DATA 609 - Final Project

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Textbook Projects from:

Giordano, F. R., Fox, W. P., & Horton, S. B. (2014). ***A first course in mathematical modeling.*** **Australia: Brooks/Cole.**

Textbook Part I:

[Christina's work]

Textbook part II: Chapter 5.3, Project 1

Construct and perform a Monte Carlo simulation of blackjack as per the following:

- Play 12 games (simulations) where each game lasts two decks. When the two decks are out, the round is completed using two fresh decks (that is the last round of the game). Everything is tgen reset for the start of the next game.
- The dealer cannot see the players cards and vice versa.
- The player wins 3 dollars with a winning hand.
- The player loses 2 dollars with a losing hand.
- No money is exchanged if there is no winner.
  - There is no winner when neither has gone bust and they stand at the same amount.
  - If the dealer goes bust, the player automatically wins.
- The dealer strategy is to stand at 17 or above.
- The player strategy is open and can be set as desired.

To enable a more informative simulation, different player strategy are generated: stand at 16 through 20. Simulation sizes of 100, 500, 1,000, and 1,500 are run and the results are compared

Create a two deck game

The below functions create the game of blackjack from a dealt card, to a hand, to a round, and to a full two-deck game. Game results are stored as a vector: 1 = dealer win; 2 = player win; 0 = no winner. The resulting vector is used to calculate player winnings/losses.

```r
# Create a hand for the dealer.
# Return a list containing the hand and remaining deck.
deal_card <- function(d_deck,decks_2){
  flag_last_round <- 0
  if (length(d_deck) == 0) {
    flag_last_round <- 1
    d_deck <- decks_2
  }
  d_card <- sample(d_deck,1,FALSE)
  d_deck <- d_deck [!d_deck %in% d_card | duplicated(d_deck)]
  return(list(d_card,d_deck,flag_last_round))
}

dealer_hand <- function(dh_deck,decks_2) {
  dh_hand <- 0
  dh_last_round_flag <- 0
  dh_count <- 1
  while (sum(dh_hand) < 17) {
    dh_deal <- deal_card(dh_deck,decks_2)
    dh_hand[dh_count] <- dh_deal[[1]]
    dh_deck <- dh_deal[[2]]
    if (dh_deal[[3]] == 1) {dh_last_round_flag <- dh_deal[[3]]}
    dh_count <- dh_count+1 # ace = 1 if hand > 21
    if(sum(dh_hand) > 21) {
      if(11 %in% dh_hand) {
        dh_hand[match(11,dh_hand)] <- 1
      }
    }
  }
  return(list(dh_hand,dh_deck,dh_last_round_flag))
}

# Create player hand
# Returns list containing hand and remaining deck.
player_hand <- function(ph_deck,decks_2,threshold_p) {
  ph_hand <- 0
  ph_count <- 1
  ph_last_round_flag <- 0
  while (sum(ph_hand) < threshold_p) {
    ph_deal <- deal_card(ph_deck,decks_2)
    ph_hand[ph_count] <- ph_deal[[1]]
    ph_deck <- ph_deal[[2]]
    if (ph_deal[[3]] == 1) {ph_last_round_flag <- ph_deal[[3]]}
    ph_count <- ph_count+1 # ace = 1 if hand > 21
    if(sum(ph_hand) > 21) {
      if(11 %in% ph_hand) {
        ph_hand[match(11,ph_hand)] <- 1
      }
    }
  }
  return(list(ph_hand,ph_deck,ph_last_round_flag))
}
```

```r
# Create a game
# Returns list containing player hands and remaining deck.
game <- function(game_deck,decks_2,threshold_p) {
  #dealer hand
  d_result <- dealer_hand(game_deck,decks_2)
  d_hand <- d_result[[1]]
  game_deck <- d_result[[2]]
  last_round_flag_d <- d_result[[3]]
  #player hand
  p_result <- player_hand(game_deck,decks_2,threshold_p)
  p_hand <- p_result[[1]]
  game_deck <- p_result[[2]]
  last_round_flag_p <- p_result[[3]]
  if (last_round_flag_d == 1 || last_round_flag_p == 1) {
