CPC 2020 – HGF Tutorial

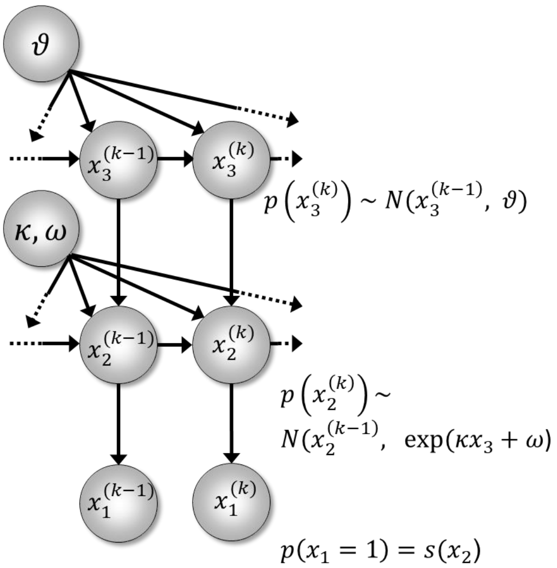

Figure 1: Generative model of the HGF and the posterior distributions of the true hidden states (from Mathys et al., 2011)

The HGF consists of learning **parameters** (maximum-a-posteriori (MAP) estimates, i.e. one estimate per parameter) and **environmental states** (i.e. true hidden states and inferred beliefs; evolve over time):

Parameters: each parameter is described by one estimate

Parameter estimation: For each parameter the mean and variance of the **prior distribution** needs to be specified. This prior distribution determines the range in which the optimization algorithm determines the MAP estimates of all parameters.

- **Perceptual parameters** (e.g. see tapas_hgf_binary_config.m):
  - ϑ: determines the step-size of state x_3, i.e. speed of learning about the log-volatility of the environment
  - ω: determines the step-size of state x_2, i.e. component of the learning rate at the second level
  - κ: coupling parameter between x_3 and x_2, i.e. determines how much environmental volatility affects learning at the second level

- **Initial values** of perceptual trajectories: determine the origin of the trajectory (e.g. see tapas_hgf_binary_config.m):

- $\mu_3^{(0)}$: initial value of mean of trajectory μ_3
- $\mu_2^{(0)}$: initial value of mean of trajectory μ_2
- $\mu_1^{(0)}$: initial value of mean of trajectory μ_1 – however,
  not specified in a binary setting, as it is determined by the second level.
- $\sigma_3^{(0)}$: initial value of variance of trajectory μ_3
- $\sigma_2^{(0)}$: initial value of variance of trajectory μ_2
- $\sigma_1^{(0)}$: initial value of variance of trajectory μ_1
  however, not specified in a binary setting, as it is determined by the second level.

- **Observational parameter** (e.g. see tapas_unitsq_sgm_config.m):
  - ζ: response noise

Trajectories/ environmental states: number of entries per state = number of trials in the task trajectory.

True hidden states (x_i):

- x_3: log-volatility of the environment (i.e. tendency of x_2); evolves as a Gaussian random walk with constant step size ϑ
- x_2: tendency towards category "1"; evolves as a Gaussian random walk with step size $exp(\kappa x_3 + \omega)$
- x_1: stimulus category; sigmoid transformation of x_2

Corresponding **inferred** hidden states (μ_i), i.e. **beliefs**:

- μ_3: log-volatility of the environment (i.e. tendency of μ_2)
- μ_2: tendency towards category "1"
- μ_1: stimulus category

References:

Mathys, C., Daunizeau, J., Friston, K.J. & Stephan, K.E. 2011. A bayesian foundation for individual learning under uncertainty. *Front Hum Neurosci*, 5, 39. 10.3389/fnhum.2011.00039

Mathys, C.D., Lomakina, E.I., Daunizeau, J., Iglesias, S., Brodersen, K.H., Friston, K.J. & Stephan, K.E. 2014. Uncertainty in perception and the Hierarchical Gaussian Filter. Front Hum Neurosci, 8, 825. 10.3389/fnhum.2014.00825