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CSC I6715
Assignment 1

Writing Assignment

1. How does an image change if the focal length of a pinhole camera is varied?

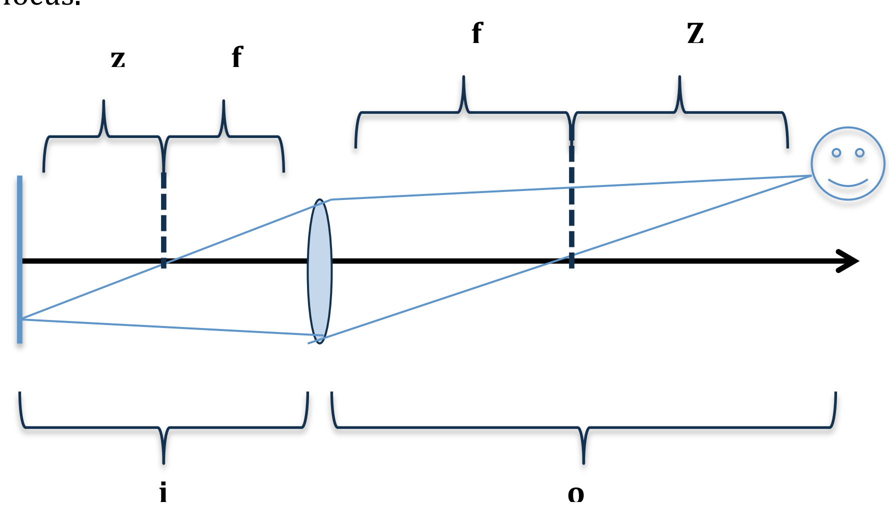
The size and shape of the image changes linearly with the focal length. The image is therefore enlarged as the focal length increases. As the focal length increases the field of view decreases, and as the focal length decreases the field of view increases.

2. Give an intuitive explanation of the reason why a pinhole camera has infinite depth of field.

Depth of field can be understood as the distance between the closest and furthest points in an image that are in good focus – meaning where they are all equally sharp regardless of whether they are close or far away. Since a pinhole camera has a hole as small as possible then only one ray can pass through and therefore be in focus (therefore an infinite depth of field).

3. In the thins lens model, $\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$, there are 3 variables, the focal length f, the object distance o, and the image distance i. If we define Z = o – f , and z = i – f, please write a few words to describe the physical meanings of Z and z, and then prove that Z*z = f*f given that $\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$.

According to thins lens model, **o** represents the distance from the center of the lens to a point on an object, and **i** represents the distance behind the lens at which the rays of the object will be in focus.



Therefore, the physical meaning of **z** is the distance past the focal length at which the rays of the object will be in focus, and the physical meaning of **Z** is the distance past the focal length to a point on an object.

Given that Z = o – f, and z = i – f, we can rewrite the thin lens model equation $\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$ as

$$\frac{1}{Z+f}+\frac{1}{z+f}=\frac{1}{f} \quad \rightarrow \quad \frac{(z+f)+(Z+f)}{(Z+f)(z+f)}=\frac{1}{f}$$

$$\rightarrow f[(z+f)+(Z+f)]=(Z+f)(z+f)$$

$$\rightarrow fz+f^2+fZ+f^2=Zz+zf+Zf+f^2$$

$$\rightarrow f^2=Zz$$

Therefore, it can be proven that Z*z = f*f.

4. Prove that, in the pinhole camera model, three collinear points in 3D space are imaged into three collinear points on the image plane.

Three points are collinear in 3D space if the world coordinates (X, Y, Z) for each point form a matrix whose determinant is 0. If we have 3 collinear world coordinates: (X1, Y1, Z1), (X2, Y2, Z2), (X3, Y3, Z3), the determinant is:

$$Det\begin{vmatrix} X1 & Y1 & Z1 \\ X2 & Y2 & Z2 \\ X3 & Y3 & Z3 \end{vmatrix}=0$$

Using the perspective projection formulas, the image coordinates can be calculated (x1,y1,f), (x2,y2,f), (x3,y3,f).

