744508659 2.269984e-01
## 412
         1.757548003 2.130721e-01
## 413
         1.770587347
                      1.997283e-01
## 414
         1.783626691
                      1.869425e-01
## 415
         1.796666035
                      1.747101e-01
## 416
         1.809705379
                     1.630360e-01
## 417
         1.822744723
                     1.519225e-01
## 418
         1.835784067
                     1.413689e-01
## 419
         1.848823411 1.313728e-01
## 420
         1.861862755
                      1.219297e-01
## 421
         1.874902099
                      1.130335e-01
## 422
         1.887941443 1.046811e-01
## 423
         1.900980787 9.691887e-02
## 424
         1.914020131 8.966836e-02
## 425
         1.927059475 8.291293e-02
## 426
         1.940098819
                      7.663306e-02
## 427
         1.953138163
                      7.080642e-02
## 428
         1.966177507
                      6.540814e-02
## 429
         1.979216851
                      6.041137e-02
## 430
         1.992256195
                      5.578788e-02
## 431
         2.005295539
                      5.150888e-02
## 432
         2.018334883
                     4.755835e-02
## 433
         2.031374227
                      4.391587e-02
## 434
                      4.053049e-02
         2.044413571
## 435
         2.057452915
                      3.738095e-02
## 436
         2.070492259 3.444894e-02
## 437
         2.083531603 3.171906e-02
## 438
         2.096570947 2.917866e-02
## 439
         2.109610291 2.681742e-02
##
  440
         2.122649635
                      2.462696e-02
## 441
         2.135688979
                      2.260027e-02
## 442
         2.148728323
                      2.074308e-02
## 443
         2.161767667
                      1.904404e-02
## 444
         2.174807011
                     1.748928e-02
## 445
         2.187846355
                      1.607254e-02
##
  446
         2.200885699
                      1.478710e-02
## 447
         2.213925043
                      1.362586e-02
## 448
         2.226964387
                      1.258134e-02
## 449
         2.240003731 1.164585e-02
## 450
         2.253043075 1.081157e-02
## 451
         2.266082419 1.007069e-02
## 452
         2.279121763 9.424141e-03
##
  453
         2.292161107
                      8.854886e-03
## 454
         2.305200451
                      8.354247e-03
## 455
         2.318239795
                      7.915003e-03
## 456
         2.331279139
                      7.530003e-03
## 457
         2.344318483
                     7.192134e-03
## 458
         2.357357827
                      6.894313e-03
## 459
         2.370397171
                      6.629500e-03
## 460
         2.383436515
                      6.390751e-03
## 461
         2.396475859
                      6.171525e-03
## 462
         2.409515203 5.965464e-03
## 463
         2.422554547
                      5.764778e-03
## 464
         2.435593891 5.564527e-03
## 465
         2.448633235 5.360644e-03
##
  466
         2.461672579
                      5.150038e-03
## 467
         2.474711923
                      4.930656e-03
## 468
         2.487751267
                      4.701499e-03
## 469
         2.500790611 4.462589e-03
## 470
                     4.214883e-03
         2.513829955
## 471
         2.526869299
                      3.959926e-03
## 472
         2.539908643
                      3.700634e-03
## 473
         2.552947987
                      3.440434e-03
## 474
         2.565987331
                      3.182305e-03
## 475
         2.579026675
                      2.929114e-03
## 476
         2.592066019 2.683473e-03
## 477
         2.605105363 2.447626e-03
## 478
         2.618144707 2.223381e-03
##
  479
         2.631184051
                      2.012070e-03
## 480
         2.644223395
                      1.814554e-03
## 481
         2.657262739
                      1.632497e-03
## 482
         2.670302083
                     1.465156e-03
## 483
         2.683341427
                     1.311539e-03
## 484
         2.696380771
                     1.171040e-03
## 485
         2.709420115
                      1.042914e-03
##
  486
         2.722459459
                      9.263361e-04
## 487
         2.735498803
                      8.204385e-04
## 488
         2.748538147 7.243530e-04
```

```
## 489
         2.761577491 6.372378e-04
## 490
         2.774616835 5.583004e-04
## 491
         2.787656179 4.876338e-04
  492
##
         2.800695523 4.236583e-04
##
   493
         2.813734867
                      3.658047e-04
## 494
         2.826774211
                      3.136267e-04
         2.839813555
## 495
                      2.667459e-04
## 496
         2.852852899 2.248394e-04
## 497
         2.865892243 1.876246e-04
## 498
         2.878931587 1.548435e-04
## 499
         2.891970931
                      1.262473e-04
## 500
         2.905010275
                      1.016864e-04
## 501
         2.918049619 8.109971e-05
## 502
         2.931088963 6.379438e-05
## 503
         2.944128307 4.945468e-05
## 504
         2.957167651 3.775319e-05
##
  505
         2.970206995
                      2.835873e-05
##
   506
         2.983246339
                      2.094505e-05
##
  507
         2.996285683
                      1.519932e-05
## 508
         3.009325027
                      1.082972e-05
##
  509
         3.022364371
                      7.571476e-06
## 510
         3.035403715
                      5.231855e-06
## 511
         3.048443059
                      3.578129e-06
## 512
         3.061482403
                      2.402234e-06
```

 $ggplot(data=df,mapping=aes(x)) + geom_histogram(aes(y=..density..),color="grey",fill="lightblue",bins=60) + geom_line(data=df.estimate1,aes(x = d.estimate1.x, y = d.estimate1.y))$



Question 3

Using the formula for the Silverman rule that is given in the notes, compute the default bandwidth by hand. Do you get the same value as returned by density()? (If you don't...you coded the formula incorrectly.)

```
#s is the sample standard deviation and is the inter-quartile range  \#s = sd(x) = 0.7736771   \#d.estimate n = 20722   \#IQR/1.34 = 0.9526457   h = 0.9*(0.7736771)*(20722^{-1/5}))  h
```

```
## [1] 0.09539287
```

Question 4

Repeat Q2, but use the unbiased cross-validation estimator, whose use is specified in the notes. Again, print the bandwidth and make the same density estimate overlaid onto histogram plot as in Q2. Stare hard at the two plots, the one here and the one in Q2: can you see any differences in the density estimates?

