Bayesian learning and the Hierarchical Gaussian Filter

Practical session CPC 2020

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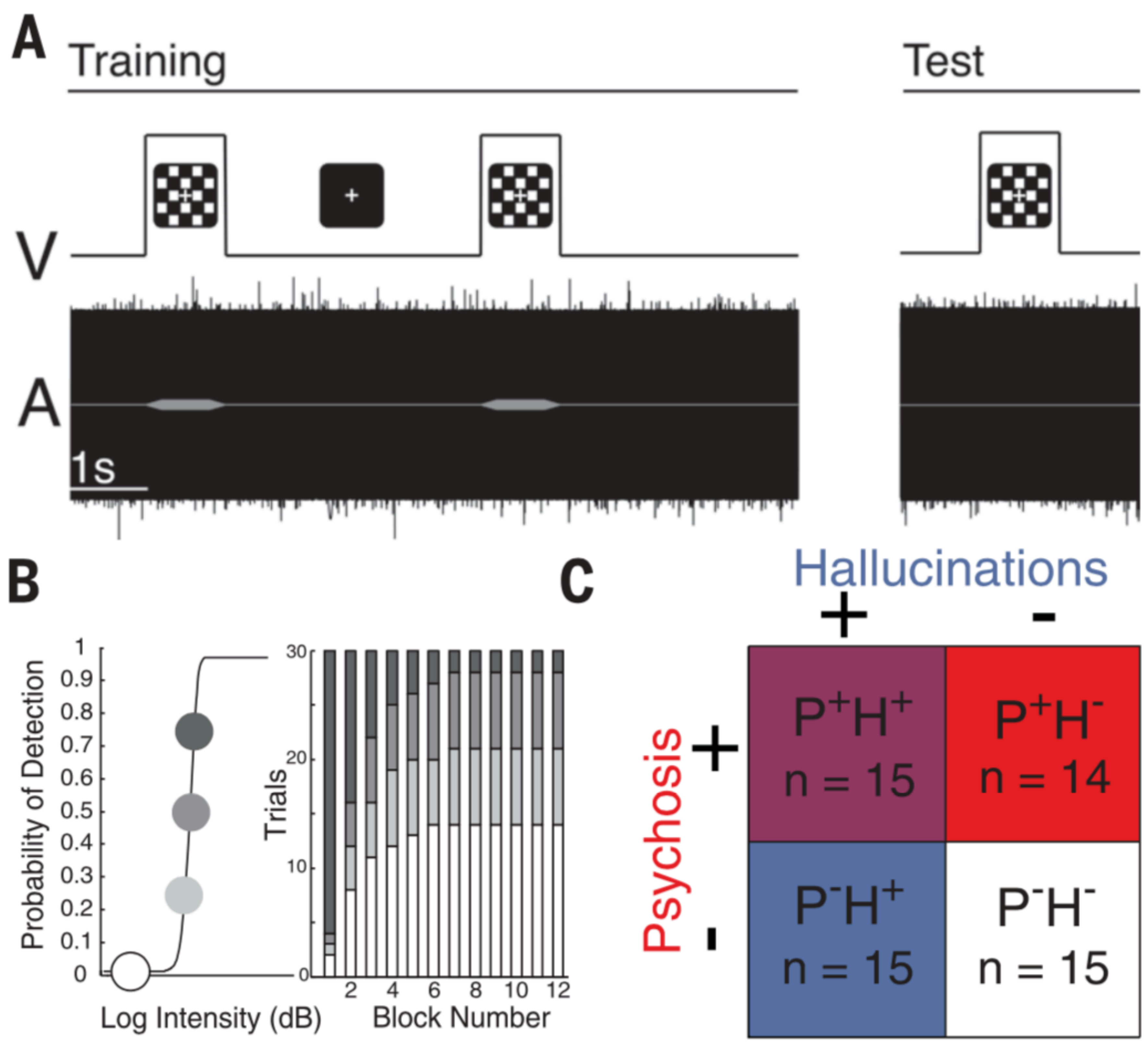
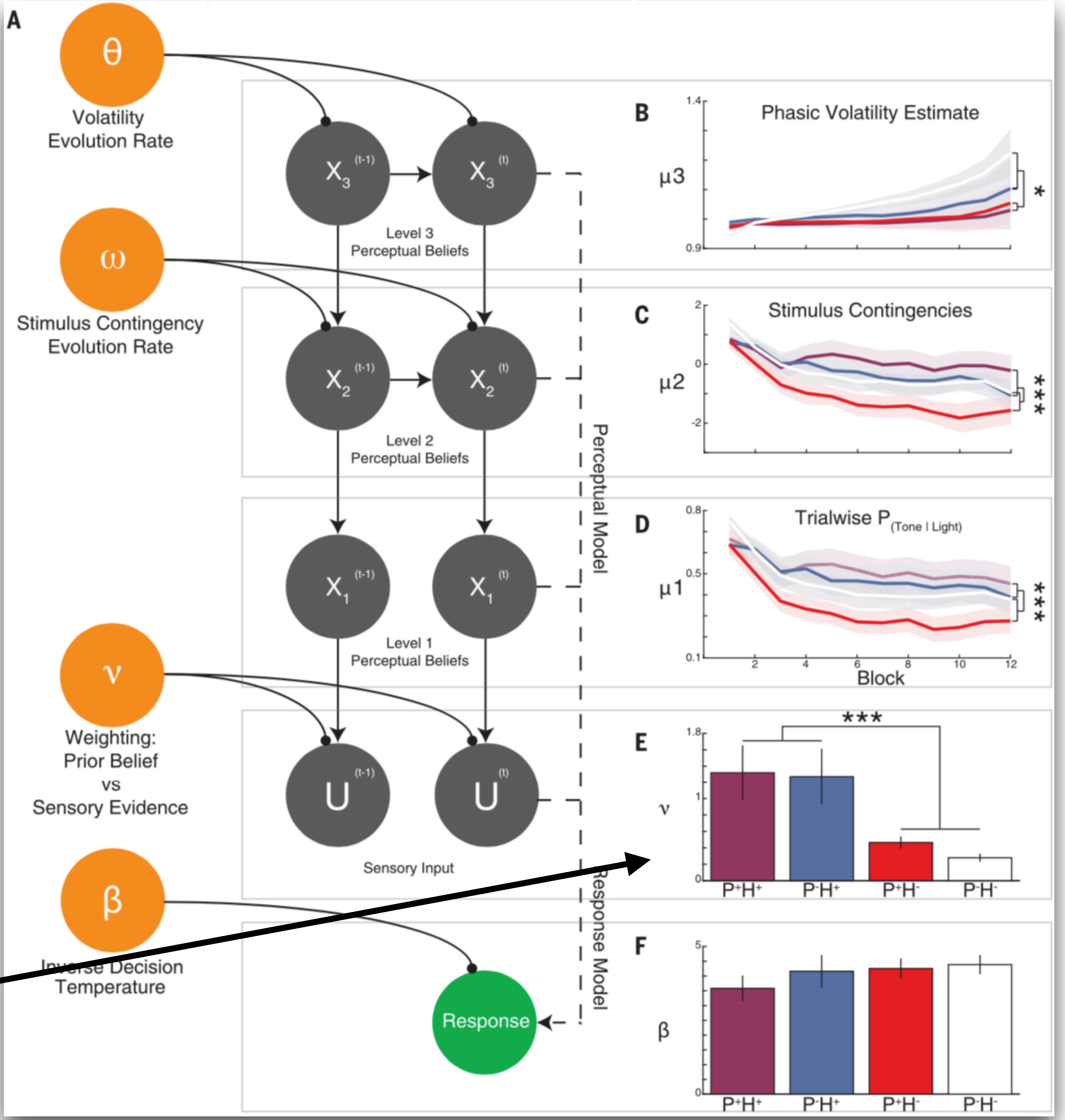
Conditioned hallucinations