$$x=f\frac{X}{Z}, y=f\frac{Y}{Z}$$

$$Det\begin{vmatrix} f\frac{X1}{Z1} & f\frac{Y1}{Z1} & f\frac{Z1}{Z1} \\ f\frac{X2}{Z2} & f\frac{Y2}{Z2} & f\frac{Z1}{Z1} \\ f\frac{X3}{Z3} & f\frac{Y3}{Z3} & f\frac{Z1}{Z1} \end{vmatrix} = Det\begin{vmatrix} f\frac{X1}{Z1} & f\frac{Y1}{Z1} & f \\ f\frac{X2}{Z2} & f\frac{Y2}{Z2} & f \\ f\frac{X3}{Z3} & f\frac{Y3}{Z3} & f \end{vmatrix}=0 \rightarrow Det\begin{vmatrix} x1 & y1 & f \\ x2 & y2 & f \\ x3 & y3 & f \end{vmatrix}=0$$

We see therefore, that by calculating the determinant of the image coordinates, which is also zero and therefore if 3 points in 3D are collinear, they are also collinear in the image plane.

Programming Assignment

1. Image Formation

a) A color image C1(x,y) = (R(x,y), G(x,y), B(x,y)) is read using the *imread* function of Matlab.

b) The three separate bands of the color image can be displayed by generating three "color" images where each image is filled with each band.

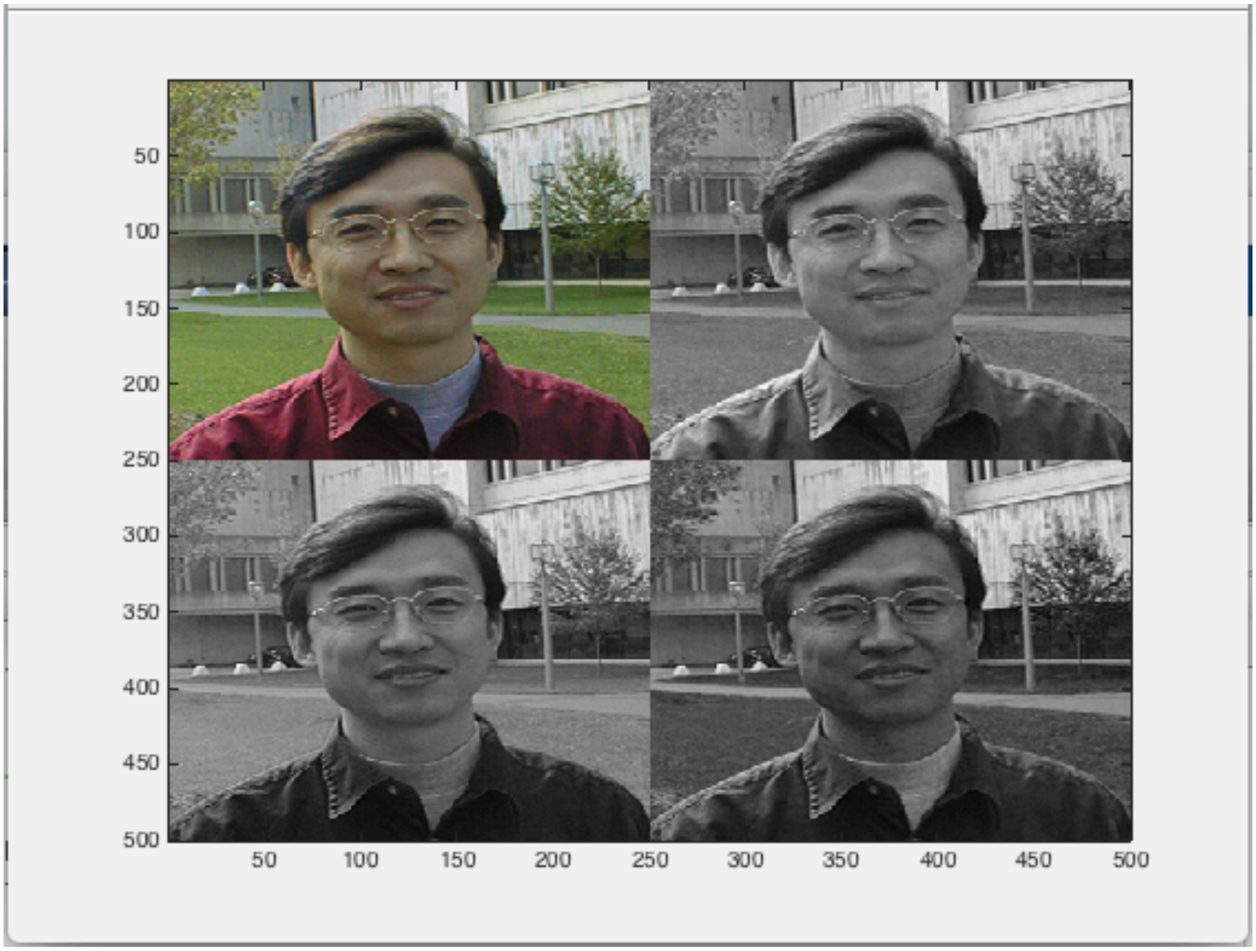


Figure 1. Original color image and R, G, and B band images

c) An intensity image, also known as grayscale, is an image where each pixel carries only intensity information. This grayscale image is stored with 8 bits per pixel, which allows for 256 intensities of gray (Black-0 to White-255).