    game_last_round_flag <- 1
  } else {
    game_last_round_flag <- 0
  }
  return(list(d_hand,p_hand,game_deck,game_last_round_flag))
}

# Go through two decks- create dealer and player hand each round
# Results returned as list of lists
play_decks_2 <- function(p2d_deck,decks_2,threshold_p) {
  round_counter <- 1
  p2d_last_round_flag <- 0
  result_list.names <- c("Dealer_Hand", "Player_Hand")
  result_list <- vector("list", length(result_list.names))
  names(result_list) <- result_list.names
  while (p2d_last_round_flag != 1) {
    round_result <- game(p2d_deck,decks_2,threshold_p)
    result_list$Dealer_Hand[round_counter] <- list(round_result[[1]])
    result_list$Player_Hand[round_counter] <- list(round_result[[2]])
    p2d_deck <- round_result[[3]]
    p2d_last_round_flag <- round_result[[4]]
    round_counter <- round_counter+1
  }
  return(result_list)
}

# Takes in list containing all hands after above running through two decks.
# Returns vector: 1 = dealer win; 2 = player win; 0 = no winner.
# If dealer goes bust the player automatically wins.
results_vec <- function(results_set) {
  winner <- numeric(length(results_set$Dealer_Hand))
  for (i in 1:length(results_set$Dealer_Hand)) {
    d_Hand <- sum(results_set$Dealer_Hand[[i]])
    p_Hand <- sum(results_set$Player_Hand[[i]])
    if(d_Hand < 22) {
      if(p_Hand < 22) {
        if(d_Hand > p_Hand) {
          winner[i] <- 1
        } else if (d_Hand < p_Hand) {
```

```
            winner[i] <- 2
        } else if (d_Hand == p_Hand) {
            winner[i] <- 0
        }
      } else if(p_Hand >= 22) {
          winner[i] <- 1
      }
    } else if(d_Hand >= 22){
            winner[i] <- 2 # if dealer goes bust player wins
      }
  }
  return(winner)
}

# Calculates the player winnings with assumptions:
# player bets $2/hand
# player wins = player receives $3
# player loss = player loses $2 bet
# tie = $0 exchanged
# dealer goes bust = win for player
calc_player_win <- function(results) {
  winnings <- numeric(length(results))
  for(i in 1:length(results)) {
    if (results[i] == 0) winnings[i] <- 0
    if (results[i] == 1) winnings[i] <- -2
    if (results[i] == 2) winnings[i] <- 3
  }
  return(winnings)
}

# Play a full two deck game, calculate winnings/losses.
# Returns total winnings/loses for the game.
play_deck2 <- function(pd2_deck,threshold_p){
  #create list of results per round after running two decks game
  one_set <- play_decks_2(pd2_deck,pd2_deck,threshold_p)
  #create vector for results of each hand
  outcome_vec <- results_vec(one_set)
  player_winnings <- 0
  player_winnings <- calc_player_win(outcome_vec)
  return(sum(player_winnings))
}

# Create function to run the simulation for two deck games.
# Takes two decks and number of simulations as params.
# Each run uses player strategies (stand at 16 to 20); dealer stands at 17.
# Returns dataframe containing winnings/losses per strategy per obs.
build_output <- function(out_deck, runs) {
  count_obs <- numeric(runs)
  vec16 <- numeric(runs)
  vec17 <- numeric(runs)
  vec18 <- numeric(runs)
  vec19 <- numeric(runs)
  vec20 <- numeric(runs)
```

```
  for (i in 1:runs){
    count_obs[i] <- i
    vec16[i] <- play_deck2(out_deck,16)
    vec17[i] <- play_deck2(out_deck,17)
    vec18[i] <- play_deck2(out_deck,18)
    vec19[i] <- play_deck2(out_deck,19)
    vec20[i] <- play_deck2(out_deck,20)
  }
  output <- data.frame("count_obs"= count_obs,
                       "stand_16"=vec16,"stand_17"=vec17,
                       "stand_18"=vec18,"stand_19"=vec19,
                       "stand_20"=vec20)
  return(output)
}
```