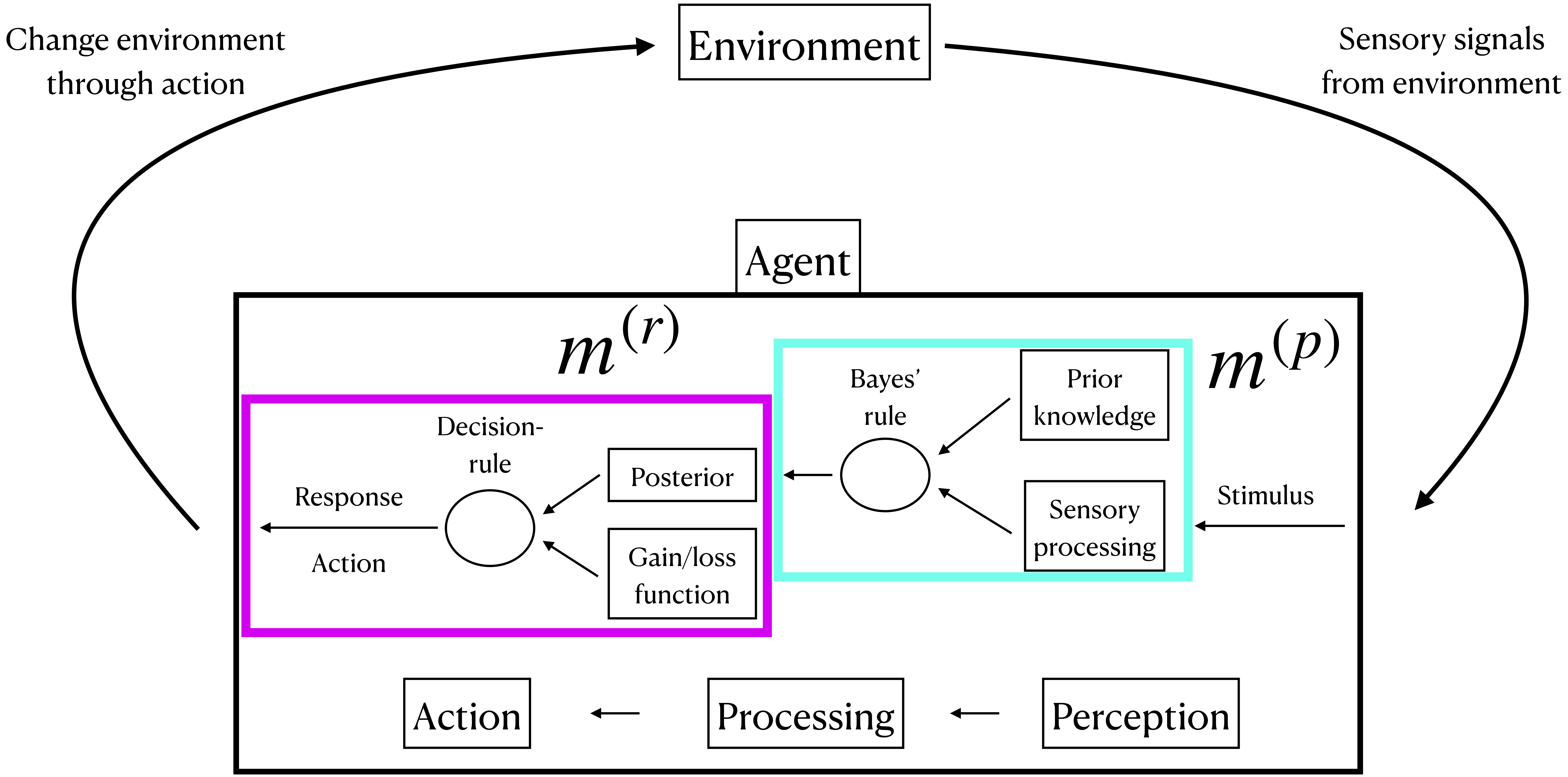
Subjects with hallucinations show higher estimates for weights on prior beliefs

Powers et al. (2017): "Pavlovian conditioning–induced hallucinations result from overweighting of perceptual priors"

Introduction

- Computational psychiatry is concerned with understanding mental disorders through formalisation and model-building
- Underlying processes can often be described in terms of inference
- And these can be studied through decision-making tasks
- **Inverse Bayesian decision theory** (see Daunizeau et al (2010)) :
    - "a meta-Bayesian procedure which allows for Bayesian inferences about subject's Bayesian inferences"

Modelling the inference process



Example: gambling task

Example: gambling task

- Two slot machines:
  For 100 trials, subjects can choose to play either machine to obtain a reward

- Generative process of task:
  At each time t one of the machines will give a reward. This can be described as a coin flip:

$$u^{(t)} \sim Ber(x)$$

Subject's response in t-th trial:

$$y^{(t)} \in \{0,1\}$$

Subject's reward in t-th trial:

$$r^{(t)} = \begin{cases} 1, & if\ u^{(t)} = y^{(t)} \\ 0, & else \end{cases}$$

Derive inference process

We assume this perceptual model:

$$m^{(p)}:\begin{cases} p\left(u^{(t)}|x\right) = Ber(x) \quad t = 1,\ldots,T \\ p\left(x\right) = Beta\left(1,1\right) \end{cases}$$

Which has this posterior:

