ASSIGNMENT 4

IS 605 FUNDAMENTALS OF COMPUTATIONAL MATHEMATICS

1. Problem Set 1

In this problem, we'll verify using R that SVD and Eigenvalues are related as worked out in the weekly module. Given a 3×3 matrix $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 5 \\ 1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \tag{1}$$

write code in R to compute $\mathbf{X} = \mathbf{A}\mathbf{A}^{\mathbf{T}}$ and $\mathbf{Y} = \mathbf{A}^{\mathbf{T}}\mathbf{A}$. Then, compute the eigenvalues and eigenvectors of $\mathbf{X}$ and $\mathbf{Y}$ using the built-in commans in R.

Then, compute the left-singular, singular values, and right-singular vectors of $\mathbf{A}$ using the *svd* command. Examine the two sets of singular vectors and show that they are indeed eigenvectors of $\mathbf{X}$ and $\mathbf{Y}$. In addition, the non-zero eigenvalues of both $\mathbf{X}$ and $\mathbf{Y}$ are the same and are squares of the non-zero singular values of $\mathbf{A}$.

Your code should compute all these vectors and scalars and store them in variables. Please add enough comments in your code to show me how to interpret your steps.

2. Problem Set 2

Using the procedure outlined in section 1 of the weekly handout, write a function to compute the inverse of a well-conditioned full-rank square matrix using co-factors. In order to compute the co-factors, you may use built-in R commands to compute the determinant. Your function should have the following signature:

```
B = myinverse(A)
```

where $\mathbf{A}$ is a matrix and $\mathbf{B}$ is its inverse and $\mathbf{A}\times\mathbf{B} = \mathbf{I}$. The off-diagonal elements of $\mathbf{I}$ should be close to zero, if not zero. Likewise, the diagonal elements should be close to 1, if not 1. Small numerical precision errors are acceptable but the function *myinverse* should be correct and must use co-factors and determinant of $\mathbf{A}$ to compute the inverse.

Please submit PS1 and PS2 in an R-markdown document with your first initial and last name.