

# Intro to Search and Matching in the Labor Market

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# Outline for the next weeks

- Week 1
  - Stigler (1961) & McCall (1970)
  - The wage distribution
- Week 2
  - Diamond, Mortensen, Pissarides
  - Labor Supply and Data
- Week 3
  - Structural Estimation: MLE
  - Structural Estimation: GMM
- Week 4
  - Structural Estimation: SMM
  - Structural Estimation: Non-Parametric
- Week 5
  - Structural Estimation: EM Algorithm
  - Spillover or Directed Search

# Outline for the next weeks

- What you need
  - Matlab: recommended, this is what I will use
  - Python
  - R
  - Julia
- Assignment 1: **Due 8 December 2025**
  - recommend you work on this throughout the course
  - I will let you know when we have covered what you need to know for each question
  - there will be some time allocated during the following lecture to ask any questions

# Two types of labor market models

- Walrasian Markets
  - wages are often marginal product
  - people don't need to look for jobs
- Market with frictions
  - models of unemployment
  - understanding how people search for jobs
  - understanding how people flow between E and U

# What are Frictions?

- **Search Frictions:** it takes time or money (or both) for buyers and seller to find each other
- **Matching Frictions:** when buyers and sellers meet they may not be a good, or the best match

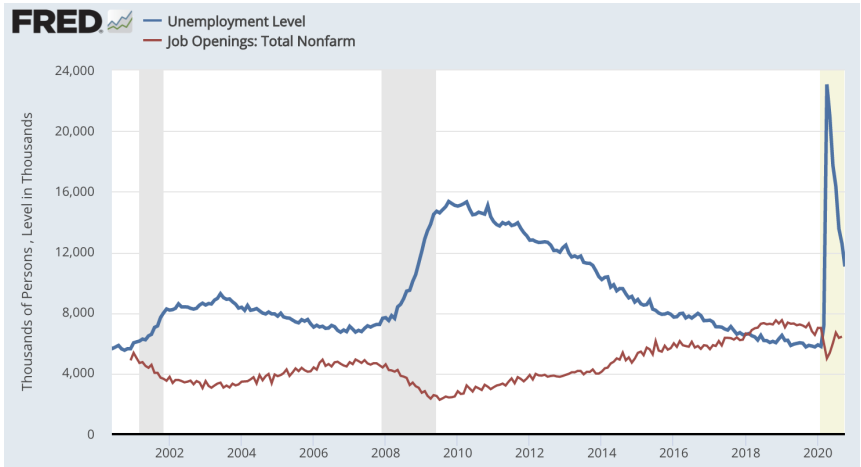
# What are some markets with frictions?

- **Labor Market:** workers are looking for jobs, it takes time to apply to jobs. Firms are looking for workers, it costs money to interview people.
- **Housing Market:** buyers are looking for homes
- **Marriage Market:** everyone's looking for someone
- **Asset Market:** Over-the-counter markets for non-routine financial assets

# Walrasian Model does not work with frictions

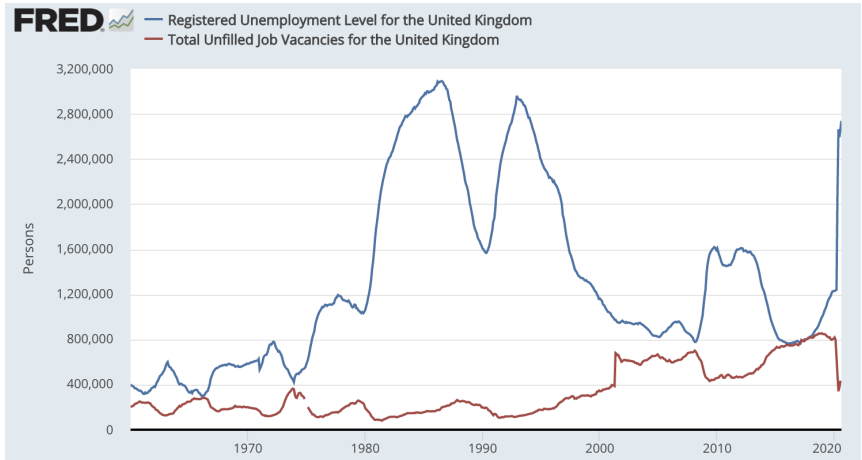
- From a Walrasian perspective
  - Excess demand: vacant jobs
  - Excess supply: unemployed workers
  - Unemployment explained by wages being too high to clear the market
  - Vacancies explained by wages being too low to clear the market
- How can excess demand and excess supply coexist?