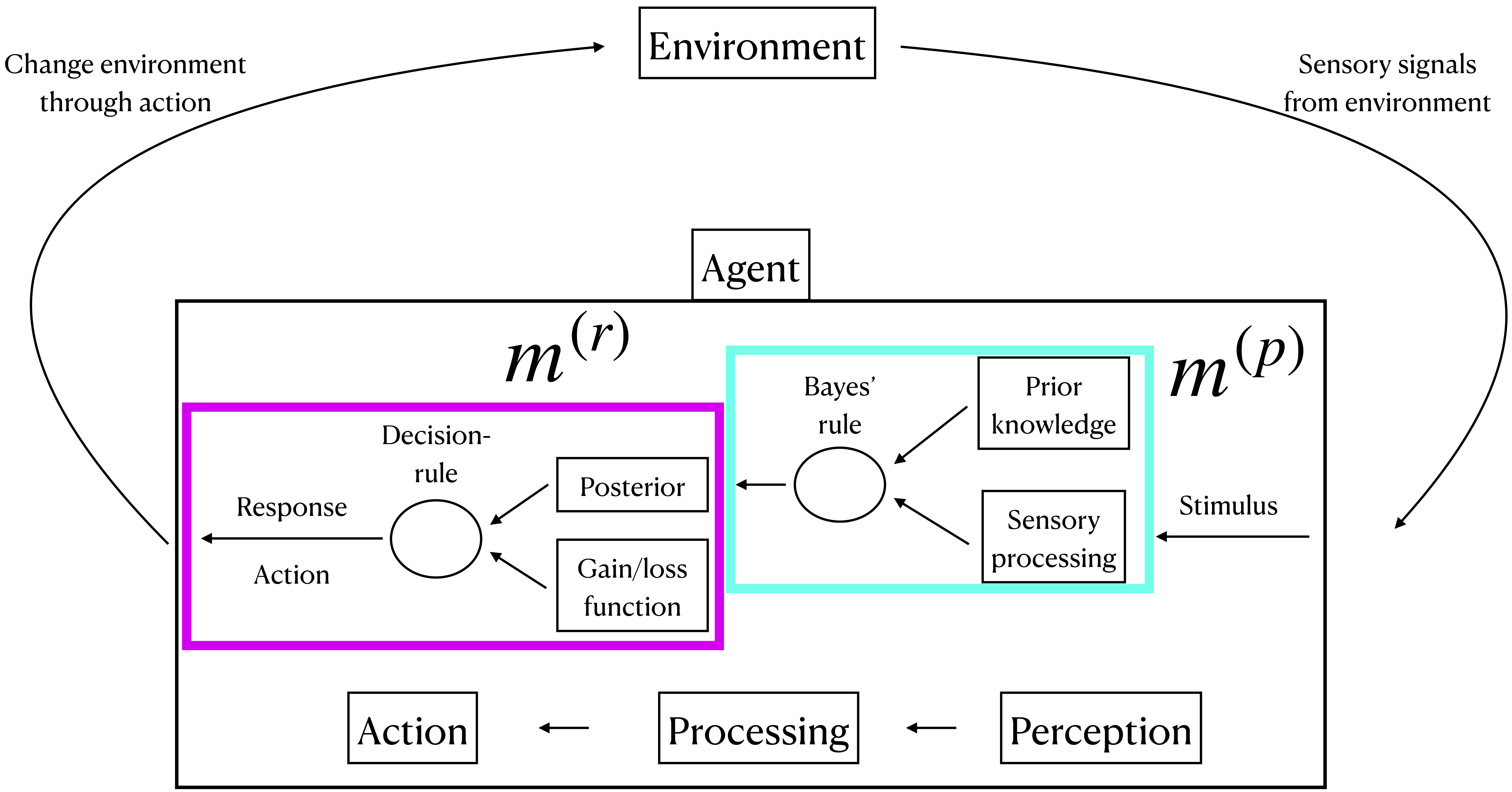
$$\pi\left(x\,|\,u^{(1)},\ldots,u^{(T)}\right) = Beta\left(a + \sum_{t=1}^{T} u^{(t)}; b + T - \sum_{t=1}^{T} u^{(t)}\right)$$

This gives the following sequence of parameters:

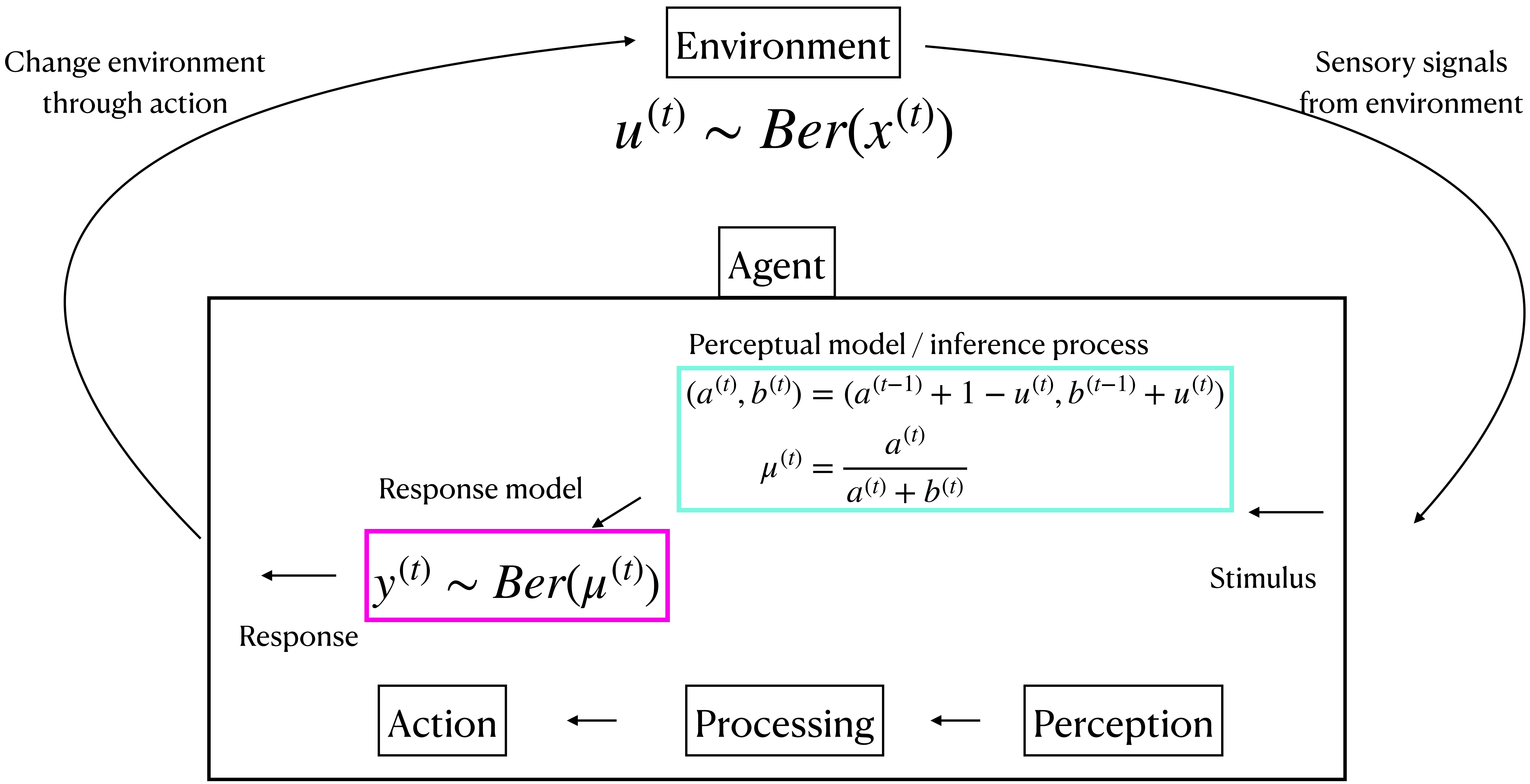
$$(a^{(t)}, b^{(t)}) = (a^{(t-1)} + u^{(t)}, b^{(t-1)} + 1 - u^{(t)})$$