The simplest method for finding the intensity image is to simply average the 3 channels:

$$I = \frac{Red + Green + Blue}{3}$$

We can see the result from using this method to find the intensity image on *Figure 2* below. Although averaging is simple and produces a reasonably good grayscale image as can be seen in the figure, it does a poor job representing shades of gray relative to how we perceive luminosity (brightness).

Another method for finding the intensity image is by the formula:

$$I = 0.299 * red + 0.587 * green + 0.114 * blue$$

which is suggested by the National Television Systems Committee for converting color feeds to black and white television sets. This formula weighs each color differently, based on how the human eye perceives it. For example, the human eye is more sensitive to green, which is why green is weighed the most in the NTSC formula. We can actually observe this difference in perception of green in the intensity image in *Figure 3* where the grass appears brighter.

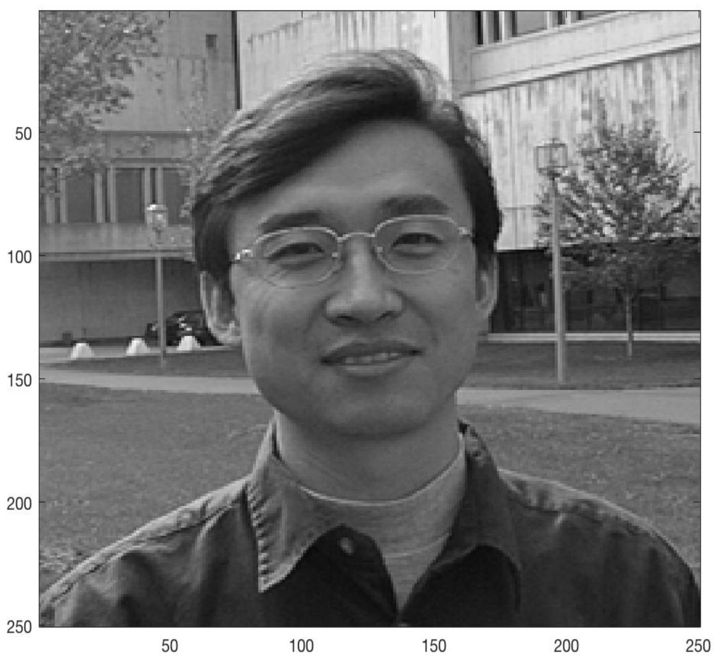


Figure 2. Intensity image calculated using average

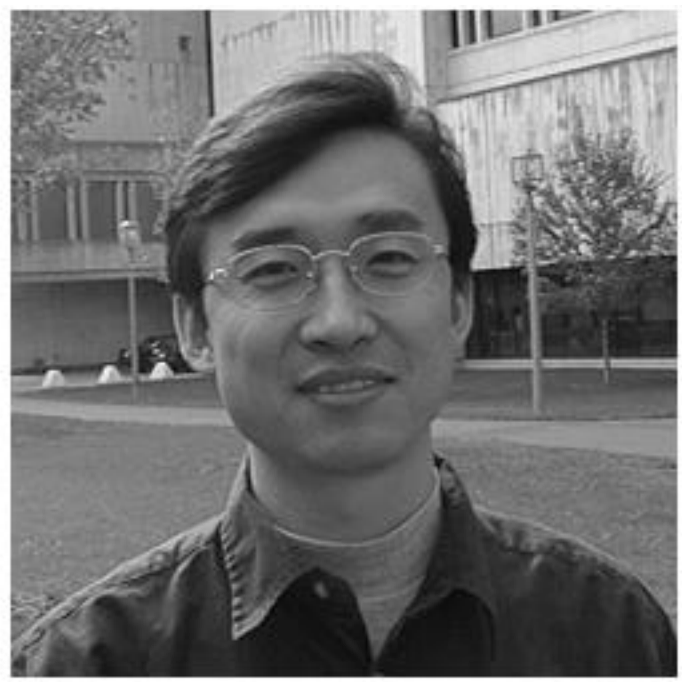


Figure 3. Intensity image calculated using NTSC formula.

