Intro to Search and Matching in the Labor Market

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Outline for the next weeks

- Week 6
  - Stigler (1961) & McCall (1970)
  - The wage distribution
- Week 7
  - Diamond, Mortensen, Pissarides
  - Labor Supply and Data
- Week 8
  - Structural Estimation: MLE
  - Structural Estimation: GMM
- Week 9
  - Structural Estimation: SMM
  - Structural Estimation: Non-Parametric
- Week 10
  - Structural Estimation: EM Algorithm
  - Spillover or Directed Search

Outline for the next weeks

- What you need
  - Matlab: recommended, this is what I will use
  - Python
  - R
  - Julia
- Assignment 1: **Due 8 January 2025**
  - recommend you work on this throughout the course
  - I will let you know when we have covered what you need to know for each question
  - there will be some time allocated during the following lecture to ask any questions

Two types of labor market models

- Walrasian Markets
  - wages are often marginal product
  - people don't need to look for jobs
- Market with frictions
  - models of unemployment
  - understanding how people search for jobs
  - understanding how people flow between E and U

What are Frictions?

- **Search Frictions:** it takes time or money (or both) for buyers and seller to find each other
- **Matching Frictions:** when buyers and sellers meet they may not be a good, or the best match

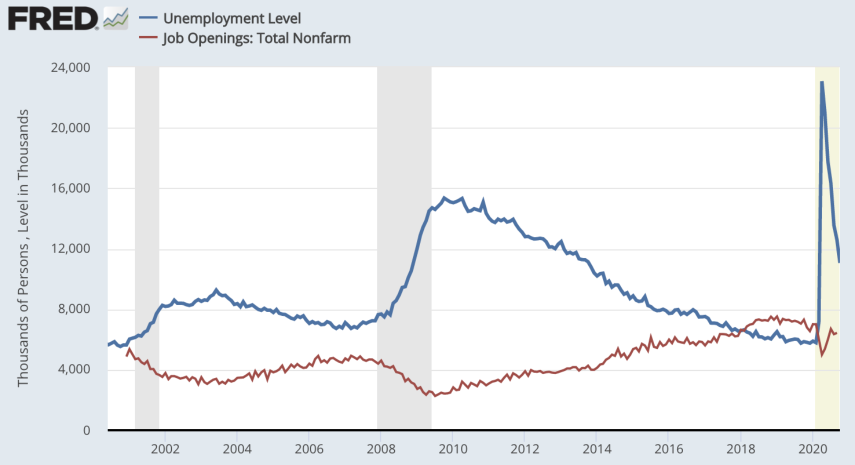
What are some markets with frictions?

- **Labor Market:** workers are looking for jobs, it takes time to apply to jobs. Firms are looking for workers, it costs money to interview people.
- **Housing Market:** buyers are looking for homes
- **Marriage Market:** everyone's looking for someone
- **Asset Market:** Over-the-counter markets for non-routine financial asses

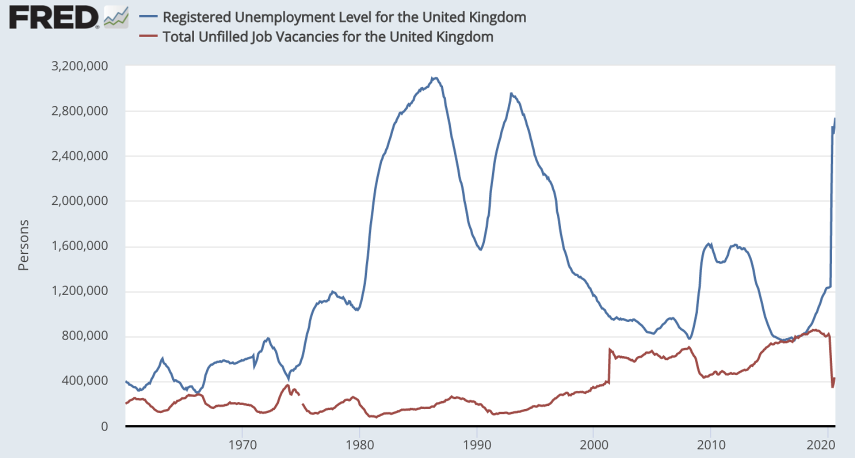
Walrasian Model does not work with frictions