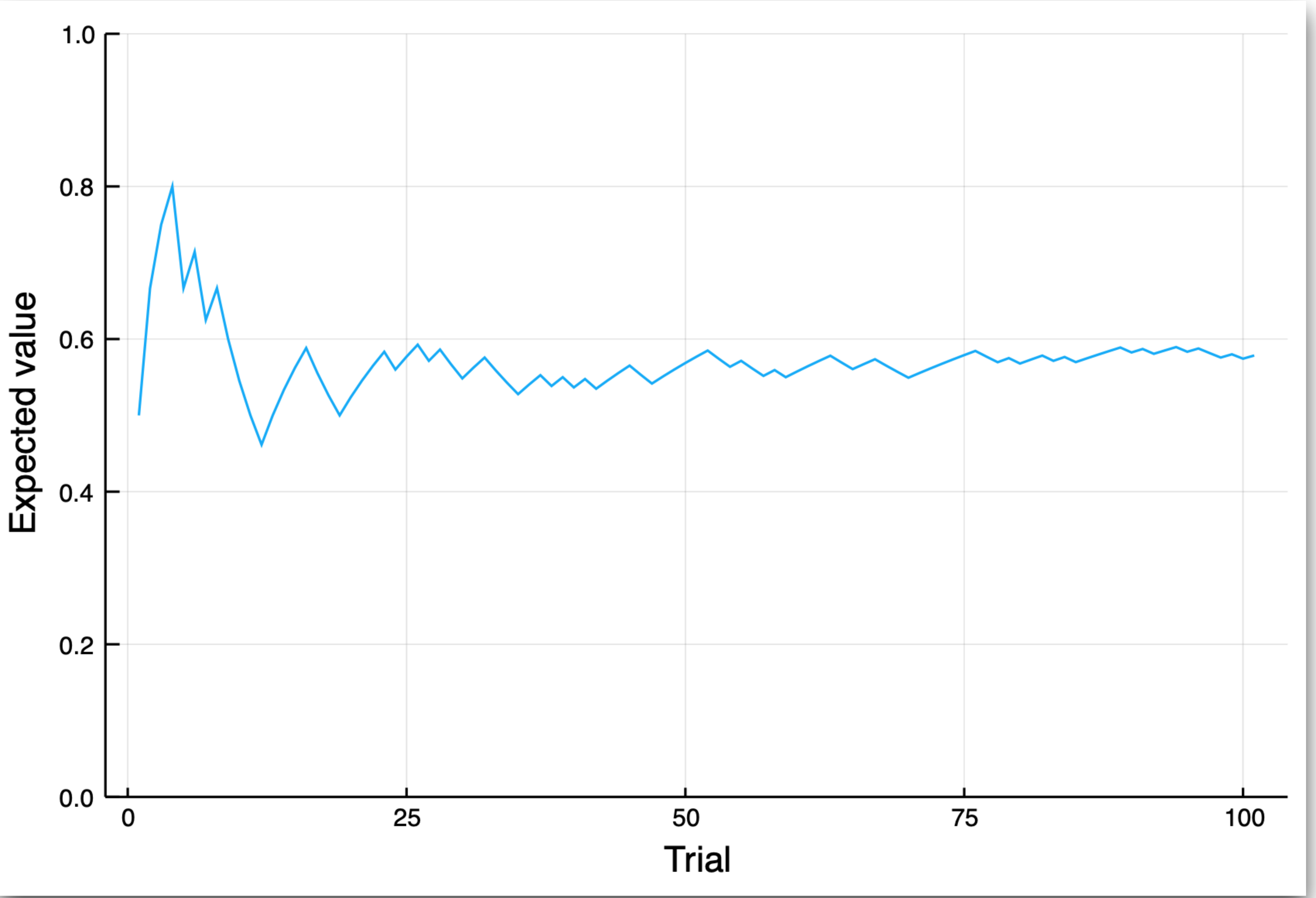
And these expectations: $\mu^{(t)} = \dfrac{a^{(t)}}{a^{(t)} + b^{(t)}}$

Modelling the inference process



Example inference process





1.0
0.8
0.6
0.4
0.2
0.0
Expected value
0
25
50
75
100
Trial

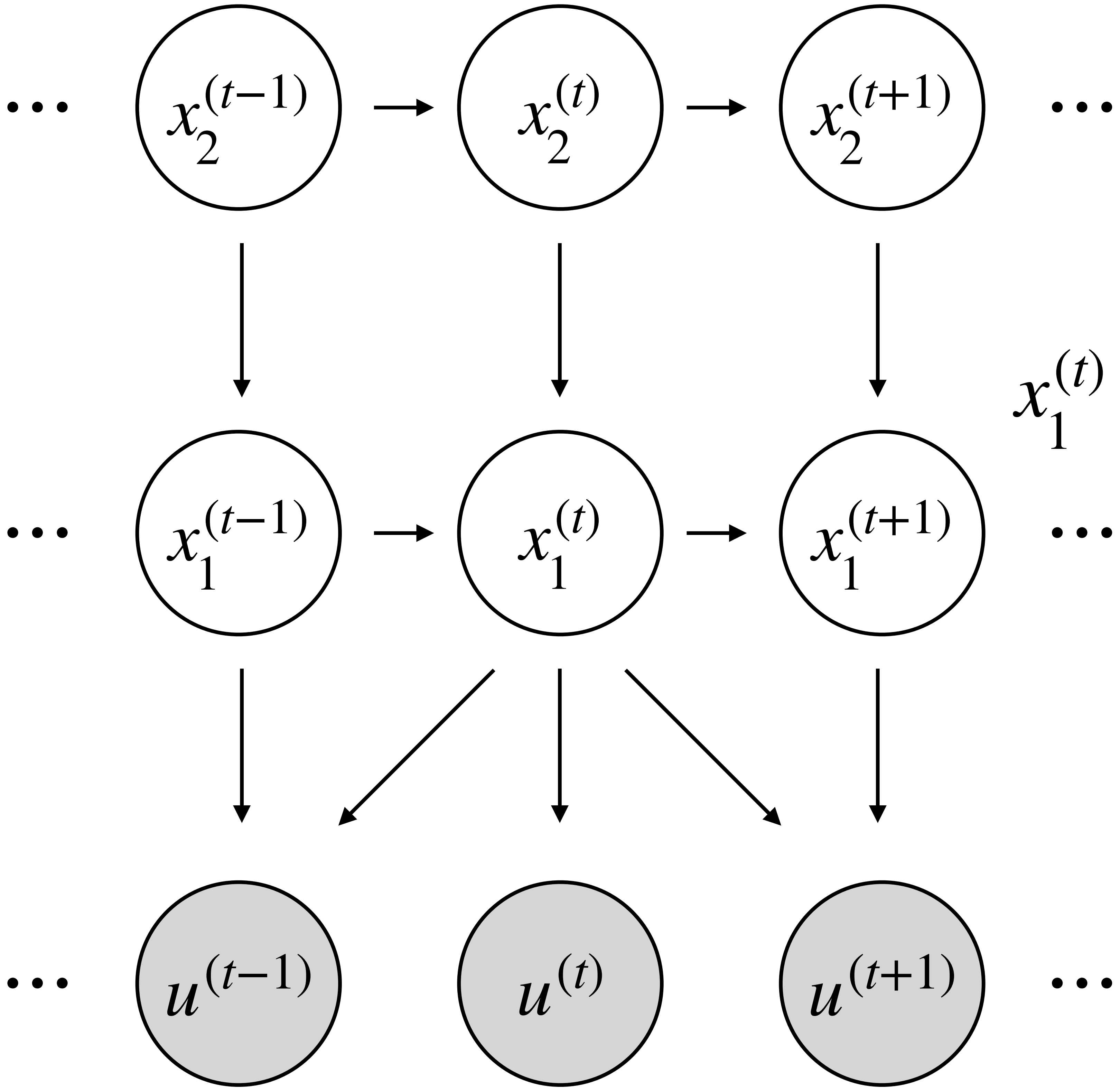
The Hierarchical Gaussian Filter (HGF)