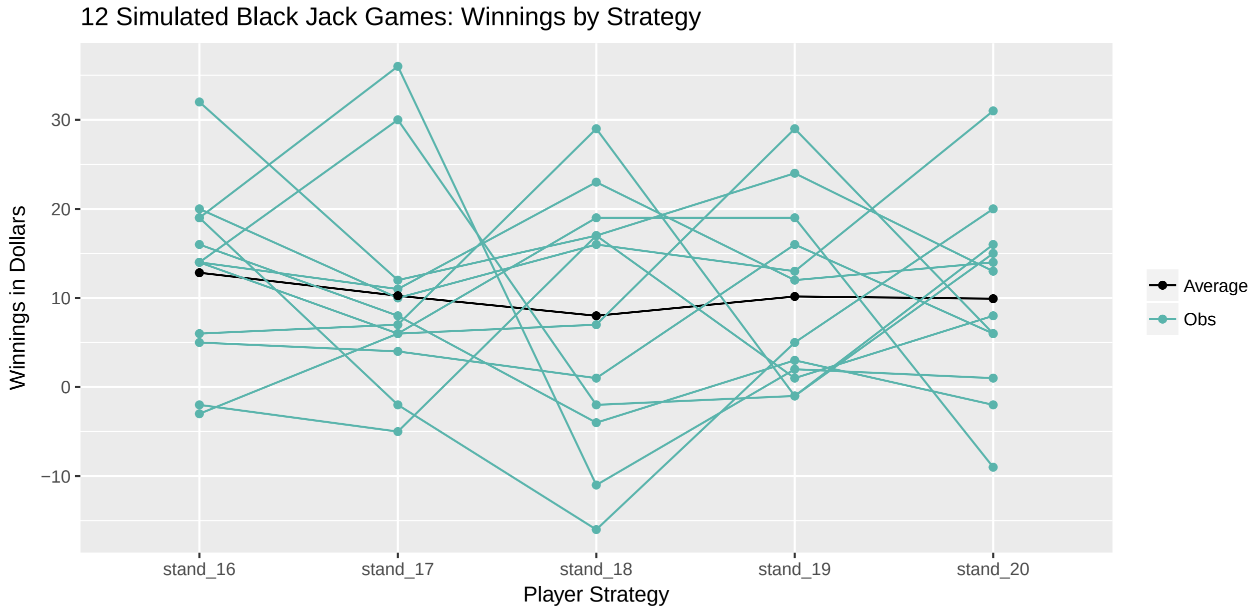
12 Simulations

```
set.seed(2345)
# Create set of two decks, simulate 12 two-deck games; run sim with diff strategies
# vector for two decks
two_deck <- c(2,2,2,2,3,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6,7,7,7,7,8,8,8,8,
              9,9,9,9,10,10,10,10,10,10,10,10,10,10,10,10,10,10,10,10,
              11,11,11,11,2,2,2,2,3,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6,
              7,7,7,7,8,8,8,8,9,9,9,9,10,10,10,10,10,10,10,10,10,10,10,
              10,10,10,10,10,11,11,11,11)

# 12 games per strategy
n12_output <- build_output(two_deck,12)
n12_output_avg <- c("stand_16"=mean(n12_output$stand_16),
                    "stand_17"=mean(n12_output$stand_17),
                    "stand_18"=mean(n12_output$stand_18),
                    "stand_19"=mean(n12_output$stand_19),
                    "stand_20"=mean(n12_output$stand_20))
round(n12_output_avg,2)
```

```
## stand_16 stand_17 stand_18 stand_19 stand_20
##    12.83    10.25     8.00    10.17     9.92
```

If you were to only look at the average winnings across 12 games, it would appear that standing at 16 is the best option when the dealer stands at 17. The below plot though illustrates just how much variation there is in results. Simulations of 100, 500, 1000, and 1,500 rounds are made below to improve decision-making.