```
d.estimate=density(x, bw="ucv", adjust =1)
d.estimate
```

```
##
## Call:
   density.default(x = x, bw = "ucv", adjust = 1)
##
                       Bandwidth 'bw' = 0.05836
## Data: x (20722 obs.):
##
##
                       У
   Min. :-3.4905 Min. :0.000004
##
   ##
   Median :-0.2701
                 Median :0.027016
##
   Mean :-0.2701
                 Mean :0.155106
##
   3rd Qu.: 1.3402
                  3rd Qu.:0.299222
   Max. : 2.9504 Max. :0.697426
```

```
#Default Bandwidth 'bw' = 0.05836

df.estimate = data.frame(d.estimate$x,d.estimate$y)
df.estimate
```

```
##
       d.estimate.x d.estimate.y
## 1
       -3.4905317238 4.612954e-06
## 2
      -3.4779271769 8.592507e-06
## 3
       -3.4653226300 1.532062e-05
## 4
       -3.4527180831 2.617532e-05
## 5
      -3.4401135362 4.289964e-05
## 6
      -3.4275089893 6.824707e-05
## 7
      -3.4149044424 1.036580e-04
## 8
      -3.4022998955 1.504487e-04
## 9
       -3.3896953485 2.088459e-04
## 10
       -3.3770908016 2.775239e-04
## 11
      -3.3644862547 3.533404e-04
## 12 -3.3518817078 4.314065e-04
## 13 -3.3392771609 5.055637e-04
## 14 -3.3266726140 5.692485e-04
## 15 -3.3140680671 6.166125e-04
## 16
      -3.3014635202 6.436739e-04
## 17
       -3.2888589733 6.492317e-04
## 18
      -3.2762544264 6.352924e-04
      -3.2636498795 6.068563e-04
## 19
## 20 -3.2510453326 5.710763e-04
## 21
      -3.2384407856 5.366062e-04
      -3.2258362387 5.104734e-04
## 22
## 23
       -3.2132316918 4.968286e-04
## 24
      -3.2006271449 4.968363e-04
## 25 -3.1880225980 5.087505e-04
## 26 -3.1754180511 5.285152e-04
## 27 -3.1628135042 5.506921e-04
## 28 -3.1502089573 5.694946e-04
## 29
       -3.1376044104 5.797326e-04
## 30
       -3.1249998635 5.775384e-04
## 31
      -3.1123953166 5.608152e-04
## 32
      -3.0997907697 5.294087e-04
## 33
      -3.0871862228 4.850355e-04
## 34
      -3.0745816758 4.310152e-04
## 35
      -3.0619771289 3.718689e-04
## 36
       -3.0493725820 3.133705e-04
## 37
       -3.0367680351 2.608108e-04
## 38
      -3.0241634882 2.185571e-04
## 39
      -3.0115589413 1.898244e-04
## 40
      -2.9989543944 1.763438e-04
## 41
      -2.9863498475 1.781622e-04
## 42
       -2.9737453006 1.935958e-04
## 43
       -2.9611407537 2.193591e-04
## 44
      -2.9485362068 2.509015e-04
      -2.9359316599 2.829609e-04
## 45
      -2.9233271129 3.103093e-04
```

```
## 47
      -2.9107225660 3.286072e-04
      -2.8981180191 3.352314e-04
## 49
       -2.8855134722 3.299137e-04
## 50
       -2.8729089253 3.148619e-04
##
  51
       -2.8603043784 2.960550e-04
## 52
       -2.8476998315 2.809269e-04
## 53
      -2.8350952846 2.773704e-04
## 54
       -2.8224907377 2.929729e-04
## 55
       -2.8098861908 3.340056e-04
## 56
      -2.7972816439 4.045236e-04
## 57
       -2.7846770970 5.056601e-04
## 58
       -2.7720725500 6.352034e-04
## 59
       -2.7594680031 7.875498e-04
## 60
      -2.7468634562 9.541009e-04
## 61
      -2.7342589093 1.124117e-03
## 62
      -2.7216543624 1.285953e-03
## 63
       -2.7090498155 1.428505e-03
## 64
       -2.6964452686 1.542631e-03
## 65
       -2.6838407217 1.619904e-03
## 66
       -2.6712361748 1.660267e-03
## 67
       -2.6586316279 1.666417e-03
## 68
       -2.6460270810 1.643390e-03
## 69
       -2.6334225341 1.597919e-03
## 70
       -2.6208179872 1.537238e-03
## 71
       -2.6082134402 1.468019e-03
## 72
       -2.5956088933 1.395647e-03
## 73
       -2.5830043464 1.323983e-03
## 74
      -2.5703997995 1.255639e-03
## 75
      -2.5577952526 1.192678e-03
## 76
       -2.5451907057 1.137478e-03
## 77
       -2.5325861588 1.093425e-03
## 78
       -2.5199816119 1.065101e-03
## 79
      -2.5073770650 1.057840e-03
## 80
      -2.4947725181 1.078332e-03
## 81
      -2.4821679712 1.126490e-03
## 82
      -2.4695634243 1.199706e-03
## 83
       -2.4569588773 1.290885e-03
## 84
       -2.4443543304 1.389112e-03
## 85
       -2.4317497835 1.481303e-03
## 86
       -2.4191452366 1.554465e-03
## 87
       -2.4065406897 1.598098e-03
## 88
      -2.3939361428 1.606204e-03
## 89
       -2.3813315959 1.578492e-03
## 90
       -2.3687270490 1.520534e-03
## 91
       -2.3561225021 1.442852e-03
## 92
       -2.3435179552 1.359127e-03
## 93
       -2.3309134083 1.283880e-03
## 94
      -2.3183088614 1.230653e-03
## 95
       -2.3057043145 1.210270e-03
## 96
       -2.2930997675 1.224062e-03
## 97
       -2.2804952206 1.269051e-03
## 98
       -2.2678906737 1.337397e-03
## 99
      -2.2552861268 1.417914e-03
## 100 -2.2426815799 1.498109e-03
## 101 -2.2300770330 1.566447e-03
## 102 -2.2174724861 1.614369e-03
## 103 -2.2048679392 1.637671e-03
## 104 -2.1922633923 1.636933e-03
## 105 -2.1796588454 1.616907e-03
## 106 -2.1670542985 1.585048e-03
## 107 -2.1544497516 1.549616e-03
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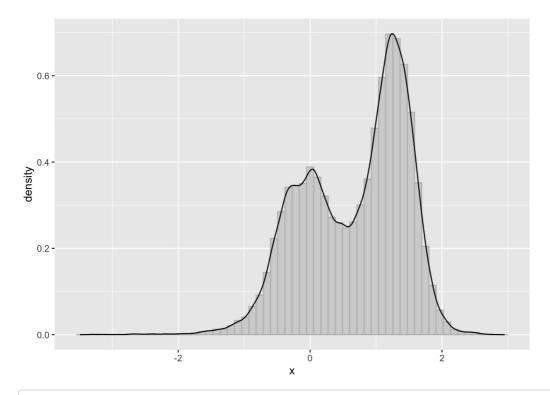
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```



I do see slight differences in the density estimates. This one (compared to the previous one in question 2) is less smoothed and closer to the shape of the histogram.