- From a Walrasian perspective
  - Excess demand: vacant jobs
  - Excess supply: unemployed workers
  - Unemployment explained by wages being too high to clear the market
  - Vacancies explained by wages being too low to clear the market
- How can excess demand and excess supply coexist?

Labor Market in the US



Labor Market in the UK



How do we model markets with frictions

- the individual's decision problem:
  - how to search for a job, search effort
  - which jobs to accept
- the firm's decision problem:
  - how many workers they need
  - how much to pay
- an equilibrium
  - number of unemployed and vacancies
  - a wage distribution

A Short Chronological Outline

- **One-sided Search:** individual decision problem, optimal stoping problem.
  - McCall (1970)
- **1-st Generation Equilibrium Search:** random search models with wage posting. Consider workers decisions of accepting jobs and firms decisions on wages.
- **Matching and Bargaining:** Diamond, Mortensen, Pissarides (DMP). Also take into consideration the firms decision to post vacancies.
- **Directed (Competitive):** Workers decide *which* jobs to apply to.

Stigler (1961): The Economics of Information

- **Observation:** Prices of homogenous goods vary across seller
  - Chevrolets
  - Coal
- **Theory:** People must visit multiple sellers to get the best price, people search
- **Question:** How do you decide how many seller to visit before you know enough of the price distribution to buy the good?
  - How many wage offers should you get before you take the best job?

Example

- Suppose there are two prices: \$2 and \$3
- Seller are split equally between the prices

Number of Prices Sampled	Probability of Minimum Price of \$2	Probability of Minimum Price of \$3	Expected Minimum Price

- How many prices should you sample if it's costless to search?
- How many prices should you sample if it's costly to search?

Example

- Suppose there are two prices: \$2 and \$3
- Seller are split equally between the prices

Number of Prices Sampled	Probability of Minimum Price of \$2	Probability of Minimum Price of \$3	Expected Minimum Price
1	0.5	0.5	2.5

- How many prices should you sample if it's costless to search?
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Example

- Suppose there are two prices: \$2 and \$3
- Seller are split equally between the prices

Number of Prices Sampled	Probability of Minimum Price of \$2	Probability of Minimum Price of \$3	Expected Minimum Price
1	0.5	0.5	2.5
2	0.75	0.25	2.25

- How many prices should you sample if it's costless to search?
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Example

- Suppose there are two prices: \$2 and \$3
- Seller are split equally between the prices

Number of Prices Sampled	Probability of Minimum Price of \$2	Probability of Minimum Price of \$3	Expected Minimum Price
1	0.5	0.5	2.5
2	0.75	0.25	2.25
3	0.875	0.125	2.125

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Example

- Suppose there are two prices: \$2 and \$3
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1	0.5	0.5	2.5
2	0.75	0.25	2.25
3	0.875	0.125	2.125
∞	1	0	2

- How many prices should you sample if it's costless to search?
- How many prices should you sample if it's costly to search?

Stigler (1961): The Economics of Information

- There exists some price distribution $F(p)$
- cost of making one draw: c
- First what's the probability that p is the minimum of n draws

$$
\begin{aligned}
H(p) = P(\min_i X_i < p) &= 1 - P(\min_i X_i > p) \\
&= 1 - [1 - F(p)]^n
\end{aligned}
$$

$$h(p) = n[1 - F(p)]^{n-1} f(p)$$

- Let M_n be the expected value of the min value of n samples

$$M_n = n \int_0^{\infty} p[1 - F(p)]^{n-1} f(p)\, dp$$

Stigler (1961): The Economics of Information

- The gain from drawing one more sample

$$
\begin{aligned}
G_n &= M_{n-1} - M_n \\
&= (n-1)\int_0^\infty p[1-F(p)]^{n-2} f(p)\ dp - n\int_0^\infty p[1-F(p)]^{n-1} f(p)\ dp \\
&= \int_0^\infty p[1-F(p)]^{n-1} f(p)[nF(p)-1]\ dp
\end{aligned}
$$