d) An image can be quantized into k uniform levels. The procedure for quantization is:

Scale = 256/levels
Output = Scale * floor (Input/Scale)

In this case, the intensity image is quantized into levels k = 4, 16, 32, 64. Image quantization is a compression technique where a range of values is assigned to a quantum value. In uniform quantization in order to determine a mapping to a K discrete levels you divide the range (0-255) into K equal sized intervals. In *Figure 4* below, it can be observed that as the K levels increase the quantized images start to resemble more the original intensity image. Because the human eye brightness resolution is limited, we can perceive an image quantized to ¼ the levels as equivalent to the original.

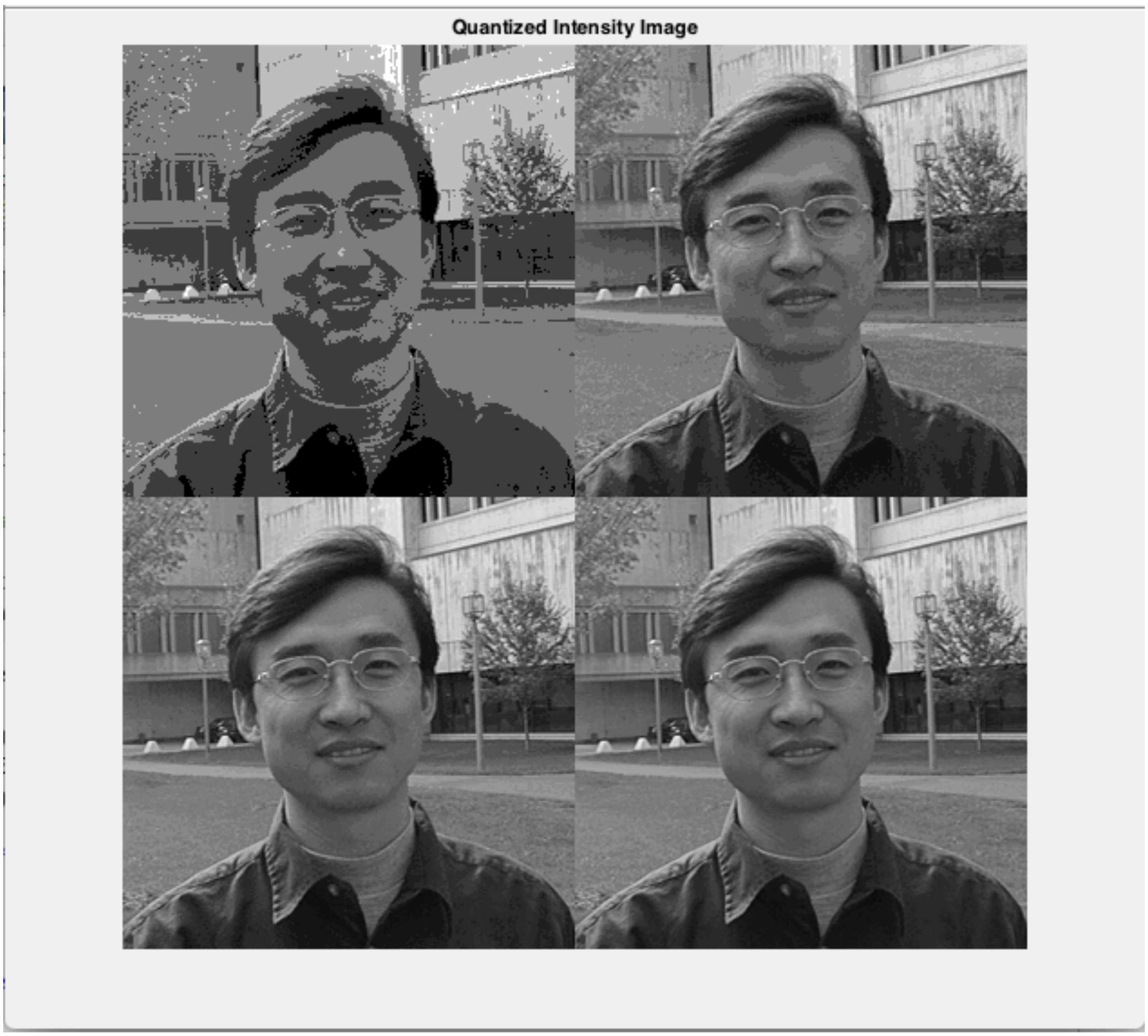


Figure 4. Quantized intensity image, k = [4 16; 32 64]

e) In this case, the color image is quantized. Color quantization reduces the number of colors used in an image. This allows for the image to be displayed on devices that support a limited amount of colors for efficient compression. In the figure below it can be observed that as the K values increase, you can start seeing more shades of color in the image. At k=64, the image looks identical to the original image.