# Labor Market in the US





# Labor Market in the UK



# How do we model markets with frictions

- the individual's decision problem:
  - how to search for a job, search effort
  - which jobs to accept
- the firm's decision problem:
  - how many workers they need
  - how much to pay
- an equilibrium
  - number of unemployed and vacancies
  - a wage distribution

# A Short Chronological Outline

- **One-sided Search:** individual decision problem, optimal stopping problem.
  - McCall (1970)
- **1-st Generation Equilibrium Search:** random search models with wage posting. Consider workers decisions of accepting jobs and firms decisions on wages.
- **Matching and Bargaining:** Diamond, Mortensen, Pissarides (DMP). Also take into consideration the firms decision to post vacancies.
- **Directed (Competitive):** Workers decide *which* jobs to apply to.

# Stigler (1961): The Economics of Information

- **Observation:** Prices of homogenous goods vary across seller
  - Chevrolets
  - Coal
- **Theory:** People must visit multiple sellers to get the best price, people search
- **Question:** How do you decide how many seller to visit before you know enough of the price distribution to buy the good?
  - How many wage offers should you get before you take the best job?

## Example

- Suppose there are two prices: \$2 and \$3
- Seller are split equally between the prices

Number of Prices Sampled	Probability of Minimum Price of		Expected Minimum Price
	\$2	\$3	

- How many prices should you sample if it's costless to search?
- How many prices should you sample if it's costly to search?

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1	0.5	0.5	2.5
2	0.75	0.25	2.25

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## Example

- Suppose there are two prices: \$2 and \$3
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1	0.5	0.5	2.5
2	0.75	0.25	2.25
3	0.875	0.125	2.125

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## Example

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3	0.875	0.125	2.125
$\infty$	1	0	2

- How many prices should you sample if it's costless to search?
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# Stigler (1961): The Economics of Information

- There exists some price distribution  $F(p)$
- cost of making one draw:  $c$
- First what's the probability that  $p$  is the minimum of  $n$  draws

$$\begin{aligned}H(p) &= P(\min_i X_i < p) = 1 - P(\min_i X_i > p) \\&= 1 - [1 - F(p)]^n\end{aligned}$$

$$h(p) = n[1 - F(p)]^{n-1}f(p)$$

- Let  $M_n$  be the expected value of the min value of  $n$  samples

$$M_n = n \int_0^{\infty} p[1 - F(p)]^{n-1}f(p) dp$$

# Stigler (1961): The Economics of Information

- The gain from drawing one more sample

$$\begin{aligned} G_n &= M_{n-1} - M_n \\ &= (n-1) \int_0^\infty p[1-F(p)]^{n-2} f(p) dp - n \int_0^\infty p[1-F(p)]^{n-1} f(p) dp \\ &= \int_0^\infty p[1-F(p)]^{n-1} f(p) [nF(p) - 1] dp \end{aligned}$$

- $G_n$  is decreasing in  $n \Rightarrow \lim_{n \rightarrow \infty} G_n = 0$
- Optimal number of draws:  $n$  s.t.  $G_{n+1} < c < G_n$

# McCall Model

- **Environment:**
  - workers search for jobs, infinitely lived, discount at  $\beta$
  - cost of search each period:  $\kappa$
  - while unemployed worker gets  $c$
  - if searching, each period they draw an offer from  $F(w)$
  - if they accept the job, it lasts forever

# McCall Model

- **Objective:**

- maximize expected discounted earnings

$$\mathbb{E} \sum_0^{\infty} \beta^t Y_t$$

- income each period depends on employment state

$$Y_t = \begin{cases} c - \kappa & \text{if unemployed} \\ w & \text{if employed} \end{cases}$$

- **Trade off:**

- Waiting too long for a good offer is costly
- Accepting too early is costly, since better offers might arrive in the future

# McCall Model

- **Solution:**

- optimal stopping problem
- the reservation wage:  $w_R$

$w \geq w_R \Rightarrow$  accept the job

$w < w_R \Rightarrow$  keep searching

- How does this relate to Stigler's optimal number of draws problem?

# McCall Model

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  - **Stigler:**  $n$  determines the expected maximum price we are going to pay, **non-sequential search**

# McCall Model

- **Solution:**

- optimal stopping problem
- the reservation wage:  $w_R$

$w \geq w_R \Rightarrow$  accept the job

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- How does this relate to Stigler's optimal number of draws problem?
  - **Stigler:**  $n$  determines the expected maximum price we are going to pay, **non-sequential search**
  - **Here:**  $w_R$  determines the expected time of unemployment, how many periods on average we have to wait before we get an acceptable wage, **sequential search**



# McCall Model - Solving Numerically

- With an offer of  $w$  in hand worker can
  - accept the job and get

$$\frac{w}{1 - \beta}$$

- reject the job and get

$$c + \beta[\text{get new offer tomorrow}]$$

- Recursive formulation of the objective:

$$V(w) = -\kappa + \max \left\{ \frac{w}{1 - \beta}, c + \beta \int V(w) dF(w) \right\}$$

# McCall Model - Solving Numerically

- Policy Function

- Let  $\sigma(w)$  be the policy function,  $\sigma(w) = 1$  if we accept the job

$$\frac{w}{1 - \beta} > c + \beta \int V(w) dF(w)$$

$$w > (1 - \beta) \left[ c + \beta \int V(w) dF(w) \right]$$

- This condition depends on the value function!
- Solve value function numerically: **QuantEcon**

# McCall Model

- Now that we have  $V(w)$  we can solve for

$$\bar{V} = c + \beta \int V(w) dF(w)$$

where  $\bar{V}$  is a constant

- The reservation wage makes us indifferent

$$\frac{w_R}{1 - \beta} = \bar{V}$$

- $w/(1 - \beta)$  is an increasing function of  $\beta$  and  $\bar{V}$  is constant so we have a solution to  $w_R$ .