HGF generative model

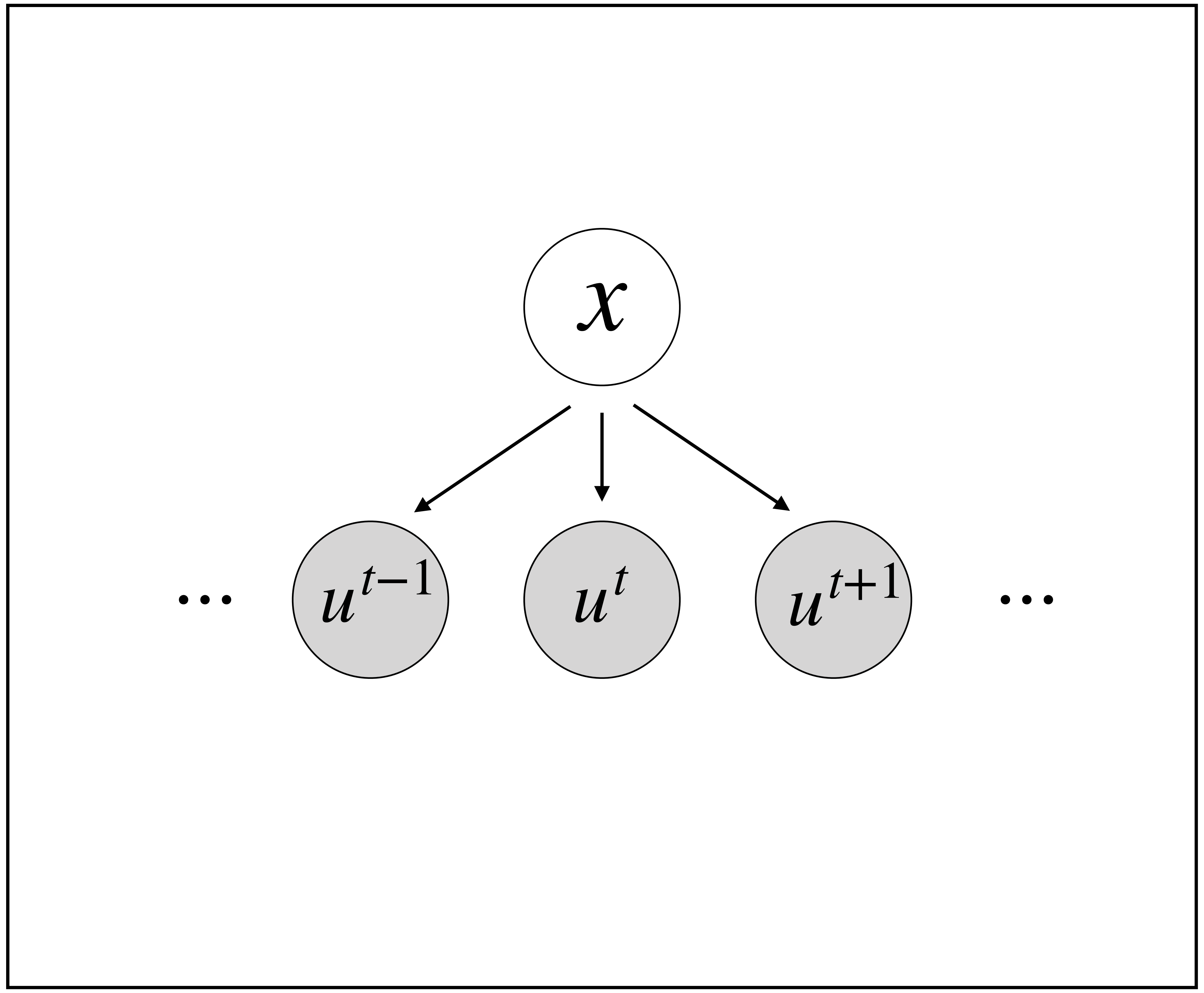
- The HGF is a model for perceptual learning defined through specific choices for the inference process:
    - Generative model: hierarchy of random walks
    - update equations derived through minimising perceptual free energy
- The HGF dynamically updates its learning rate with every observation

$$x_2^{(t)} \sim \mathcal{N}\left(x_2^{(t-1)}, \vartheta\right)$$

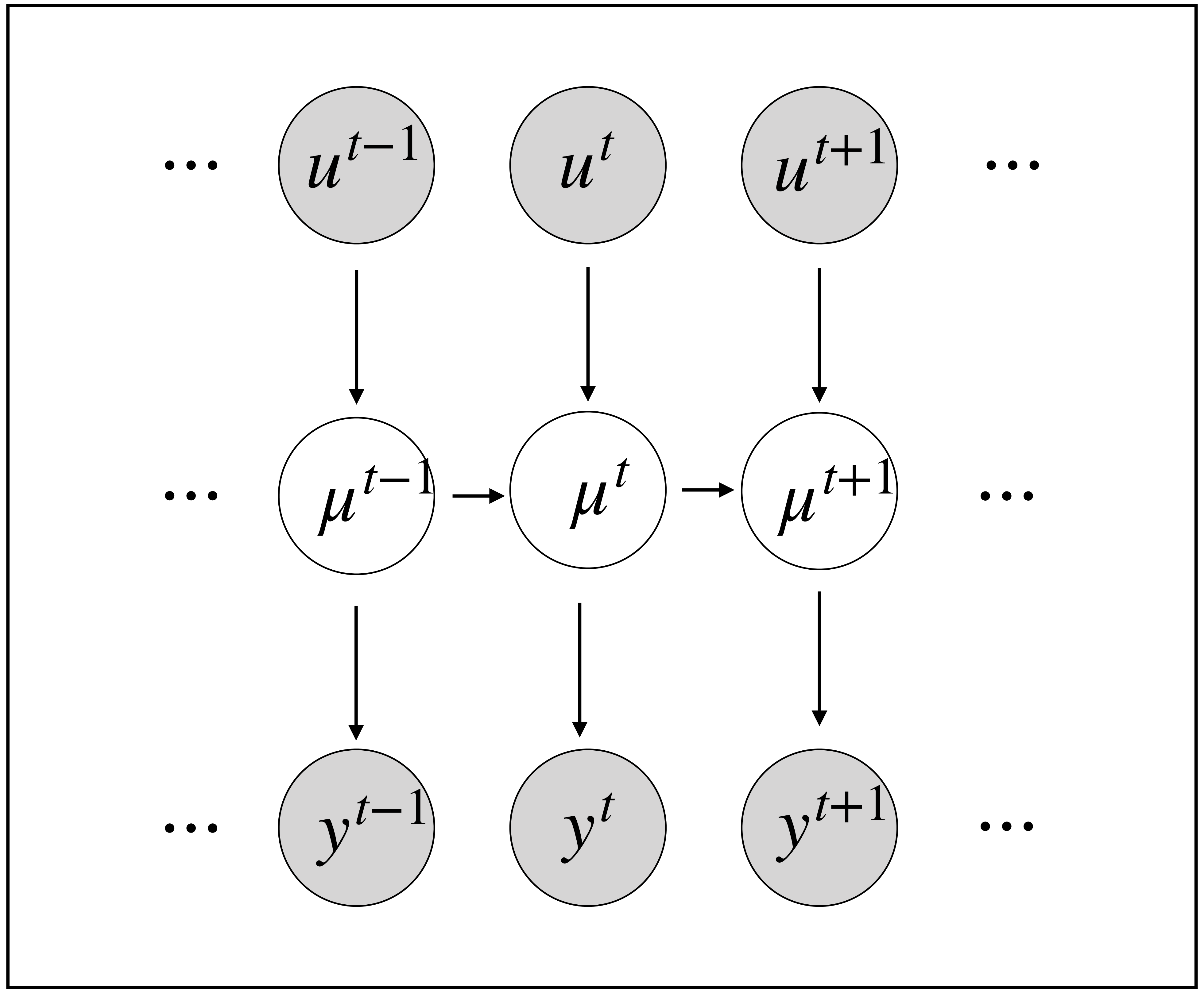
$$x_1^{(t)} \sim \mathcal{N}\left(x_1^{(t-1)}, f\left(x_2^{(t)}\right)\right)$$