Question 5

Density estimates tend to work fine with unbounded data, but can exhibit boundary bias if the data values are bounded on either or both sides. Repeat Q4, except run the code for only *x* values between 0 and 1, and set the bandwidth manually to 0.1. What do you observe?

```
df.new = filter(df, x<=1 & x>=0)

d.estimate2=density(df.new$x, bw=0.1, adjust =1)
d.estimate2
```

```
##
##
   Call:
##
    density.default(x = df.new$x, bw = 0.1, adjust = 1)
##
##
  Data: df.new$x (6524 obs.); Bandwidth 'bw' = 0.1
##
##
##
    Min.
          :-0.30000
                             :0.001655
##
    1st Qu.: 0.09997
                       1st Qu.:0.203632
##
    Median : 0.49994
                       Median :0.821325
    Mean : 0.49994
                       Mean :0.624380
    3rd Qu.: 0.89991
                       3rd Qu.:0.931110
                             :1.031557
##
          : 1.29987
                       Max.
```

```
df.estimate2 = data.frame(d.estimate2$x,d.estimate2$y)
df.estimate2
```

```
d.estimate2.x d.estimate2.y
##
## 1
       -0.2999950817
                       0.001654691
## 2
       -0.2968642260
                       0.001832877
## 3
                       0.002031211
       -0.2937333704
## 4
       -0.2906025147
                       0.002243739
## 5
       -0.2874716590
                       0.002478610
## 6
       -0.2843408033
                       0.002739134
##
  7
       -0.2812099477
                       0.003017683
## 8
       -0.2780790920
                       0.003324545
## 9
       -0.2749482363
                       0.003663745
## 10
       -0.2718173806
                       0.004025609
## 11
       -0.2686865250
                       0.004422995
## 12
       -0.2655556693
                       0.004860741
## 13
       -0.2624248136
                       0.005326697
##
       -0.2592939580
  14
                       0.005836773
## 15
       -0.2561631023
                        0.006396714
## 16
       -0.2530322466
                       0.006991417
```

```
## 17
       -0.2499013909
                         0.007640364
## 18
       -0.2467705353
                         0.008350292
##
   19
       -0.2436396796
                         0.009102618
##
   20
       -0.2405088239
                         0.009920954
##
   21
       -0.2373779683
                         0.010813100
##
   22
       -0.2342471126
                         0.011756424
## 23
       -0.2311162569
                         0.012779245
##
  24
       -0.2279854012
                         0.013890474
## 25
       -0.2248545456
                         0.015062834
##
  26
       -0.2217236899
                         0.016329933
##
   27
       -0.2185928342
                         0.017701805
##
  28
       -0.2154619786
                         0.019145926
## 29
       -0.2123311229
                         0.020701748
## 30
       -0.2092002672
                         0.022380408
## 31
       -0.2060694115
                         0.024143537
## 32
       -0.2029385559
                         0.026036943
##
                         0.028072798
   33
       -0.1998077002
##
   34
       -0.1966768445
                         0.030206327
##
  35
       -0.1935459889
                         0.032490126
## 36
       -0.1904151332
                         0.034937278
##
  37
       -0.1872842775
                         0.037496113
## 38
       -0.1841534218
                         0.040226330
##
  39
       -0.1810225662
                         0.043141742
##
   40
       -0.1778917105
                         0.046183394
## 41
       -0.1747608548
                         0.049418260
## 42
       -0.1716299991
                         0.052860635
## 43
       -0.1684991435
                         0.056444032
## 44
       -0.1653682878
                         0.060242677
## 45
       -0.1622374321
                         0.064271045
##
   46
       -0.1591065765
                         0.068455042
##
   47
       -0.1559757208
                         0.072875912
## 48
       -0.1528448651
                         0.077547943
## 49
       -0.1497140094
                         0.082389572
## 50
       -0.1465831538
                         0.087488579
## 51
       -0.1434522981
                         0.092858644
## 52
       -0.1403214424
                         0.098411125
##
   53
       -0.1371905868
                         0.104239579
##
   54
       -0.1340597311
                         0.110356624
##
  55
       -0.1309288754
                         0.116667193
## 56
       -0.1277980197
                         0.123269585
##
  57
       -0.1246671641
                         0.130174868
## 58
       -0.1215363084
                         0.137282503
##
   59
       -0.1184054527
                         0.144694207
##
   60
       -0.1152745971
                         0.152419010
##
  61
       -0.1121437414
                         0.160352144
## 62
       -0.1090128857
                         0.168597150
## 63
       -0.1058820300
                         0.177160544
## 64
       -0.1027511744
                         0.185934868
## 65
       -0.0996203187
                         0.195023644
##
   66
       -0.0964894630
                         0.204430443
##
  67
       -0.0933586074
                         0.214046916
## 68
       -0.0902277517
                         0.223974527
## 69
       -0.0870968960
                         0.234213528
##
  70
       -0.0839660403
                         0.244656718
## 71
       -0.0808351847
                         0.255401308
## 72
       -0.0777043290
                         0.266443950
##
   73
       -0.0745734733
                         0.277680804
##
   74
       -0.0714426176
                         0.289202558
##
  75
       -0.0683117620
                         0.301002087
##
   76
       -0.0651809063
                         0.312981246
## 77
       -0.0620500506
                         0.325221922
## 78
       -0.0589191950
                         0.337713161
##
   79
       -0.0557883393
                         0.350364887
##
   80
       -0.0526574836
                         0.363247988
## 81
       -0.0495266279
                         0.376347765
## 82
       -0.0463957723
                         0.389584538
## 83
       -0.0432649166
                         0.403016167
## 84
       -0.0401340609
                         0.416624431
## 85
       -0.0370032053
                         0.430342240
##
   86
       -0.0338723496
                         0.444212644
##
  87
       -0.0307414939
                         0.458214262
## 88
       -0.0276106382
                         0.472294564
## 89
       -0.0244797826
                         0.486480336
##
  90
       -0.0213489269
                         0.500747526
## 91
       -0.0182180712
                         0.515059828
##
   92
       -0.0150872156
                         0.529426705
##
   93
       -0.0119563599
                         0.543822026
## 94
       -0.0088255042
                         0.558227000
## 95
       -0.0056946485
                        0.572633143
```

```
## 96
       -0.0025637929
                         0.587012930
## 97
        0.0005670628
                         0.601365930
##
   98
        0.0036979185
                         0.615665569
   99
##
        0.0068287741
                         0.629883664
##
   100
        0.0099596298
                         0.644038517
##
   101
        0.0130904855
                         0.658085796
##
   102
        0.0162213412
                         0.671997425
##
   103
        0.0193521968
                         0.685810309
##
  104
        0.0224830525
                         0.699463170
##
   105
        0.0256139082
                         0.712928791
##
   106
        0.0287447639
                         0.726262054
##
  107
        0.0318756195
                         0.739385987
## 108
        0.0350064752
                         0.752274940
## 109
        0.0381373309
                         0.765000675
## 110
        0.0412681865
                         0.777472167
## 111
        0.0443990422
                         0.789665971
##
        0.0475298979
   112
                         0.801669202
##
   113
        0.0506607536
                         0.813378747
##
   114
        0.0537916092
                         0.824773909
##
  115
        0.0569224649
                         0.835955256
##
  116
        0.0600533206
                         0.846809803
## 117
        0.0631841762
                         0.857320022
##
  118
        0.0663150319
                         0.867597748
##
   119
        0.0694458876
                         0.877522495
##
   120
        0.0725767433
                         0.887080198
##
  121
