EC9A2 Problem Set 1

1 Finite Horizon Consumption-Savings

Consider a consumer who lives for $T = 3$ periods with initial wealth $W_0 = 100$ and no income. The consumer has CRRA utility $u(c) = \frac{c^{1-\theta}}{1-\theta}$ with $\theta = 2$, discount factor $\beta = 0.9$, and faces interest rate $r = 0.05$.

(a) Using the Lagrangian method, derive the present value budget constraint and set up the optimization problem.

(b) Derive the Euler equation and solve for the consumption growth rate $\frac{c_{t+1}}{c_t}$.

(c) Calculate the consumption level in each period (c_0, c_1, c_2) and verify that the budget constraint is satisfied.

2 Infinite Horizon Analysis

Consider an infinite horizon consumption-savings problem with the following setup:

$$\max \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to:

$$\begin{aligned} a_{t+1} &= (1+r)a_t + y - c_t \\ c_t &\geq 0 \\ a_t &\geq \underline{\mathrm{a}} \\ \lim_{t\to\infty} \beta^t u'(c_t) a_t &= 0 \end{aligned}$$

(a) Explain the economic meaning of each constraint.

(b) Write down the Bellman equation and derive the first-order condition, envelope condition, and Euler equation.

(c) Suppose $\beta(1+r) = 1$. Show that the transversality condition becomes $\lim_{t\to\infty} \left(\frac{1}{1+r}\right)^t a_t = 0$. What does this imply about the agent's asset holding behavior in the long run?

(d) Now suppose $\beta(1+r) > 1$. Analyze what happens to the consumption path over time. Is this economically reasonable? Explain why the transversality condition is necessary to rule out explosive behavior.

3 MATLAB Implementation

Implement value function iteration to solve the infinite horizon consumption-savings problem numerically using the following parameters:

- Utility: $u(c) = \ln(c)$
- $\beta = 0.9$, $r = 0.05$, $y = 1$
- Asset grid: 500 points from $a_{min} = 0$ to $a_{max} = 50$
- Convergence tolerance: 10^{-6}

(a) Write MATLAB code that implements the value function iteration algorithm. Your code should:

- Set up the asset grid and initial value function guess
- Implement the Bellman operator
- Iterate until convergence
- Store both the value function and policy functions

(b) Create plots showing:

- The converged value function $V(a)$
- The optimal policy functions $g_c(a)$ and $g_a(a)$