Beta-bernoulli model: generative model and inference process

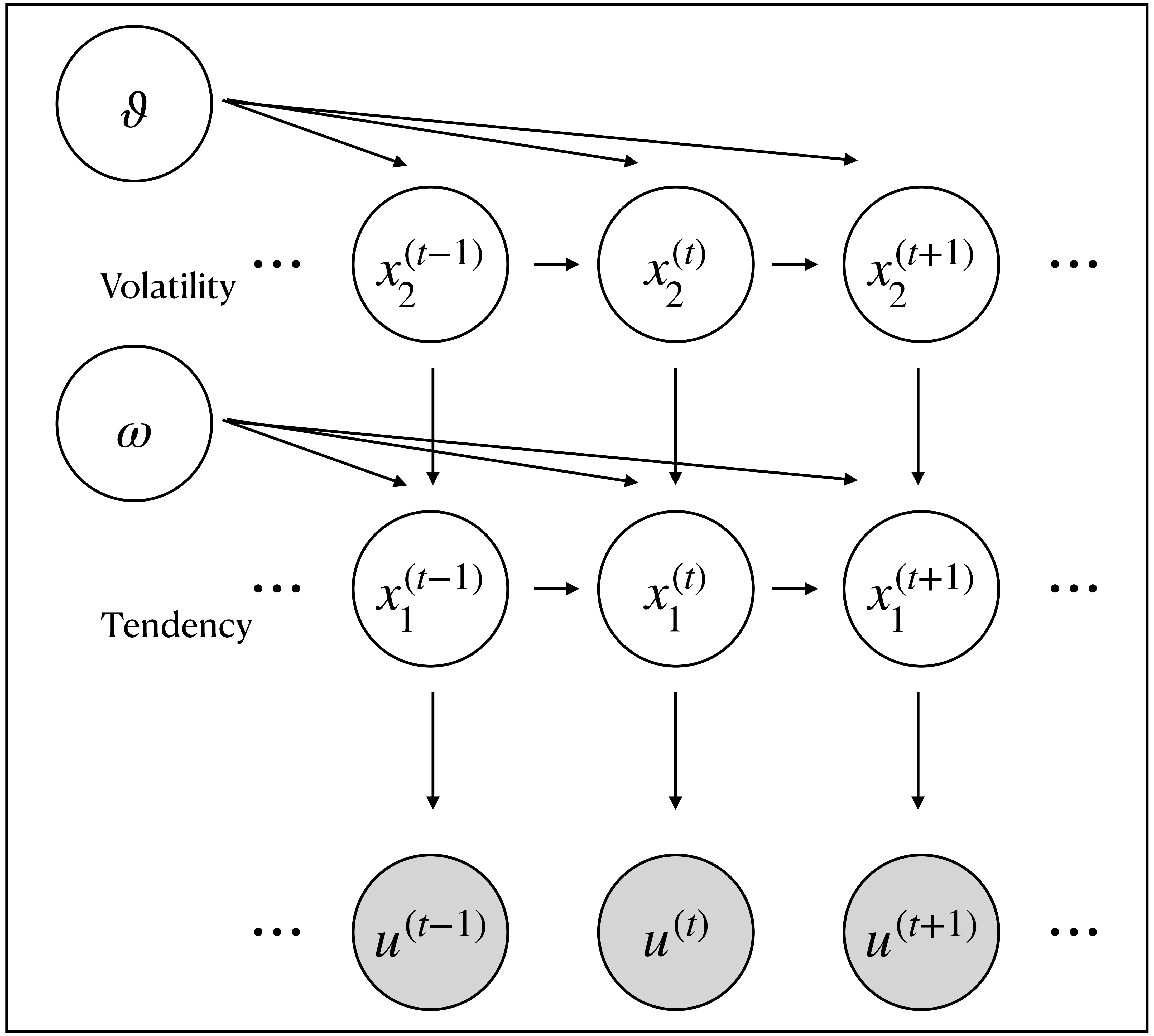


Generative model

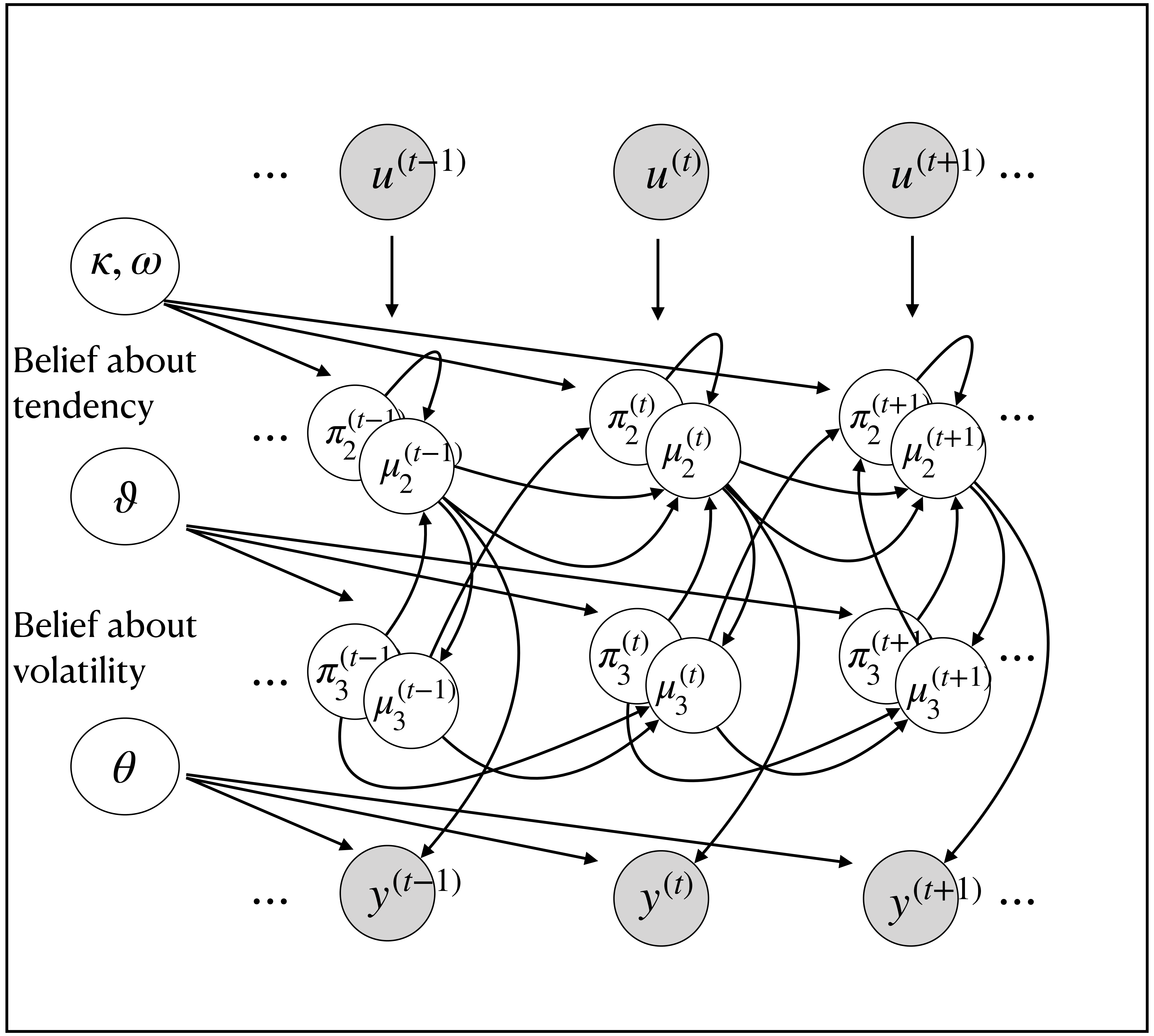


Inference model

HGF: generative model and inference process



Generative model



Inference model

Precision weights and types of uncertainty

The learners observations are generated by:

$$u^{(t)} \sim \mathrm{Ber}\left(x_1^{(t)}\right)$$

which leads to these updates for the belief about the latent process:

$$\mu_2^{(t)} = \mu_2^{(t-1)} + \frac{1}{\pi_2^{(t)}}\delta_1^{(t)} \qquad \hat{\mu}_1^{(t)} = s\left(\mu_2^{(t)}\right)$$

The precision weight can be decomposed into factors corresponding to different kinds of uncertainty:

$$\frac{1}{\pi_2^{(t)}} = \frac{1}{\sigma_2^{(t-1)}\exp(\kappa\mu_3^{(t-1)} + \omega)} + \frac{1}{\hat{\mu}_1^{(t)}(1 - \hat{\mu}_1^{(t)})}$$