# McCall Model

- But for this simple model we can do better than this
- Rewrite the problem with two value function:  $E(w)$ ,  $U$ 
  - suppose  $\kappa = 0$
- The value of employment at wage  $w$

$$E(w) = w + \beta E(w) \tag{1}$$

$$E(w) = \frac{w}{1 - \beta}$$

# McCall Model

- Value of unemployment

$$U = c + \beta \int \max\{U, E(w)\} dF(w) \quad (2)$$

- Reservation wage

$$E(w_R) = U$$

$$\frac{w_R}{1 - \beta} = U$$

# McCall Model

$$\frac{w_R}{1-\beta} = c + \beta \int_0^{w_R} \frac{w_R}{1-\beta} dF(w) + \beta \int_{w_R}^{\infty} \frac{w}{1-\beta} dF(w)$$

$$\begin{aligned} \int_0^{w_R} \frac{w_R}{1-\beta} + \int_{w_R}^{\infty} \frac{w_R}{1-\beta} \\ = c + \beta \int_0^{w_R} \frac{w_R}{1-\beta} dF(w) + \beta \int_{w_R}^{\infty} \frac{w}{1-\beta} dF(w) \end{aligned}$$

$$w_R \int_0^{w_R} dF(w) - c = \int_{w_R}^{\infty} \frac{\beta w - w_R}{1-\beta} dF(w)$$

# McCall Model

- Adding  $w_R \int_{w_R}^{\infty} dF(w)$  to both sides

$$w_R - c = \frac{\beta}{1 - \beta} \int_{w_R}^{\infty} (w - w_R) dF(w)$$

- Integration by parts:

$$\int_{w_R}^{\infty} (w - w_R) dF(w) = \int_{w_R}^{\infty} [1 - F(w)] dw$$

- So finally we have

$$w_R - c = \frac{\beta}{1 - \beta} \int_{w_R}^{\infty} [1 - F(w)] dw$$

# McCall Model - Unemployment Duration

- The probability of getting a job in a given period

$$H = 1 - F(w_R)$$

- $H$  is called the hazard function
- What is the probability of being unemployed for  $n$  periods

$$P(\text{dur} = n) = (1 - H)^{n-1}H$$

- The expected unemployment duration

$$\mathbb{E}[\text{dur}] = \sum_{n=0}^{\infty} n(1 - H)^{n-1}H = \frac{1}{H}$$



# Moving to Continuous Time

- McCall model is written in discrete time
  - each period you get one offer
  - each period you decide to accept or reject
- A lot of labor search models are written in continuous time
  - since we don't have periods we need an arrival rate of job offers  $\alpha$
- $\alpha$  is a Poisson arrival rate

# Poisson Arrival Process

- In an infinitesimal unit of time  $dt$  only one arrival will occur with probability  $\alpha dt$
- The number of arrivals  $N(t)$  in a finite time period  $t$  has a poisson distribution

$$P(N(t) = n) = \frac{(\alpha t)^n}{n!} e^{-\alpha t}$$

- Arrival times are independent and the time until arrival has an exponential distribution

$$P(T > t) = e^{-\alpha t}$$

# Poisson Arrival Process - Two Properties

- **Memoryless:** for  $t_1 \geq 0$  and  $t_2 \geq 0$

$$P(T > t_1 + t_2) = P(T > t_1)P(T > t_2)$$

$$P(T > t_1 + t_2 | t_1) = P(T > t_2)$$

- Is unemployment a memoryless process?
- **Random Selection:** if each arrival is selected with probability  $p$ , independent of other arrivals, then the resulting process is a poisson process with intensity  $\alpha p$ 
  - The reservation wage is independent of the number of offers you have received

# Moving to Continuous Time

- Environment
  - $w, b$  are an instantaneous flows
  - $\alpha$  is a poisson arrival rate of jobs
  - $r$  is the discount rate
- Value of Employment a period of length  $dt$

$$E(w) = \frac{w dt + E(w)}{1 + r dt}$$

- Take the limit as  $dt \rightarrow 0$

$$rE(w) = w$$

# For next time

- **Homework:**

1. Write down the value function for employment (1) and unemployment (2) if you have a probability  $\delta$  of losing your job every period.
2. Derive the continuous time value functions if  $\delta$  is the poisson rate of losing your job

- **Think About:** Where does the wage distribution come from?