        0.0757075989
                         0.896391565
## 122
        0.0788384546
                         0.905331058
## 123
        0.0819693103
                         0.913888226
## 124
        0.0851001659
                         0.922190147
##
   125
        0.0882310216
                         0.930108659
##
   126
        0.0913618773
                         0.937636950
##
   127
        0.0944927330
                         0.944905937
## 128
        0.0976235886
                         0.951787168
## 129
        0.1007544443
                         0.958277367
## 130
        0.1038853000
                         0.964508827
##
  131
        0.1070161556
                         0.970354973
##
   132
        0.1101470113
                         0.975815851
##
   133
        0.1132778670
                         0.981022808
##
  134
        0.1164087227
                         0.985853089
## 135
        0.1195395783
                         0.990309756
##
  136
        0.1226704340
                         0.994521098
##
  137
        0.1258012897
                         0.998369854
##
   138
        0.1289321454
                        1.001861728
##
   139
        0.1320630010
                         1.005120094
##
   140
        0.1351938567
                         1.008034569
##
   141
        0.1383247124
                         1.010613075
##
        0.1414555680
  142
                         1.012972515
## 143
        0.1445864237
                         1.015010453
##
  144
        0.1477172794
                         1.016736586
##
   145
        0.1508481351
                         1.018260100
##
   146
        0.1539789907
                         1.019487275
## 147
        0.1571098464
                         1.020429139
## 148
        0.1602407021
                        1.021186229
## 149
        0.1633715577
                         1.021674021
## 150
        0.1665024134
                         1.021904448
##
   151
        0.1696332691
                        1.021968774
##
   152
        0.1727641248
                        1.021791886
##
   153
        0.1758949804
                         1.021386226
##
   154
        0.1790258361
                         1.020833451
##
  155
        0.1821566918
                         1.020067840
##
  156
        0.1852875474
                         1.019101994
## 157
        0.1884184031
                         1.018007883
##
                         1.016728960
   158
        0.1915492588
##
   159
        0.1946801145
                         1.015277693
##
   160
        0.1978109701
                         1.013716500
##
  161
        0.2009418258
                         1.011997640
##
   162
        0.2040726815
                         1.010133202
## 163
        0.2072035371
                         1.008176371
##
  164
        0.2103343928
                        1.006087724
##
   165
        0.2134652485
                         1.003878788
##
        0.2165961042
                         1.001593964
   166
##
   167
        0.2197269598
                         0.999201582
##
  168
        0.2228578155
                         0.996712471
##
   169
        0.2259886712
                         0.994162807
## 170
        0.2291195269
                         0.991528050
##
   171
        0.2322503825
                         0.988818245
##
   172
        0.2353812382
                         0.986061964
## 173
        0.2385120939
                         0.983241154
## 174
        0.2416429495
                        0.980365034
```

```
## 175
        0.2447738052
                        0.977455219
## 176
        0.2479046609
                         0.974499507
##
  177
        0.2510355166
                        0.971506274
##
   178
                         0.968490843
        0.2541663722
##
   179
        0.2572972279
                         0.965446237
##
   180
        0.2604280836
                         0.962379997
##
  181
        0.2635589392
                         0.959301806
##
   182
        0.2666897949
                         0.956209315
##
  183
        0.2698206506
                         0.953109255
##
   184
        0.2729515063
                         0.950006300
##
   185
        0.2760823619
                         0.946902162
##
  186
        0.2792132176
                         0.943802796
## 187
        0.2823440733
                         0.940708465
## 188
        0.2854749289
                         0.937624411
## 189
                         0.934555855
        0.2886057846
## 190
        0.2917366403
                        0.931499209
##
   191
        0.2948674960
                         0.928462750
##
   192
        0.2979983516
                         0.925451002
##
   193
        0.3011292073
                         0.922457052
##
  194
        0.3042600630
                         0.919491750
##
  195
        0.3073909186
                         0.916558961
## 196
        0.3105217743
                         0.913648935
##
  197
        0.3136526300
                         0.910774662
##
   198
        0.3167834857
                         0.907939382
##
   199
        0.3199143413
                         0.905130967
##
   200
        0.3230451970
                         0.902364139
##
   201
        0.3261760527
                         0.899641545
##
   202
        0.3293069084
                         0.896949108
##
  203
        0.3324377640
                         0.894302898
##
   204
        0.3355686197
                         0.891705000
##
   205
        0.3386994754
                         0.889139789
##
   206
        0.3418303310
                         0.886624325
##
   207
        0.3449611867
                         0.884160161
##
   208
        0.3480920424
                         0.881730502
##
  209
        0.3512228981
                         0.879353062
##
  210
        0.3543537537
                         0.877028894
##
                         0.874740388
   211
        0.3574846094
##
   212
        0.3606154651
                         0.872505602
## 213
        0.3637463207
                         0.870325125
## 214
        0.3668771764
                         0.868180860
##
  215
        0.3700080321
                         0.866090929
## 216
        0.3731388878
                         0.864055504
##
   217
        0.3762697434
                         0.862056296
##
   218
        0.3794005991
                         0.860111247
##
   219
        0.3825314548
                         0.858220161
##
   220
        0.3856623104
                         0.856364825
##
        0.3887931661
   221
                         0.854562795
##
        0.3919240218
                         0.852813565
  222
##
  223
        0.3950548775
                         0.851099229
##
   224
        0.3981857331
                         0.849436791
##
  225
        0.4013165888
                         0.847825505
## 226
        0.4044474445
                         0.846247957
## 227
        0.4075783001
                         0.844720486
## 228
        0.4107091558
                         0.843242178
## 229
        0.4138400115
                         0.841796251
##
  230
        0.4169708672
                         0.840398315
##
   231
        0.4201017228
                         0.839047371
##
   232
        0.4232325785
                         0.837727352
##
   233
        0.4263634342
                         0.836453135
##
   234
        0.4294942899
                         0.835223709
##
  235
        0.4326251455
                         0.834023762
##
  236
        0.4357560012
                         0.832867477
##
        0.4388868569
                         0.831753915
   237
##
   238
        0.4420177125
                         0.830668499
##
   239
        0.4451485682
                         0.829624808
## 240
        0.4482794239
                         0.828622050
##
  241
        0.4514102796
                         0.827646311
## 242
        0.4545411352
                         0.826710708
## 243
        0.4576719909
                         0.825814657
##
   244
        0.4608028466
                         0.824944791
##
   245
        0.4639337022
                         0.824113938
##
   246
        0.4670645579
                         0.823321777
##
   247
        0.4701954136
                         0.822555328
##
   248
        0.4733262693
                         0.821827334
## 249
        0.4764571249
                         0.821137772
##
   250
        0.4795879806
                         0.820473861
##
   251
        0.4827188363
                         0.819848476
## 252
        0.4858496919
                         0.819261919
## 253
        0.4889805476
                        0.818701389
```