Increasing the Number of Simulations

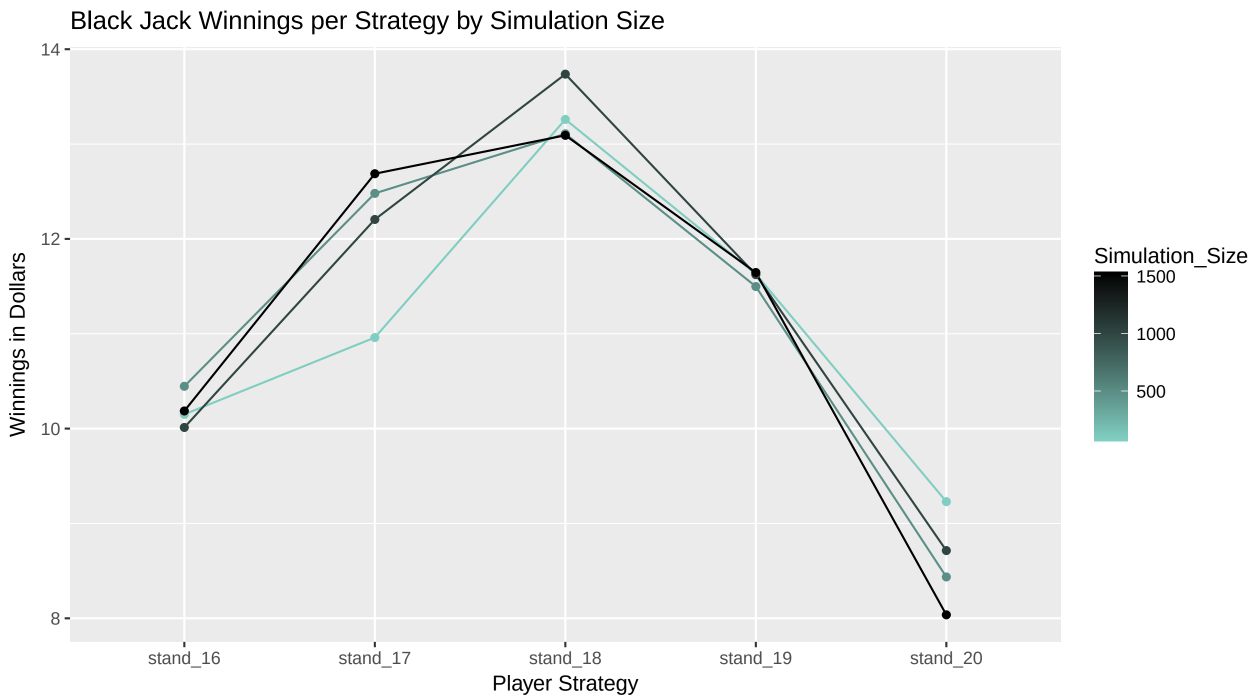
```
set.seed(2345)
# 100 simulations
n100_output <- build_output(two_deck,100)
# average winnings/loses
n100_output_avg <- c("stand_16"=mean(n100_output$stand_16),
                     "stand_17"=mean(n100_output$stand_17),
                     "stand_18"=mean(n100_output$stand_18),
                     "stand_19"=mean(n100_output$stand_19),
                     "stand_20"=mean(n100_output$stand_20))

# 500 simulations
n500_output <- build_output(two_deck,500)
# average winnings/loses
n500_output_avg <- c("stand_16"=mean(n500_output$stand_16),
                     "stand_17"=mean(n500_output$stand_17),
                     "stand_18"=mean(n500_output$stand_18),
                     "stand_19"=mean(n500_output$stand_19),
                     "stand_20"=mean(n500_output$stand_20))

# 1000 simulations
n1000_output <- build_output(two_deck,1000)
# average winnings/loses
n1000_output_avg <- c("stand_16"=mean(n1000_output$stand_16),
                      "stand_17"=mean(n1000_output$stand_17),
                      "stand_18"=mean(n1000_output$stand_18),
                      "stand_19"=mean(n1000_output$stand_19),
                      "stand_20"=mean(n1000_output$stand_20))

# 1500 simulations
n1500_output <- build_output(two_deck,1500)
# average winnings/loses
```

```r
n1500_output_avg <- c("stand_16"=mean(n1500_output$stand_16),
                      "stand_17"=mean(n1500_output$stand_17),
                      "stand_18"=mean(n1500_output$stand_18),
                      "stand_19"=mean(n1500_output$stand_19),
                      "stand_20"=mean(n1500_output$stand_20))
```



As the number of simulations increases, the results converge. It appears to be that standing at 18 is the best option, as standing at 19 leaves the player with fewer winnings. Similarly, standing at 16, 17, or 20 results in fewer winnings overall. Therefore if the dealer stands at 17, the player should stand at 18.

Textbook Part III:

[Christophe's work]