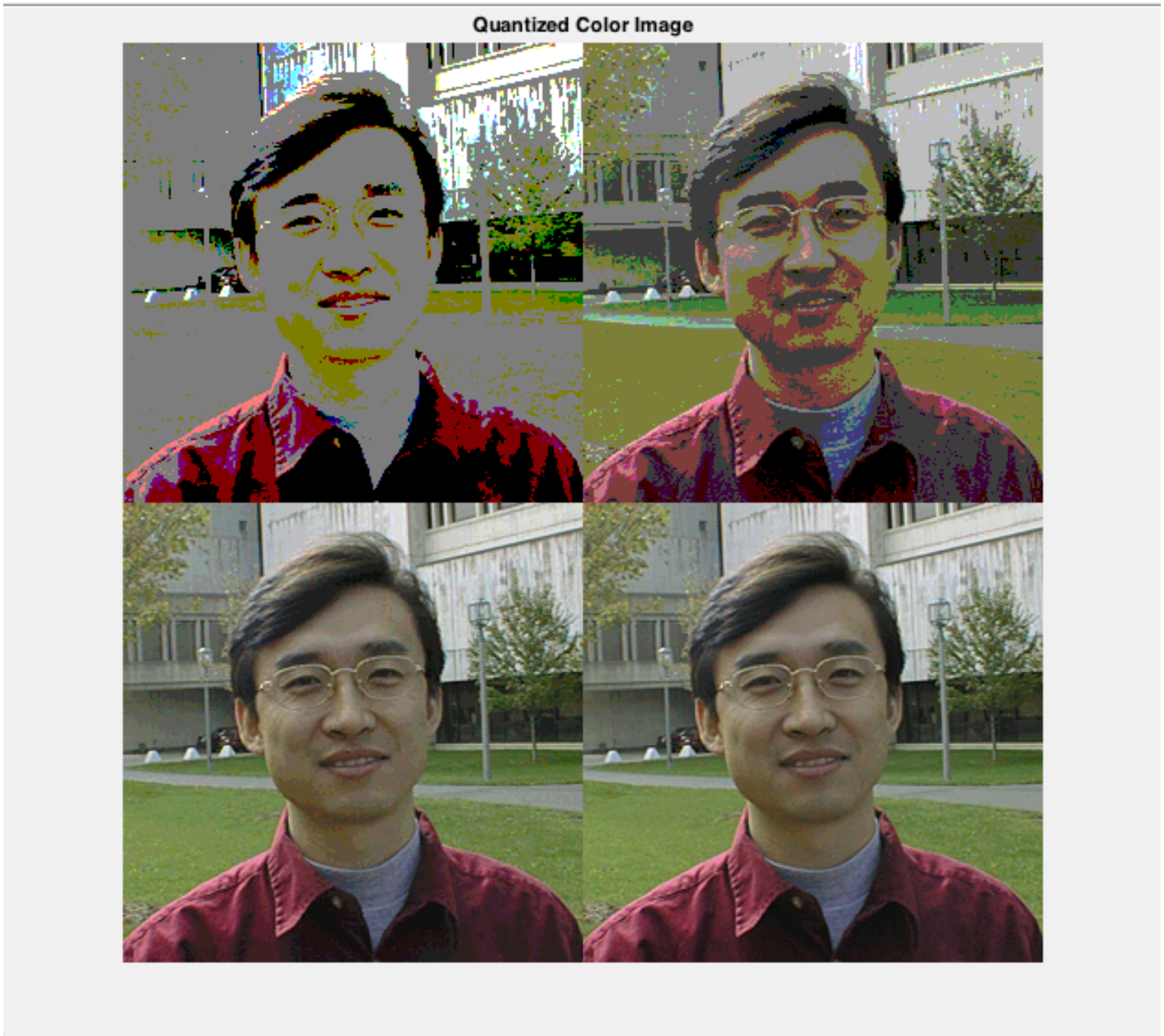


Figure 5. Color quantization

f) In this case, the color image is quantized with a logarithmic function. Logarithmic quantization enhances detail in the low signal values but loses detail in the high signal values. In this figure the formula I' = cln(I+1) is used. The constant c= 8 is chosen as appropriate for scaling the quantization level, and values of I =2,4,8,256. It can be observed that for low values of I such as 2,4, 8 the quantized image is identical to the original image in comparison to uniform quantization where the image at such low levels was of a very low resolution. It can be observed that in very high values the image starts to lose detail and become too bright.

