EC9C5 Topics in Labour Economics Assignment 1
Frictional Markets and Estimation

Instructions: The due date is 12 April 2022. Turn in on Tabula in one single pdf. You should include all script files you used.

1.) **The Model:** Consider a world where workers receive a flow value of leisure b. For now let's consider the contract rate of job offers to be fixed at λ. When a firm and a worker match they both observe a match specific productivity draw θ from an exogenous distribution $G(\theta)$ with support $[0, \infty)$, that is output from a job is equal to θ. After observing the match specific productivity the wage is determined by Nash Bargaining in which the worker has a bargaining power of β. Existing matches exogenously break up at rate δ. Workers and firms discount the future at rate r.

   a. Write down the value functions (V_u and V_e), for the worker and the expected discounted value of a filled match.

   b. Solve for the Nash Bargained wage and the reservation wage as a function of the value of unemployment (you will not be able to solve for the reservation wage as a function of the primitive parameters). Are these a function of the match specific productivity? Why or why not? Also solve for the reservation match productivity, i.e., the productivity value that leads to the reservation wage, denote it θ^*.

   c. Solve for the accepted wage distribution in terms of the productivity distribution. What is the support of the accepted wage distribution?

   d. Solve for the steady state unemployment and employment rate.

2.) **The Policy:** Now let's impose a minimum wage into the model. A minimum wage m is the lowest possible wage that can be paid. Denote $V_u(m)$ as the new value of unemployment with a minimum wage m.

   a. A minimum wage is considered to be binding if workers would be willing to accept jobs at wages below the minimum wage. In the model, when is a minimum wage binding?

   b. Firms are not willing to have negative profits, with a binding minimum wage, which matches, i.e. which productivity levels, are no longer feasible?

   c. Solve for the productivity level for which the Nash Bargained wage is exactly equal to the minimum wage, call it $\hat{\theta}$. When is $\hat{\theta} > m$?

   d. For a binding minimum wage, are matches that have a productivity $\theta \in [m, \hat{\theta}]$ feasible? If so what is the wage at these matches be? What is the wage for matches with $\theta > \hat{\theta}$? Solve for the accepted wage distribution. What is the support of the excepted wage distribution?

   e. Write down the new value of unemployment $V_u(m)$ for a binding minimum wage.

   f. Solve for the steady state unemployment and employment rate for a binding minimum wage.

3.) **The Likelihood Function:** The data you have contains observations of worker outcomes under two different minimum wages. You can observe if a worker is unemployed or employed, what their wage is if they are employed, and the unemployment duration of unemployed workers.

    a. *The unemployed's contribution:* Derive the joint probability of observing an unemployed worker with duration t. Assume that the arrival rate of job offers is Poisson.

    b. *The employed's contribution:* Derive the joint probability of observing an employed worker at wage $w > m$ and the joint probability of observing an employed worker working at the minimum wage.

    c. Write down the likelihood of observing the data at both minimum wages allowing the arrival rate and reservation wage at each minimum wage to differ. Denote these values λ_0, λ_1 and w_0^r, w_1^r.

4.) **The Estimation:** For this part you will have to assume that the workers bargaining power is $\beta = 0.5$ and invariant the the minimum wage. To estimate the bargaining power you would need matched worker-firm data. Assume that the productivity distribution is log-normal with mean μ and standard deviation σ.

    a. Maximize the likelihood function allowing for different arrival rates and reservation wage for each minimum wage. Report the point estimates, standard errors, and likelihood value.

    b. Maximize the likelihood function allowing for a different reservation wage for each minimum wage. Report point estimates, standard errors, and the likelihood value.

    c. Maximize the likelihood function allowing for a different arrival rate for each minimum wage. Report point estimates, standard errors, and the likelihood value.

    d. Using the likelihood values from parts a. - c., test if the contact rate and reservation wage change when the minimum wage changes.

    e. Explain why we can identify the reservation wage and the arrival rate in this model.