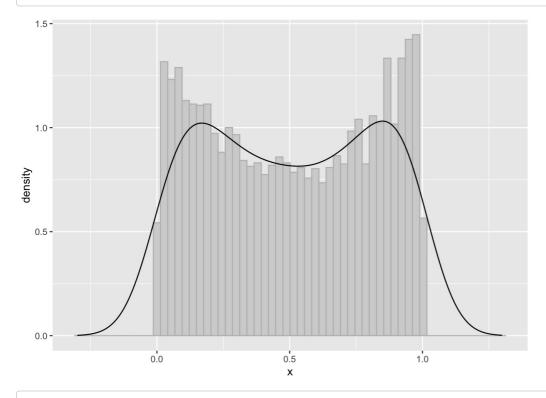
```
## 254
        0.4921114033
                         0.818180123
## 255
        0.4952422590
                         0.817698746
   256
##
        0.4983731146
                         0.817244218
##
   257
        0.5015039703
                         0.816830353
##
   258
        0.5046348260
                         0.816458089
##
   259
        0.5077656816
                         0.816113920
##
   260
        0.5108965373
                         0.815812442
##
   261
        0.5140273930
                         0.815554871
##
   262
        0.5171582487
                         0.815327026
##
   263
        0.5202891043
                         0.815144457
##
   264
        0.5234199600
                         0.815008616
##
   265
        0.5265508157
                         0.814904453
## 266
        0.5296816714
                         0.814848615
## 267
        0.5328125270
                         0.814842723
##
  268
                         0.814870713
        0.5359433827
## 269
        0.5390742384
                         0.814950414
##
   270
        0.5422050940
                         0.815083555
##
   271
        0.5453359497
                         0.815252937
##
   272
        0.5484668054
                         0.815477621
##
  273
        0.5515976611
                         0.815759367
##
   274
        0.5547285167
                         0.816079777
##
  275
        0.5578593724
                         0.816459138
##
   276
        0.5609902281
                         0.816899164
##
   277
        0.5641210837
                         0.817380243
##
   278
        0.5672519394
                         0.817923831
##
  279
        0.5703827951
                         0.818531527
##
   280
        0.5735136508
                         0.819182532
##
   281
        0.5766445064
                         0.819899374
##
  282
        0.5797753621
                         0.820683469
##
   283
        0.5829062178
                         0.821512910
##
   284
        0.5860370734
                         0.822411158
##
   285
        0.5891679291
                         0.823379385
##
   286
                         0.824394701
        0.5922987848
##
   287
        0.5954296405
                         0.825481323
##
   288
        0.5985604961
                         0.826640132
##
   289
        0.6016913518
                         0.827847412
##
   290
                         0.829127939
        0.6048222075
##
   291
        0.6079530631
                         0.830482265
##
   292
        0.6110839188
                         0.831886038
##
   293
        0.6142147745
                         0.833364369
##
   294
        0.6173456302
                         0.834917461
##
   295
        0.6204764858
                         0.836520533
##
   296
        0.6236073415
                         0.838198803
##
   297
        0.6267381972
                         0.839952110
##
   298
        0.6298690529
                         0.841755469
##
   299
        0.6329999085
                         0.843633968
##
   300
        0.6361307642
                         0.845587074
##
        0.6392616199
                         0.847589835
   301
##
   302
        0.6423924755
                         0.849666966
##
   303
        0.6455233312
                         0.851817567
##
   304
        0.6486541869
                         0.854016951
##
   305
        0.6517850426
                         0.856289224
## 306
        0.6549158982
                         0.858633118
##
  307
        0.6580467539
                         0.861024448
## 308
        0.6611776096
                         0.863486468
##
   309
                         0.866017547
        0.6643084652
##
   310
        0.6674393209
                         0.868594233
##
   311
        0.6705701766
                         0.871238688
##
   312
        0.6737010323
                         0.873948904
##
  313
        0.6768318879
                         0.876702405
##
  314
        0.6799627436
                         0.879520000
## 315
        0.6830935993
                         0.882399291
                         0.885319021
##
        0.6862244549
   316
##
   317
        0.6893553106
                         0.888298368
##
   318
        0.6924861663
                         0.891334514
##
  319
        0.6956170220
                         0.894407682
##
  320
        0.6987478776
                         0.897535107
## 321
        0.7018787333
                         0.900713505
  322
##
        0.7050095890
                         0.903924867
##
   323
        0.7081404446
                         0.907184130
##
        0.7112713003
   324
                         0.910487480
##
   325
        0.7144021560
                         0.913819002
##
  326
        0.7175330117
                         0.917190923
##
   327
        0.7206638673
                         0.920598820
## 328
        0.7237947230
                         0.924029246
##
   329
        0.7269255787
                         0.927491239
##
   330
        0.7300564344
                         0.930979677
## 331
        0.7331872900
                         0.934484013
## 332
        0.7363181457
                        0.938009544
```

```
## 333
        0.7394490014
                         0.941550348
## 334
        0.7425798570
                         0.945099284
##
   335
        0.7457107127
                        0.948657271
##
   336
        0.7488415684
                         0.952217489
##
   337
        0.7519724241
                         0.955776781
##
   338
        0.7551032797
                         0.959330978
##
   339
        0.7582341354
                         0.962872264
##
   340
        0.7613649911
                         0.966402127
##
   341
        0.7644958467
                         0.969910524
##
   342
        0.7676267024
                         0.973388565
##
   343
        0.7707575581
                         0.976843114
##
   344
        0.7738884138
                         0.980257414
## 345
        0.7770192694
                         0.983621451
## 346
        0.7801501251
                         0.986948259
## 347
                         0.990213490
        0.7832809808
## 348
        0.7864118364
                         0.993405992
##
                         0.996545811
   349
        0.7895426921
##
   350
        0.7926735478
                         0.999600125
##
   351
        0.7958044035
                         1.002556685
##
   352
        0.7989352591
                         1.005443403
##
   353
        0.8020661148
                         1.008218138
##
   354
        0.8051969705
                         1.010867639
##
   355
        0.8083278261
                         1.013428528
##
   356
        0.8114586818
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##
   357
        0.8145895375
                         1.018113690
##
  358
        0.8177203932
                         1.020269992
##
   359
        0.8208512488
                         1.022254519
##
   360
        0.8239821045
                         1.024052582
##
   361
        0.8271129602
                         1.025720481
##
   362
                        1.027184057
        0.8302438159
##
   363
        0.8333746715
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##
   364
        0.8365055272
                         1.029520322
##
   365
        0.8396363829
                         1.030374438
##
   366
        0.8427672385
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##
        0.8458980942
                         1.031402464
  367
##
   368
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                         1.031557466
##
   369
        0.8521598056
                        1.031426464
   370
##
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                         1.031098640
##
  371
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                         1.030466023
## 372
        0.8615523726
                        1.029515307
##
   373
        0.8646832282
                         1.028346585
## 374
        0.8678140839
                        1.026841604
##
   375
        0.8709449396
                        1.024988634
##
   376
        0.8740757953
                         1.022898118
##
   377
        0.8772066509
                         1.020442639
##
   378
        0.8803375066
                         1.017612579
##
   379
        0.8834683623
                         1.014527827
##
  380
        0.8865992179
