

# Homework - Ch. 13

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## 1

*What conditions might argue for allowing a temporarily out of stock policy?*

Extremely high storage costs would act as a disincentive for storing the stock at high enough levels to prevent stock-outs.

*What effect does this policy have on storage costs?*

It reduces storage costs, especially if there is a high level of variability in demand that would necessitate potentially storing a lot more stock than you would actually sell in short time intervals.

*Should costs be assigned to stock-outs? Why?*

Stock-outs will result fewer orders being filled, since customers may seek out a competitor or substitute instead of waiting for the restock.

*How would you make such an assignment?*

There would need to be a loss-factor I would call ‘missed sales = m’. The loss-factor m would be a cost like s and d.

*What assumptions are implied by the model in Fig. 13.7?*

Stock-outs will be remedied at a certain level and be built back up

*Suppose a “loss of goodwill cost” of w dollars per day is assigned to each stock-out. Compute the optimal order quantity Q and interpret your model.*

The optimal quantity is when the cost from stock-outs is less than the storage costs of keeping enough stock to prevent stockouts. The delivery cost is constant in either scenario. Therefore the model is:

$$\text{minimize}_c = \frac{d}{t} + \frac{srtm}{2}$$

subject to:

$$m < s$$

## 2

$$f(x, y) = 3x^2 + 6xy + 7y^2 - 2x - 4y$$

$$\frac{\partial}{\partial x} = 6x + 6y + 0 - 2 - 0 \quad \frac{\partial}{\partial y} = 0 + 6x + 14y - 0 + 4 \\ = 6x + 6y - 2 \quad = 6x + 14y + 4$$

$$6x + 6y - 2 = 6x + 14y + 4 \\ -6x \qquad \qquad -6x$$

$$6y - 2 = 14y + 4 \\ -6y \qquad \qquad -6y$$

$$-2 = 8y + 4 \\ -4 \qquad \qquad -4$$

$$-6 = 8y \\ -\frac{3}{4} = y$$

$$0 = 6x + 14(-\frac{3}{4}) + 4$$

$$0 = 6x + (-10.5) + 4$$

$$0 = 6x - 6.5$$

$$6.5 = 6x$$

$$1.08\bar{3} = x$$

$$\begin{aligned} \min f(x, y) &= 3(1.083)^2 + 6(1.083)(-\frac{3}{4}) + 7(-\frac{3}{4})^2 - 2(1.083) + 4(-\frac{3}{4}) \\ &= 3.52 + (-4.87) + (-3.94) + (-2.17) + (-3) \\ &= -2.58 \end{aligned}$$

```
min.soln <- function(x,y){  
  min <- 3*x^2 + 6*x*y + 7*y^2 - 2*x + 4*y  
  return(min)  
}  
  
min.soln(1.083, -0.75)
```

```
## [1] -2.583333
```

3

$$\text{Max: } T(x, y, z) = 8x^2 + 4yz - 16z + 600$$

$$\text{subject to: } g(x, y, z) = 4x^2 + y^2 + 4z^2 - 16 = 0$$

$$= 4x^2 + y^2 + 4z^2 - 16 = 0$$

$$F(x, y, z, \lambda) = 8x^2 + 4yz - 16z + 600 + \lambda [4x^2 + y^2 + 4z^2 - 16]$$

$$= 8x^2 + 4yz - 16z + 600 + 4x^2\lambda + y^2\lambda + 4z^2\lambda - 16\lambda$$

$$\frac{\partial F}{\partial x} = 16x + 8x\lambda = 0 \quad \left\{ \begin{array}{l} 8x\lambda = -16x \\ \lambda = -2 \end{array} \right.$$

$$\frac{\partial F}{\partial y} = 4z + 2y\lambda = 0 \quad \left\{ \begin{array}{l} 4z + 2y(-2) = 0 \\ 4z - 4y = 0 \\ 4z = 4y \end{array} \right.$$

$$\frac{\partial F}{\partial z} = 4y - 16 + 8z\lambda = 0 \quad \left\{ \begin{array}{l} 4y - 16 + 8z(-2) = 0 \\ 4y - 16 - 16z = 0 \\ 4y = 16 + 16z \end{array} \right.$$

$$\frac{\partial F}{\partial \lambda} = 4x^2 + y^2 + 4z^2 - 16 = 0 \quad \left\{ \begin{array}{l} 4y - 16 + 8z = 0 \\ 4y = 16 + 16z \end{array} \right.$$

$$4x^2 + y^2 + 4z^2 - 16 = 0$$

$$4x^2 + \left(\frac{4}{3}\right)^2 + 4\left(-\frac{4}{3}\right)^2 - 16 = 0$$

$$4x^2 + \frac{16}{9} + \frac{64}{9} - \frac{144}{9} = 0$$

$$4x^2 + \frac{-64}{9} = 0$$

$$\frac{4x^2}{\frac{1}{4}} = \frac{64}{\frac{9}{4}}$$

$$x^2 = \frac{64}{36} = \frac{16}{9}$$

$$\sqrt{x^2} = \sqrt{\frac{16}{9}}$$

$$x = \frac{4}{3}$$

$$4y + 8z = 16$$

$$4y + 8y(-2) = 16$$

$$4y - 16y = 16$$

$$-12y = 16$$

$$y = -\frac{16}{12} = -\frac{4}{3}$$

$$\text{and } z = -\frac{4}{3}$$

**TIVIX**

The hottest point on the elliptical orbit is:  $(\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3})$

## 4

*“...if a quota for the season is established based on the estimated growth rate, then the fish population can be maintained, increased, or decreased as desired.”*

*Discuss the difficulties in determining reproduction models precise enough to be used in this advantage.*

So many factors affect the reproduction models: - food supply

- competition with other species
- climate
- parasites and predators at all stages of the lifecycle

That changes in these factors may result in an unsteady growth rate.

The reproduction models are essential to the stability of the system. If the they are imprecise, the quota may be set too high and the population may crash in a short amount of time.

*How would you estimate the population level?*

Ecological population sampling is a entire subject unto itself. I have actually done fish population sampling in rivers before. Generally, the estimate method is: - select a standard sampling area size

- randomly select a location to sample, or visit sites which have historic data
- establish standardized sampling methods
- sample the area for a standardized time limit. This can be done with nets in deep water, or electro-shocking in shallow, fresh water
- identify, count, and measure the fish sampled
- use previous studies of estimated error rates for the sampling method (large predator fish are typically harder to catch, therefore their error rate is higher than slower fish)
- extrapolate from the sampling areas based on habitate, water volume, etc. to the entire habitate area to estimate the population

Over time:

- resample during different seasons depending on lifecycle
- reample on multi-year time scales

*What are the disadvantages of varying the quota from year to year? What are the political challenges?*

If the quota varies from year to year, then the fleet of fishermen do not know whether they should invest in more equipment and employees or contract. Fishing can be capital intensive, so the fleet would be unwilling to buy things like boats if the quota could render them worthless. The varied supply may result in a volatile pricing, which could affect consumer demand and the favorability of the catch.

Politically, a variable quota would be very difficult to implement because the fishing industry would find it very difficult to make investment decisions.