IS 604 – Homework #2

1. Suppose that X is a discrete random variable having probability function $\Pr(X = k) = ck^2$ for $k = 1,2,3$. Find c, $\Pr(X \leq 2)$, E[X], and Var(X).

2. Suppose that X is a continuous random variable having p.d.f. $f(x) = cx^2$ for $1 \leq x \leq 2$. Find c, $\Pr(X \geq 1)$, E[X], and Var(X).

3. Suppose that X and Y are jointly continuous random variables with

$$\begin{cases} y - x & \text{for } 0 < x < 1 \text{ and } 1 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

   a. Compute and plot $f_X(x)$ and $f_Y(y)$.
   b. Are X and Y independent?
   c. Compute $F_X(x)$ and $F_Y(y)$.
   d. Compute E[X], Var(X), E[Y], Var(Y), Cov(X,Y), and Corr(X,Y).

4. Suppose that the following 10 observations come from some distribution (not highly skewed) with unknown mean μ.

   7.3 6.1 3.8 8.4 6.9 7.1 5.3 8.2 4.9 5.8

   Compute $\bar{X}$, S^2, and an approximate 95% confidence interval for μ.

5. A random variable X has the *memoryless property* if, for all s,t > 0,

$$\Pr(X > t + s | X > t) = \Pr(X > s)$$

   Show that the exponential distribution has the memoryless property.

6. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. Exp(λ=1). Use the Central Limit Theorem to find the approximate value of $\Pr(100 \leq \sum_{i=1}^{100} X_i \leq 110)$.

7. *DES* textbook problems: 5.13, 5.14, 5.39.