                         1.011053288
##
   381
        0.8897300736
                        1.007181993
##
   382
        0.8928609293
                         1.003041994
##
   383
        0.8959917850
                         0.998492380
##
   384
        0.8991226406
                         0.993529325
##
  385
                         0.988287393
        0.9022534963
##
   386
        0.9053843520
                         0.982622062
##
  387
        0.9085152076
                         0.976533076
##
   388
        0.9116460633
                         0.970159514
##
   389
        0.9147769190
                         0.963355739
##
   390
        0.9179077747
                         0.956125395
##
   391
        0.9210386303
                         0.948609781
##
   392
        0.9241694860
                         0.940664856
##
   393
        0.9273003417
                         0.932298382
##
  394
        0.9304311974
                         0.923651330
##
        0.9335620530
   395
                         0.914584112
##
   396
        0.9366929087
                         0.905108684
##
   397
        0.9398237644
                         0.895362974
##
   398
        0.9429546200
                         0.885214744
##
   399
        0.9460854757
                         0.874680066
##
  400
        0.9492163314
                         0.863891031
##
   401
        0.9523471871
                         0.852725585
##
   402
        0.9554780427
                         0.841203685
##
   403
        0.9586088984
                         0.829448801
##
  404
        0.9617397541
                         0.817351731
   405
##
        0.9648706097
                         0.804935929
##
   406
        0.9680014654
                         0.792313582
##
  407
        0.9711323211
                         0.779390733
##
   408
        0.9742631768
                         0.766193805
##
   409
        0.9773940324
                         0.752821252
## 410
        0.9805248881
                         0.739196399
## 411
        0.9836557438
                        0.725347973
```

```
## 412
       0.9867865994
                        0.711358569
## 413
        0.9899174551
                        0.697170378
## 414
        0.9930483108
                        0.682813673
##
   415
        0.9961791665
                        0.668353460
##
   416
        0.9993100221
                        0.653751873
##
  417
        1.0024408778
                        0.639039904
## 418
        1.0055717335
                        0.624263706
##
   419
        1.0087025891
                        0.609405886
## 420
        1.0118334448
                        0.594497298
## 421
                        0.579564492
        1.0149643005
##
   422
        1.0180951562
                        0.564610516
## 423
        1.0212260118
                        0.549665234
## 424
        1.0243568675
                        0.534735386
## 425
        1.0274877232
                        0.519843872
## 426
        1.0306185789
                        0.505018756
## 427
        1.0337494345
                        0.490247305
## 428
                        0.475571162
        1.0368802902
##
   429
        1.0400111459
                        0.461015866
## 430
        1.0431420015
                        0.446550059
##
  431
        1.0462728572
                        0.432232557
##
  432
        1.0494037129
                        0.418085750
## 433
        1.0525345686
                        0.404060987
## 434
        1.0556654242
                        0.390232291
##
   435
        1.0587962799
                        0.376618419
## 436
        1.0619271356
                        0.363155158
## 437
        1.0650579912
                        0.349929477
## 438
        1.0681888469
                        0.336956158
## 439
        1.0713197026
                        0.324157494
## 440
        1.0744505583
                        0.311630927
##
   441
        1.0775814139
                        0.299387080
##
   442
        1.0807122696
                        0.287337059
##
  443
        1.0838431253
                        0.275586200
## 444
        1.0869739809
                        0.264140941
## 445
        1.0901048366
                        0.252903649
## 446
        1.0932356923
                        0.241984966
## 447
        1.0963665480
                        0.231387263
##
   448
        1.0994974036
                        0.221006665
##
  449
        1.1026282593
                        0.210956620
## 450
        1.1057591150
                        0.201235674
## 451
        1.1088899706
                        0.191736172
##
  452
        1.1120208263
                        0.182572016
## 453
        1.1151516820
                        0.173738292
                        0.165125892
##
   454
        1.1182825377
##
   455
        1.1214133933
                        0.156847061
##
  456
        1.1245442490
                        0.148893859
## 457
        1.1276751047
                        0.141157873
## 458
        1.1308059604
                        0.133747840
## 459
        1.1339368160
                        0.126653301
## 460
        1.1370676717
                        0.119768427
##
   461
        1.1401985274
                        0.113196916
##
  462
        1.1433293830
                        0.106926318
## 463
        1.1464602387
                        0.100854999
## 464
        1.1495910944
                        0.095080405
## 465
        1.1527219501
                        0.089588630
## 466
        1.1558528057
                        0.084283540
##
  467
        1.1589836614
                        0.079255440
##
   468
        1.1621145171
                        0.074489484
        1.1652453727
##
  469
                        0.069896034
## 470
        1.1683762284
                        0.065557668
## 471
        1.1715070841
                        0.061459087
## 472
        1.1746379398
                        0.057517835
## 473
        1.1777687954
                        0.053808467
##
  474
        1.1808996511
                        0.050315659
##
   475
        1.1840305068
                        0.046964541
## 476
        1.1871613624
                        0.043821603
## 477
        1.1902922181
                        0.040871863
## 478
        1.1934230738
                        0.038048185
## 479
        1.1965539295
                        0.035409160
## 48A
        1.1996847851
                        0.032940452
##
   481
        1.2028156408
                        0.030582584
   482
        1.2059464965
                        0.028386589
##
## 483
        1.2090773521
                        0.026339010
##
   484
        1.2122082078
                        0.024387770
##
   485
        1.2153390635
                        0.022576818
##
  486
        1.2184699192
                        0.020893739
##
  487
        1.2216007748
                        0.019293461
##
   488
        1.2247316305
                        0.017813407
## 489
        1.2278624862
                        0.016442313
## 490
        1.2309933418
                        0.015141601
```

```
## 491
        1.2341241975
                        0.013942796
## 492
        1.2372550532
                        0.012835827
        1.2403859089
                       0.011788040
##
  493
  494
        1.2435167645
                        0.010825704
##
  495
        1.2466476202
                        0.009939949
##
##
  496
        1.2497784759
                        0.009103428
        1.2529093316
## 497
                        0.008337802
                        0.007635364
##
  498
        1.2560401872
## 499
        1.2591710429
                        0.006973458
                        0.006369755
##
  500
        1.2623018986
##
  501
        1.2654327542
                        0.005817650
##
  502
        1.2685636099
                        0.005298566
## 503
        1.2716944656
                        0.004826774
## 504
        1.2748253213
                        0.004396683
## 505
        1.2779561769
                        0.003993220
## 506
        1.2810870326
                       0.003627792
##
  507
        1.2842178883
                        0.003295723
##
  508
        1.2873487439
                        0.002984911
##
  509
        1.2904795996
                        0.002704379
## 510
        1.2936104553
                        0.002450267
## 511
        1.2967413110
                        0.002212952
## 512
        1.2998721666
                        0.001999504
```