- G_n is decreasing in $n \Rightarrow \lim_{n\to\infty} G_n = 0$

- Optimal number of draws: n s.t. $G_{n+1} > c > G_n$

McCall Model

- **Environment:**
  - workers search for jobs, infinitely lived, discount at β
  - cost of search each period: κ
  - while unemployed worker gets c
  - if searching, each period she draws an offer from $F(w)$
  - if she accepts the job, it lasts forever

McCall Model

- **Objective:**
  - maximize expected discounted earnings

$$\mathbb{E} \sum_{0}^{\infty} \beta^t Y_t$$

  - income each period depends on employment state

$$Y_t = \begin{cases} c - \kappa & \text{if unemployed} \\ w & \text{if employed} \end{cases}$$

- **Trade off:**
  - Waiting too long for a good offer is costly
  - Accepting too early is costly, since better offers might arrive in the future

McCall Model

- **Solution:**
  - optimal stoping problem
  - the reservation wage: w_R

$$w \geq w_R \Rightarrow \text{ accept the job}$$
$$w < w_R \Rightarrow \text{ keep searching}$$

- How does this relate to Stigler's optimal number of draws problem?

McCall Model

- **Solution:**
  - optimal stoping problem
  - the reservation wage: w_R

$$w \geq w_R \Rightarrow \text{ accept the job}$$
$$w < w_R \Rightarrow \text{ keep searching}$$

- How does this relate to Stigler's optimal number of draws problem?
  - **Stigler:** n determines the expected maximum price we are going to pay, non-sequential search

McCall Model

- **Solution:**
  - optimal stoping problem
  - the reservation wage: w_R

$$\begin{aligned} w \geq w_R &\Rightarrow \text{ accept the job} \\ w < w_R &\Rightarrow \text{ keep searching} \end{aligned}$$

- How does this relate to Stigler's optimal number of draws problem?
  - **Stigler:** n determines the expected maximum price we are going to pay, non-sequential search
  - **Here:** w_R determines the expected time of unemployment, how many periods on average we have to wait before we get an acceptable wage, sequential search

McCall Model - Solving Numerically

- With an offer of w in hand worker can
    - accept the job and get

$$\frac{w}{1-\beta}$$

    - reject the job and get

$$c + \beta[\text{get new offer tomorrow}]$$

- Recursive formulation of the objective:

$$V(w) = -\kappa + \max\left\{\frac{w}{1-\beta}\ ,\ c + \beta \int V(w) dF(w)\right\}$$

McCall Model - Solving Numerically

- Policy Function
  - Let $\sigma(w)$ be the policy function, $\sigma(w) = 1$ if we accept the job

$$\frac{w}{1-\beta} > c + \beta \int V(w)\, dF(w)$$

$$w > (1-\beta)\left[c + \beta \int V(w)\, dF(w)\right]$$

  - This condition depends on the value function!
  - Solve value function numerically: QuantEcon

McCall Model

- Now that we have $V(w)$ we can solve for

$$\bar{V} = c + \beta \int V(w)\ dF(w)$$

  where $\bar{V}$ is a constant

- The reservation wage makes us indifferent

$$\frac{w_R}{1-\beta} = \bar{V}$$

- $w/(1-\beta)$ is an increasing function of β and $\bar{V}$ is constant so we have a solution to w_R.