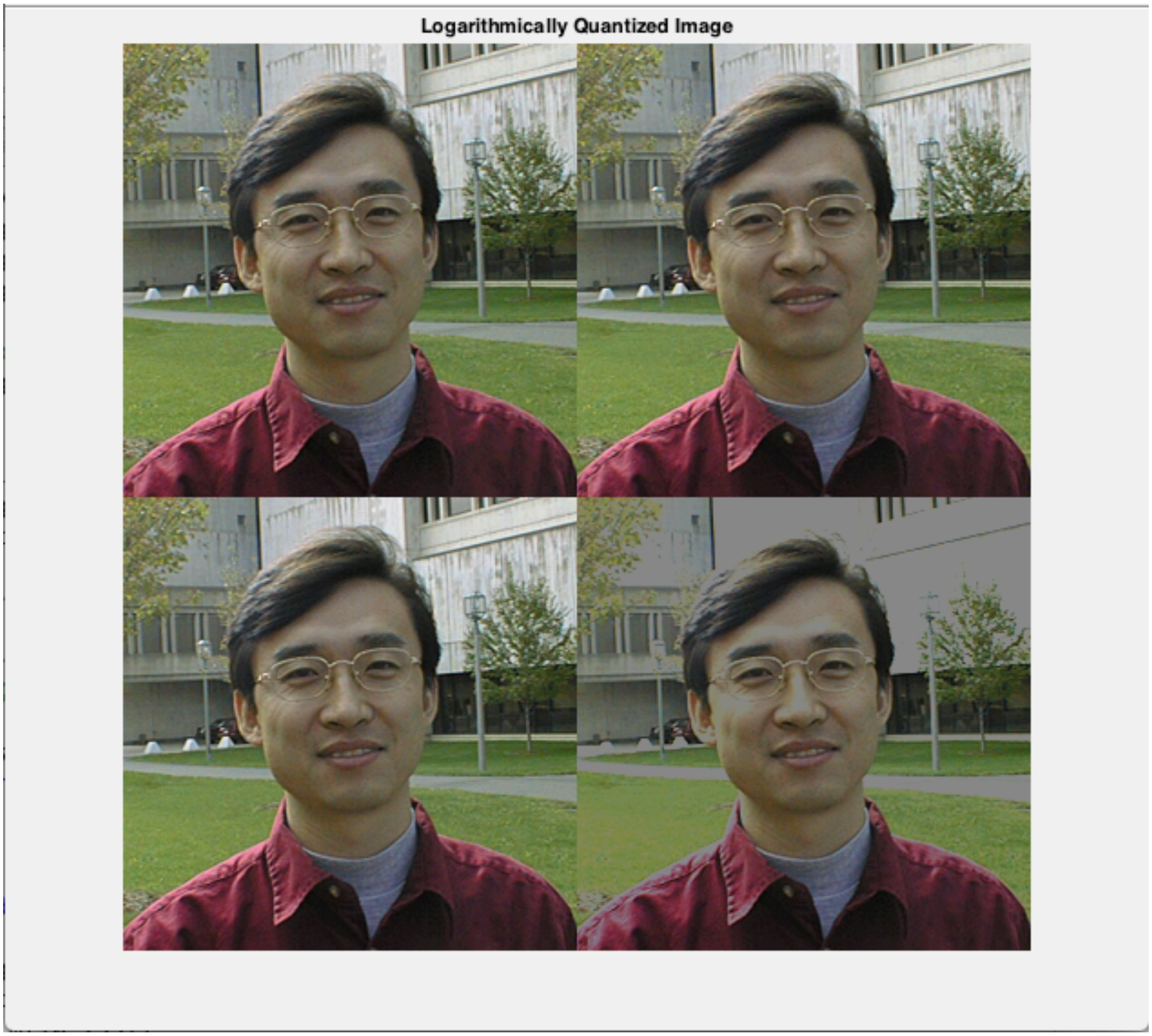


Figure 6. Color quantization and logarithmic