 $ggplot(data=df.new, mapping=aes(x)) + geom_histogram(aes(y=..density..), color="grey", fill="lightgrey", bins=60) + geom_line(data=df.estimate2, aes(x = d.estimate2.x), y = d.estimate2.y))$



The estimate line does not match the histogram at all, and it is very removed from the actual shape of the histogram. It also extends past the range of values that x takes.

Question 6

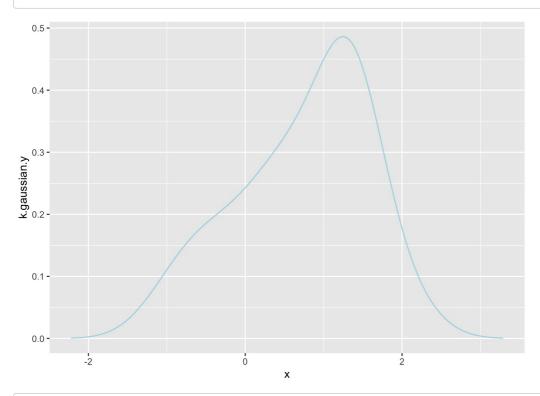
Pick 20 points at random from the initial, unbounded *x* sample. Perform density estimates with "gaussian", "triangular", and "epanechnikov" kernels. Use ggplot() to draw the three density estimates (without the histogram). Do you see any significant differences in the estimates? Change the number of randomly sampled points to 500 and redo the plot...are there still any discernible differences?

```
set.seed(100)
s=sample(x, 20)
k.gaussian =density(s, bw="nrd0", adjust =1, kernel="gaussian")
k.triangular=density(s, bw="nrd0", adjust =1, kernel="triangular")
epanechnikov= density(s, bw="nrd0", adjust =1, kernel="epanechnikov")

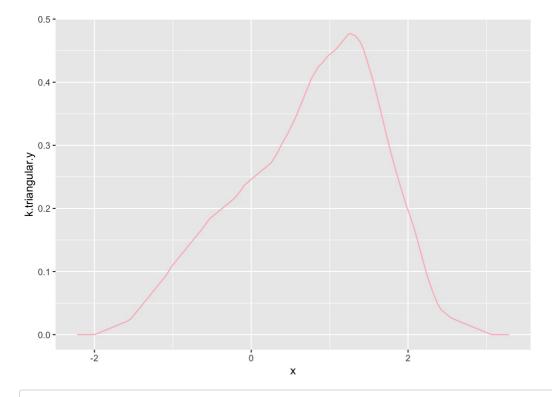
k.gaussian = data.frame(k.gaussian$x,k.gaussian$y)
k.triangular=data.frame(k.triangular$x,k.triangular$y)
epanechnikov=data.frame(epanechnikov$x,epanechnikov$y)

k.gaussian_plot =ggplot(data=df,mapping=aes(x)) + geom_line(data=k.gaussian,aes(x = k.gaussian.x, y =k.gaussian.y),
color="lightblue")
k.triangular_plot =ggplot(data=df,mapping=aes(x)) + geom_line(data=k.triangular,aes(x = k.triangular.x, y =k.triangular.y),color="lightpink")
epanechnikov_plot =ggplot(data=df,mapping=aes(x)) + geom_line(data=epanechnikov,aes(x = epanechnikov.x, y =epanechnikov.y),color="lightgreen")
```

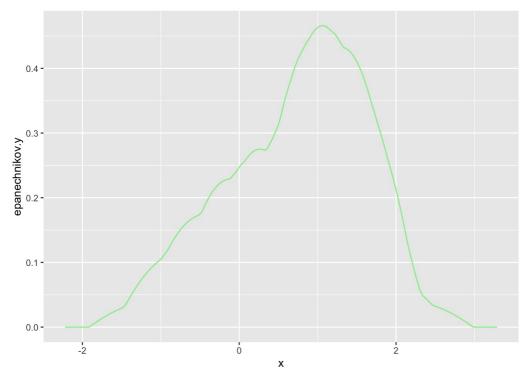
k.gaussian_plot



k.triangular_plot



epanechnikov_plot



```
set.seed(100)
s.500=sample(x, 500)

k5.gaussian =density(s.500, bw="nrd0", adjust =1, kernel="gaussian")
k5.triangular=density(s.500, bw="nrd0", adjust =1, kernel="triangular")
epanechnikov.5= density(s.500, bw="nrd0", adjust =1, kernel="epanechnikov")

k5.gaussian = data.frame(k5.gaussian$x,k5.gaussian$y)
k5.triangular=data.frame(k5.triangular$x,k5.triangular$y)
epanechnikov.5=data.frame(epanechnikov.5$x,epanechnikov.5$y)
epanechnikov.5
```

```
## epanechnikov.5.x epanechnikov.5.y
## 1 -3.215811722 3.733932e-17
## 2 -3.203853202 3.704854e-17
```

## 3	-3.191894681	4.120951e-18
## 4	-3.179936160	2.365626e-17
## 5	-3.167977639	2.439574e-17
## 6	-3.156019118	3.337741e-17
## 7	-3.144060598	3.366554e-17
## 8	-3.132102077	1.882897e-17
## 9	-3.120143556	2.294166e-17
## 10	-3.108185035	1.432978e-17
## 11	-3.096226514	4.984219e-17
## 12	-3.084267993	7.848339e-17
## 13	-3.072309473	3.326943e-06
## 14	-3.060350952	4.121114e-05
## 15	-3.048392431	1.863174e-04
## 16	-3.036433910	3.524732e-04
## 17	-3.024475389	5.152617e-04
## 18	-3.012516869	6.737768e-04
## 19	-3.000558348	8.266275e-04
## 20	-2.988599827	9.750521e-04
## 21	-2.976641306	1.119203e-03
==		
## 22	-2.964682785	1.259081e-03
## 23	-2.952724265	1.394480e-03
## 24	-2.940765744	1.524268e-03
## 25	-2.928807223	1.649782e-03
## 26	-2.916848702	1.771023e-03
## 27	-2.904890181	1.887990e-03
## 28	-2.892931661	2.000121e-03
## 29	-2.880973140	2.106997e-03
## 30	-2.869014619	2.209601e-03
## 31	-2.857056098	2.307931e-03
## 32	-2.845097577	2.401988e-03
## 33	-2.833139057	2.490850e-03
## 34	-2.821180536	2.574816e-03
## 35	-2.809222015	2.654509e-03
## 36	-2.797263494	2.729929e-03
## 37	-2.785304973	2.801075e-03
## 38	-2.773346452	2.866668e-03
## 39	-2.761387932	2.927724e-03
## 40	-2.749429411	2.984507e-03
## 41	-2.737470890	3.037016e-03
## 42	-2.725512369	3.085158e-03
## 43	-2.713553848	3.127576e-03
## 44	-2.701595328	3.165721e-03
## 45	-2.689636807	3.199593e-03
## 46	-2.677678286	3.238250e-03
## 47	-2.665719765	3.311394e-03
## 48	-2.653761244	3.489650e-03
## 49	-2.641802724	3.670362e-03
## 50	-2.629844203	3.843174e-03
## 51	-2.617885682	4.007439e-03
## 52	-2.605927161	4.161536e-03
## 53	-2.593968640	4.305620e-03
## 54	-2.582010120	4.441157e-03
## 55	-2.570051599	4.568148e-03
## 56	-2.558093078	4.686591e-03
## 57	-2.546134557	4.794152e-03
## 58	-2.534176036	4.892414e-03
## 59	-2.522217516	4.982130e-03
## 60	-2.510258995	5.063300e-03
## 61	-2.498300474	5.135923e-03
## 62	-2.486341953	5.196945e-03
	-2.474383432	5.249387e-03
## 64	-2.462424911	5.293282e-03
## 65	-2.450466391	5.328631e-03
## 66	-2.438507870	5.354750e-03
## 67	-2.426549349	5.369918e-03
## 68	-2.414590828	5.376538e-03
## 69	-2.402632307	5.374612e-03
## 70	-2.390673787	5.364140e-03
	-2.378715266	5.343722e-03
## 71		
## 72	-2.366756745	5.313068e-03
## 73	-2.354798224	5.273868e-03
## 74	-2.342839703	5.226121e-03
## 75		E 1600276 02
	-2.330881183	5.169827e-03
## 76	-2.330881183 -2.318922662	5.109827e-03 5.102872e-03
## 76 ## 77		
-	-2.318922662	5.102872e-03
## 77	-2.318922662 -2.306964141	5.102872e-03 5.026397e-03
## 77 ## 78 ## 79	-2.318922662 -2.306964141 -2.295005620 -2.283047099	5.102872e-03 5.026397e-03 4.941376e-03 4.847808e-03
## 77 ## 78	-2.318922662 -2.306964141 -2.295005620	5.102872e-03 5.026397e-03 4.941376e-03