McCall Model

- But for this simple model we can do better than this
- Rewrite the problem with two value function: $E(w)$, U
  - suppose $\kappa = 0$
- The value of employment at wage w

$$E(w) = w + \beta E(w) \tag{1}$$

$$E(w) = \frac{w}{1-\beta}$$

McCall Model

- Value of unemployment

$$U = c + \beta \int \max\{U, E(w)\}\ dF(w) \tag{2}$$

- Reservation wage

$$E(w_R) = U$$
$$\frac{w_R}{1-\beta} = U$$

McCall Model

$$\frac{w_R}{1-\beta} = c + \beta \int_0^{w_R} \frac{w_R}{1-\beta}\ dF(w) + \beta \int_{w_R}^{\infty} \frac{w}{1-\beta}\ dF(w)$$

$$\begin{aligned}\int_0^{w_R} \frac{w_R}{1-\beta} + \int_{w_R}^{\infty} \frac{w_R}{1-\beta} \\ &= c + \beta \int_0^{w_R} \frac{w_R}{1-\beta}\ dF(w) + \beta \int_{w_R}^{\infty} \frac{w}{1-\beta}\ dF(w)\end{aligned}$$

$$w_R \int_0^{w_R}\ dF(w) - c = \int_{w_R}^{\infty} \frac{\beta w - w_R}{1-\beta}\ dF(w)$$

McCall Model

- Adding $w_R \int_{w_R}^{\infty} \, dF(w)$ to both sides

$$w_R - c = \frac{\beta}{1-\beta} \int_{w_R}^{\infty} (w - w_R) \, dF(w)$$

- Integration by parts:

$$\int_{w_R}^{\infty} (w - w_R) \, dF(w) = \int_{w_R}^{\infty} [1 - F(w)] \, dw$$

- So finally we have

$$w_R - c = \frac{\beta}{1-\beta} \int_{w_R}^{\infty} [1 - F(w)] \, dw$$

McCall Model - Unemployment Duration

- The probability of getting a job in a given period

$$H = 1 - F(w_R)$$

- H is called the hazard function
- What is the probability of being unemployed for n periods

$$P(\text{dur} = n) = (1 - H)^{n-1} H$$

- The expected unemployment duration

$$\mathbb{E}[\text{dur}] = \sum_{n=0}^{\infty} n(1 - H)^{n-1} H = \frac{1}{H}$$

Moving to Continuous Time

- McCall model is written in discrete time
  - each period you get one offer
  - each period you decide to accept or reject
- A lot of labor search models are written in continuous time
  - since we don't have periods we need an arrival rate of job offers α
- α is a Poisson arrival rate

Poisson Arrival Process

- In an infinitesimal unit of time dt only one arrival will occur with probability αdt

- The number of arrivals $N(t)$ in a finite time period t has a poisson distribution

$$P(N(t) = n) = \frac{(\alpha t)^n}{n!} e^{-\alpha t}$$

- Arrival times are independent and the time until arrival has an exponential distribution

$$P(T > t) = e^{-\alpha t}$$

Poisson Arrival Process - Two Properties

- **Memoryless**: for $t_1 \geq 0$ and $t_2 \geq 0$

$$P(T > t_1 + t_2) = P(T > t_1)P(T > t_2)$$

$$P(T > t_1 + t_2 | t_1) = P(T > t_2)$$

  - Is unemployment a memoryless process?

- **Random Selection**: if each arrival is selected with probability p, independent of other arrivals, then the resulting process is a poisson process with intensity αp

  - The reservation wage is independent of the number of offers you have received

Moving to Continuous Time

- Environment
  - w, b are an instantaneous flows
  - α is a poisson arrival rate of jobs
  - r is the discount rate
- Value of Employment a period of length dt

$$E(w) = \frac{wdt + E(w)}{1 + rdt}$$

- Take the limit as $dt \to 0$

$$rE(w) = w$$

For next time

- **Homework:**
  1. Write down the value function for employment (1) and unemployment (2) if you have a probability δ of losing your job every period.
  2. Derive the continuous time value functions if δ is the poisson rate of losing your job
- **Think About:** Where does the wage distribution come from?