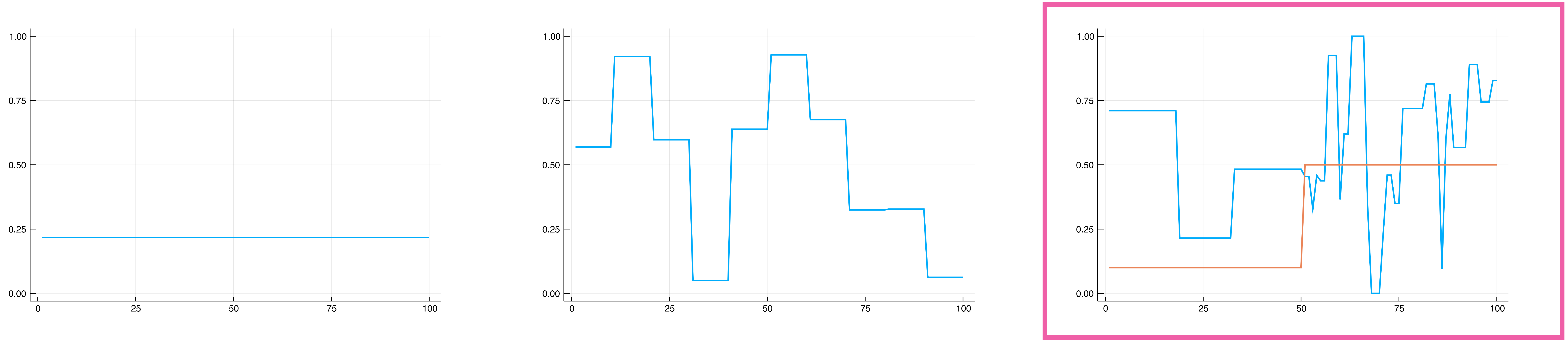
Estimation uncertainty

Estimated volatility of the environment

Irreducible uncertainty about the outcome

Generative process of gambling task

$x^{(t)}$

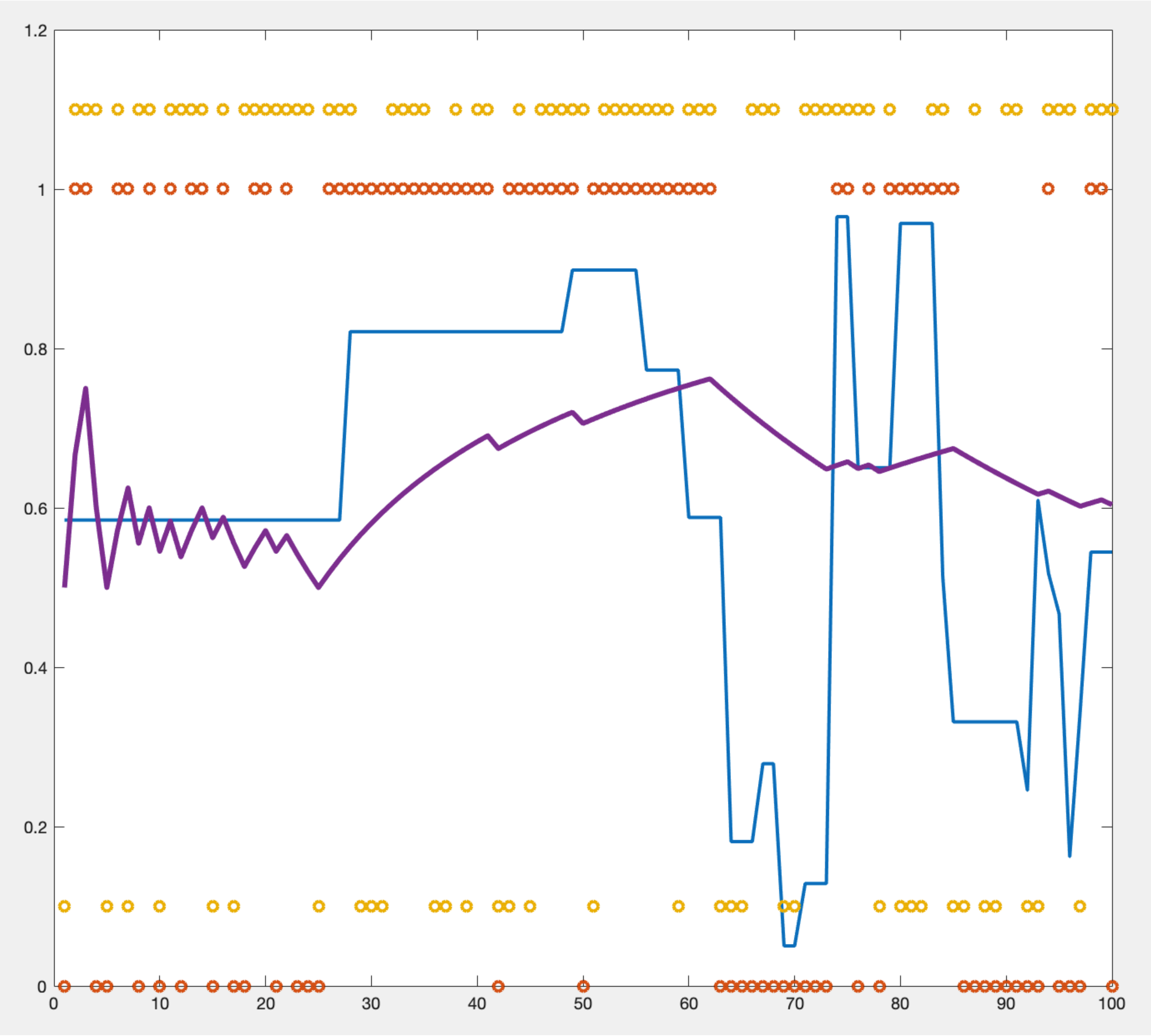


Types of uncertainty:

- Expected and irreducible: noise
- Unexpected and reducible: estimation error
- Unexpected and irreducible: state changes (volatility)

Simulation of Beta-Bernoulli model

$x^{(t)}$ **and** $\hat{\mu}_1^{(t)}$



Simulated responses

Inputs

Simulated belief (mean of posterior)

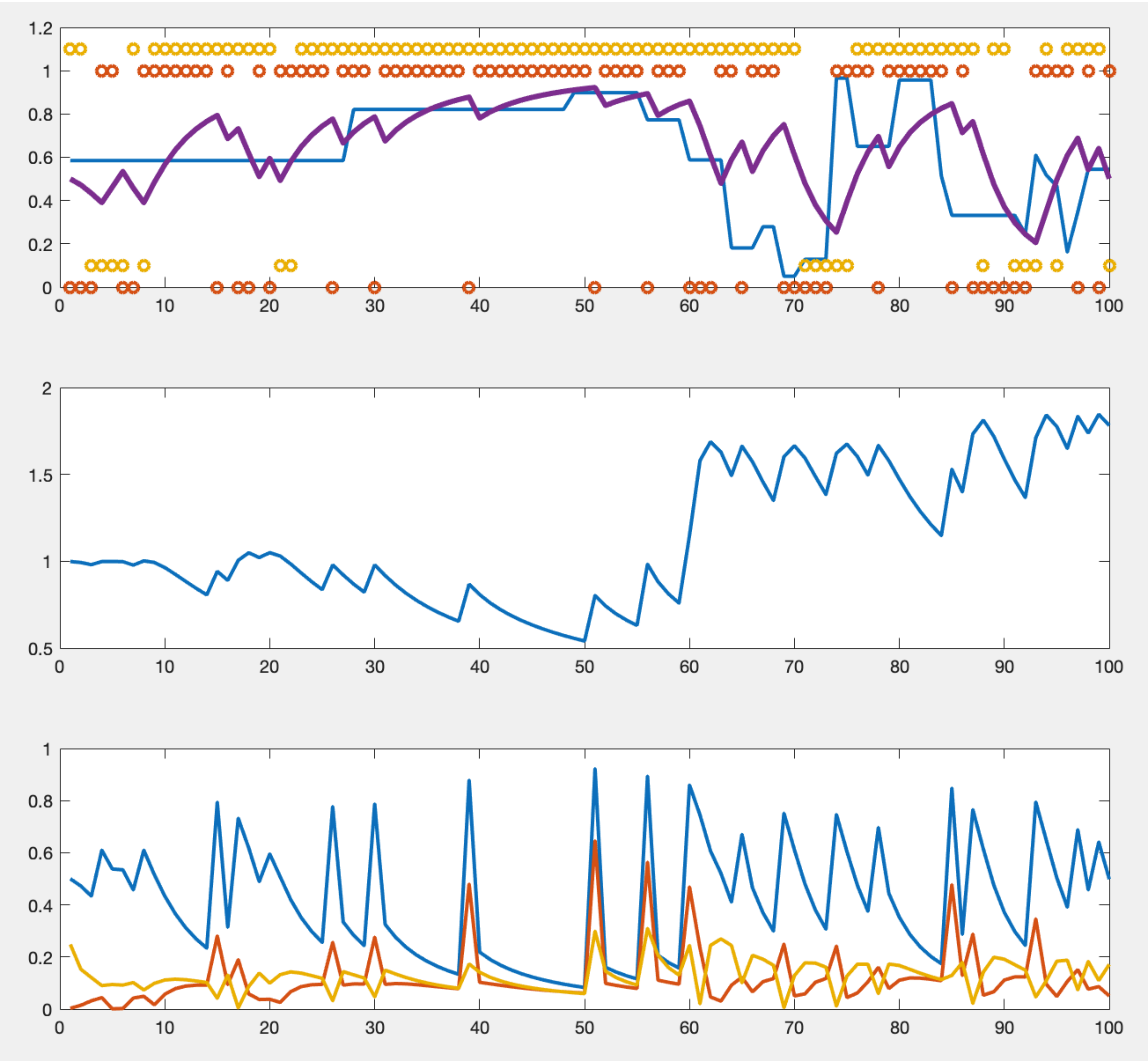
True value of hidden variable

Simulation of HGF

$x^{(t)}$ and $\hat{\mu}_1^{(t)}$

Belief about volatility, $\hat{\mu}_3^{(t)}$

Prediction errors, $\delta_{1:3}^{(t)}$



Simulated responses

Inputs

Simulated belief (mean of posterior)

True value of hidden variable

$\delta_1^{(t)}$

$\delta_2^{(t)}$

$\delta_3^{(t)}$

References / further reading

- Theory
  - “A reading list on Bayesian methods": http://cocosci.princeton.edu/tom/bayes.html
  - Mathys et al. (2011): “A Bayesian foundation for individual learning under uncertainty”
  - Mathys et al. (2014): “Uncertainty in perception and the Hierarchical Gaussian Filter”
  - Daunizeau et al. (2010): “Observing the Observer (I): Meta-Bayesian Models of Learning and Decision-Making”
  - Maia and Frank (2011): “From Reinforcement Learning Models to Psychiatric and Neurological Disorders”
- Applications
  - Iglesias et al. (2013): “Hierarchical Prediction Errors in Midbrain and Basal Forebrain during Sensory Learning”
  - de Berker et al. (2015): “Computations of uncertainty mediate acute stress responses in humans”
  - Powers et al. (2017): “Pavlovian conditioning–induced hallucinations result from overweighting of perceptual priors”
- General modelling
  - Wilson and Collins (2019): “Ten simple rules for the computational modeling of behavioral data”
  - Palminteri et al. (2017): “The Importance of Falsification in Computational Cognitive Modeling”