## 82	-2.247171537	4.509905e-03
## 83	-2.235213016	4.379062e-03
## 84	-2.223254495	4.239673e-03
## 85	-2.211295975	4.091278e-03
## 86	-2.199337454	3.931708e-03
## 87	-2.187378933	3.763591e-03
## 88	-2.175420412	3.587181e-03
## 89	-2.163461891	3.409339e-03
## 90	-2.151503370	3.256231e-03
## 91	-2.139544850	3.215734e-03
## 92	-2.127586329	3.184209e-03
## 93	-2.115627808	3.148411e-03
## 94	-2.103669287	3.108340e-03
## 95	-2.091710766	3.063049e-03
	-2.031710700	
		3.012887e-03
## 97	-2.067793725	2.958452e-03
## 98	-2.055835204	2.899744e-03
## 99	-2.043876683	2.836762e-03
## 100	-2.031918162	2.768203e-03
## 101	-2.019959642	2.695130e-03
## 102	-2.008001121	2.617785e-03
## 103	-1.996042600	2.559914e-03
## 104	-1.984084079	2.570819e-03
## 104	-1.972125558	2.644826e-03
	-1.960167038	2.712637e-03
## 107	-1.948208517	2.772222e-03
## 108	-1.936249996	2.823261e-03
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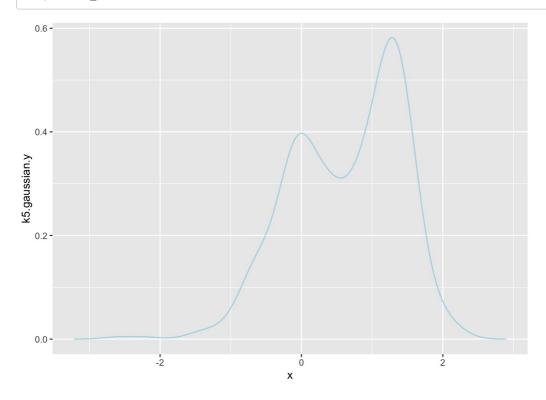
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## 505
             2.811282765
                             0.000000e+00
                             0.000000e+00
## 506
            2.823241286
##
   507
            2.835199806
                              0.000000e+00
## 508
             2.847158327
                             0.000000e+00
## 509
                             0.000000e+00
            2.859116848
## 510
             2.871075369
                              0.000000e+00
## 511
             2.883033890
                              0.000000e+00
                              0.0000000+00
## 512
             2.894992410
```

 $k5. gaussian_plot = ggplot(data=df, mapping=aes(x)) + geom_line(data=k5. gaussian, aes(x = k5. gaussian.x, y = k5. gaussian.y), color="lightblue")$

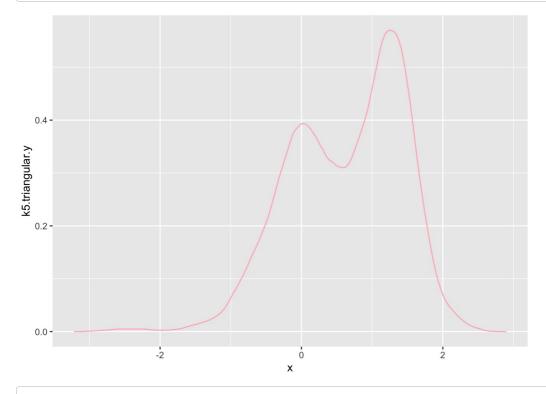
 $k5.triangular_plot = ggplot(data=df,mapping=aes(x)) + geom_line(data=k5.triangular,aes(x = k5.triangular.x, y = k5.triangular.y), color="lightpink")$

epanechnikov.5_plot =ggplot(data=df,mapping=aes(x)) + geom_line(data=epanechnikov.5,aes(x = epanechnikov.5.x, y =epanechnikov.5.y),color="lightgreen")

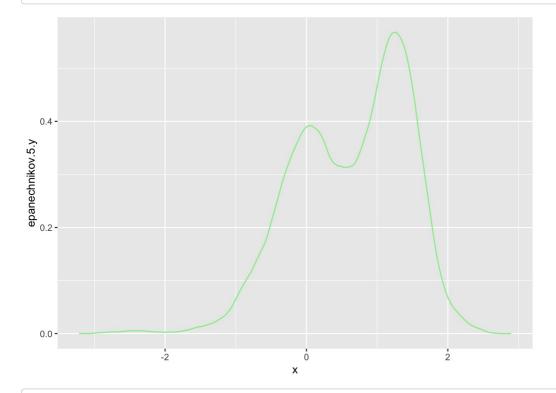
k5.gaussian plot



k5.triangular_plot



epanechnikov.5_plot



There are fairly similar, gaussian tends to be smoother where the curved peaks are but generally all three seem to take on similar shapes. When we resample to 500 points, the graph seems to have two peaks instead of one.

Question 7

Estimate galaxy mass from emission-line strength using the Nadaraya-Watson kernel estimator.

In the normal model learning paradigm, you split the data and learn the model using the training data, then apply the model to predict response values for the test data. You then compute the MSE.

For Nadaraya-Watson, the way this would play out is that we would split the data, then perform, e.g., cross-validation on the *training* set to determine the optimal value of h. We would then apply this value of h when working with the test data, and when computing the MSE.

Here, we are going to keep things simple: do not split the data, and compute a plug-in value of h using one of the bandwidth functions in the base stats package. (Type, e.g., ?bw.nrd0 at the prompt in the Console pane.) Estimate \hat{y} for all the data using a Gaussian kernel, then plot the predicted response vs. the observed response. (Note that this is a little tricky! First, you have to specify x.points=x in the call to ksmooth(), so that the model is actually evaluated at the input points x rather than along a default grid. Then you have to compare out\$y

versus y[order(x)] in the diagnostic plot, because ksmooth() sorts the x values in ascending order. This is all a bit painful to figure out. Your final diagnostic plot won't look great...but that's OK, because we've really simplified the regression here [only one predictor variable, not 10].)

```
bw.nrd0(x)
```

[1] 0.09539287

Processing math: 100%